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SU(2) and SU(3) chiral perturbation theory analyses on baryon masses in 2 + 1 flavor lattice QCD

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We investigate the quark mass dependence of baryon masses in 2 + 1 flavor lattice QCD using SU(3) heavy baryon chiral perturbation theory up to one-loop order. The baryon mass data used for the analyses are obtained for the degenerate up-down quark mass of 3 to 24 MeV and two choices of the strange quark mass around the physical value. We find that the SU(3) chiral expansion fails to describe both the octet and the decuplet baryon data if phenomenological values are employed for the meson-baryon couplings. The SU(2) case is also examined for the nucleon. We observe that higher order terms are controlled only around the physical point. We also evaluate finite size effects using SU(3) heavy baryon chiral perturbation theory, finding small values of order 1% even at the physical point.

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I. INTRODUCTION

The aim of the PACS-CS project is full QCD calculations on the physical point avoiding any contamination due to chiral extrapolations. At the first stage of this project [1] we have succeeded in reducing the up-down quark mass from 67 MeV, the minimum value reached by the previous CP-PACS/JLQCD work [2], to 3 MeV, corresponding to the decrease of pion mass from 702 to 156 MeV. This work allowed us to make detailed chiral analyses on the pseudoscalar meson sector with the use of chiral perturbation theory (ChPT). An important finding is that the strange quark mass is not small enough to be treated by the SU(3) ChPT up to the next-to-leading order (NLO). For the octet and decuplet baryon masses we simply employed a linear chiral extrapolation to the physical point assuming isospin symmetry and analyticity of the strange quark contribution around its physical mass.

In this article we reinvestigate the quark mass dependence of the octet and decuplet baryon masses employing the SU(3) heavy baryon chiral perturbation theory (HBChPT) up to NLO [3–5]. The results are compared with those of the linear chiral extrapolation obtained in Ref. [1]. We also examine the convergence property of HBChPT fits to the lattice results. For the nucleon mass we examine the SU(2) HBChPT with an analytic expansion of the low energy constants (LECs) in terms of the strange quark mass around its physical value. The SU(2) BChPT analyses in two flavor lattice QCD were previously made by other collaborations and reported in Refs. [6,7]. We also discuss the magnitude of finite size effects based on the NLO SU(3) HBChPT.

This paper is organized as follows. In Sec. II we briefly review the results obtained in Ref. [1] to make the paper self-contained. In Sec. III we apply SU(3) HBChPT analyses to the octet baryon masses. We present the fit results for the LECs and discuss the convergence behavior up to NLO. The same analysis is repeated for the decuplet baryon masses. Section IV describes the fit results of the nucleon mass with the SU(2) HBChPT. In Sec. V we discuss the magnitude of finite size effects for the baryon masses based on the SU(3) HBChPT. Our conclusions are summarized in Sec. VI.

II. SU(2) LINEAR CHIRAL FIT FOR BARYON MASSES

We give a quick review of the chiral analyses employed in Ref. [1]. The extrapolation to the physical point is performed with the following fit formula:

$$m = \alpha + \beta m_{\rm ud}^{\rm AWI} + \gamma m_{\rm s}^{\rm AWI}, \qquad (1)$$

where $m_{\rm ud}^{\rm AWI}$ denotes the axial Ward identity quark mass for the up-down quark and $m_{\rm s}^{\rm AWI}$ for the strange quark. Since we choose $\kappa_{\rm s}$ around the physical strange quark mass, the above formula is essentially an SU(2) chiral expansion with the strange quark contribution analytically expanded around its physical value. Actually we can rewrite the formula (1) as

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$$m = \alpha' + \beta m_{\rm ud}^{\rm AWI} + \gamma(m_{\rm s}^{\rm AWI} - m_{\rm s,ph}^{\rm AWI}), \tag{2}$$

where $\alpha' = \alpha + \gamma m_{\rm s,ph}^{\rm AWI}$ with $m_{\rm s,ph}^{\rm AWI}$ the physical value of the strange quark mass. This is a linear expansion of the baryon mass around $(m_{\rm ud}^{\rm AWI}, m_{\rm s}^{\rm AWI}) = (0, m_{\rm s,ph}^{\rm AWI})$.

The simulation points and the measured hadron masses are given in Tables I and III of Ref. [1], respectively. The fit range is the lightest four points at $\kappa_{\rm ud} \geq 0.137\,54$, where the pion mass varies from 156 to 410 MeV and $m_{\rm ud}^{\rm AWI}$ from 3 to 24 MeV in the $\overline{\rm MS}$ scheme. In Figs. 1 and 2 we present the fit results for the octet and the decuplet baryon masses, respectively. Star symbols denote the extrapolated values at the physical point whose numerical values are listed in Tables I and II. The physical point together with the lattice cutoff is determined with m_π, m_K, and m_Ω inputs by applying the NLO SU(2) ChPT fit to the pseudoscalar meson sector [1]. The data are reasonably described by the linear function [Eq. (1)]. The values for α, β, γ, and χ^2/dof are summarized in Tables III and IV. In order to investigate the convergence behavior the contribution of each term in Eq. (2) is drawn in Fig. 3 for the octet baryon masses, where $m_{\rm s}^{\rm AWI}$ is fixed at the measured value for $(\kappa_{\rm ud}, \kappa_{\rm s}) = (0.137\,54, 0.136\,40)$. We observe that the $O(m_{\rm s} - m_{\rm s,ph})$ terms are small, and that $O(m_{\rm ud})$ contributions are less than 20% of that of α' for $m_{\rm ud}^{\rm AWI} \lesssim 0.01$. This suggests that higher order terms in $m_{\rm ud}$ and $m_{\rm s} - m_{\rm s,ph}$ would be small. Similar trends are found for the decuplet baryon masses in Fig. 4.

