

Gauge invariance of color confinement due to the dual Meissner effect caused by Abelian monopoles

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The mechanism of non-Abelian color confinement is studied in SU(2) lattice gauge theory in terms of the Abelian fields and monopoles extracted from non-Abelian link variables *without adopting gauge fixing*. First, the static quark-antiquark potential and force are computed with the Abelian and monopole Polyakov loop correlators, and the resulting string tensions are found to be identical to the non-Abelian string tension. These potentials also show the scaling behavior with respect to the change of lattice spacing. Second, the profile of the color-electric field between a quark and an antiquark is investigated with the Abelian and monopole Wilson loops. The color-electric field is squeezed into a flux tube due to monopole supercurrent with the same Abelian color direction. The parameters corresponding to the penetration and coherence lengths show the scaling behavior, and the ratio of these lengths, i.e., the Ginzburg-Landau parameter, indicates that the vacuum type is near the border of the type 1 and type 2 (dual) superconductors. These results are summarized in which the Abelian fundamental charge defined in an arbitrary color direction is confined inside a hadronic state by the dual Meissner effect. As the color-neutral state in any Abelian color direction corresponds to the physical color-singlet state, this effect explains non-Abelian color confinement and supports the existence of a gauge-invariant mechanism of color confinement due to the dual Meissner effect caused by Abelian monopoles.

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I. INTRODUCTION

Color confinement in quantum chromodynamics (QCD) is still an important unsolved problem [1]. 't Hooft [2] and Mandelstam [3] conjectured that the QCD vacuum is a kind of dual superconducting state caused by condensation of magnetic monopoles. The color charges are then confined inside hadrons due to the formation of the color-electric flux tube through the dual Meissner effect. However, in contrast to the Georgi-Glashow model [4,5] or supersymmetric QCD [6] with scalar fields, it is not straightforward to identify the color-magnetic monopoles in QCD.

An interesting idea to realize this conjecture is proposed by 't Hooft [7], such that SU(3) QCD can be reduced to an Abelian $[U(1)]^2$ theory by adopting a partial gauge fixing, and the color-magnetic monopoles appear according to $\pi_2(SU(3)/[U(1)]^2) = \mathbb{Z}^2$. The role of monopoles for the confinement mechanism is investigated extensively on the lattice by applying Abelian projection in the maximally Abelian (MA) gauge [8,9], where monopoles are extracted *a la* DeGrand-Toussaint [10] as in compact U(1) lattice gauge theory. It is then found that the results strongly support the dual superconducting scenario [11–18]. The confining properties are dominated by the Abelian fields [11,13,19] and monopoles [13,20–24], which are called Abelian dominance and monopole dominance, respectively. The color-electric flux is squeezed by the dual

Meissner effect [12,15,17,18,25]. Moreover, monopole condensation is confirmed by the energy-entropy balance of the monopole trajectories [22,26–28]. These results indicate that there must exist a dual Ginzburg-Landau (GL) type theory as an infrared effective theory of QCD [29–31].

However, there are still serious problems to prove this scenario. First, there are infinite ways of the partial gauge fixing. Since the behavior of the monopoles can depend on the gauge choice, it is not clear if the lattice results in the MA gauge are universal. Note that in the Polyakov (PL) gauge, 't Hooft's color-magnetic monopoles [7] propagate only in the time direction, which cannot confine static color charges [32]. Second, as the 't Hooft scheme essentially uses the Abelian degrees of freedom, it is not explained how non-Abelian color charges are confined.

Recently, we obtained clear numerical evidences of Abelian dominance and the dual Meissner effect in local unitary gauges such as the F_{12} and the PL gauges in SU(2) lattice gauge theory [33], where we have used the DeGrand-Toussaint monopoles [10] as in the MA gauge. These results provide us with the following ideas.

- (1) The DeGrand-Toussaint monopoles on the lattice [10] can be different from the 't Hooft color-magnetic monopoles [7].
- (2) There must exist a gauge-invariant mechanism of color confinement due to Abelian monopoles [34,35].

In this paper, we aim to show detailed numerical evidence of how these ideas are realized. We investigate the confining properties in SU(2) lattice gauge theory in terms of the gauge-variant Abelian fields and monopoles extracted from non-Abelian link variables *without adopting any spatially local or nonlocal gauge fixing*. We find that a gauge-invariant Abelian mechanism of color confinement due to Abelian monopoles works even in the continuum limit of SU(2) QCD, although we consider gauge-variant Abelian operators. The results may also apply to SU(3) gauge theory, since the essential features are not altered.

The paper is organized as follows. In Sec. II, we explain how to extract the Abelian fields and the monopoles from non-Abelian link variables without gauge fixing. In Secs. III and IV, we compute the static quark-antiquark potential and the force with the Abelian and monopole Polyakov loop correlators, and find that the string tensions exhibit Abelian dominance and monopole dominance. These potentials also show the scaling behavior with respect to the change of lattice spacing. In Sec. V, we investigate the correlation function between the Abelian operators and the Wilson loop. We observe that the color-electric field is squeezed into a flux tube due to monopole supercurrent with the same Abelian color direction. The parameters corresponding to the penetration depth and the coherence length show the scaling behavior, and the ratio of these lengths, i.e., the GL parameter, indicates that the vacuum type is near the border of the type 1 and type 2 (dual) superconductors. In Sec. VI, we discuss implications of our results, i.e., the Abelian fundamental charge defined in an arbitrary color direction is confined by the dual Meissner effect. As the color-neutral state in any Abelian color direction corresponds to the physical color-singlet state, the dual Meissner effect for the Abelian fundamental charge can also explain confinement of non-Abelian color charges. The final section VII is devoted to conclusion and remarks. Our preliminary results are already published in Ref. [36].

II. ABELIAN PROJECTION AND EXTRACTION OF MONOPOLES

We explain how to extract the Abelian fields and the color-magnetic monopoles from the thermalized non-Abelian SU(2) link variables,

$$U_\mu(s) = U_\mu^0(s) + i\vec{\sigma} \cdot \vec{U}_\mu(s), \quad (1)$$

where $\vec{\sigma} = (\sigma^1, \sigma^2, \sigma^3)$ is the Pauli matrix. Abelian link variables in one of the color directions, for example, in the σ^1 direction, are defined as

$$u_\mu(s) = \cos\theta_\mu(s) + i\sigma^1 \sin\theta_\mu(s), \quad (2)$$

where

$$\theta_\mu(s) = \arctan\left(\frac{U_\mu^1(s)}{U_\mu^0(s)}\right) \quad (3)$$

correspond to the Abelian fields. Without gauge fixing the Abelian fields in any color directions should be equivalent.

We then define the Abelian field strength tensors as

$$\begin{aligned} \Theta_{\mu\nu}(s) &= \theta_\mu(s) + \theta_\nu(s + \hat{\mu}) - \theta_\mu(s + \hat{\nu}) - \theta_\nu(s) \\ &= \bar{\Theta}_{\mu\nu}(s) + 2\pi n_{\mu\nu}(s), \end{aligned} \quad (4)$$

where $\bar{\Theta}_{\mu\nu} \in [-\pi, \pi]$ and $n_{\mu\nu}(s)$ is an integer corresponding to the number of the Dirac strings piercing the plaquette. The monopole currents are then defined by [10]

$$\begin{aligned} k_\nu(s) &= \frac{1}{4\pi} \epsilon_{\mu\nu\rho\sigma} \partial_\mu \bar{\Theta}_{\rho\sigma}(s + \hat{\nu}) \\ &= -\frac{1}{2} \epsilon_{\mu\nu\rho\sigma} \partial_\mu n_{\rho\sigma}(s + \hat{\nu}) \in \mathbb{Z}, \end{aligned} \quad (5)$$

where ∂_μ is regarded as a forward difference.

