

Phenomenology of 10^{32} dark sectorsGia Dvali^{1,2,3,*} and Michele Redi^{4,†}¹*CERN, Theory Division, CH-1211 Geneva 23, Switzerland*²*CCPP, Department of Physics, New York University, 4 Washington Place, New York, New York 10003, USA*³*Max-Planck-Institute for Physics, Foehringer Ring 6, D-80805 Muenchen, Germany*⁴*ITPP, EPFL, CH-1015, Lausanne, Switzerland*

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We postulate an exact permutation symmetry acting on 10^{32} standard model copies as the largest possible symmetry extension of the standard model. This setup automatically lowers the fundamental gravity cutoff down to TeV, and thus, accounts for the quantum stability of the weak scale. We study the phenomenology of this framework and show that below TeV energies the copies are well hidden, obeying all the existing observational bounds. Nevertheless, we identify a potential low energy window into the hidden world, the oscillation of the neutron into its dark copies. At the same time, proton decay can be suppressed by gauging the diagonal baryon number of the different copies. This framework offers an alternative approach to several particle physics questions. For example, we suggest a novel mechanism for generating naturally small neutrino masses that are suppressed by the number of neutrino species. The mirror copies of the standard model naturally house dark matter candidates. The general experimentally observable prediction of this scenario is an emergence of strong gravitational effects at the LHC. The low energy permutation symmetry powerfully constrains the form of this new gravitational physics and allows to make observational predictions, such as, production of micro black holes with very peculiar properties.

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I. INTRODUCTION

Large numbers have always fascinated physicists at least since Dirac times [1]. One incarnation is the hierarchy problem of particle physics, the huge ratio between the weak scale (TeV) and the Planck scale, $M_p^2/\text{TeV}^2 = 10^{32}$. Within the standard model (SM) this ratio is set by the VEV of the Higgs field which is unnatural due to the quadratic divergences in the Higgs mass. The hierarchy problem is therefore not about the big number *per se*, but about its inexplicable stability in the light of its quadratic sensitivity towards the ultraviolet cutoff of the theory.

A radical approach to the problem is to assume that the fundamental scale of gravity is TeV and that M_p is a derived scale. In this way the hierarchy problem is erased because the quadratic divergences to the Higgs mass are cut off at TeV, making its value around the weak scale natural. This point of view, originally introduced in the large extra dimensions scenario [2], has been recently revived in the framework of theories with large number of particles species [3,4]. There it was shown that in *any* theory with N particle species,

$$M_p^2 \geq NM_*^2, \quad (1.1)$$

where M_* is the fundamental scale of gravity. This bound follows from the consistency of large distance black hole physics [3–7]. Notice that the simple naturalness arguments based on perturbation theory point in the same

direction [8,9]. Interestingly, the same bound on N also holds in theories where N labels elements of a discrete symmetry group, e.g., Z_N [3,10] suggesting that both large symmetries as well as many species are incompatible with a high cutoff. In the case of many species the bound implies that $N \sim 10^{32}$ elementary particle species automatically lower the cutoff to $M_* \sim \text{TeV}$, thus explaining the quantum stability of the hierarchy between the weak and the Planck scales. The important point is that the above bound holds independently of the nature of the particles. For example the large extra dimensions proposal automatically falls in this class. This is because the relation (1.1) can be viewed as the relation between the higher dimensional (M_*) and four-dimensional Planck scales, where N is the number of Kaluza-Klein (KK) modes. Indeed, the volume of the compactified extra space, measured in units of the fundamental Planck length $1/M_*$, counts the number of KK species, irrespective of the precise shape of the manifold or the number of dimensions. Thus, whenever $M_* \sim \text{TeV}$, from a 4D point of view there are 10^{32} Kaluza-Klein resonances below the fundamental gravity scale.

In this paper, we shall focus on another extreme case, in which the species are not KK states, but identical copies of the SM. The idea of identical copies of the SM was already introduced in [11,12] in different contexts. In the case of two copies parity symmetric worlds this was also considered in [13] (see also [14] for earlier discussions). In our case we postulate the existence of 10^{32} mirror copies of the SM coupled only through gravity as this would automatically explain the hierarchy. At this level of the discussion, the large number $N = 10^{32}$ is an UV-insensitive input,

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which is shared by other approaches to the hierarchy problem. For example, in the simplest realizations of the softly broken supersymmetric SM [15] an equally large input number is the ratio between the Planck mass and the soft supersymmetry-breaking masses.

However, in the present case there is a new rationale for the emergence of a large number $N \sim 10^{32}$ from the symmetry requirement. Neglecting the hierarchy problem, let us ask the following question. What is the largest possible exact symmetry extension of the SM group under which the SM particles transform nontrivially? Existence of a continuous symmetry is excluded for the following reasons. First, the nonobservation of the corresponding massless gauge bosons, immediately rules out continuous gauge symmetries, unless the corresponding gauge coupling is extraordinarily small. For example, already the bounds on long-distance gravity-competing forces (see [16]), constrain the strength of the new gauge forces at most to $\alpha_{\text{new}} \sim 10^{-50}$ or so. In such a case, the symmetry in question for all practical purposes is essentially a global symmetry which, however, is forbidden in quantum gravity theories. In fact it was shown in [10] that from the consistency of the BH physics, any continuous global symmetry must be at most broken down to a discrete subgroup with maximum $N \equiv M_P^2/M_*^2$ commuting elements, where as before M_* is the fundamental scale where gravity gets strong. Since, irrespective of the hierarchy problem, the ultimate phenomenological bound on M_* is around TeV, we automatically get the upper bound on N being 10^{32} or so. From purely large distance considerations, we are then left with discrete symmetries as possible unbroken extensions of the SM group. The largest of these is permutation symmetry acting on 10^{32} SM copies. Thus, the requirement of a phenomenologically acceptable maximal exact symmetry, automatically solves the hierarchy problem. Another motivation is provided by the strong CP problem [17].

In this paper we shall investigate several phenomenological aspects of the above proposal. The physics of our interest comes from two types of considerations. First, restricting the strength of all possible low energy interactions among the SM copies solely by symmetries and consistency requirements, such as unitarity, we uncover new phenomenologically interesting phenomena. One result is a novel mechanism of small neutrino mass generation, which comes out to be $1/\sqrt{N}$ -suppressed in our scenario. Another interesting effect is a potentially observable oscillation of neutron into its hidden copies. The novelty in comparison to previous studies lies in the large number of “neighboring” copies into which neutron can oscillate. Second, we discuss the phenomenology of the short distance completion of the gravitational sector that is imposed upon us by consistency. In this regard our approach is different from the large extra-dimensional scenario, in which geometry is an input. In the present case,

the underlying gravitational physics is an outcome of consistency and of the well-established large distance dynamics, such as thermodynamics of macroscopic black holes. Inevitability of the new gravitational dynamics is revealed when the above well-known large distance properties are combined with the field theoretic consistency requirements, such as unitarity. By supplementing the above knowledge with symmetry, one can go surprisingly far in understanding the properties of the new short-distance gravitational physics.

One model independent aspect of the present framework is that existence of low energy species generically implies extra-dimension-type modification of gravity at distances parametrically larger than the cutoff length $l_* \equiv M_*^{-1}$ [6]. This modification is not necessarily reducible to a smooth geometry, however it does exhibit two important characteristics of extra dimensions: emergent locality in the space of species, and inevitability of a distance scale $R \lesssim l_*$ below which gravity changes classically. The scale R plays a role qualitatively similar to the radius of the extra dimensions, beyond which the gravitational force law gets modified. An interesting fact is that the correspondence to classical geometry-type description can be classified in terms of the subgroups of the full permutation group. In particular, the full permutation symmetry $P(N)$ is associated to the maximal possible departure from an extra-dimensional picture.

The connection between symmetry and geometry arises through black-hole (BH) physics. An essential feature of the framework, hinging solely on unitarity, is that small BHs cannot decay democratically into all the species but predominantly into the specie that produced them in the collision process. This feature is in stark contrast with the standard macroscopic BH physics, but is characteristic to the small BHs in the large extra dimensions scenario. In the latter case, a BH that is smaller than the size of the extra dimensions predominantly decays in the four-dimensional species that are localized within its reach in the extra space, while it cannot decay into the distant ones (this phenomenon and its reconciliation with the known universal thermal properties of the quasiclassical black holes was discussed in detail in [18]). As we increase the mass of the BH more species become available for the BH evaporation and eventually large BHs decay universally (thermally) into all the species as dictated by semiclassical gravity. This can only happen for BHs whose mass is larger than $M_P\sqrt{N}$ because, as was shown in [4,6,18] below this mass BHs never behave as four-dimensional Einsteinian (classical) BHs. Thus, from the point of view of the quasiclassical BH physics and gravity, species behave as if they are indeed separated by an extra dimension. That is, the BH evaporation allows to define a metric in space of species: the distance between specie i and j can be related to the critical size of the BH of specie i to decay into particles from specie j . In other words,

UNITARITY + BLACK HOLE CONSISTENCY

$$\rightarrow \text{LOCAL GEOMETRY.} \quad (1.2)$$

Depending on the degree of symmetry the geometrical description can be in fact real [6]. For example requiring cyclic symmetry between the species, the gravitational dynamics becomes describable in terms of classical extra-dimension of size $R \gg 1/M_*$.

The maximal departure from the smooth geometric interpretation is obtained considering the limit in which all the 10^{32} copies of the SM are related by an exact permutation symmetry. In this limit any point in the space of species becomes “equidistant” from any other. Correspondingly, the phenomenology of the gravitational sector of the cyclically symmetric extension of the SM shares some qualitative features with the one of the large extra dimensions [19], but there are crucial differences, for example, the evaporation rate of the microscopic BHs. The case of full permutation symmetry on the other hand presents rather different and novel features.

Having removed the crutches of the smooth geometry, the permutation symmetric SM becomes very predictive since the couplings are highly constrained by the full permutation symmetry. This is in contrast with geometrical models where the details of the geometry (such as number of dimensions, warping, location of the branes, etc.) are a crucial input. Unitarity requires interspecies coupling to be strongly suppressed. This allows us to automatically satisfy all astrophysical and cosmological constraints. Many mechanisms introduced in the context of large extra dimensions find the corresponding implementations here, leading, for example, to interesting scenarios for dark matter (see the complementary work [20]) and neutrino masses.

The paper is organized as follows. In Sec. II we consider the connection between black holes and geometry emphasizing the role of symmetries in the space of species. In the rest of the paper we focus on the phenomenology of the permutation symmetric SM. In Sec. III we consider astrophysical, cosmological, and laboratory bounds on the model. The possibility of mixing between neutrinos from different copies is discussed in Sec. IV. In Sec. V we present possible mechanisms for neutrino masses and dark matter. We conclude in Sec. VI.