III. SU(3) HBCHPT ANALYSES ON BARYON MASSES

A. Lagrangian to leading order

We use the continuum HBChPT in this article, leaving the development of the Wilson HBChPT, which incorporates the chiral symmetry breaking effects of the Wilson-type quark action, for future work. We follow the notation

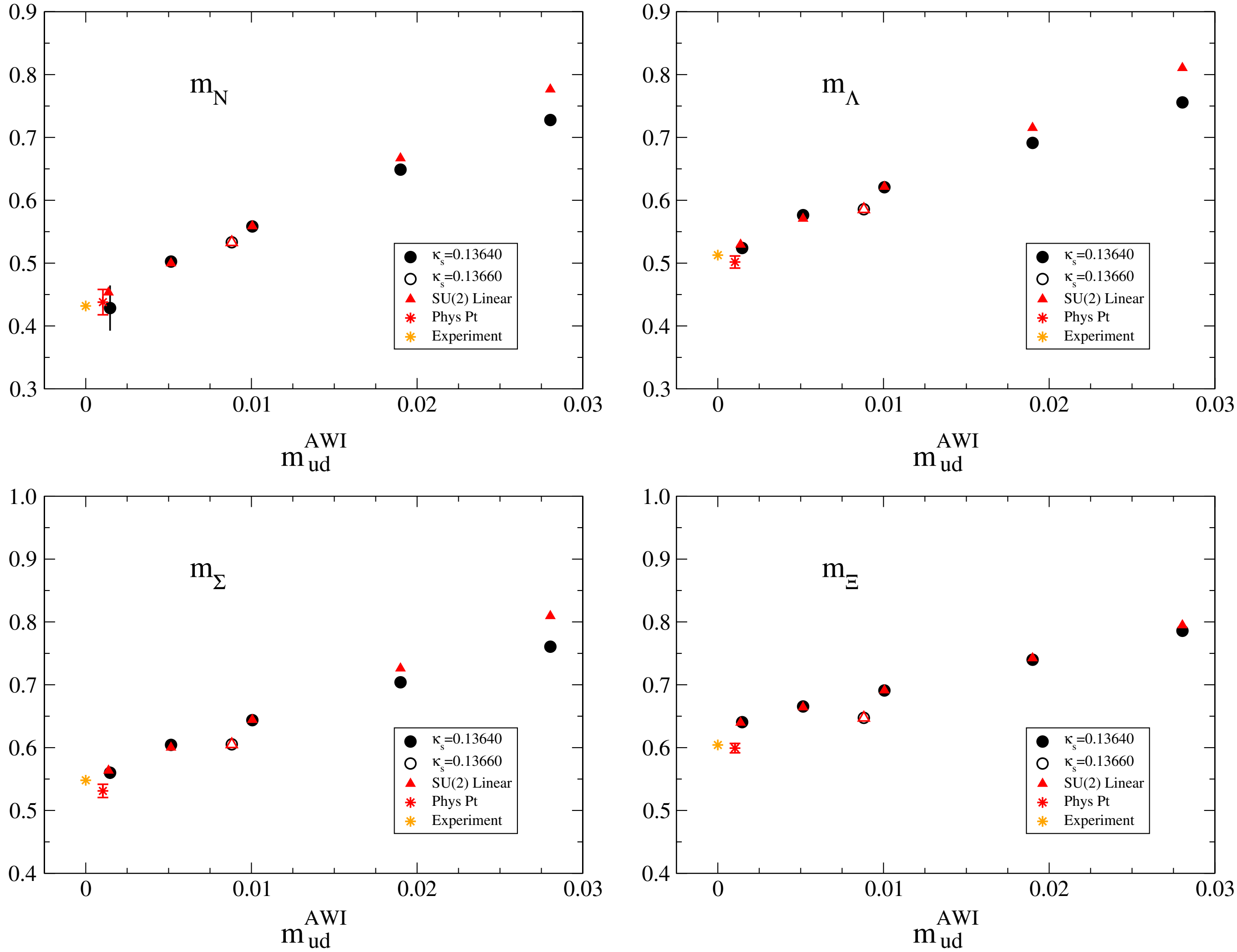

FIG. 1 (color online). Fit results with the linear formula [Eq. (1)] for the octet baryon masses. Experimental values are given in lattice units with $a^{-1} = 2.176$ GeV in Ref. [1].