III. ABELIAN DOMINANCE

We show the result of the Abelian static potential [36]. We generate thermalized gauge configurations using the SU(2) Wilson action at a coupling constant $\beta = 2.5$ on the lattice $N_s^3 \times N_t = 24^3 \times 24$, where the lattice spacing $a(\beta = 2.5) = 0.0836(8)$ (fm) is fixed by assuming $\sqrt{\sigma} = 440$ (MeV).

By using the multilevel noise reduction method [37], we evaluate the Abelian static potential V_A from the correlation function of the Abelian Polyakov loop operator

$$P_A = \exp\left[i \sum_{k=0}^{N_t-1} \theta_4(s + k\hat{4})\right], \quad (6)$$

separated at a distance R as

$$V_A(R_I) = -\frac{1}{aN_t} \ln\langle P_A(0)P_A^*(R) \rangle. \quad (7)$$

The q - \bar{q} distance R is improved to $R_I = (4\pi G(R))^{-1}$ in order to reduce the lattice artifact due to finite-lattice spacing, where $G(R)$ is the Green function of the lattice Laplacian in three dimensions [38,39]. For the multilevel method, the number of sublattices adopted is 6 and the sublattice size is $4a$.

The result is plotted in Fig. 1, where the non-Abelian static potential computed from the ordinary Polyakov loop correlation function is also plotted for comparison. The number of independent gauge configurations is $N_{\text{conf}} = 10$ in both cases, but the number of internal updates in the multilevel method is 15 000 for the non-Abelian case and 160 000 for the Abelian case. The statistical errors are determined by the jackknife method.

We fit the potential to the usual functional form

$$V_{\text{fit}}(R) = \sigma R - c/R + \mu, \quad (8)$$

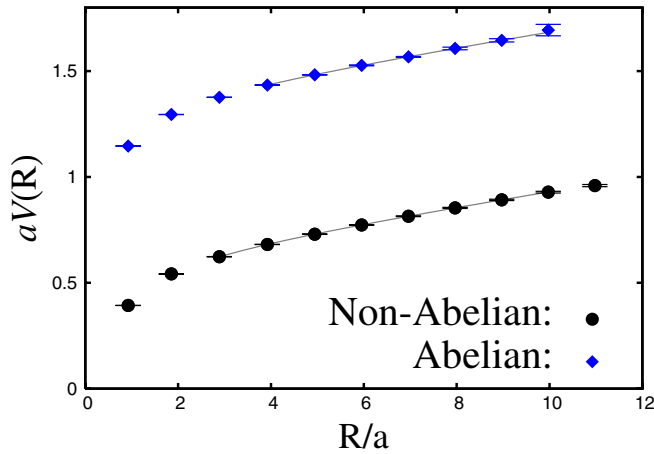


FIG. 1 (color online). The Abelian static potential in comparison with the non-Abelian one. The lines denote the best fitting curve to the function $V_{\text{fit}}(R)$.

where σ denotes the string tension, c the Coulombic coefficient, and μ the constant. The result is summarized in Table I. We find Abelian dominance such that the Abelian string tension is the same as the non-Abelian one.

Here we comment on the theoretical observations of the Abelian dominance of the string tension using the character expansion [40,41]. The authors of [41] say that they have proved exactly the Abelian dominance without gauge fixing based on a relation for any two half-integer representations $j_1 > j_2$

$$\lim_{T \rightarrow \infty} \frac{W_{j_1}[R, T]}{W_{j_2}[R, T]} = 0, \quad (9)$$

where

$$W_j[C] = \frac{1}{2j+1} \langle \chi_j[U(C)] \rangle \quad (10)$$

and $\chi_j[g]$ is the SU(2) group character in representation j . In Ref. [41], the above relation is derived on some considerations of screening effects and glue-lump energy. But nobody knows an exact method of analytic calculations in the infrared nonperturbative region of QCD, so that their considerations about the screening effects and glue-lump energy are not exact theoretical, although very plausible.

TABLE I. Best fitted values of the string tension σa^2 , the Coulombic coefficient c , and the constant μa . V_{NA} denotes the non-Abelian static potential. N_{iup} is the number of internal updates in the multilevel method. FR means the fitting range. The χ^2 for the central value is $\chi^2/N_{\text{df}} < 0.1$.

	σa^2	c	μa	FR (R/a)	N_{iup}
V_{NA}	0.0348(7)	0.243(6)	0.607(4)	3.92–9.97	15 000
V_{A}	0.0352(16)	0.231(39)	1.357(17)	4.94–9.97	160 000

Our numerical observations here are hence nontrivial and they suggest the above relation (9) is actually exact.

IV. MONOPOLE DOMINANCE

A. The monopole Polyakov loop

We investigate the monopole contribution to the static potential in order to examine the role of monopoles for confinement. The monopole part of the Polyakov loop operator is extracted as follows. Using the lattice Coulomb propagator $D(s - s')$, which satisfies $\partial_\nu \partial'_\nu D(s - s') = -\delta_{ss'}$ with a forward (backward) difference ∂_ν (∂'_ν), the temporal components of the Abelian fields $\theta_4(s)$ are written as

$$\theta_4(s) = -\sum_{s'} D(s - s') [\partial'_\nu \Theta_{\nu 4}(s') + \partial_4(\partial'_\nu \theta_\nu(s'))]. \quad (11)$$

Inserting Eq. (11) [and then Eq. (4)] to the Abelian Polyakov loop (6), we obtain

$$\begin{aligned} P_A &= P_{\text{ph}} \cdot P_{\text{mon}}, \\ P_{\text{ph}} &= \exp \left\{ -i \sum_{k=0}^{N_t-1} \sum_{s'} D(s + k\hat{4} - s') \partial'_\nu \bar{\Theta}_{\nu 4}(s') \right\}, \\ P_{\text{mon}} &= \exp \left\{ -2\pi i \sum_{k=0}^{N_t-1} \sum_{s'} D(s + k\hat{4} - s') \partial'_\nu n_{\nu 4}(s') \right\}. \end{aligned} \quad (12)$$

We call P_{ph} the photon and P_{mon} the monopole parts of the Abelian Polyakov loop, respectively [23]. The latter is due to the fact that the Dirac strings $n_{\nu 4}(s)$ lead to the monopole currents in Eq. (5) [10]. Note that the second term of Eq. (11) does not contribute to the Abelian Polyakov loop in Eq. (6).