II. BLACK HOLES AND GEOMETRY IN THE SPACE OF SPECIES

In this section we wish to recall the close relationship between geometry and BH evaporation. Material related to the one here appeared in [4,6] and further properties of small black holes in theories with large number of species are discussed in [18].

We will consider a theory space with 10^{32} exact copies of the SM coupled to gravity. According to Eq. (1.1) the presence of the N species lowers the quantum gravity scale

at $M_* \sim M_p/\sqrt{N} \sim \text{TeV}$. Since gravity becomes strong around TeV, the model independent prediction of this framework is the existence of microscopic black holes with mass $M_{\text{BH}} \geq M_*$. Such micro-BHs would be produced in particle collisions at energies above M_* . Thus, if the hierarchy problem is solved by the low quantum gravity scale, this will be directly probed at the LHC.

The existence of large number of species in the low energy theory, dramatically affects the physical properties of the micro-BHs. The absolute lower bound on the BH mass, below which these objects can no longer be treated as normal Schwarzschild BHs, is given by [6],

$$M_{\text{Schwarzschild BH}} \geq M_p \sqrt{N}. \quad (2.1)$$

For this value of the mass the curvature at the horizon of a classical BH is of the order of M_* so BHs lighter than $M_p \sqrt{N}$ must be non-Einsteinian. The departure from the Einsteinian regime can, however, start at even larger distances already at the classical level and delay the quantum regime up to energies $\sim M_*$. As we shall see on the example of our present framework, this feature is model dependent and closely related to symmetry properties of the theory.

The other major difference concerns the properties of the BH “hair”. It is well known that in Einsteinian gravity classical BH satisfy no-hair theorems [21]. The absence of hairs implies that BHs can only be labeled by charges that can be measured at infinity, either classically or quantum mechanically. As a result, in the absence of such charges, the Hawking evaporation process of a large Schwarzschild black hole is thermal and is completely democratic between the species. In the evaporation process of a BH of temperature T_H all the thermally accessible species (with masses $m \lesssim T_H$) will be produced at the same rate $\Gamma \sim T_H$, whereas the production of the heavy species ($m \gg T_H$) will be Boltzmann suppressed by the factor $e^{-m/T}$.

However, the nature of the micro black holes in theories with large number of fields is dramatically different from their macro counterparts [4,6,18]. Unitarity implies that the decay of the smallest BHs is maximally asymmetric in species. In order to see this, let us follow the argument of [4] and consider a production of a lightest micro BH in the collision of particles and antiparticles in i th SM copy. Such a BH will be produced with probability of order one at a center of mass energy $\sim M_*$, and will have a characteristic mass $M_{\text{BH}} \sim M_*$. As a reference to the SM copy of origin, we shall endow such a BH by an index i . This process then can expressed as

$$\Phi_i + \bar{\Phi}_i \rightarrow \text{BH}_i. \quad (2.2)$$

For a macroscopic Schwarzschildian BH such a label would be redundant, because of the absence of hair, but for the microscopic ones it is not, as it is evident from the following reasoning. For the center of mass energy $\sim M_*$ the only scale in the problem is M_* , and the production rate

of the lightest BH is $\Gamma \sim M_*$. Since the BH_i was produced in particle-antiparticle collision in the i th SM copy, it does not carry any internal quantum number, and by symmetries alone could easily decay into particle-antiparticle pairs of any other j th copy. For a classical Einsteinian BH this decay rate would indeed be j -independent. However, for the microscopic BH in question, this is impossible. By *CPT* invariance, the BH_i should be able to decay back into a pair of i -th species with the rate $\Gamma_{ii} \sim M_*$. But then by unitarity, it cannot decay into all N other individual copies of SM with the same partial rates. The decay rate into the majority of the SM copies must be $\sim M_*/N$, or else unitarity would be violated much below M_* energies.

In the limit of exact permutation symmetry, the BH_i decay rates are extremely restricted, since decay rates into all $j \neq i$ copies must be strictly equal, for any values of i, j . That is, the decay rates split into the diagonal and nondiagonal ones,

$$\Gamma_{ii} = \Gamma_{\text{diag}}(M_{\text{BH}}), \quad \Gamma_{ij} = \Gamma_{\text{nondiag}}(M_{\text{BH}}). \quad (2.3)$$

where both diagonal and nondiagonal rates are the functions of the BH mass. So far, what we know from unitarity is that, for $M_{\text{BH}} \sim M_*$,

$$\Gamma_{\text{diag}}(M_*) \sim M_*, \quad \Gamma_{\text{nondiag}}(M_*) \lesssim \frac{M_*}{N} \quad (2.4)$$

As we will see, this suppression of the interspecies couplings (induced by BHs or not) is the crucial feature of the permutations symmetric scenario that makes its phenomenology rather constrained.

Notice that had we limited the symmetry group to the cyclic permutations of the SM copies, the decay rates Γ_{ij} could have been nontrivial periodic functions of i, j . The general unitarity requirement is

$$\sum_j \Gamma_{ij}(M_*) \lesssim M_* \quad (2.5)$$

In such a case one could introduce a notion of nearest neighbor(s) in the space of species, for example,

$$\Gamma_{ij}(M_*) \propto M_* e^{-N \sin(2\pi((i-j)/N))}. \quad (2.6)$$

This is the case which naturally admits an extra-dimensional interpretation where the copies of the SM are localized on the branes uniformly spaced around an extra dimension. Consider a small BH produced in particle collision on i th brane. If the high-dimensional gravitational radius is smaller than the interbrane distance, the BH_i cannot evaporate into a SM copy that is localized on a distant brane. In this example, the nondemocracy has a clear geometric meaning, and is a consequence of locality in the extra space. In contrast, the case of the full permutation symmetry is an extreme case, which maximally departs from any conventional geometric picture, since there is no notion of a nearest neighbor there.

A. The classical crossover length scale

As shown in [6], the nondemocratic evaporation of the smallest BHs, inevitably leads to the existence of the second length scale, which we shall denote by R . The key point is that R is larger or of the order of the fundamental Planck length M_*^{-1} , and marks the crossover distance, beyond which the BHs become normal Einsteinian BHs. That is, for BHs larger than R the index j becomes redundant and their evaporation becomes democratic in all the species.

At this point the BH horizon is related to the mass of the BH via the usual Schwarzschild relation

$$R = 2M_{\text{BH}}G_{\text{Newton}}. \quad (2.7)$$

At the intermediate distances

$$M_*^{-1} \ll r \ll R, \quad (2.8)$$

gravity is still classical, but the micro-BHs of gravitational radius within this interval cannot be Einsteinian BH. In particular in their evaporation spectrum there is an r_g -dependent bias with respect to some species. The black holes of sizes $r_g \sim M_*^{-1}$ and $r_g \geq R$ are extreme cases, corresponding to the maximally biased and unbiased cases.

In the case of N standard model copies, we can give to the distance R a simple geometric meaning if we think of a label j as of the coordinate of the localization site in the extra dimension of radius R . This setup would reproduce all the general properties of the microscopic BHs that were displayed above. For example, BHs of gravitational radius smaller than the size of the extra dimension, but still larger than the fundamental scale, will be classical objects. However, their evaporation process will be undemocratic in copies of SM since the small BH will not be able to reach out to all the localization sites. On the other hand, BHs larger than the size of the extra dimension would be normal, and thus completely democratic with respect to all the species. It is, however, important to stress that the interpretation of the underlying gravitational dynamics in terms of the conventional extra dimensions may not be possible in general. In particular this is the case if we require the exact full permutation symmetry between the SM copies. This setup corresponds to the maximal departure from a smooth geometry.

To investigate the properties of microscopic BHs in our case we can follow the general strategy in [6], focusing on the theory with N exact SM copies. We require that the physics, including the gravitational one, is identical as seen from each copy. In symmetry terms, this can be guaranteed for instance by requiring the cyclic permutation symmetry under which j th SM copy is replaced by $j - 1$ th one (this operation, of course, requires an obvious periodicity in numbering $j \equiv j + N$). This symmetry, still leaves room for further restriction, since we may or may not require the symmetry under full permutation group. We keep such an option open at the moment.

Since at distances $r < R$ gravitational interaction among the different SM copies is no longer universal, we should keep the labels explicit. Consider a Newtonian interaction between the two pointlike sources, belonging to the i th SM copy, of masses M_i and m_i , and assume the separation by a distance r . Following [6], this interaction can be parametrized by the following gravitational potential (below everywhere we shall ignore numbers of order one)

$$V(r) = \frac{M_i m_i}{M_*^2} \frac{1}{r \nu(M_* r)}, \quad (2.9)$$

where $\nu(M_* r)$ is some smooth function, such that

$$\nu \sim 1 \quad \text{for } r \sim M_*^{-1} \quad \text{and} \quad \nu \simeq \frac{M_P^2}{M_*^2} \quad \text{for } r > R. \quad (2.10)$$

The latter boundary conditions follow from the fact that by changing r from R to M_*^{-1} , the above interaction must interpolate between the usual Newtonian force and the strong gravity.

Since at the intermediate distances gravity is weakly coupled, the effective gravitational radius (r_g^{ii}) of the source of mass $M_{\text{BH}i}$ seen by the particles of the same i th species can be estimated from the equation,

$$1 = \frac{M_{\text{BH}i}}{M_*^2} \frac{1}{r_g^{ii} \nu(M_* r_g^{ii})}, \quad (2.11)$$

that is from the condition that the Newtonian potential becomes order one. Although, r_g^{ii} represents the BH horizon for the same species, r_g^{ii} is not necessarily a horizon for all the other species, since only the species that can be produced in the evaporation process of a given BH, can see its horizon.

For any BH belonging to i th SM copy the number of species to which it can evaporate is a function of the black hole mass (r_g^{ii}), $\mathcal{N}(M_* r_g^{ii})$. Because of the symmetry, this function is the same for all the copies, and its boundary properties are similar to (2.10)

$$\mathcal{N}(1) \sim 1, \quad \mathcal{N}(M_* R) = \frac{M_P^2}{M_*^2} \quad (2.12)$$

In contrast the gravitational potential between two sources of masses M_i and M_j belonging to two different i th and j th SM copies, will be set by the index-dependent function $\nu(M_* r)_{ij}$, with the boundary condition $\nu(MR)_{ij} = M_P^2/M_*^2 = N$, which follows from the definition of R . The necessary condition for the particles of the j th copy to be produced in the evaporation process of the BH of mass $M_{\text{BH}i}$ made out of the i th species, is that the j -particles see the horizon of the i th BH. The corresponding horizon we shall call r_g^{ij} . Generalizing the notion of r_g^{ii} , the nondiagonal gravitational radius can be estimated from

$$\frac{M_{\text{BH}i}}{M_*^2} \frac{1}{r_g^{ij} \nu(M_* r_g^{ij})_{ij}} = 1. \quad (2.13)$$

Then r_g^{ij} represents the horizon of i th BH with respect to the particles from the j th species. Thus, the function \mathcal{N} counts the solutions of (2.13).