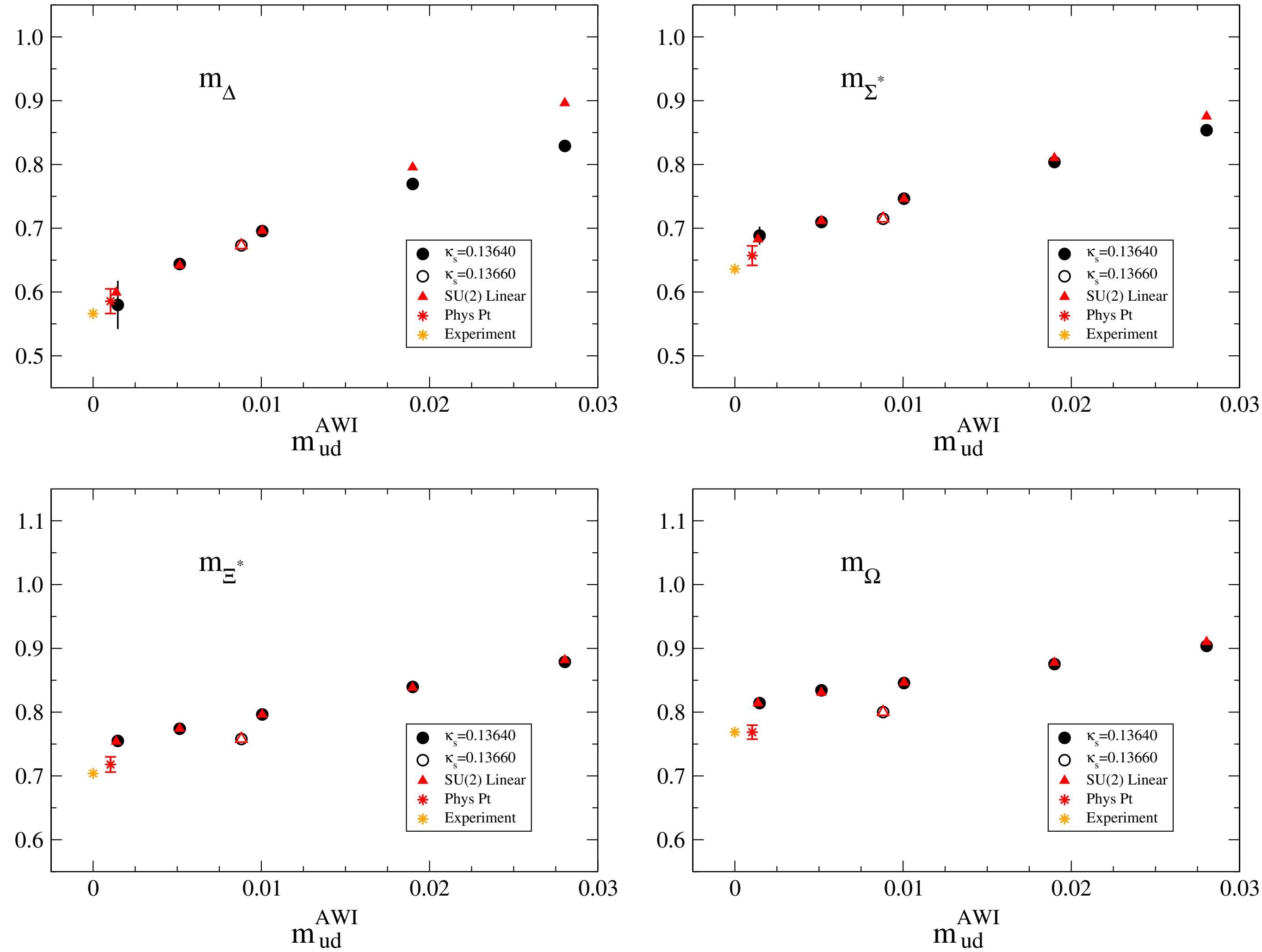

FIG. 2 (color online). Fit results with the linear formula [Eq. (1)] for the decuplet baryon masses. Experimental values are given in lattice units with $a^{-1} = 2.176$ GeV in Ref. [1].

TABLE I. Octet baryon masses at the physical point. NLO results are obtained without (case 1) and with (case 2) fixing D, F, and $\mathcal{C}$ at the phenomenological estimate.

			NLO	
	Linear	LO	case 1	case 2
N	0.438(20)	0.4532(75)	0.447(15)	0.322(32)
Λ	0.502(10)	0.5199(72)	0.517(11)	0.387(82)
Σ	0.531(11)	0.5439(77)	0.546(12)	0.53(16)
Ξ	0.5992(75)	0.5987(80)	0.600(14)	0.62(18)

TABLE II. Decuplet baryon masses at the physical point. NLO results are obtained without (case 1) and with (case 2) fixing $\mathcal{C}$ at the phenomenological estimate.

			NLO	
	Linear	LO	case 1	case 2
Δ	0.586(19)	0.612(9)	0.604(19)	0.545(20)
Σ^*	0.657(15)	0.666(9)	0.663(14)	0.604(17)
Ξ^*	0.718(12)	0.720(9)	0.721(14)	0.694(17)
Ω	0.769(11)	0.774(11)	0.777(17)	0.815(21)

TABLE III. Fit results with the linear formula [Eq. (1)] for the octet baryon masses.

	N	Λ	Σ	Ξ
α	0.371(50)	0.375(28)	0.381(33)	0.408(21)
β	11.6(2.4)	9.6(1.0)	8.0(1.0)	4.24(46)
γ	1.8(1.3)	3.90(76)	4.7(9)	6.24(58)
χ^2/dof	0.63(2.5)	1.3(2.3)	0.6(1.4)	0.09(59)

TABLE IV. Same as Table III for the decuplet baryon masses.

	Δ	Σ^*	Ξ^*	Ω
α	0.527(60)	0.538(48)	0.548(40)	0.552(33)
β	10.7(2.1)	6.3(1.5)	3.41(82)	1.80(57)
γ	1.6(1.6)	3.8(1.3)	5.5(1.1)	7.2(9)
χ^2/dof	0.4(1.4)	0.2(1.3)	0.00(21)	0.6(1.7)