B. Simulation parameters

We then compute the static potential from the monopole Polyakov loop correlation function. However, since Eq. (12) contains the nonlocal Coulomb propagator $D(s - s')$ and the Polyakov loop is not written as a product of local operators along the time direction, the multilevel method cannot be applied. Without such a powerful noise reduction method, it is hard to measure the Polyakov loop correlation function at zero temperature with the present available computer resource. Thus we consider a finite temperature $T \neq 0$ system in the confinement phase. We set $T = 0.8T_c$. In order to examine the scaling behavior of the potential, we simulate the Wilson action on the $24^3 \times (N_t = 4, 6, 8)$ lattices. We choose the gauge coupling for each N_t so as to keep the same temperature. We also investigate the spatial volume dependence of the potential for the $N_t = 6$ case. Simulation parameters are summarized in Table II. The lattice spacing $a(\beta)$ is determined by using the Sommer scale [$r_0 = 0.5$ (fm)] at zero temperature.

TABLE II. Simulation parameters for the measurement of the static potential and the force from P_A , P_{ph} , and P_{mon} . N_{RGT} is the number of random gauge transformations.

β	$N_s^3 \times N_t$	$a(\beta)$ (fm)	N_{conf}	N_{RGT}
2.20	$24^3 \times 4$	0.211(7)	6000	1000
2.35	$24^3 \times 6$	0.137(9)	4000	2000
2.35	$36^3 \times 6$	0.137(9)	5000	1000
2.43	$24^3 \times 8$	0.1029(4)	7000	4000

C. Noise reduction by gauge averaging

Since the signal-to-noise ratio of the correlation functions of P_A , P_{ph} , and P_{mon} are still very small with no gauge fixing, we adopt a new noise reduction method [36]. For a thermalized gauge configuration $U_\mu(s)$, we produce many gauge copies $U_\mu^g(s)$ applying random gauge transformations. Then we compute the operator for each copy and take the average over all copies. When we consider a

gauge-invariant operator $O(U)$, the value of the operator is the same for both ensembles, i.e., $O(U) = O(U^g)$. Hence random gauge transformations are not useful for statistical error reduction. But when we consider a gauge-variant operator $O_A(U)$, we get $O_A(U) \neq O_A(U^g)$, since it contains a gauge-variant piece. But using the gauge invariance of the Haar measure and the action, we can show the expectation value is the same:

$$\frac{1}{Z} \int DU e^{-\beta S} O_A(U) = \frac{1}{Z} \int DU e^{-\beta S} O_A(U^g).$$

Hence random gauge transformations are useful to reduce the statistical errors and to get the gauge-invariant expectation value. The results obtained with this method are gauge averaged, thus, gauge invariant.

In practice, we prepare a few thousand gauge copies for each independent gauge configuration (see Table II). We also apply one-step hypercubic blocking (HYP) [42] to the

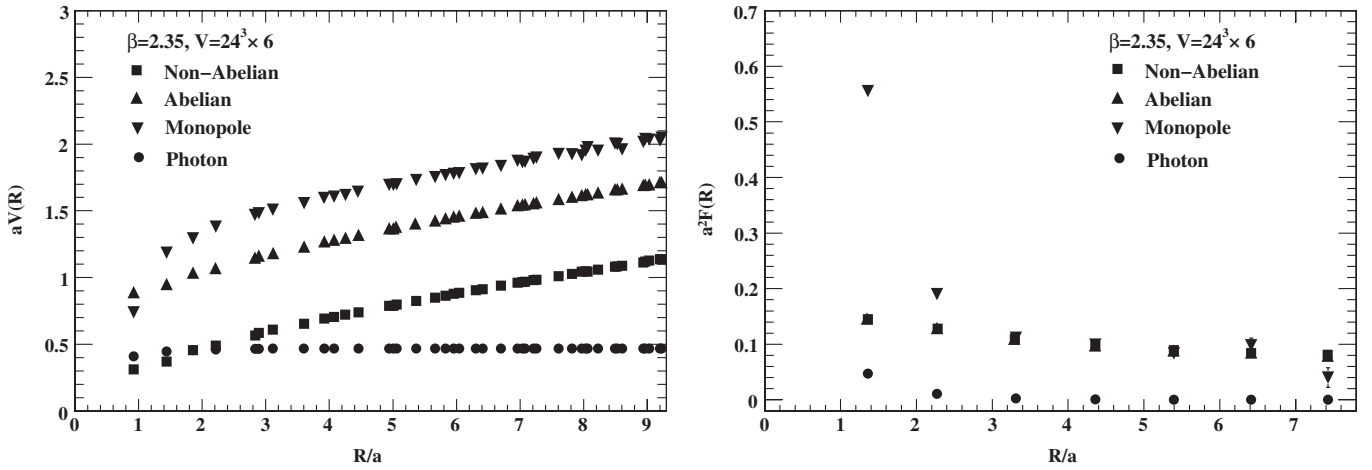


FIG. 2. The static potential (left panel) and the force (right panel) from the non-Abelian, the Abelian, the monopole, and the photon Polyakov loop correlation function at $\beta = 2.35$ on the $24^3 \times 6$ lattice.

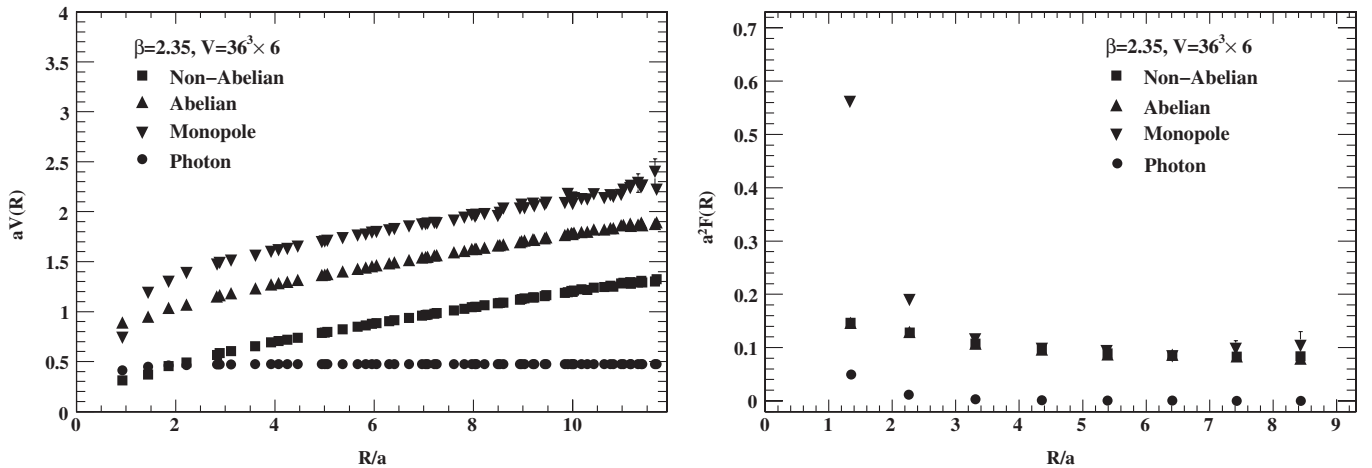
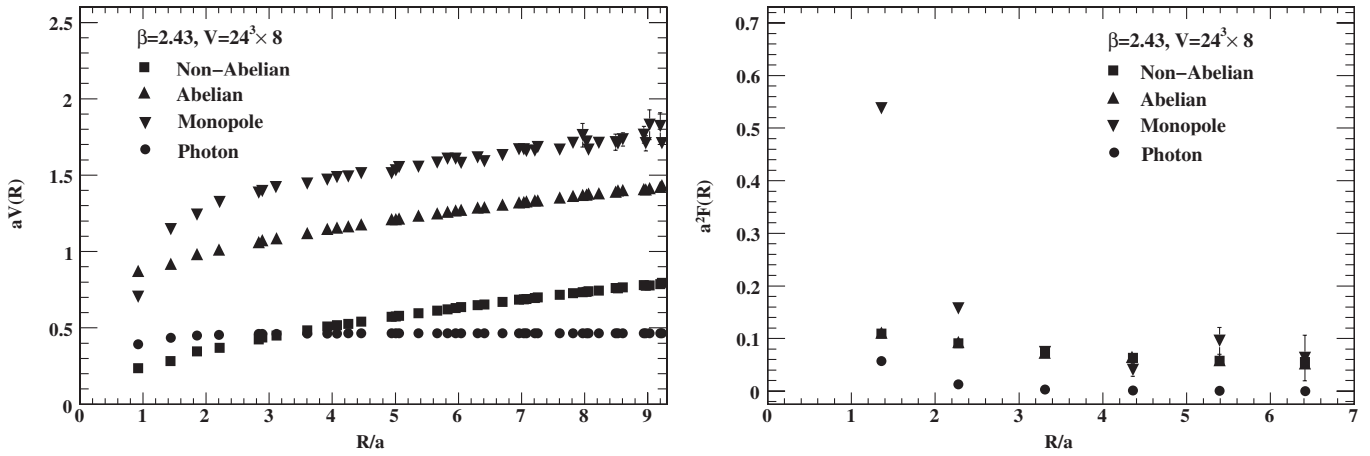