In the above ‘‘geometric’’ example, in which SM copies are displaced in the extra-dimensional space, the nonuniversality of the function ν_{ij} is explicit and follows from the locality in the extra space. The important point however is that locality in the space of species emerges automatically regardless of any input geometrical assumptions. The resulting properties of the space of species, can be parametrized by an effective ‘‘metric’’ in this space. Depending on the level of the intercopies symmetry, this metric can exhibit different level of correspondence with the one of conventional compact dimensions. In the extreme case of full perturbation symmetry, the departure from any resemblance of the classical geometry will be maximal.

B. Cyclic symmetry of the standard model copies

So far, our only symmetry requirement was, that the laws of physics, as seen by an observers from any SM copy, must be identical. This requirement guarantees that the quantities r_g^{ii} , ν and \mathcal{N} are independent of i . The analog of flat geometry in the space of species would then correspond to the following approximate form of the functions ν and \mathcal{N} ,

$$\nu = \mathcal{N} = \left(\frac{r_g}{R}\right)^n \frac{M_P^2}{M_*^2}, \quad (2.14)$$

where n is an arbitrary number. The boundary condition $\mathcal{N}(1) = 1$ fixes $R = (M_P^2/M_*^2)^{(1/n)} M_*^{-1}$, and thus,

$$\nu = \mathcal{N} = (r_g M_*)^n. \quad (2.15)$$

Can this form be guaranteed by some discrete symmetry among the SM copies? For $n = 1$, this form is indeed guaranteed by the cyclic symmetry under which $i \rightarrow i + 1$ for any $i \neq N$ and $(i = N) \rightarrow (i = 1)$. We can then generalize to the required symmetry group for arbitrary n in the following way. Let us label copies by a group of n indexes as follows,

$$\Phi_{i_1 i_2 \dots i_n}, \quad (2.16)$$

where each index takes the value $i_\alpha = 1, 2, \dots, N_\alpha$, with $\alpha = 1, 2, \dots, n$. Since the total number of copies is N , the numbers N_α must satisfy

$$\prod_\alpha N_\alpha = N. \quad (2.17)$$

We then require invariance under permutations of copies that are obtained by independent cyclic permutations of indexes within each group i_α ,

$$i_\alpha \rightarrow i_\alpha + 1 \quad \text{for any } i_\alpha, \quad \text{and} \quad (i_\alpha = N_\alpha) \rightarrow (i_\alpha = 1). \quad (2.18)$$

For all N_α being equal, this symmetry uniquely fixes the form of the functions ν and \mathcal{N} to be given by (2.15). This is remarkable, since a simple requirement of symmetry under cyclic permutation of the SM copies, forces the short-distance gravity to behave as if n -extra dimensions with the sizes $R = M_*^{-1} \sqrt[n]{N}$ open up, although nothing like this has ever been postulated. Making N_α 's arbitrary, simply changes the sizes of this dimensions to $R_\alpha = M_*^{-1} N_\alpha$.

C. Full permutation symmetry of the standard model copies

We are now ready to consider the case which corresponds to a maximal departure from the classical geometry, by postulating the full permutation symmetry group of the standard model copies. That is, we require physics to be invariant under the exchange of arbitrary copies. This requirement immediately fixes all the nondiagonal quantities to have the same value $\nu_{ij} = \nu'$. The consequences for BH evaporation are pretty dramatic, since all nondiagonal decay channels must also have the same rate, by symmetry. In this case formally the crossover scale $R \sim M_*$ and gravity jumps directly from the classical 4D description to the full quantum gravity regime at that scale.

To such interspecies relations it is hard to give any sensible geometric meaning because of the following reason. The permutation symmetry group implies that, if there is any notion of metric in the space of species, the copies must be equidistant in this space. This is impossible unless the space of species has effectively $n = N - 1$ dimensionality! The only geometric space that could imitate such an extreme democracy is the $N - 1$ -dimensional space (see Fig. 1), however this is precisely the case where any geometrical notion is lost since only the first KK modes of the theory would be well described by the effective theory.

The fact above can also be understood from the observation that the full permutation symmetry group can be obtained as the limit of the above-introduced n -cyclic permutations for which $n = N - 1$. Indeed, in such a limit the number of indexes i_α becomes $N - 1$, and each index can take only two possible values, which can be taken to be 0, 1. Moreover, in each sequence there can only be a single distinct index, e.g., $\Phi_{00001000,\dots,000}$. The species then are simply identified by the position of 1 in this sequence. Making the cyclic permutation in any i_α then simply reduces to changing $0 \rightarrow 1$ or vice versa at that position. But because 1 can only appear once in the whole sequence, we have to make the opposite flip in one of the other indexes. This effectively is equivalent to permuting a single pair of SM copies, that had index 1 in these two locations.

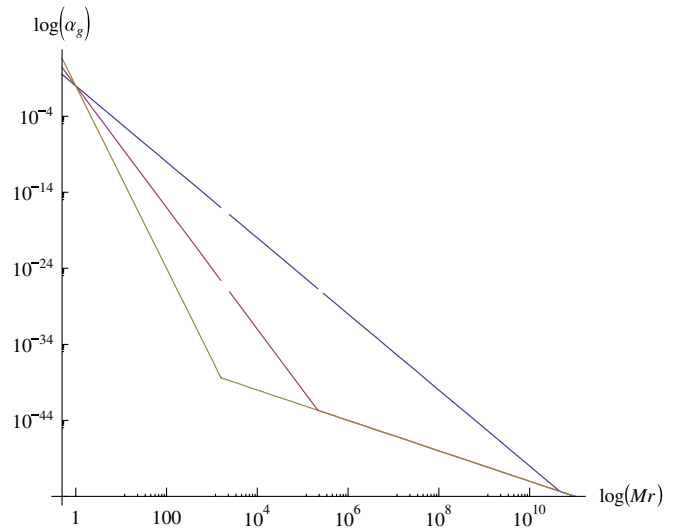


FIG. 1 (color online). Gravitational force of two point particles with mass M_* as function of distance with 3 (blue, upper), 6 (red, intermediate), 10 (green, lower) extra dimensions. The limit $n \rightarrow \infty$ formally resembles the permutation symmetric scenario.

D. Democratization of micro-black holes

Before concluding this section let us discuss an important property of microscopic BHs that reconciles their nonuniversality with respect to different species with the known thermal and democratic properties of the classical BHs. Indeed, at first glance, such a nonuniversality of BH evaporation looks puzzling because of the following reason. On one hand, the BHs in question are quasiclassical thermal objects and thus are expected to radiate all the light species universally. On the other hand, this is impossible by unitarity, and their nondemocratic evaporation inevitably suggests that different species see different horizons. How can the two seemingly interexclusive properties of thermality and nonuniversality be reconciled?

Both of the above properties seem to be explicitly supported by the extra-dimensional considerations, in which even quasiclassical BHs that are smaller than the compactification radius must evaporate nondemocratically in various four-dimensional species simply by locality in extra dimensions. Nevertheless, such a situation would be problematic, since it would imply that BHs can be labeled by their ability to belong to a particular species. This label will not be associated with any exactly-conserved quantum number measurable at large distances by a four-dimensional observer, in contrast to known no-hair properties [21].

The resolution of the puzzle was given in [18]. The outcome of these studies suggests that microscopic BHs on top of the quantum Hawking evaporation time, possess another intrinsically classical time scale, a “democratization” time. During the latter time any given nondemocratic BH is classically unstable, and evolves in time until it becomes fully democratic in species. Thus, the nondemocratic

racy is never a property of a classically stable neutral BH. The balance between the two time scales depends on the BH mass, the number of species and geometry in the species space. However, for the smallest BHs evaporation always wins and they never reach a fully democratic state. For such BHs the democratization process can be ignored and all the analysis given above is fully applicable. Applying this consideration to our present context, the importance of the democratic transition will depend on the subgroups of $P(N)$.

For example, for the full $P(N)$ -symmetry case, the BHs that can be potentially observed at LHC will evaporate way before the democratization time. Thus, for such light BHs, the democratization time scale is irrelevant. However, for the cyclic symmetric case, the existence of the neighboring copies gives important correction even in the dynamics of the relatively light BHs. First, the existence of extra copies, should increase the evaporation rate substantially [6],

$$\frac{dM_{\text{BH}}}{dt} = M_*^2 \left(\frac{M_*}{M_{\text{BH}}} \right)^{(2-n)/(1+n)}, \quad (2.19)$$

which for the BH lifetime gives

$$\tau_{\text{BH}} \simeq M_*^{-1} \left(\frac{M_{\text{BH}}}{M_*} \right)^{(3)/(n+1)} \frac{n+1}{3}. \quad (2.20)$$

The above lifetime is by a factor $\left(\frac{M_{\text{BH}}}{M_*} \right)^{(n)/(n+1)}$ shorter than the lifetime of a microscopic BH localized within n flat extra dimensions. The picture instead is such, as if the BH is localized in n flat extra dimensions in which there are N 4-dimensional light species localized at uniformly distributed sites (3-branes). With the growing mass, the BH horizon grows in the space of species according to $n+4$ -dimensional law, and captures more and more sites.

Correspondingly, the branching ratio of BH evaporation also changes. For a BH of some mass $M_{\text{BH}} \gg M_*$, produced by particles of i th copy, there are $\mathcal{N} = \left(\frac{M_{\text{BH}}}{M_*} \right)^{(n)/(n+1)}$ invisible decay channels. Thus, for such a BH only a fraction

$$\frac{E_{\text{BH} \rightarrow i\text{th copy}}}{E_{\text{BH} \rightarrow \text{all copies}}} \sim \left(\frac{M_*}{M_{\text{BH}}} \right)^{(n)/(n+1)} \quad (2.21)$$

of the total energy will be released back in i th species. The rest will be distributed over the extra copies.

Second, the (partial) democratization process for such BHs will also be important and be comparable with the evaporation rate. In the other words, the number of available evaporation channels can change substantially within the BH lifetime, and this in return will decrease the lifetime even further. Thus, the micro-BHs in the theory with many SM copies, possess the observable properties very distinct from the standard large extra-dimensional case, and these differences can potentially be tested at LHC.