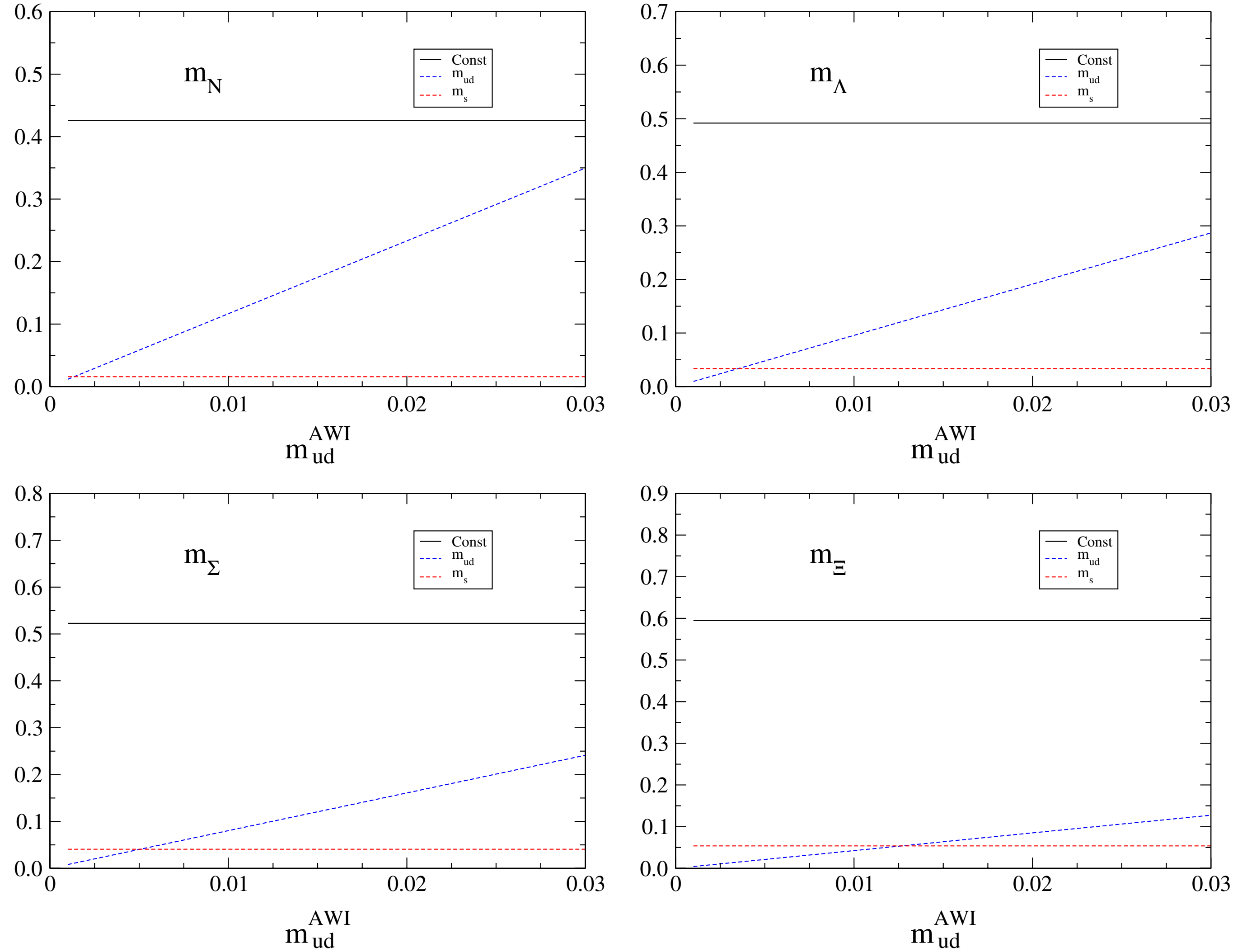

FIG. 3 (color online). Convergence behavior for the octet baryon masses with the linear formula [Eq. (1)]. m_s is fixed at the measured value at $(\kappa_{ud}, \kappa_s) = (0.137\,54, 0.136\,40)$.

described in Ref. [5]. The Lagrangian of HBChPT is written in terms of velocity-dependent baryon fields with a perturbative derivative expansion. To fix the notation we present the leading order terms:

$$\begin{aligned}\mathcal{L} = {} & (\bar{B} i v \cdot D B) + 2\alpha_M(\bar{B} B \mathcal{M}_+) + 2\beta_M(\bar{B}\mathcal{M}_+ B) \\ & + 2\sigma_M(\bar{B}B)\,\mathrm{tr}(\mathcal{M}_+) - (\bar{T}^\mu\{iv\cdot D - \Delta\}T_\mu) \\ & + 2\gamma_M(\bar{T}^\mu \mathcal{M}_+ T_\mu) - 2\bar{\sigma}_M(\bar{T}^\mu T_\mu)\,\mathrm{tr}(\mathcal{M}_+) \\ & + 2\alpha(\bar{B}S^\mu B A_\mu) + 2\beta(\bar{B}S^\mu A_\mu B) \\ & + 2\mathcal{H}(\bar{T}^\nu S^\mu A_\mu T_\nu) + \sqrt{\tfrac{3}{2}}\mathcal{C}[(\bar{T}^\nu A_\nu B) + (\bar{B}A_\nu T^\nu)],\end{aligned} \tag{3}$$

where B and T represent the velocity-dependent octet and decuplet baryon fields with the four-velocity v_μ, respectively. This Lagrangian contains nine LECs: α_M, β_M, σ_M, γ_M, $\bar{\sigma}_M$, α, β, $\mathcal{H}$, and $\mathcal{C}$. Hereafter we use the axial couplings F, D instead of α, β. They are related with

$$\alpha = \tfrac{2}{3}D + 2F, \tag{4}$$

$$\beta = -\tfrac{5}{3}D + F. \tag{5}$$

The decuplet-octet mass difference denoted by $\Delta = m_T - m_B$ has a comparable magnitude to intermultiplet mass splittings of both the octet and the decuplet. We are allowed to treat it as a perturbation. The octet pseudoscalar mesons couple derivatively to the baryon fields through the vector combination V_μ, the axial vector one A_μ, and the chiral covariant derivative D_μ. The light quark masses m_u, m_d, and m_s are contained in $\mathcal{M}_+$. We refer to Ref. [5] for the explicit expression of V_μ, A_μ, D_μ, and $\mathcal{M}_+$.

In this section the physical m_{ud} and m_s are determined by the SU(3) ChPT analyses in the pseudoscalar meson sector including m_π and m_K; m_Ω is an additional input for the lattice cutoff [1].