FIG. 3. The same plot as in Fig. 2 at $\beta = 2.35$ on the $36^3 \times 6$ lattice.

FIG. 4. The same plot as in Fig. 2 at $\beta = 2.43$ on the $24^3 \times 8$ lattice.

temporal links for further noise reduction. The short-distance part of the potential may be affected by HYP.

D. Results

We obtain very good signals for the potentials and the forces defined by differentiating the potential with respect to R . The results at $\beta = 2.35$ on the $24^3 \times 6$ lattice and on the $36^3 \times 6$ lattice, and at $\beta = 2.43$ on the $24^3 \times 8$ lattice are plotted in Figs. 2–4, respectively. The q - \bar{q} distances R of the potentials and the forces are improved to R_l and $\bar{R} = (\frac{4\pi}{a}\{G(R-a) - G(R)\})^{-1/2}$, respectively. We fit these potentials to the function $V_{\text{fit}}(R)$ in Eq. (8) and extract the string tension and the Coulombic coefficient, which are summarized in Table III. Since the potential and the

TABLE III. Best fitted values of the string tension σa^2 , the Coulombic coefficient c , and the constant μa for the potentials V_{NA} , V_A , V_{mon} , and V_{ph} .

Lattice	σa^2	c	μa	FR (R/a)	χ^2/N_{df}
$24^3 \times 4$					
V_{NA}	0.181(8)	0.25(15)	0.54(7)	3.9–8.5	1.00
V_A	0.183(8)	0.20(15)	0.98(7)	3.9–8.2	1.00
V_{mon}	0.183(6)	0.25(11)	1.31(5)	3.9–6.7	0.98
V_{ph}	$-2(1) \times 10^{-4}$	0.010(1)	0.48(1)	4.9–9.4	1.02
$24^3 \times 6$					
V_{NA}	0.072(3)	0.49(6)	0.53(3)	4.0–9.0	0.99
V_A	0.073(4)	0.41(7)	1.09(3)	3.7–10.9	1.00
V_{mon}	0.073(4)	0.44(10)	1.41(4)	3.9–9.3	1.00
V_{ph}	$-1.7(3) \times 10^{-4}$	0.0131(1)	0.4717(3)	5.1–9.4	0.99
$36^3 \times 6$					
V_{NA}	0.072(3)	0.48(9)	0.53(3)	4.6–12.1	1.03
V_A	0.073(2)	0.47(6)	1.10(2)	4.3–11.2	1.03
V_{mon}	0.073(3)	0.46(7)	1.43(3)	4.0–11.8	1.01
V_{ph}	$-1.0(1) \times 10^{-4}$	0.0132(1)	0.4770(2)	6.4–11.5	1.03
$24^3 \times 8$					
V_{NA}	0.0415(9)	0.47(2)	0.46(8)	4.1–7.8	0.99
V_A	0.041(2)	0.47(6)	1.10(3)	4.5–8.5	1.00
V_{mon}	0.043(3)	0.37(4)	1.39(2)	2.1–7.5	0.99
V_{ph}	$-6.0(3) \times 10^{-5}$	0.0059(3)	0.46649(6)	7.7–11.5	1.02

force at $\beta = 2.20$ on the $24^3 \times 4$ lattice are already published in Ref. [36], we only present the fitting result for this data set.

Abelian dominance is seen again as in Sec. III. Moreover, we observe monopole dominance, i.e., the string tension of the static potential from the monopole Polyakov loop correlation function, is identical to that of the non-Abelian static potential, while the potential from the photon Polyakov loop correlation function contains no linear part. It is remarkable that Abelian dominance and monopole dominance for the string tension are almost perfect as explicitly shown in Fig. 5, which also shows the good scaling behavior with respect to the change of lattice spacing. We do not see the volume dependence of the string tension as shown in Fig. 6.

These results suggest that although the lattice monopoles defined in Eq. (5) are gauge dependent, they contain

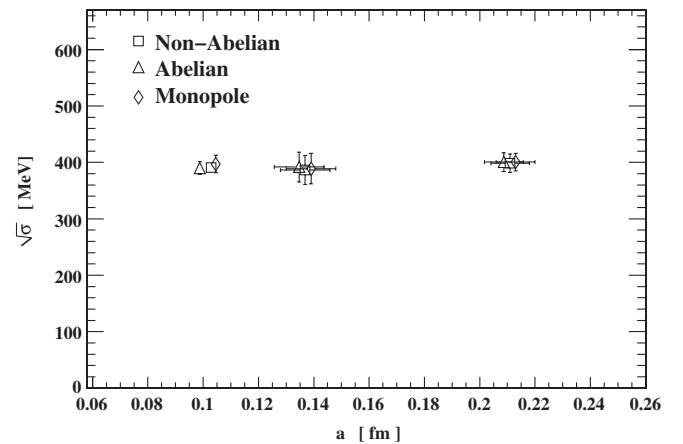


FIG. 5. The $a(\beta)$ dependence of the square root of the non-Abelian, Abelian, and monopole string tensions for the same temperature $T = 0.8T_c$. The bottom axis for a set of three data points at the same lattice spacing is slightly shifted to distinguish each other.