III. BOUNDS

In this section we study phenomenological bounds on our theory with $N = 10^{32}$ identical SM copies. As we will see these are very mild and never worse than in the large extra dimensions scenario. We focus on the maximally symmetric version of the model by requiring invariance under the full permutation group $P(N)$. The phenomenological bounds come from two sources: (1) First, the new effective interactions generated among the SM particles that are subject to the precision electroweak constraints; (2) second, from the production of new states in the collision processes of the SM particles. The role of the new states, that are either produced in SM collisions or are mediators of the new interactions, can be played either by the hidden SM copies or by new gravitational degrees of freedom that modify gravity at the scale M_* .

A. Graviton induced processes

We start considering processes induced by the standard massless graviton, which is the only field mediating interactions among the different species at low energies. Phenomenologically, the crucial difference with large extra-dimensions is due to the fact that the hidden SM copies do not couple directly to our SM fields, but only through the graviton exchange as in Fig. 2. On the other hand the KK gravitons of the large extra dimensions do couple directly to the SM particles at tree level with $1/M_P$ strength. The cross section for the production of the other species in the collision of SM states due to graviton exchange is enormously suppressed. By simple dimensional analysis, we can display the relative scalings of the latter cross section versus the individual KK production in the LED case, as follows,

$$\begin{aligned} \sigma_{e^+e^- \rightarrow jj} &\sim \frac{E^2}{M_P^4} && 10^{32} \text{ SM copies} \\ \sigma_{e^+e^- \rightarrow KK+\gamma} &\sim \frac{\alpha}{M_P^2} && \text{Large Extra} - D. \end{aligned} \quad (3.1)$$

This fact ameliorates or completely removes the cosmological constraints on the processes mediated by the graviton exchange, despite the fact that each sector contains

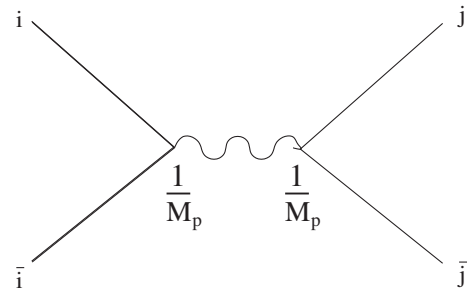


FIG. 2. Interactions between visible and hidden sector mediated through graviton exchange.

massless photons and other light states. For example, as in every theory with extra light particles, one immediate worry is that if these states were produced in the early Universe they would destroy the successes of standard cosmology. In particular, since big bang nucleosynthesis (BBN) predicts the right abundances of light elements we certainly do not want to change the cosmology at least until BBN, which corresponds to temperatures ≈ 1 MeV. This requires that at most one extra relativistic degree of freedom contributes to the Hubble expansion in the early Universe during BBN.

We can easily see that this condition is automatically satisfied by the above graviton-mediated interactions. Let us estimate the critical temperature (T_C) above which the cosmological evolution would radically deviate from the standard one, due to the production of the extra species (in [19] this temperature was called the *normalcy* temperature). For this, we assume that at some initial state Universe is populated only with our SM particles at temperature T . In the radiation dominated phase the expansion rate is given by

$$H \approx \sqrt{g^*} \frac{T^2}{M_P}, \quad (3.2)$$

where g^* is the number of active degrees of freedom which is of order one, since by assumption only the SM particles are initially in thermal equilibrium.

If the production of states from other sectors arises through the graviton exchange diagrams such as in Fig. 2, the total rate is roughly,

$$\Gamma_{\text{TOT}} \sim \frac{T^5}{M_P^4} N \sim \frac{T^5}{M_P^2 M_*^2}. \quad (3.3)$$

As long as this rate is less than the Hubble expansion rate (3.2), the observable sector cools down predominantly due to the Universe's expansion, and cosmology is normal. Thus, the normalcy temperature is defined from the condition. $\Gamma = H$, which implies

$$T_C \sim (M_P M_W^2)^{1/3} \sim 10^8 \text{ GeV}. \quad (3.4)$$

This temperature is so high that it is completely safe to assume that the other sectors have no visible effects at temperatures much above BBN. In fact, since the permutation symmetric SM is an effective theory valid only up to $M_* \sim \text{TeV}$, no temperatures above this value can be considered within our framework and any such discussion should await the UV completion of the theory. Notice that below M_* temperatures the rate (3.3) is even less than the rate of normal graviton emission. Already this fact indicates that such processes can be safely ignored within the validity of our effective field theory description. Therefore we conclude that below the cutoff temperatures, the thermal production of other copies via graviton exchange is negligible, and cosmology is standard.

From the above it also follows as no surprise that astrophysical processes such as cooling of stars due to graviton-mediated emissions of hidden species give no constraints on the model. Since all the known astrophysical objects are much cooler than TeV, the inclusive rate of production of the hidden species within such objects [which is again given by (3.3)] is much less than the graviton emission rate and is totally negligible. This should be compared with the analogous result in the case of large extra dimensions. In that case the production of light Kaluza-Klein gravitons by supernovae provides one of the strongest constraints on the model. Finally, we wish to note that because of the presence of 10^{32} exactly massless photons, the evaporation of astrophysical BHs is 10^{32} times faster. This brings the mass of the smallest primordial BHs that could survive till today, to approximately 10^{26} g.

B. Constraints from new gravitational states

We now turn to the new states that must appear at the cutoff of our effective description (assuming that a field theoretic description is possible for such states). While the coupling between the species induced by the graviton is totally negligible, one might wonder whether the effective operators induced by the new gravitational states at the cutoff are more dangerous phenomenologically. Let the new gravitational degrees of freedom that modify gravity around the scale M_* be h_α . Contrary to the graviton coupling, the coupling of these states is not uniquely determined. However, the full permutation symmetry greatly restricts the number of possibilities. Since these states should also obey the bound (1.1), their number cannot exceed N . We can classify these gravitational species by their transformation properties under the permutation group.

If the gravitational states at the cutoff are singlets of the symmetry $P(N)$ (just as the graviton is), then by permutation symmetry the coupling of such gravitational species to the different copies of the SM has the following form,

$$h^{(\alpha)} \lambda_\alpha \sum_j \bar{\Phi}_j \Phi_j, \quad (3.5)$$

where the parameter λ_α defines the form and the strength of the coupling. The spin of the $h^{(\alpha)}$ states is not specified. So the λ_α -symbols represent operators rather than simple coupling constants. For example, they can depend on the derivatives that act on the different fields entering the vertex. The total emission rate of h_α in SM collision processes is given by

$$\Gamma_{\text{SM} \rightarrow h} \sim \sum_\alpha E |\lambda_\alpha(E)|^2 \quad (3.6)$$

where the summation is over the energetically available states. Unitarity implies that for $E \lesssim M_*$,

$$\sum_\alpha |\lambda_\alpha(E)|^2 \lesssim 1. \quad (3.7)$$

The direct emission of gravitational species becomes relevant only at the energies comparable to M_* , but their virtual exchange can generate effective interactions between the different SM copies. For example, the effective four-fermion couplings among the fermions of i th and j th copies can be generated via exchange of $h^{(\alpha)}$ species as in Fig. 2, and at low energies will have the following form,

$$\bar{\Psi}_i \Psi_i \bar{\Psi}_j \Psi_j \sum_{\alpha} \frac{\lambda_{\alpha} \lambda_{\alpha}^*}{M_{\alpha}^2}. \quad (3.8)$$

Here $M_{\alpha} \sim M_*$ are the masses of the gravitational species. Through this effective operator the particles of the SM can annihilate into the hidden sectors. The total rate of annihilation at energy E goes as

$$\Gamma_{\text{SM} \rightarrow \text{hidden SM}} \sim E^5 \left(\sum_{\alpha} \frac{\lambda_{\alpha} \lambda_{\alpha}^*}{M_{\alpha}^2} \right)^2 N, \quad (3.9)$$

where the factor of N comes from the summation over all the final copies. Low energy unitarity below the scale M_* again guarantees that the above rate is compatible with all the existing phenomenological bounds. This is because the above rate is bounded by E at any energy below the cutoff M_* . Thus, for $E \lesssim M_*$, we have $\Gamma_{\text{SM} \rightarrow \text{hidden SM}} \lesssim E$. In the worst case scenario in which λ_{α} 's do not scale with energy and we neglect possible cancellations, we obtain that the rate scales as

$$\Gamma_{\text{SM} \rightarrow \text{hidden SM}} \lesssim E^5 / M_*^4. \quad (3.10)$$

In reality, for any sensible gravitational theory, in which h_{α} couple to the conserved sources, the scaling of the rate will be much more rapid. For example, for the states coupled to the energy-momentum tensors of the SM species, λ_{α} -s scale as E , and correspondingly $\Gamma_{\text{SM} \rightarrow \text{hidden SM}} \lesssim E^9 / M_*^8$. However, to make our arguments stronger we can use the rate (3.10) as the maximal possible rate. In fact, this maximal possible rate coincides with the production rate of KK gravitons in high energy processes in theories with two large extra dimensions. Although this fact is completely accidental, we can apply the results of [19], to derive phenomenological bounds, since the latter analysis solely relied on the above rate. We thus see, that thanks to permutation symmetry and unitarity even without knowing the precise details of underlying gravity theory, we are able to show that the phenomenological bounds are never more severe than a large extra-dimensional scenario.

The other option is that the gravitational states transform under $P(N)$, i.e. are interchanged along with the SM copies (other representations are excluded by unitarity due to the growth of their dimension with N). In such a case, the gravitational states can be labeled by the same indexes as the given SM copies. By unitarity each gravitational degree of freedom h_i can only couple with the i -th SM copy with the maximal (in units of M_*) strength while with all the other copies the strength of the coupling must be $1/N$ suppressed. The emission rate of all $j \neq i$ gravitational

species, in the collision of particles of i th SM copy, will go as

$$\sum \Gamma_{i \rightarrow j \neq i} \sim EN^{-1}, \quad (3.11)$$

where E is the energy in the collision, assumed to be above the mass of the gravitational species. This rate is negligible, and the total emission rate will be dominated by the emission of the gravitational species ‘‘belonging’’ to the same SM copy. The rate at energies above their mass goes as $\sim E^{n+1}/M_*^n$, where n is determined by the type of the interaction. For example, for the emission of the spin-2 state that couples to a conserved energy-momentum tensor, $n = 2$. Since, in the case of exact $P(N)$ symmetry, change of gravitational laws start around the scale $\sim M_*$, the mass of the corresponding gravitational species responsible for this change is of order M_* . Hence, their emission becomes important only close to the cutoff scale, which puts the phenomenological bound around $M_* \sim \text{TeV}$. A similar bound comes from the effective high-dimensional operators generated by the exchange of the gravitational species. Notice that from this point of view the phenomenology of gravitational species is very similar to the one of the smallest BHs. This is not surprising, since as we have argued above the smallest BHs are quantum objects, and distinction between them and particles is rather blurry. In certain sense, the smallest BHs themselves can be treated as new gravitational species that are necessary for modifying laws of gravity at the scale M_* .