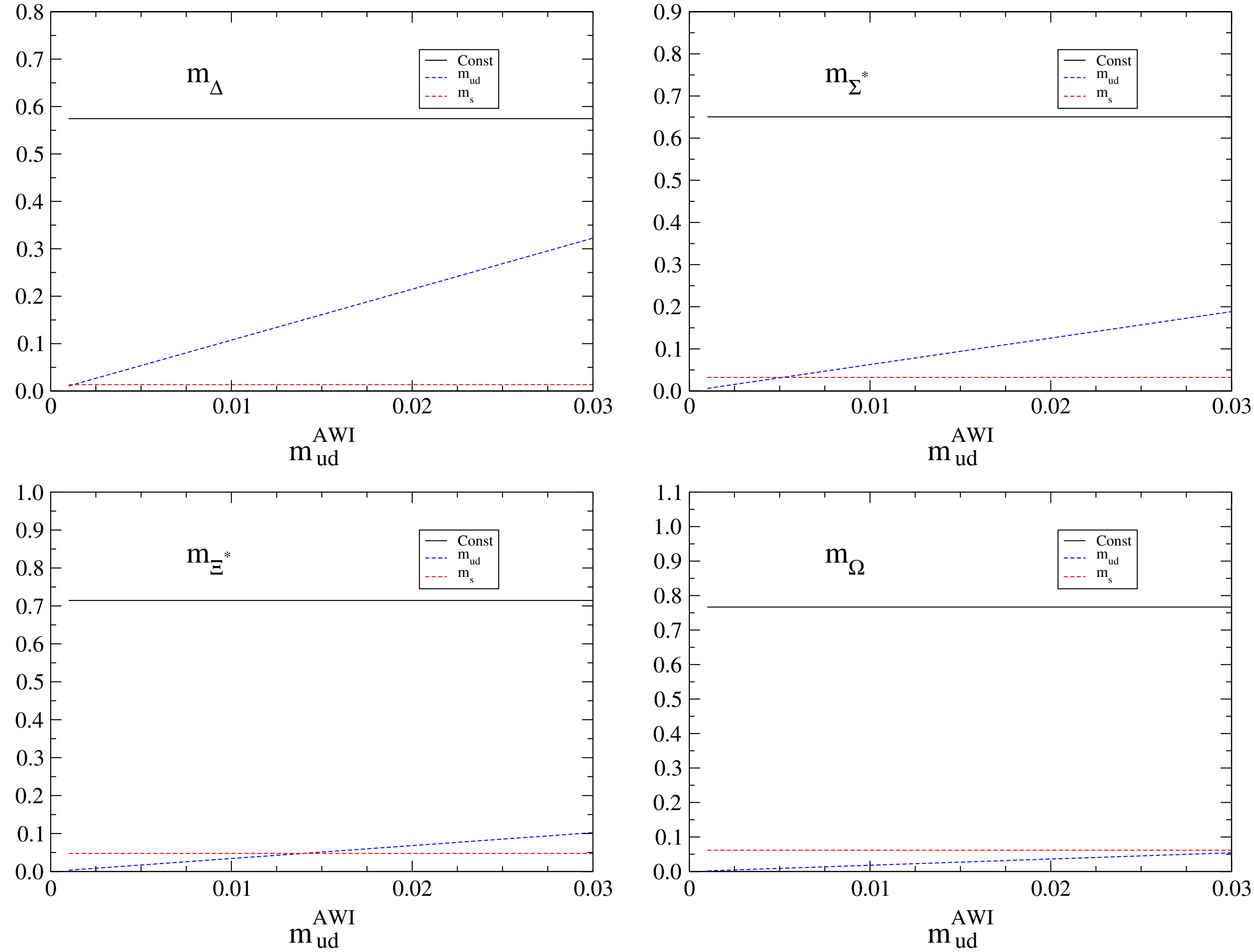

FIG. 4 (color online). Convergence behavior for the decuplet baryon masses with the linear formula [Eq. (1)]. m_s is fixed at the measured value at $(\kappa_{ud}, \kappa_s) = (0.137\,54, 0.136\,40)$.

B. Octet baryons

In the SU(3) HBChPT the renormalized mass of the ith octet baryon is expanded as

$$m_{B_i} = m_B - m_{B_i}^{(1)} - m_{B_i}^{(3/2)} + \cdots, \quad (6)$$

where m_B is the octet baryon mass in the chiral limit under the SU(3) flavor symmetry, and $m_{B_i}^{(n)}$ is the $O(m_q^{(n)})$ contribution to the ith octet baryon. The LO corrections are given by

$$m_N^{(1)} = (2\alpha_M + 2\beta_M + 4\sigma_M)m_{ud} + 2\sigma_M m_s, \quad (7)$$

$$m_\Lambda^{(1)} = (\alpha_M + 2\beta_M + 4\sigma_M)m_{ud} + (\alpha_M + 2\sigma_M)m_s, \quad (8)$$

$$m_\Sigma^{(1)} = (\tfrac{5}{3}\alpha_M + \tfrac{2}{3}\beta_M + 4\sigma_M)m_{ud} + (\tfrac{1}{3}\alpha_M + \tfrac{4}{3}\beta_M + 2\sigma_M)m_s, \quad (9)$$

TABLE V. Coefficients for the octet baryons $A_\phi^{B_i}$ and $B_\phi^{B_i}$.

	$A_\phi^{B_i}$			$B_\phi^{B_i}$		
ϕ	π	K	η	π	K	η
N	$\frac{3}{2}(D+F)^2$	$\frac{1}{3}(5D^2 - 6DF + 9F^2)$	$\frac{1}{6}(D-3F)^2$	$\frac{4}{3}$	$\frac{1}{3}$	0
Λ	$2D^2$	$\frac{2}{3}(D^2 + 9F^2)$	$\frac{2}{3}D^2$	1	$\frac{2}{3}$	0
Σ	$\frac{2}{3}(D^2 + 6F^2)$	$2(D^2 + F^2)$	$\frac{2}{3}D^2$	$\frac{2}{9}$	$\frac{10}{9}$	$\frac{1}{3}$
Ξ	$\frac{3}{2}(D-F)^2$	$\frac{1}{3}(5D^2 + 6DF + F^2)$	$\frac{1}{6}(D+3F)^2$	$\frac{1}{3}$	1	$\frac{1}{3}$

TABLE VI. Fit results with the SU(3) HBChPT for the octet baryon masses. NLO results are obtained with (case 2) and without (case 1) fixing D, F, and $\mathcal{C}$ at the phenomenological estimate.