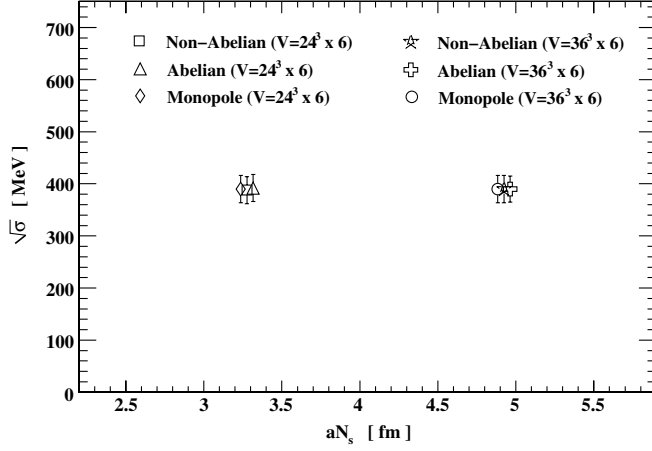


FIG. 6 (color online). The volume dependence of the square root of the string tensions on the $24^3 \times 6$ and $36^3 \times 6$ lattices at $\beta = 2.35$.

physical gauge-invariant pieces responsible for confinement, which show up after taking the gauge average.

V. THE ABELIAN DUAL MEISSNER EFFECT

A. Correlation function for the field profile around the q - \bar{q} system

We investigate the correlation function [43,44] between a Wilson loop W and a local Abelian operator \mathcal{O} connected by a product of non-Abelian link variables (Schwinger line) L ,

$$\langle \mathcal{O}(r) \rangle_W = \frac{\langle \text{Tr}[LW(R, T)L^\dagger \sigma^1 \mathcal{O}(r)] \rangle}{\langle \text{Tr}[W(R, T)] \rangle}. \quad (13)$$

A schematic figure is depicted in Fig. 7.

We shall use the cylindrical coordinate (r, ϕ, z) to parametrize the q - \bar{q} system, where the z axis corresponds to the q - \bar{q} axis and r to the transverse distance as shown in Fig. 8. We are interested in the field profile as a function of r on the midplane of the q - \bar{q} system.

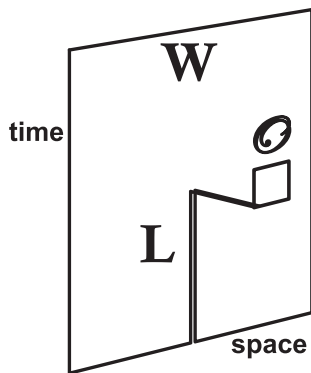


FIG. 7. A schematic figure for the connected correlation function.

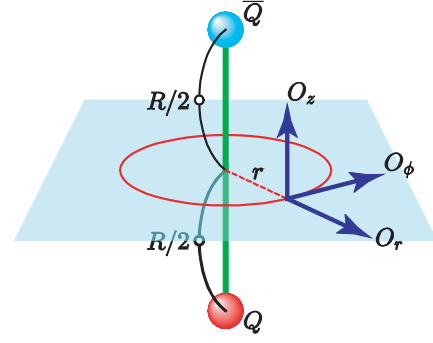


FIG. 8 (color online). Definition of the cylindrical coordinate (r, ϕ, z) along the q - \bar{q} axis.

B. Simulation parameters

In this computation, we employ the improved Iwasaki gauge action [45] with the coupling constants $\beta = 1.10$ and 1.28 on the 32^4 lattice, and $\beta = 1.40$ on the 40^4 lattice in order to investigate the scaling behavior of the correlation functions with less finite-lattice cutoff effects. Simulation parameters are listed in Table IV. The lattice spacings are determined so as to reproduce the physical string tension $\sqrt{\sigma} = 440$ (MeV). To improve the signal-to-noise ratio, the APE smearing is applied to the spatial links of the Wilson loop [46]. We use the Wilson loop $W(R = 3, T = 5)$ at $\beta = 1.10$, $W(R = 5, T = 5)$ at $\beta = 1.28$, and $W(R = 7, T = 7)$ at $\beta = 1.40$. Note that the physical q - \bar{q} distance is the same [$R = 0.32$ (fm)] for these Wilson loops.

C. The penetration depth

We measure all cylindrical components of the color-electric fields $\mathcal{O}(s) = E_{Ai}(s) = \Theta_{4i}(s)$. The results are plotted in Fig. 9. We find that only E_{Az} has correlation with the Wilson loop. We then fit $\langle E_{Az}(r) \rangle_W$ to a function $f(r) = c_1 \exp(-r/\lambda) + c_0$ and find that the profile of $\langle E_{Az}(r) \rangle_W$ is well described by this functional form, i.e., the color-electric field is exponentially squeezed. The fitting curves are also plotted in Fig. 9. The parameter λ corresponds to the penetration depth and the values for three gauge couplings are summarized in Table V and plotted in Fig. 10 as a function of lattice spacing $a(\beta)$. We find that the penetration depth λ shows the good scaling behavior.

TABLE IV. Simulation parameters for the measurement of the field profile. n and α are the number of smearing steps and the smearing parameter, which are optimized to obtain reasonable signals.

β	V	$a(\beta)$ [fm]	N_{conf}	n	α
1.10	32^4	0.1069(8)	5000	80	0.2
1.28	32^4	0.0635(5)	6000	80	0.2
1.40	40^4	0.0465(2)	7996	80	0.2