C. Baryon number violation

In theories with low gravitational cutoff, one of the most pressing phenomenological questions is the strength of the baryon number violating operators. The source of such operators can be virtual BH exchange. For example, consider a process in which two u -quarks collapse into a virtual BH, which then evaporates into an anti- d and a positron,

$$u + u \rightarrow \text{BH} \rightarrow d_c + e_c. \quad (3.12)$$

At low energies, below the BH mass this results into an effective baryon number violating operator of the form

$$uud_c e_c \frac{f_{uud_c e_c}}{M_{\text{BH}}^2}. \quad (3.13)$$

Here, we have parametrized the strength of the effective operator by a form-factor $f_{uud_c e_c}$ which encodes the probability that a BH produced by two u -quarks can evaporate into d_c and e_c . The problem in drawing definite phenomenological conclusions is due to the fact that not much is known about this form factor for the smallest BHs, which are the most relevant ones for the above process. For the large and heavy BHs, which decay universally into all the species, this form factor is tremendously suppressed, because the heavy BHs are classical. As we have argued

above, the smallest BHs, on the other hand, are not democratic in species. All the potential problems for baryon number violating operators, come from the fact that the democracy is blindly assumed for the micro-BHs. Although, we know, that for the coupling with large number of species, the unitarity arguments indicate precisely the opposite, unfortunately, for small number of species we cannot prove the suppression of the nondiagonal interactions rigorously.

What is the implication of the above analysis for the current framework of the N -copies of SM? First, here we can be sure that the baryon number violating (or any other type) of copy-non-diagonal processes are suppressed by powers of N . However, for the processes inside each copy, we do not have such a powerful statement. Therefore, in the worst case scenario, we shall assume that unless additional measures are taken, for the smallest BHs the form factors are order one. Then the small virtual BHs will mediate the baryon number violating processes at an unacceptable rate. The additional measure that we shall invoke, is the *common* gauging of the baryon number symmetry of all the different copies. That is, we shall postulate that the $U(1)_B$ symmetry, that corresponds to the simultaneous baryon number rotations of all the SM copies, is gauged. The corresponding gauge boson, B_μ , we shall call the bary-photon. By unitarity and $P(N)$ symmetry, the $U(1)_B$ gauge coupling g_B is suppressed by $1/N$ and thus, is of the gravitational strength, $g_B \sim M_*/M_P$.

The question that arises immediately in the case of gauging the baryon number symmetry, is the anomaly cancellation. In the present case, the anomalies generate a gauge-noninvariant shift of the Lagrangian density

$$q_W \alpha \sum_j \text{Tr}(\tilde{F}F)_j, \quad (3.14)$$

where α is the gauge shift parameter, $B_\mu \rightarrow \frac{1}{g_B} \partial_\mu \alpha$, where j labels the dual $SU(2)$ -field strength of different copies and q_W is the usual bary-weak mixed anomaly coefficient, which by permutation symmetry is common for all the copies. Similarly, there is also a gravitational anomaly and a mixed gravitational anomaly (see [22]),

$$q_G \alpha \tilde{R}R. \quad (3.15)$$

In order to cancel the anomalies, we can implement a Green-Schwarz type mechanism [23]. We shall achieve this, by introducing the bary-axion, b , with the respective gauge and gravitational couplings,

$$\frac{b}{M_b} \left(q_W \sum_j \text{Tr}(\tilde{F}F)_j + q_G \tilde{R}R \right), \quad (3.16)$$

that shifts appropriately under the baryon number symmetry,

$$\frac{b}{M_b} \rightarrow \frac{b}{M_b} - \alpha \quad (3.17)$$

As usual, for simultaneous cancellation of all the anomalies the coefficients q_G and q_B must be equal. This can be always arranged by introducing extra fermions with the appropriate baryon number and no charges under the SM gauge fields. Again, by unitarity the axion decay constant must be $1/N$ -suppressed and thus be gravitational, $M_b \sim M_P$.

Anomaly cancellation by GS mechanism automatically implies that the bary-photon is a massive (Proca) field, with the bary-axion being its longitudinal component $B_\mu \rightarrow B_\mu + \frac{\partial_\mu b}{g_B M_b}$. The mass of the bary-photon is $m_B = g_B M_b \sim M_*$. The gauging of the baryon number forbids all the local baryon number violating operators that could lead to the proton decay. The nonlocal ones, however, can be written down. These are of the following form,

$$e^{-ic g_B ((\partial_\mu B^\mu)/(\square))} u u d_c e_c \frac{1}{M^2}, \quad (3.18)$$

where c is the total $U(1)_B$ charge of the fermions entering in the operator. Equivalently, we can have the following operator,

$$e^{-ic((b)/(M_b))} u u d_c e_c \frac{1}{M^2}. \quad (3.19)$$

Notice, that if it were possible to identify b with the phase of a local scalar field, then the operator of the above form could have been written in a local way. This, however, is impossible, since going above the vacuum expectation value of the scalar field, we should have recovered an anomaly free theory even without b , which would contradict to the existence of the anomaly in the baryonic current. So the above operators cannot be generated by the exchange of the smallest virtual BH of mass $\sim M_*$, since effective operators generated in this way must be local. At best, such operators must pay the price of nonperturbative suppression, by the exponential factors $\exp(\frac{1}{g_B^2})$ and is probably negligible.

IV. NEUTRON OSCILLATIONS

Gauging of the diagonal baryon number still allows mixing between neutral states of different species. One phenomenologically relevant possibility is the mixing of neutrons. Indeed, gauge symmetries allow the generation of the following dimension-9 operators

$$\kappa_{ij} \frac{u_i d_i \bar{u}_j \bar{d}_j}{M_*^5} \quad (4.1)$$

where κ_{ij} parametrize the strength of these operators. The structure of the matrix κ_{ij} is determined by the permutation symmetry. For any system of N SM copies invariant under the full $P(N)$ group, we have $\kappa_{ij} = \kappa$.

The above operator induces a mixing between neutrons of the different SM copies

$$\delta m_n \sum_{i \neq j} n_i n_j, \quad (4.2)$$

with

$$\delta m_n \sim 10^{-4} m_n \kappa \left(\frac{m_n}{M_*} \right)^5, \quad (4.3)$$

where m_n is the neutron mass, and the extra suppression factor of order 10^{-4} comes from the three-quark-neutron matrix elements. The constraints on the parameter κ are the following.

The requirement of the three-level unitarity of the quark scattering processes below the scale M_* , gives the constraint $\kappa \lesssim \frac{1}{\sqrt{N}}$. However, taking into account loops with intermediate quarks from different species, the constraint gets more severe. Summing over intermediate species, the loop expansion goes as series in $N\kappa$, and this demands $\kappa \sim 1/N$. Notice, that the latter value is compatible with the expected suppression of the interspecies operators generated by the virtual BH exchange. We shall, therefore, adopt this value.

This mixing can lead to the transition of our neutron into the hidden ones. Such a mixing with a single mirror copy was already studied in [24] and as we shall see the case with many copies introduces some qualitatively distinct features. In the case of many species an analog mixing for the Higgs was considered in [12].

Of course, due to exact degeneracy of neutron masses, the transition is normally forbidden in nuclei due to energy conservation (a neutron that oscillates in a hidden copy would not be bound and so would have higher energy), and therefore will not be in conflict with the usual neutron disappearance bounds inside nuclei [25]. But this would not apply to free neutrons which could indeed oscillate into the hidden ones. We shall see, that the transitions can take place and lead to some potentially observable effects.

A. Vacuum oscillations

We now wish to give a detailed description of this effect. By taking into the account the mixing between the neutrons of the different SM copies, the full $N \times N$ neutron mass-matrix takes the following form (see also [12]),

$$m_{ij} = \begin{pmatrix} m_n & \delta m_n & \delta m_n & \cdots \\ \delta m_n & m_n & \delta m_n & \cdots \\ \delta m_n & \delta m_n & m_n & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{pmatrix} \quad (4.4)$$

Notice, that since due to exact permutation symmetry all the off-diagonal entries are equal, this matrix can be written in a very simple form

$$(m_n - \delta m_n) \sum_j n_j n_j + \delta m_n \left(\sum_j n_j \right)^2. \quad (4.5)$$

We can rewrite the above as

$$m_n n_1 n_1 + \delta m_n \left(n_1 \sum_{j \neq 1} n_j + n_1 \sum_{j \neq 1} n_j \right) + \delta m_n \left(\sum_{j \neq 1} n_j \right)^2 + (m_n - \delta m_n) \sum_{j \neq 1} n_j n_j. \quad (4.6)$$

where n_1 denotes the neutron from our copy. Introducing the notation $n_h = \frac{1}{\sqrt{N-1}} \sum_{j \neq 1} n_j$, the problem reduces to the following 2×2 mixing between the two states n_1 and n_h ,

$$\begin{pmatrix} m_n & \delta m_n \sqrt{N-1} \\ \delta m_n \sqrt{N-1} & m_n + \delta m_n (N-2) \end{pmatrix} \quad (4.7)$$

In addition there are $N-2$ exactly degenerate orthogonal states with mass $m_n - \delta m$, which do not mix. The above matrix has two mass eigenstates: $n_H \equiv \frac{1}{\sqrt{N}} n_1 + \frac{\sqrt{N-1}}{\sqrt{N}} n_h$ and $n'_1 \equiv -\frac{1}{\sqrt{N}} n_h + \frac{\sqrt{N-1}}{\sqrt{N}} n_1$, with masses $m_H = m_n + (N-1)\delta m_n$ and $m'_1 = m_n - \delta m_n$, respectively. Thus, in total there are $N-1$ exactly degenerate states of mass $m_n - \delta m_n$, and a single state of mass m_H . In the other words, the mass eigenstates decompose into the $N-1$ dimensional representation of the permutation group $P(N)$ plus a singlet. Each particular neutron mixes with a single common state, in which all the other neutrons enter democratically. Thus, our neutron will oscillate into the other neutrons through n_H -state, and evolve in time as

$$n_1(t) = \sqrt{\frac{N-1}{N}} n'_1 + \frac{1}{\sqrt{N}} n_H e^{-iN\delta m_n t}. \quad (4.8)$$

The inclusive probability of disappearance is easily obtained to be

$$P(t) \approx \frac{4}{N} \sin^2 \left(\frac{N\delta m_n t}{2} \right). \quad (4.9)$$

The frequency of the oscillations is $1/\tau \sim \delta m_n N$, and can be quite large. Using (4.3) and setting $\kappa \sim 1/N$, we get $1/\tau \sim 10^{-10} (\text{TeV}/M_*)^5 \text{ eV}$, implying the oscillation period of approximately

$$\tau \sim 10^{-5} \left(\frac{M_*}{\text{TeV}} \right)^5 \text{ sec}. \quad (4.10)$$

However, the important point is that the amplitude of the oscillation is suppressed by $\frac{1}{\sqrt{N}}$, independently of δm_n . This gives the total disappearance probability of neutrons (in our SM copy) in the vacuum $P_{\text{dis}} \approx 2/N$.