		NLO		
	LO	Case 1	Case 2	Phenomenological
m_B	0.410(14)	0.391(39)	−0.15(9)	
α_M	−2.262(62)	−2.62(62)	−15.3(2.0)	
β_M	−1.740(58)	−2.6(1.5)	−21.3(3.0)	
σ_M	−0.53(12)	−0.71(34)	−9.6(1.4)	
D		$0.000(16) \times 10^{-8}$	0.80 fixed	0.80
F		$0.000(9) \times 10^{-8}$	0.47 fixed	0.47
$\mathcal{C}$		0.36(30)	1.5 fixed	1.5
χ^2/dof	1.10(63)	1.39(77)	153(82)	

$$m_\Xi^{(1)} = (\tfrac{1}{3}\alpha_M + \tfrac{4}{3}\beta_M + 4\sigma_M)m_{\rm ud} + (\tfrac{5}{3}\alpha_M + \tfrac{2}{3}\beta_M + 2\sigma_M)m_{\rm s} \quad (10)$$

with $m_{\rm ud}$ the averaged up-down quark mass, $m_{\rm s}$ is the strange quark mass, and α_M, β_M, and σ_M are LECs in the Lagrangian (3).

The NLO contributions, which are obtained from one-loop diagrams, are written as

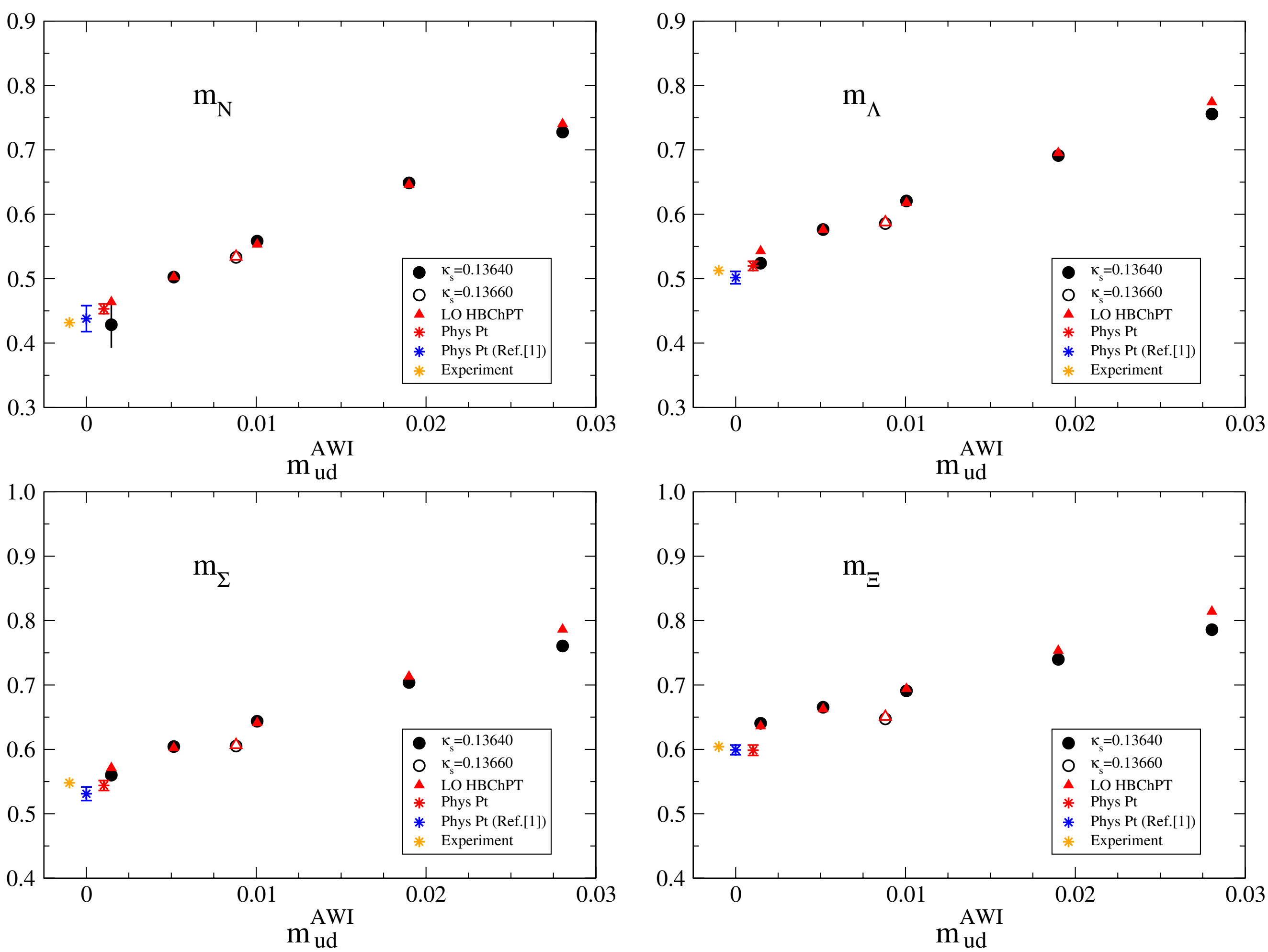

FIG. 5 (color online). Fit results with the SU(3) HBChPT up to LO for the octet baryon masses. Experimental values are given in lattice units with $a^{-1} = 2.176$ GeV in Ref. [1].