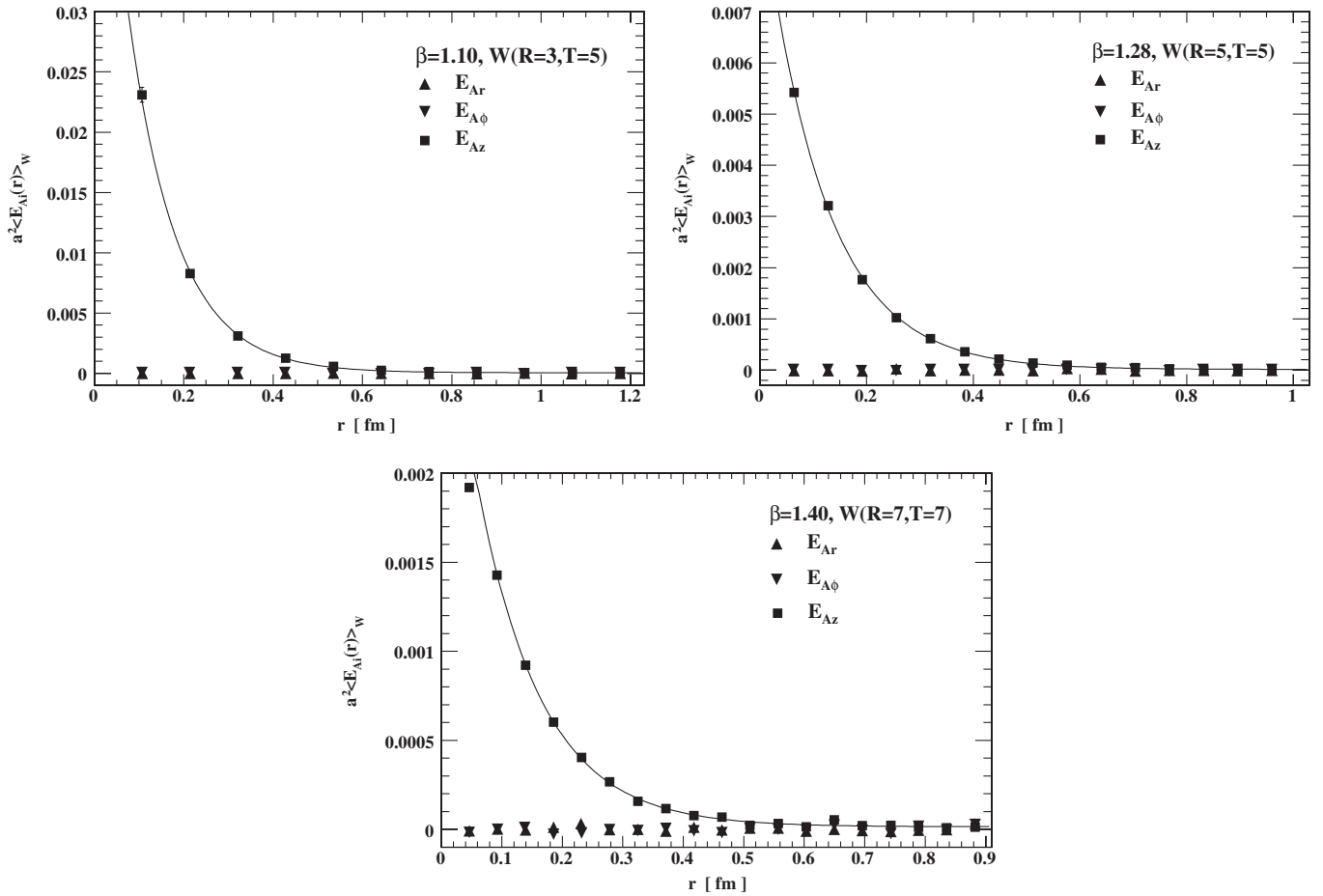


FIG. 9. The profile of the color-electric field \vec{E}_A at $\beta = 1.10$ (left panel), $\beta = 1.28$ (right panel), and $\beta = 1.40$ (lower panel).

D. The dual Ampère law

To see what squeezes the color-electric field, we study the Abelian (dual) Ampère law derived from the definition of the monopole current in Eq. (5),

$$\vec{\nabla} \times \vec{E}_A = \partial_4 \vec{B}_A + 2\pi \vec{k}, \quad (14)$$

where $B_{Ai}(s) = (1/2)\epsilon_{ijk}\bar{\Theta}_{jk}(s)$. The correlation of each term with the Wilson loop is evaluated on the same mid-plane of the $q\bar{q}$ system as for the profile measurements of the color-electric field. We find that only the azimuthal components are nonvanishing, which are plotted in Fig. 11. Note that if the color-electric field is purely of the Coulomb type, the curl of the electric field is zero. On the contrary, the curl of the electric field is nonvanishing and is repro-

duced mostly by the monopole currents. In any case, the dual Ampère law is satisfied, which is a clear signal of the Abelian dual Meissner effect. This result is quite the same as that observed in the MA gauge [17,18].

TABLE V. The parameter λ corresponding to the penetration depth.

β	$W(R, T)$	λ (fm)	c_1	c_0
1.10	$W(3, 5)$	0.1075(13)	$6.09(18) \times 10^{-2}$	$9(2) \times 10^{-5}$
1.28	$W(5, 5)$	0.1077(14)	$1.024(14) \times 10^{-2}$	$4.6(8) \times 10^{-6}$
1.40	$W(7, 7)$	0.106(4)	$3.40(17) \times 10^{-3}$	$1.6(8) \times 10^{-5}$

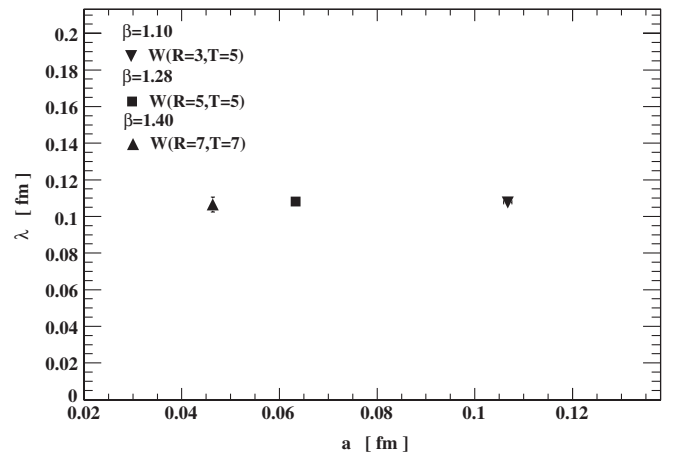


FIG. 10. The penetration depth λ as a function of lattice spacing $a(\beta)$.

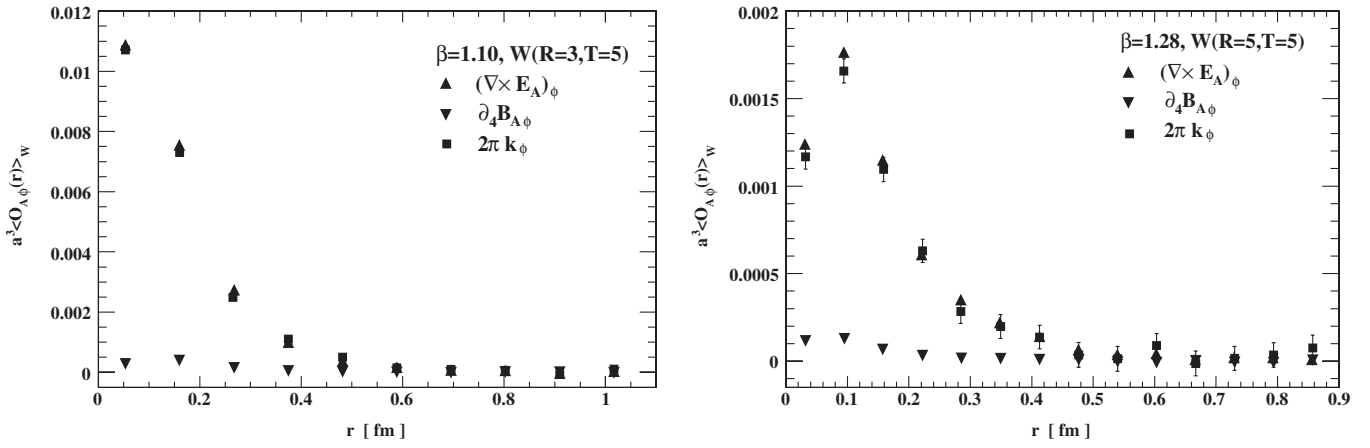


FIG. 11. Tests of the dual Ampère law at $\beta = 1.10$ for $W(R = 3, T = 5)$ (left panel) and at $\beta = 1.28$ for $W(R = 5, T = 5)$ (right panel).