B. Oscillations in the magnetic field

Typically our neutrons do not propagate in the vacuum, and this fact can change the transition probability dramatically. In particular, the presence of the magnetic field can trigger a resonant transition. The previous analysis can be readily generalized in the presence of an additional potential ϵ (e.g., due to magnetic field) for our neutron n_1 . Since the potential is only experienced by our neutron, its pres-

ence affects only 2×2 mixing matrix (4.7) of n_1 and n_h states, which in this case becomes,

$$m_{ij} = \begin{pmatrix} m_n + \epsilon & \delta m_n \sqrt{N-1} \\ \delta m_n \sqrt{N-1} & m_n + \delta m_n (N-2) \end{pmatrix} \quad (4.11)$$

The presence of the extra potential can resonantly increase the oscillation amplitude if the two states become nearly degenerate. This happens when

$$\epsilon \approx \delta m_n (N-2). \quad (4.12)$$

In such a case, the mixing angle becomes 45° and the resonant oscillation period is

$$\tau_r \approx (\delta m_n \sqrt{N-1})^{-1}. \quad (4.13)$$

In particular, the source of the potential can be a magnetic field B . In this case $\epsilon = \vec{\mu} \cdot \vec{B}$, where μ is the neutron magnetic moment. We are now ready to consider some phenomenologically interesting regimes.

C. Different regimes

Let us note, that since the transition amplitude in the nonresonant case is suppressed by $1/\sqrt{N}$, the phenomenologically most interesting situation is when N is not extremely large. In the present context this implies that the symmetry should be less than the full permutation group $P(10^{32})$, in such a way that each neutron has a permutation symmetric mixing with the subset of the copies, less than the total number. This smaller subset then will define a set of “nearest neighbors” in the space of species. In such a case, the number N in our previous calculations has to be understood as the number of such neighbors, as opposed to the total number of species, which we shall temporarily denote by $N_{\text{TOTAL}} \sim 10^{32}$. The symmetry justification for such a situation is straightforward. We can split all N_{TOTAL} SM copies into $n = N_{\text{TOTAL}}/N$ subgroups, with $N \ll N_{\text{TOTAL}}$ members in each subgroup. We then require the full permutation symmetry within each group, plus symmetry of all possible permutations among the groups. The resulting symmetry group thus is, $P(n) \times (P(N)_1 \times P(N)_2 \times \dots \times P(N)_n)$, where it is understood that the elements of $P(n)$ exchange groups among each other. Requiring such a symmetry, the neutron mass matrix splits into the $N \times N$ blocks. The blocks on the diagonal have the form (4.4). The mixing entries in these blocks are much larger, since the six-quark operators (4.1) that mix neutrons from the same group, are only restricted to be suppressed by $\kappa \sim 1/N$. All the other entries of the full $N_{\text{TOTAL}} \times N_{\text{TOTAL}}$ matrix, that mix neutrons from different subgroups, as before, must be suppressed by $\kappa \sim 1/N_{\text{TOTAL}}$, and are negligible. Therefore, for each given neutron, the influence of all the other subgroups can be ignored, and the problem reduces to $N \times N$ mixing. Thus, we can simply use all our previous results, keeping in mind that N is now

not the total number of copies but only of our close neighbors.

Having freed the parameter N , we can now discuss some experimentally interesting regimes of neutron oscillation.

1. Negligible magnetic field

The first regime takes place when the neutron potential created by the magnetic field is negligible with respect the mass difference, $\epsilon \ll \delta m_n (N_q - 2)$. In this case oscillations proceed as in the vacuum, and the oscillation period is given by (4.10). So in any measurement performed with a time resolution $\Delta t \gg \tau$, the neutron will appear in the hidden state with probability $2/N$. In experimental measurements this will manifest itself in form of the unaccounted neutrons.

The relevant experiments are the ones on neutron lifetime measurements [26] and the cold neutron experiments that directly test oscillations of neutrons into the mirror copies [27,28]. In order to place the constraints on our parameters the experimental results must be compared with the present model. For example, let us consider the effect of the above oscillations on the measurement of the neutron lifetime. The neutron decay can be easily accounted by giving an imaginary part i/τ_n to the diagonal Hamiltonian. Since the copies of SM are identical, the decay rates of all neutron into their own SM species are the same. In the usual case of $N = 1$, this leads to the familiar decay law,

$$N_n(t) = N_n(0)e^{-t/\tau_n}, \quad (4.14)$$

where N_n is the number of neutrons. However, for many copies, the law changes. In particular in this case when $\tau \ll \tau_n$ the number of neutrons gets modulated by the rapid oscillations,

$$N_n(t) = N_n(0) \left(1 - \frac{4}{N} \sin^2\left(\frac{t}{\tau}\right) \right) e^{-t/\tau_n}. \quad (4.15)$$

For the measurements produced with the time resolution $\Delta t \gg \tau$, one sees an average value of the prefactor

$$N_n(t) = N_n(0) \left(1 - \frac{2}{N} \right) e^{-t/\tau_n} \quad (4.16)$$

Thus, the modulation cannot be captured in the measurements that are sensitive only to relative decrease of the neutron number, if the time resolution of the measurement is $\Delta t \gg \tau$. In such a case in order to place the bound on N , the mechanisms responsible for systematic neutron losses as well as the impact of the neutron-detection measurement on the oscillation process have to be well understood.

2. Resonant oscillations

The second interesting regime is the resonant transition in the magnetic field. As said above, this happens when the condition (4.12) is satisfied. In such a case, the mixing

angle becomes 45° and the resonant oscillation period is given by

$$\tau_r \sim 10^{-5} \sqrt{N-1} \left(\frac{M_*}{\text{TeV}} \right)^5 \text{ sec.} \quad (4.17)$$

For example, for the earth magnetic field, $.5G$, one has $\epsilon \sim 10^{-12}$ eV. For $N > 2$ and $M_* = \text{TeV}$, the resonance happens in a strong magnetic field $\sim 10^2 G$ or so.

However, both the resonant value of the magnetic field as well as the oscillation period are very sensitive to the value of M_* in TeV units. So taking M_* only few TeV decreases the resonant value of the magnetic field, and increases the oscillation time dramatically. To illustrate the point, notice that for $M_* = 5$ TeV or so, the value of the resonant magnetic field would be the one of the experiments [27,28]. The lower bound on the oscillation time derived from these experiments ($\tau > 103$ sec from [27] and $\tau > 448$ sec from [28], respectively), would translate into the bound on the number of neighboring copies of approximately $N \gtrsim 10^9$. From the above consideration it is clear that for testing this framework, one has to scan entire portion of the parameter space corresponding to the values of M_* within the interval of 1–10 TeV, dictated by the hierarchy problem. For this purpose, performing the experiments of this type for the different values of the magnetic field would be extremely important [29]. In such a case, in the experiment with ultracold neutrons, one should see a resonant increase of the missing neutron number for certain values of the magnetic field.

V. NEUTRINOS AND DARK MATTER

Experimentally the two most concrete indications that the SM is incomplete are provided by the presence of dark matter in the Universe and by neutrino masses. In this section we wish to show that our framework offers a novel way of generating the small neutrino masses as well as possible dark matter candidates.

A. Small neutrino masses from large number of species

Observation of neutrino oscillations has shown that neutrinos have masses of the order 10^{-1} eV (for a review see [30]). In trying to understand the underlying physics responsible for the small neutrino mass, one can think in terms of lepton number symmetry. If lepton number is only an approximate symmetry of the low energy world and is not respected by physics at some high scale, the small neutrino mass can arise as a result of the lepton number violating effective operator suppressed by the cutoff scale. In such a case, neutrinos can be effectively Majorana particles. This scenario is explicitly realized by the seesaw mechanism [31]. In this case, the lepton number is maximally violated by a large mass of the right-handed neutrino. Since this state carries no gauge charge under the SM group, it is not prevented by the latter gauge symmetry from having an arbitrarily high mass. After integrating out

this heavy fermion, one is left with the small Majorana mass of the SM neutrino.

The large extra dimensions scenario, offers an alternative explanation for the smallness of the neutrino mass [32]. In contrast to the seesaw, in the minimal version, the lepton number is assumed to be a good symmetry of the low energy world (this can be achieved, e.g., by gauging $B - L$ symmetry). In this case, neutrino is bounded to be a Dirac particle. The smallness of its mass is then guaranteed by the fact that the right-handed neutrino, very much like the graviton, is delocalized and lives in the extra-dimensional bulk. For this reason, the extra dimensional neutrino is forced to be extremely weakly coupled to its left-handed gauge nonsinglet partner. Notice, that although the two scenarios are dramatically different, they both employ the crucial property that the right-handed neutrino is a SM gauge singlet. It is this very property that allows the right-handed state to either be very heavy or to be freely spread out into the extra-dimensional bulk.

We now wish to suggest an alternative mechanism for generating the small neutrino mass, in the framework of large number of the SM copies. Just like in the extra-dimensional scenarios, the new mechanism crucially relies on the fact that the right-handed neutrino is a gauge singlet, and like the graviton can share its couplings with the different SM copies. As a result of this democracy, the couplings and the resulting neutrino masses are suppressed by $1/\sqrt{N}$ factor.

For simplicity of illustration, we present our mechanism for the case of single lepton generation in each SM copy. Generalization to three generations of neutrinos, is trivial. In order to generate neutrino masses we assume that each copy of the SM is endowed with a right-handed neutrino ν_{Rj} , which we shall label by the same index j . However, since right-handed neutrinos share no SM gauge charges, the notion of belonging to a given copy is only defined in terms of transformation properties with respect to the permutation group. The right-handed neutrino sharing the same label with the given SM copy is defined as the one that under $P(N)$ permutation symmetry transforms simultaneously with the rest of the particles belonging to this copy.

As for the neutrons, the right-handed neutrinos do not have SM charges so they can mix with states in the other sectors. In a sense the neutrinos live in the “bulk” of the space of species just as the graviton does. With each SM copy the right-handed neutrinos interact through the dimension-4 gauge-invariant operators,

$$(HL)_i \lambda_{ij} \nu_{Rj} + \text{H.c.} \quad (5.1)$$

where λ_{ij} are Yukawa coupling constants, that represent $N \times N$ matrix in the space of species. As said above, we assume that no right-handed neutrino Majorana mass arises (as a consequence, for example, of $B - L$ number conservation).