$$m_{B_i}^{(3/2)} = \frac{2}{(4\pi f_0)^2} \sum_{\phi=\pi,K,\eta} [A_\phi^{B_i} \mathcal{F}(m_\phi, 0, \mu) + \mathcal{C}^2 B_\phi^{B_i} \mathcal{F}(m_\phi, \Delta, \mu)], \quad (11)$$

where Δ is the octet-decuplet baryon mass difference in the SU(3) chiral limit. The pion decay constant f_0 is also defined in the SU(3) chiral limit with the convention of $f_\pi = 130.4$ MeV. The function $\mathcal{F}$ is expressed as

$$\mathcal{F}(m_\phi, \Delta, \mu) = (m_\phi^2 - \Delta^2)\left[\sqrt{\Delta^2 - m_\phi^2 + i\epsilon} \times \ln\left(\frac{\Delta - \sqrt{\Delta^2 - m_\phi^2 + i\epsilon}}{\Delta + \sqrt{\Delta^2 - m_\phi^2 + i\epsilon}}\right) - \Delta \ln\left(\frac{m_\phi^2}{\mu^2}\right)\right] - \frac{1}{2}\Delta m_\phi^2 \ln\left(\frac{m_\phi^2}{\mu^2}\right) \quad (12)$$

with $\mathcal{F}(m_\phi, 0, \mu) = \pi m_\phi^3$, and μ the renormalization scale. This formula assumes $\Delta \geq m_\phi$. For $\Delta < m_\phi$ we apply the analytic continuation:

$$\sqrt{\Delta^2 - m_\phi^2 + i\epsilon} \ln\left(\frac{\Delta - \sqrt{\Delta^2 - m_\phi^2 + i\epsilon}}{\Delta + \sqrt{\Delta^2 - m_\phi^2 + i\epsilon}}\right) \to \arccos\left(\frac{\Delta}{m_\phi}\right). \quad (13)$$

We revisit the function $\mathcal{F}$ later in Sec. V to discuss finite size effects. The contributions from the octet-octet- and the decuplet-octet-axial couplings are factored out by $A_\phi^{B_i}$ and $\mathcal{C}^2 B_\phi^{B_i}$, respectively. We summarize their values in Table V. The LECs are phenomenologically estimated as [8–10]

$$D = 0.80, \qquad F = 0.47, \qquad \mathcal{C} = 1.5. \quad (14)$$

We first present the fit results up to leading order employing the formula $m_{B_i} = m_B - m_{B_i}^{(1)}$. The values for m_B, α_M, β_M, and σ_M are given in Table VI together with

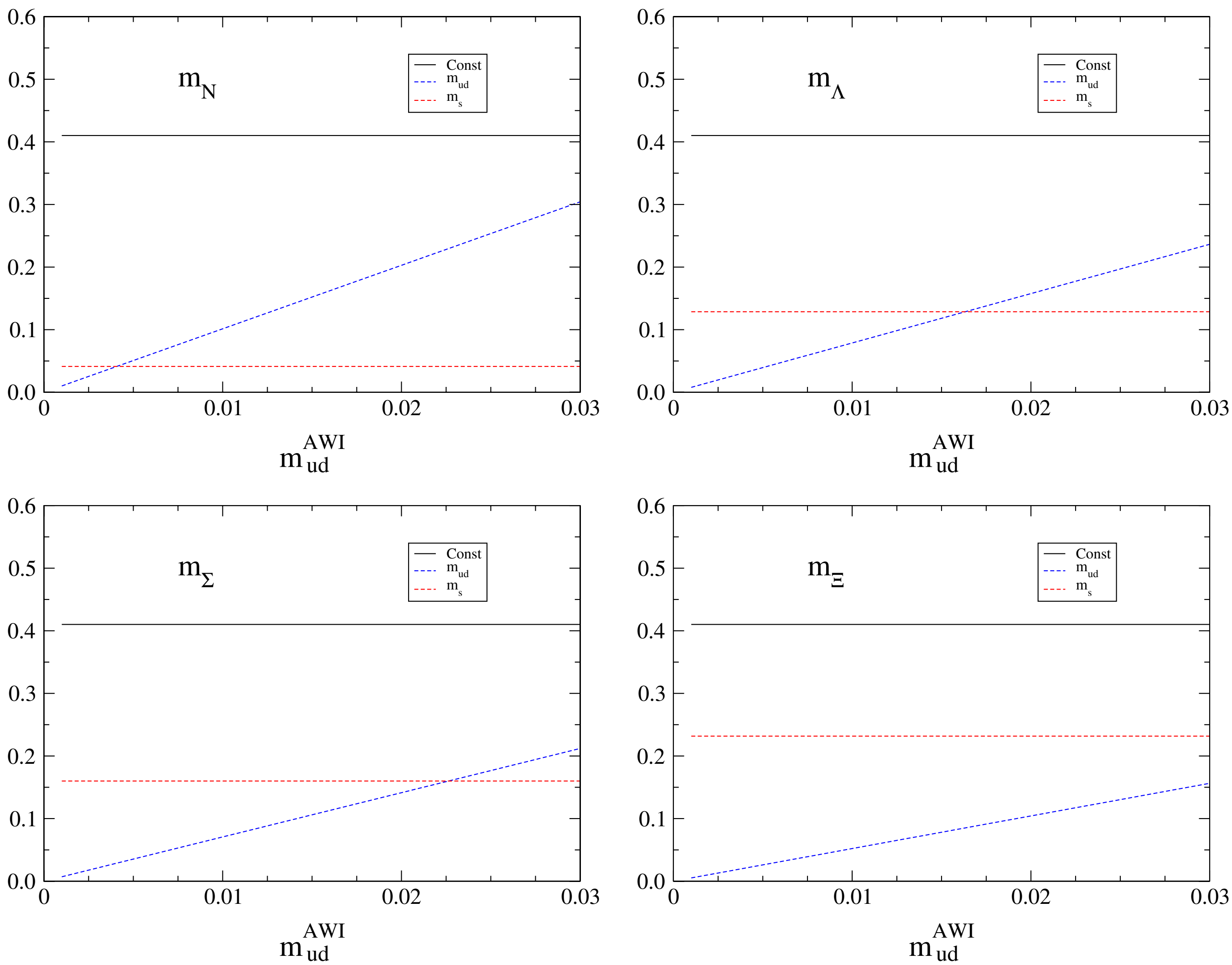

FIG. 6 (color online). Convergence behavior for the octet masses with the SU(3) HBChPT up to LO. m_s is fixed with the measured value at $(\kappa_{ud}, \kappa_s) = (0.137\,54, 0.136\,40)$.