E. The coherence length

Let us estimate the coherence length by evaluating the correlation function between the squared monopole density $\mathcal{O}(s) = k_\mu^2(s)$ and the Wilson loop [47]. To measure such a correlation function, we use the disconnected cor-

relation function, since the Schwinger lines are canceled and the connected correlation functions for the squared monopole currents are automatically reduced to the disconnected ones. Simulation parameters, the lattice volume, and the gauge couplings are the same as the measurements

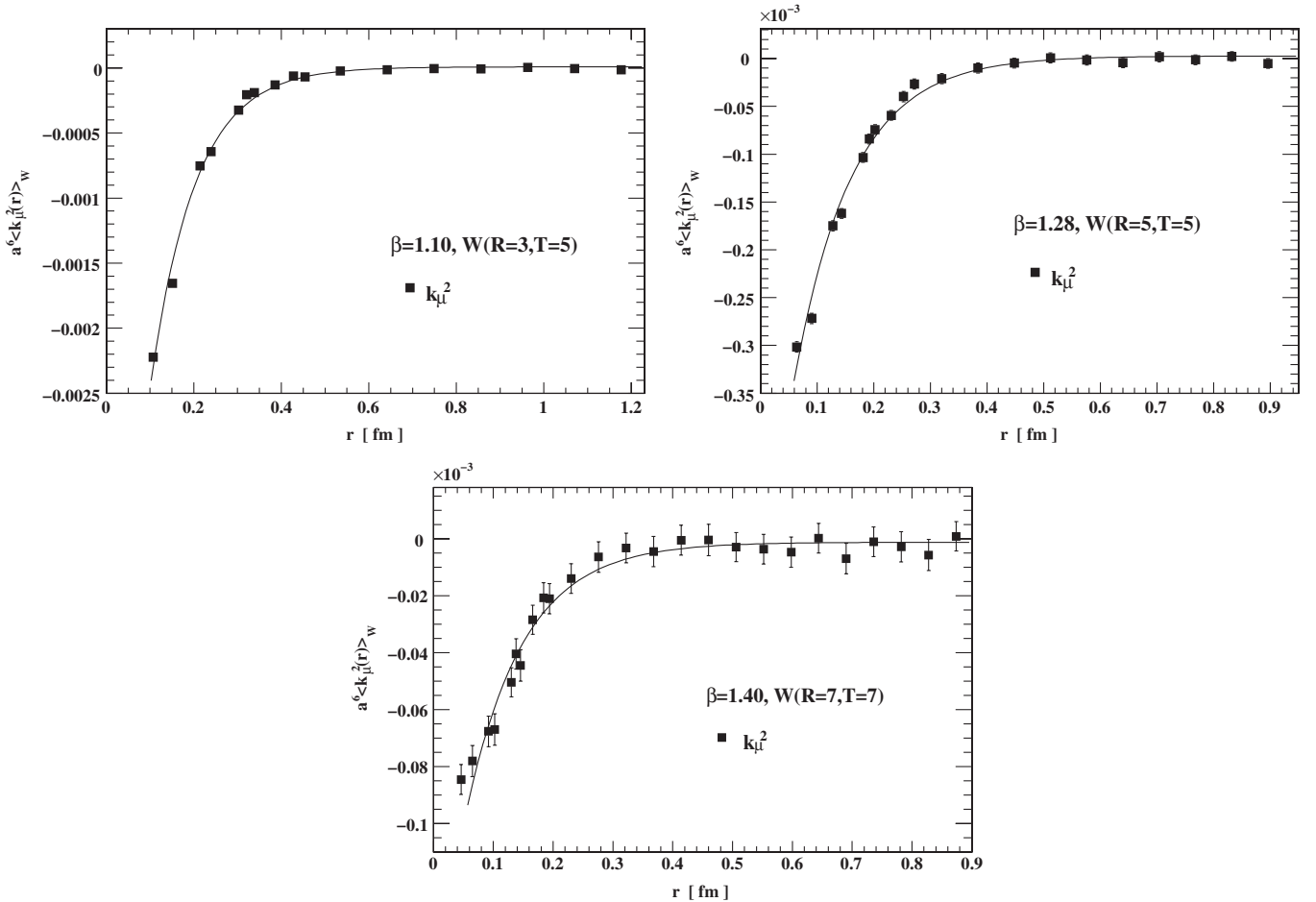


FIG. 12. The profile of the squared monopole currents at $\beta = 1.10$ (left panel), 1.28 (right panel), and 1.40 (lower panel).

TABLE VI. The parameter $\xi/\sqrt{2}$ corresponding to the coherence length.

β	$W(R, T)$	$\xi/\sqrt{2}$ (fm)	c'_1	c'_0
1.10	$W(3, 5)$	0.103(7)	$-4.7(11) \times 10^{-3}$	$-2(2) \times 10^{-6}$
1.28	$W(5, 5)$	0.090(4)	$-7.5(3) \times 10^{-4}$	$2(3) \times 10^{-6}$
1.40	$W(7, 7)$	0.097(7)	$-1.68(16) \times 10^{-4}$	$-1(3) \times 10^{-6}$

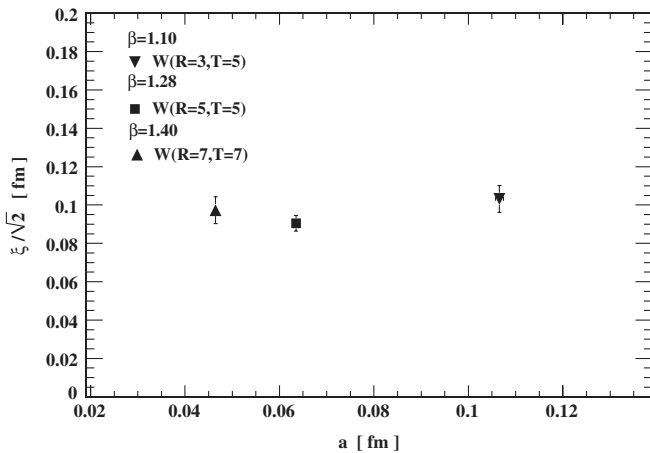
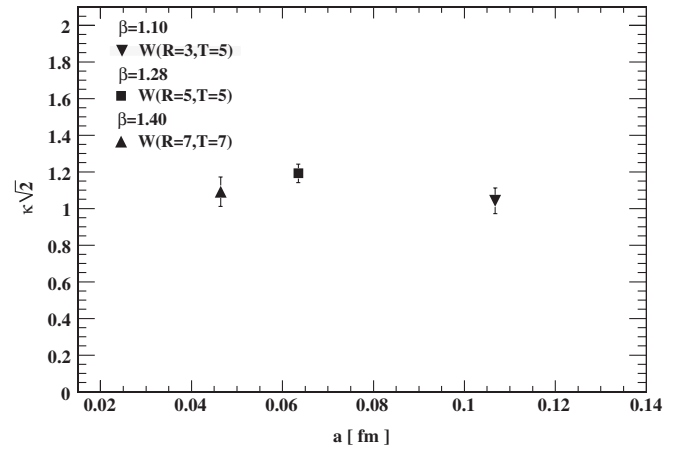
of the color-electric field profile, but the number of gauge configurations is increased, $N_{\text{conf}} = 5500$ for $\beta = 1.10$ and $N_{\text{conf}} = 11887$ for $\beta = 1.40$. For $\beta = 1.28$ we use the same number of configurations $N_{\text{conf}} = 6000$. The physical $q-\bar{q}$ distance is again fixed to $R = 0.32$ (fm). To reduce the noise, we further produce $N_{\text{RGT}} = 100$ gauge copies for each independent configuration by applying the random gauge transformations and take gauge averaging.