The form of the Yukawa coupling matrix between the different species is restricted by the permutation symmetry to be of the following form,

$$\lambda_{ij} = \begin{pmatrix} a & b & b & \cdots \\ b & a & b & \cdots \\ b & b & a & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{pmatrix} \quad (5.2)$$

After taking into the account the nonzero expectation values of the SM Higgs fields, the above Yukawa matrix translates into the mass matrix for neutrinos. Here we make the minimal assumption that the permutation symmetry is not broken by the Higgs VEVs, and thus, they are all equal $\langle H_j \rangle = v$. Then, the mass matrix is $m_{ij} = \lambda_{ij}v$. This is very similar to the one of neutrons (4.4). However, since unlike neutrons, the right-handed neutrinos are introduced as elementary fermions, the diagonal and nondiagonal entries have no reason to be as different, as they were in the case of neutrons. So, it is natural to assume that, to the leading order in $1/N$, the coefficients a and b are roughly similar.

In order to find the resulting neutrino masses, notice that the coefficient b cannot be large because of unitarity. For example, consider a process of right-handed neutrino production in the scattering of the SM states. The inclusive rate for this process goes as

$$\Gamma_{\text{tot}} \approx Nb^2E, \quad (5.3)$$

where the factor N comes from the summation over the different ν_{Rj} species in the final state. Unitarity below the cutoff scale then demands

$$b \leq \frac{1}{\sqrt{N}}. \quad (5.4)$$

On the other hand the coefficient a , the coupling of a the right-handed neutrino to its ‘‘own’’ specie is not so restricted by unitarity and could be taken to be larger. However, the two couplings are exactly the same in nature, we shall not require any strong hierarchy between them.

We are now ready to compute the neutrino masses. When the Higgs in each sector acquires a VEV, from (5.1) one obtains Dirac masses for the neutrinos. The structure of the mixing matrix is identical to the one of neutrons, studied in the previous section in detail, so we can just use the results there. The mass eigenvalues are given by

$$m_i = (a - b)v \quad i = 1, \dots, N - 1 \quad (5.5)$$

$$m_N = [a + (N - 1)b]v \quad (5.6)$$

As a consequence of the permutation symmetry all neutrinos are degenerate while one has a mass of the order of the Planck scale. The hierarchy between the heavy and the light neutrino masses is $\sim N$. Notice, that the latter fact can be considered as an independent (from unitarity) reason for

the bound $b \lesssim 1/\sqrt{N}$. In the opposite case, effectively we would be forced to introduce an elementary particle heavier than the Planck mass, which would make a little sense. Hence, in our approach, we can equally say that the SM neutrinos are forced to be light in order to avoid the appearance of the state with the super-Planckian mass. The latter argument bares no reference to unitarity.

Assuming that $b \approx 1/\sqrt{N}$ the neutrino masses in our scenario are

$$m_\nu \approx (a/b - 1) \frac{v}{\sqrt{N}} \sim (a/b - 1) \frac{vM_*}{M_P} \quad (5.7)$$

Depending on the precise value of the M_* , the right magnitude of neutrino masses requires a mild hierarchy $a \approx 1000b$. Notice that, the above small hierarchy between a and b corresponds to a right-handed neutrinos being somewhat ‘‘localized’’ in the space of species.

One might be worried that due to the large number of light neutrino eigenstates there might be very large oscillations into the hidden neutrinos which would be inconsistent with observation. This however does not happen as can be seen by simply adopting the results of the neutron oscillations derived earlier. Exactly by the same reasons as in neutron case, any given neutrino (call it ν_1) oscillates into a single common heavy state ν_H , according to the following rule,

$$\nu_1 \approx \left(1 - \frac{1}{2N}\right)\nu'_1 + \frac{1}{\sqrt{N}}\nu_H e^{-iNbv t}. \quad (5.8)$$

The only difference from the neutron case is that the state ν_H is ultra heavy, so there cannot be oscillations into the neutrinos of the hidden sectors. The oscillations will be purely between neutrinos within the same SM copy, just as in the usual case. This fact should be compared with the analog mechanism in large extra dimensions [32]. In that case the neutrinos can oscillate into the states of the KK tower which behave as sterile neutrinos. This needs to be suppressed because such oscillations are disfavored by the present data [30].

Having introduced renormalizable couplings between different copies one might wonder about other renormalizable operators connecting the species. In fact there are only two operators of this kind which are gauge invariant, the Higgs quartic coupling and the mixing between the $U(1)_{\text{hypercharge}}$,

$$\lambda_{ij}^H |H_i|^2 |H_j|^2, \quad \alpha_{ij} F_{\mu\nu}^i F^{j\mu\nu} \quad (5.9)$$

The kinetic mixing of photon with a hidden sector gauge field is known to induce the effective charge-shifts [33] (for a stringy realization see [34] and also [35] for a more phenomenological discussion). This remains true in the present case also. So the second coupling above effectively gives a charge to the SM fermions of species i under the gauge group j (see [36]). For the Higgs quartic coupling (and similarly for α_{ij}) the model is perturbative when,

$$\lambda_{ij}^H \leq \frac{1}{N} \quad i \neq j \quad (5.10)$$

and $\lambda_{ii}^H \leq 1$ which coincides with the value demanded by naturalness. This induces a coupling between different species comparable to gravity and therefore is completely negligible for all practical purposes.

B. Dark matter

The fact that the cross section for processes between the different species are so weak, makes the baryons in the other sectors into the viable dark matter candidates. In this section we would like to discuss such a possibility. The key point that can make the dark matter effectively collisionless is the large number of copies with very low individual densities.

This idea was introduced in Ref. [11] in the context of large extra dimensions. The authors considered a brane folded in the extra dimension proposing that dark matter could just be ordinary matter from a different fold. Such a matter is electromagnetically invisible because a ray of light should travel a distance larger than the horizon size in order to reach us. On the other hand, such matter has an usual gravitational effect on the Universe since the four-dimensional gravity is common. Hence the baryons from the other folds are the natural candidates for the dark matter.

This picture becomes natural in our framework due to the fact that we do not need to postulate extra dynamics that produces the extra copies, such as the folded brane-world. Moreover, the huge number of copies automatically takes care of the needed dark matter properties. Cosmological observations require that the Universe is made of 74% dark energy, 22% dark matter, and 4% visible baryons. If only n copies are populated, we need a total baryon density from the SM copies which is about 5 times the one of visible baryons,

$$nn_B^{\text{dark}} \approx 5n_B^{\text{visible}}. \quad (5.11)$$

The minimal number is constrained by the fact that the star formation should not have started in the other copies or else this would be inconsistent with observations,

$$n_B^{\text{dark}} \leq \frac{1}{4}. \quad (5.12)$$

Close to the bound star formation would have just started in the other copies and exotic objects such as MACHOS might have been formed in the other sectors. However, when put in the cosmological context, the actual number of the required weakly-populated copies may be much higher. The outcome depends on the concrete scenario of how in the post inflationary Universe the degeneracy between the copies is broken, and how the reheating takes place. One necessary requirement is that our copy is the maximally

populated one. On the other hand, the degeneracy among the other copies may be lifted in a different ways.

A crucial question thus is how the asymmetric initial conditions on the visible and dark matter abundance can be produced. A possible mechanism was introduced in [20] where it was shown that this can be obtained through the modulated reheating mechanism of Ref. [37] within low scale inflationary models. In this scenario the equal underpopulation of all the hidden sectors is an automatic consequence of the density perturbation mechanism in which the perturbations are imprinted during the reheating, via a common modulating field. All the other sectors then have equal tiny densities, and are never thermalized. The advantage of this picture is that dark matter abundance is calculable in terms of known cosmological parameters, such as baryon to entropy ratio and density perturbations. Very low individual densities also make the outcome independent of the concrete mechanism of baryon asymmetry generation, since this mechanism is not operative in the hidden sectors anyway due to their very low densities. Thus, regardless of this mechanism, other copies effectively contain equal number of baryons and antibaryons.

In general, other scenarios are also possible, in which asymmetry among the observable and visible sectors is more involved. For example, we can have a situation in which not all the hidden sectors are equally populated, and some are having densities comparable to ours. In such a case, such sectors cannot include an equal number of hidden baryons and antibaryons, due to their effective annihilation, and the details of baryogenesis become relevant. In addition, as we mentioned earlier, in order to evade BBN constraints there should not be any extra massless degrees of freedom which contribute to the expansion of the Universe. This implies that the temperature of the other copies is much smaller than the one measured in cosmic microwave background,

$$T_{\text{dark}} \ll 3K. \quad (5.13)$$

This puts an extra burden on the cases when densities in the hidden copies are relatively high, since in such a case the species there have to be produced very cold. In general the cosmological evolution will be determined by the matter density and temperature of each SM copy, n_B^i , T^i . and a detailed analysis is needed.

Finally, we wish to quote that some constraints of many species inflationary cosmology, applicable to the regimes that differ from the ones of our interest above, were considered in [38].

VI. DISCUSSION

In this paper we have investigated various phenomenological aspects of the large N species solution of the hierarchy problem proposed recently, focusing on the permutation symmetric scenario obtained by adding 10^{32} mirror copies of the standard model.

The key point in addressing the hierarchy problem is played by the intrinsic connection between the number of species and short distance gravitational physics. According to this connection, the existence of N species in the low energy world automatically lowers the gravitational cutoff (the quantum gravity scale) according to the Eq. (1.1). In other words, the existence of species in the low energy theory implies extra dimensional type modification of gravity at distances much larger than the Planck length.

We have shown, that some of the properties of this new gravitational physics can be constrained by symmetries acting on the low energy species (in our case SM copies). In particular, cyclic symmetry among the copies, gives rise to gravitational physics strikingly similar to the one of extra dimensions. The existence of many SM copies, however, makes phenomenology different from the large extra-dimensional framework. The maximal departure from the smooth geometric picture takes place in the limit of full permutation symmetry acting among the copies. In this case the hidden copies behave as perfect dark worlds. Exact permutation symmetry combined with unitarity turns out to be a remarkably powerful principle so, not surprisingly, this version of the theory is maximally restrictive.

The basic result of our analysis is that when confronted with the existing cosmological and particle physics data below the TeV scale, the scenario easily passes all possible tests, but at the same time opens up new experimental avenues for the search of new physics. One model-independent prediction is strong gravity around TeV energies. This, among other things, should manifest itself in the production of microscopic BHs in particle collisions. However, the observable properties of these BHs (such as the evaporation rate to mass ratio, or branching ratios into the visible versus invisible channels) are very different from the canonical large extra-dimensional picture. For example, unlike the usual large extra-dimensional black holes, the branching ratio for evaporation into the hidden channels increases with the BH mass according to (2.21).