The results are plotted in Fig. 12. We then fit the profile of $\langle k_\mu^2(r) \rangle_W$ to the functional form $g(r) = c'_1 \exp(-\sqrt{2}r/\xi) + c'_0$, where the parameter ξ corresponds to the coherence length. We obtain the values for ξ as summarized in Table VI. The coherence length shows the scaling behavior as demonstrated in Fig. 13 as a function of lattice spacing $a(\beta)$.

F. The vacuum type

Taking the ratio of the penetration depth and the coherence length, the GL parameter $\sqrt{2}\kappa = \lambda/\xi$ can be estimated, which characterizes the type of the superconducting vacuum. The results are plotted in Fig. 14 against lattice spacing $a(\beta)$. We obtain $\sqrt{2}\kappa = 1.04(7)$, $1.19(5)$, and $1.09(8)$ for $\beta = 1.10$, 1.28 , and 1.40 , respectively.

We find that the GL parameter shows the scaling behavior and the value is about 1. This means that the vacuum type is near the border between the types 1 and 2 dual superconductors. However, we note that the physical spatial size of the Wilson loop used in the present simulations

FIG. 13. The coherence length ξ as a function of the lattice spacing $a(\beta)$.FIG. 14. The GL parameters as a function of the lattice spacing $a(\beta)$.

is still small [$R = 0.32$ (fm)]. Clearly, further quantitative studies with larger Wilson loops are needed to determine the definite value.

VI. NON-ABELIAN COLOR CONFINEMENT

Let us consider what is induced from the above numerical results.

Since gauge fixing is not applied in these computations, Abelian fields in any color directions are equivalent. Thus, our result is interpreted as that the color-electric fields in all color directions are squeezed and the Abelian (monopole) string tensions in all color directions are the same as the non-Abelian string tension. This indicates that QCD contains a gauge-invariant Abelian mechanism of confinement which is not related to the specific gauge fixing. Namely, Abelian monopoles in three color directions are condensed in the vacuum of the confinement phase of $SU(2)$ QCD.

Let us denote quark fields having charge $1/2$ and $-1/2$ in the σ_3 direction, respectively, as u_3 and d_3 . Then local mesonic states, $u_3\bar{u}_3$ and $d_3\bar{d}_3$, are Abelian color neutral in the σ_3 direction. Consider next

$$\begin{aligned} u_1 &= \frac{u_3 + d_3}{\sqrt{2}}, & d_1 &= \frac{u_3 - d_3}{\sqrt{2}}, \\ u_2 &= \frac{iu_3 + d_3}{\sqrt{2}}, & d_2 &= \frac{iu_3 - d_3}{\sqrt{2}}. \end{aligned}$$

u_1 and d_1 (u_2 and d_2) are quark fields having charge $1/2$ and $-1/2$ in the σ_1 ($\sigma_2/2$) direction. Using these expressions, the quark-gluon coupling term is written as

$$\begin{aligned} \bar{\psi} \gamma^\mu \frac{\sigma^a}{2} \psi A_\mu^a &= \frac{1}{2} (\bar{u}_3 \gamma_\mu d_3 + \bar{d}_3 \gamma_\mu u_3) A_\mu^1 \\ &\quad - i \frac{1}{2} (\bar{u}_3 \gamma_\mu d_3 - \bar{d}_3 \gamma_\mu u_3) A_\mu^2 \\ &\quad + \frac{1}{2} (\bar{u}_3 \gamma_\mu u_3 - \bar{d}_3 \gamma_\mu d_3) A_\mu^3 \end{aligned} \quad (15)$$

$$\begin{aligned}
&= \frac{1}{2}(\bar{u}_1\gamma_\mu u_1 - \bar{d}_1\gamma_\mu d_1)A_\mu^1 + \frac{1}{2}(\bar{u}_2\gamma_\mu u_2 - \bar{d}_2\gamma_\mu d_2)A_\mu^2 \\
&\quad + \frac{1}{2}(\bar{u}_3\gamma_\mu u_3 - \bar{d}_3\gamma_\mu d_3)A_\mu^3, \tag{16}
\end{aligned}$$

where the first equation (15) is expressed in terms of u_3 and d_3 alone. Consider local mesonic states $u_1\bar{u}_1$ and $d_1\bar{d}_1$ ($u_2\bar{u}_2$ and $d_2\bar{d}_2$) which are Abelian color neutral in the σ_1 (σ_2) direction. When we look at the states $u_1\bar{u}_1$ and $d_1\bar{d}_1$ in the σ_3 direction, they are written as the sum of color-neutral and color-charged states:

$$u_1\bar{u}_1 = \frac{1}{2}(u_3\bar{u}_3 + d_3\bar{d}_3 + u_3\bar{d}_3 + d_3\bar{u}_3), \tag{17}$$

$$d_1\bar{d}_1 = \frac{1}{2}(u_3\bar{u}_3 + d_3\bar{d}_3 - u_3\bar{d}_3 - d_3\bar{u}_3). \tag{18}$$

The same observation applies to the color-neutral states $u_2\bar{u}_2$ and $d_2\bar{d}_2$ in the σ_2 direction. However, we find that

$$u_1\bar{u}_1 + d_1\bar{d}_1 = u_2\bar{u}_2 + d_2\bar{d}_2 = u_3\bar{u}_3 + d_3\bar{d}_3 \tag{19}$$

are Abelian color neutral in all color directions. The state (19) is nothing but the non-Abelian color-singlet state.

This example tells us that the Abelian color-neutral state in any color directions corresponds to the physical non-Abelian color-singlet state. Hence, the confinement of non-Abelian color charges can be explained in terms of the Abelian dual Meissner effect due to Abelian monopoles. To the authors knowledge, this is the first paper that explains the confinement of non-Abelian color charges only in terms of the Abelian dual Meissner effect.

VII. CONCLUDING REMARKS

We make some concluding remarks. The Abelian gauge fields extracted from the thermalized non-Abelian link fields contain originally topological monopoles responsible for the confinement mechanism of non-Abelian color charges even in the continuum limit. Our results presented in this paper are almost the same as those obtained in the

maximally Abelian gauge. This suggests that the MA gauge fixing is the easiest method to extract the physical ingredients of the monopoles, since we do not need very precise time-consuming simulations in the MA gauge as done here.

In the lattice Landau gauge, it is known that no monopoles exist [48] if one uses the DeGrand-Toussaint definition and the magnetic displacement current takes a role of monopoles in the dual Ampère law. How to interpret the existence of a gauge-invariant Abelian confinement mechanism in the framework of the Landau gauge? Abelian monopoles are as a whole gauge variant without gauge fixing, but they may contain a gauge-invariant physical component and a gauge-variant unphysical one. The compatible interpretation would be that the Landau gauge is a special gauge in which the unphysical gauge-variant piece apparently cancels the physical one in the DeGrand-Toussaint monopoles, but at the same time, the role of physical monopoles is carried by the color-magnetic displacement current, which is just a matter of definition of monopoles on the lattice. On the other hand, in the MA gauge, the main part of the DeGrand-Toussaint monopoles is a physical component.

If there exist physical gauge-invariant ingredients of Abelian monopoles, one could observe them in the real experiment [49]. To find the effect in the confinement and also in the deconfinement phases is a very interesting topic in the future.

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