Second, there is an interesting low-energy window into the hidden world in the form of the oscillations of the neutron into its hidden copies. The existence of many potential mixing partners brings a qualitatively new experimentally interesting features, such as the possibility of rapid oscillations with a suppressed amplitude, or the resonant transition in a nonzero magnetic field. This opens up an exciting new possibilities of searching for an imprint of the dark sector physics in cold neutron experiments of the type [27,28].

We have also shown that some of the mechanisms introduced in the context of large extra dimensions can be generalized naturally to the case of many species and do not rely on the existence of an actual underlying geometry. In particular proton stability can be guaranteed by the gauging of the common baryon number between the spe-

cies. (This still allows the above mentioned mixing between the neutral states from different species, particularly neutrons). The neutrinos can acquire a small, $1/\sqrt{N}$ -suppressed, Dirac mass without a seesaw mechanism. Baryons from the hidden copies are viable candidates for dark matter. It is rather clear that many of the features studied in the context of large extra dimensions do not rely on the existence of a geometry but on a more general notion of unitarity and locality.

The most relevant open theoretical question to be understood is the UV completion of this scenario. As we have seen in Sec. III B the permutation symmetry already restricts strongly the possible physics at the cutoff. In the case of cyclic symmetry, which implies UV completion in terms of smooth extra dimension type geometry, the possibility of embedding in string theory, along the lines of [39], seems not to exhibit any difficulty of principle. However, UV completion of the full permutation symmetric scenario appears much more mysterious because the departure from geometry is maximal. Is there a stringy realization of this scenario? While we are not aware of any explicit construction it seems natural to look for this type models in the nongeometrical compactifications of the landscape of string theory (see, for example, [40]). These constructions may not contain Kähler moduli describing the size of physical dimensions and so appear as the appropriate starting point. In general, consistent propagation of strings requires a two dimensional conformal field theory of the appropriate central charge. In the standard case the conformal field theory is a free theory and has as low energy limit 10D supergravity. However it is possible to replace the 6-extra dimensions with a conformal field theory of equal central charge which does not admit a geometrical description at low energy. The large number of light fields could be perhaps obtained by introducing branes in this setup. Alternatively it might be possible to construct our nongeometrical setup considering compactifications with complicated topology (see, for example, [41]). In this way it might be possible to obtain an exponential proliferation of light modes which in turn creates a large hierarchy between the Planck scale and the fundamental scale. We leave a detailed investigation of these ideas to future work.

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Note Added.—Beside neutron oscillations, mixing between other neutral states might also be of interest experimentally. In particular, it was pointed out to us that measurements of positronium oscillation have been performed [42] which excluded oscillations with a precision of 4×10^{-7} . Unfortunately, in our case the mixing between these states is strongly suppressed by the size of the object so that we do not expect this to be in the experimentally interesting region due to the long period of the oscillation. Another interesting window might, however, appear in $K_0 - \bar{K}_0$ oscillations. Dimension 6 operators of the form,

$$\frac{1}{N} \frac{u_i \bar{s}_i \bar{u}_j s_j}{M_*^2} \quad (6.1)$$

would induce oscillations between Kaons of different copies (we assume as for the neutrons that mixing within the same copy are suppressed). With $M_* \approx \text{TeV}$ the oscillation time, $\tau^{-1} = O(m_K^3/M_*^2)$, is much shorter than the experimental bound from flavor physics. Given that the transition $d\bar{s} \rightarrow s\bar{d}$ is measured with a 1% precision this could already imply a rough bound $N \approx 100$ on the minimum number of copies.

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- [1] P. A. M. Dirac, *Nature (London)* **139**, 323 (1937).
 [2] N. Arkani-Hamed, S. Dimopoulos, and G. R. Dvali, *Phys. Lett. B* **429**, 263 (1998).
 [3] G. Dvali, arXiv:0706.2050.
 [4] G. Dvali and M. Redi, *Phys. Rev. D* **77**, 045027 (2008).
 [5] For a generalization of the bound to the deSitter spaces, see, G. Dvali and D. Lust, *J. High Energy Phys.* 06 (2008) 047.
 [6] G. Dvali, arXiv:0806.3801.
 [7] G. Dvali and C. Gomez, *Phys. Lett. B* **674**, 303 (2009).
 [8] G. Dvali and G. Gabadadze, *Phys. Rev. D* **63**, 065007 (2001).
 [9] G. Veneziano, *J. High Energy Phys.* 06 (2002) 051.
 [10] G. Dvali, M. Redi, S. Sibiryakov, and A. Vainshtein, *Phys. Rev. Lett.* **101**, 151603 (2008).
 [11] N. Arkani-Hamed, S. Dimopoulos, G. R. Dvali, and N. Kaloper, *J. High Energy Phys.* 12 (2000) 010.
 [12] R. Foot, *Phys. Lett. B* **632**, 467 (2006); R. Foot and R. R. Volkas, *Phys. Lett. B* **645**, 75 (2007).
 [13] R. Foot, H. Lew, and R. R. Volkas, *Phys. Lett. B* **272**, 67 (1991).
 [14] T. D. Lee and C. N. Yang, *Phys. Rev.* **104**, 254 (1956); I. Y. Kobzarev, L. B. Okun, and L. Y. Pomeranchuk, *Sov. J. Nucl. Phys.* **3**, 837 (1966); For a review see L. B. Okun, *Phys. Usp.* **50**, 380 (2007).
 [15] S. Dimopoulos and H. Georgi, *Nucl. Phys.* **B193**, 150 (1981).
 [16] C. M. Will, *Living Rev. Relativity* **4**, 4 (2001).
 [17] G. Dvali and G. R. Farrar, *Phys. Rev. Lett.* **101**, 011801 (2008).
 [18] G. Dvali and O. Pujolas, *Phys. Rev. D* **79**, 064032 (2009).
 [19] N. Arkani-Hamed, S. Dimopoulos, and G. R. Dvali, *Phys. Rev. D* **59**, 086004 (1999).
 [20] G. Dvali, I. Sawicki, and A. Vikman, arXiv:0903.0660.
 [21] W. Israel, *Phys. Rev.* **164**, 1776 (1967); *Commun. Math. Phys.* **8**, 245 (1968); B. Carter, *Phys. Rev. Lett.* **26**, 331 (1971); J. Hartle, *Phys. Rev. D* **3**, 2938 (1971); J. Bekenstein, *Phys. Rev. D* **5**, 1239 (1972); **5**, 2403 (1972); *Phys. Rev. Lett.* **28**, 452 (1972); C. Teitelboim, *Phys. Rev. D* **5**, 2941 (1972).
 [22] L. Alvarez-Gaume and E. Witten, *Nucl. Phys.* **B234**, 269 (1984).
 [23] M. B. Green and J. H. Schwarz, *Phys. Lett.* **149B**, 117 (1984); M. Dine, N. Seiberg, and E. Witten, *Nucl. Phys.* **289**, 589 (1987); J. J. Atick, L. J. Dixon, and A. Sen, *Nucl. Phys.* **B292**, 109 (1987); M. Dine, I. Ichinose, and N. Seiberg, *Nucl. Phys.* **B293**, 253 (1987).
 [24] Z. Berezhiani and L. Bento, *Phys. Rev. Lett.* **96**, 081801 (2006); Z. Berezhiani, arXiv:0804.2088.
 [25] Kamiokande Collaboration, Report No. ICRR-Report-298-93-10.
 [26] The most accurate measurement of the neutron lifetime ($886.8 \pm 1.2_{\text{stat}} \pm 3.2_{\text{sys}}$ sec) in the beam type experiments was performed in, M. S. Dewey *et al.*, *Phys. Rev. Lett.* **91**, 152302 (2003); The most accurate measurement of the neutron lifetime ($878.5 \pm 0.7_{\text{stat}} \pm 0.3_{\text{sys}}$ sec) to date, using gravitationally trapped ultracold neutrons, was performed in A. Serebrov *et al.*, *Phys. Lett. B* **605**, 72 (2005); A. P. Serebrov *et al.*, *Phys. Rev. C* **78**, 035505 (2008).
 [27] G. Ban *et al.*, *Phys. Rev. Lett.* **99**, 161603 (2007).
 [28] A. P. Serebrov *et al.*, *Phys. Lett.* **663**, 181 (2008); arXiv:0809.4902.
 [29] I. Altarev *et al.* (nEDM collaboration) (unpublished).
 [30] A. Strumia and F. Vissani, arXiv:hep-ph/0606054.
 [31] M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity*, edited by D. Freedman *et al.* (North Holland, Amsterdam, 1979); T. Yanagida, *Prog. Theor. Phys.* **64**, 1870 (1980); R. N. Mohapatra and G. Senjanovic, *Phys. Rev. Lett.* **44**, 912 (1980).
 [32] N. Arkani-Hamed, S. Dimopoulos, G. R. Dvali, and J. March-Russell, *Phys. Rev. D* **65**, 024032 (2001); G. R. Dvali and A. Y. Smirnov, *Nucl. Phys.* **B563**, 63 (1999).
 [33] B. Holdom, *Phys. Lett.* **166B**, 196 (1986).
 [34] K. R. Dienes, C. F. Kolda, and J. March-Russell, *Nucl. Phys.* **B492**, 104 (1997); D. Lust and S. Stieberger, *Fortschr. Phys.* **55**, 427 (2007).
 [35] S. A. Abel, M. D. Goodsell, J. Jaeckel, V. V. Khoze, and A. Ringwald, *J. High Energy Phys.* 07 (2008) 124.
 [36] R. Rahman, arXiv:0901.2111.
 [37] G. Dvali, A. Gruzinov, and M. Zaldarriaga, *Phys. Rev. D* **69**, 023505 (2004).
 [38] Y. Watanabe and E. Komatsu, *Phys. Rev. D* **77**, 043514 (2008); P. Adshead, R. Easther, and E. A. Lim, *Phys. Rev.*

- D **79**, 063504 (2009); L. Leblond and S. Shandera, J. Cosmol. Astropart. Phys. 08 (2008) 007.
- [39] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, and G. R. Dvali, Phys. Lett. B **436**, 257 (1998).
- [40] B. Wecht, Classical Quantum Gravity **24**, S773 (2007).
- [41] S. Dimopoulos, S. Kachru, J. McGreevy, and J. G. Wacker, J. Cosmol. Astropart. Phys. 08 (2008) 003.
- [42] A. Badertscher *et al.*, Phys. Rev. D **75**, 032004 (2007).