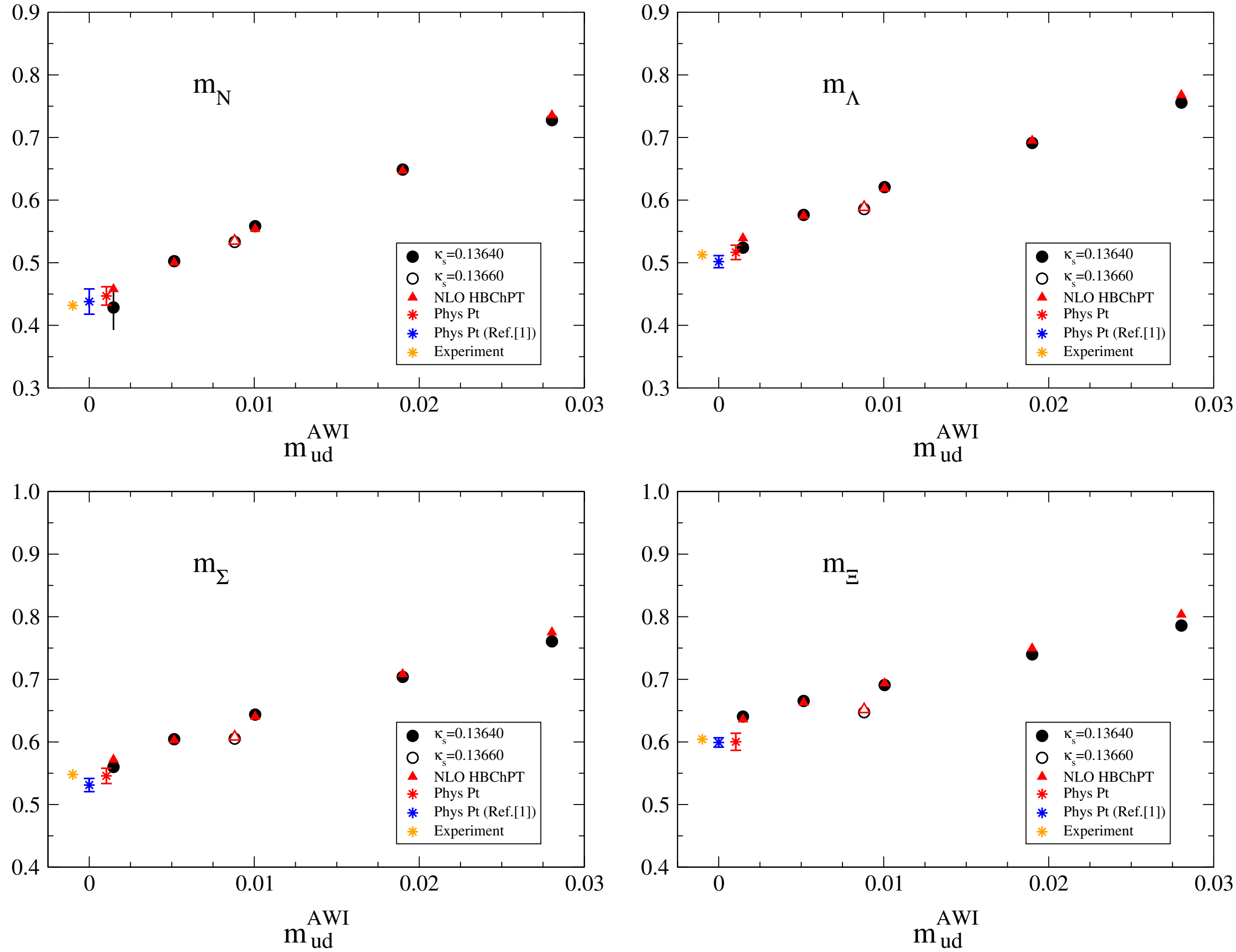

FIG. 7 (color online). Fit results with the SU(3) HBChPT up to NLO for the octet baryon masses. Experimental values are given in lattice units with $a^{-1} = 2.176$ GeV in Ref. [1].

χ^2/dof. We also present the extrapolated values at the physical point in Table I in comparison with the results of the SU(2) linear fit [Eq. (1)]. The two sets of results are consistent within 2σ error. Figure 5 shows that the data are reasonably described by the formula. The convergence behavior in Fig. 6, however, is disappointing because of the sizable contribution of the $O(m_s)$ term. This point is in a sharp contrast to the SU(2) linear expansion of Sec. II, where the contributions of the $O(m_{ud}, m_s - m_{s,ph})$ terms are well controlled in the range of $m_{ud} \lesssim 0.01$.

In the NLO fit the number of LECs increases up to seven. We give the fit results in Fig. 7 and Tables I and VI. Although the fit works in a reasonable manner, a critical observation is that the results for D, F, and $\mathcal{C}$ are essentially consistent with zero showing a significant deviation from the phenomenological estimates in Eq. (14). To examine the contributions of the NLO terms, we make a fit with D, F, and $\mathcal{C}$ fixed at the phenomenological estimates. Figure 8 and Table VI show that this fit assumption is strongly disfavored because of a prohibitively large value of χ^2/dof. The reason is found in Fig. 9: If D, F, and $\mathcal{C}$ are fixed at the phenomenological estimates, the magnitude of the NLO contribution is 2–5 times larger than that of the data even at $m_{ud} = 0$. We also remark that the same situation holds if either of D, F, $\mathcal{C}$ is fixed at the phenomenological estimate.

A similar finding is obtained in a rather different simulation setup [11]: The mixed action with the domain-wall valence quarks on the $2 + 1$ flavor asqtad sea quarks is employed with the choice of only one value for the strange quark mass which is heavier than the physical one. The conclusion of Ref. [11] is that the LECs from the NLO SU(3) fits are inconsistent with the phenomenological values.

C. Decuplet baryons

Let us turn to the decuplet baryons. The chiral expansion of the ith decuplet baryon mass is written as

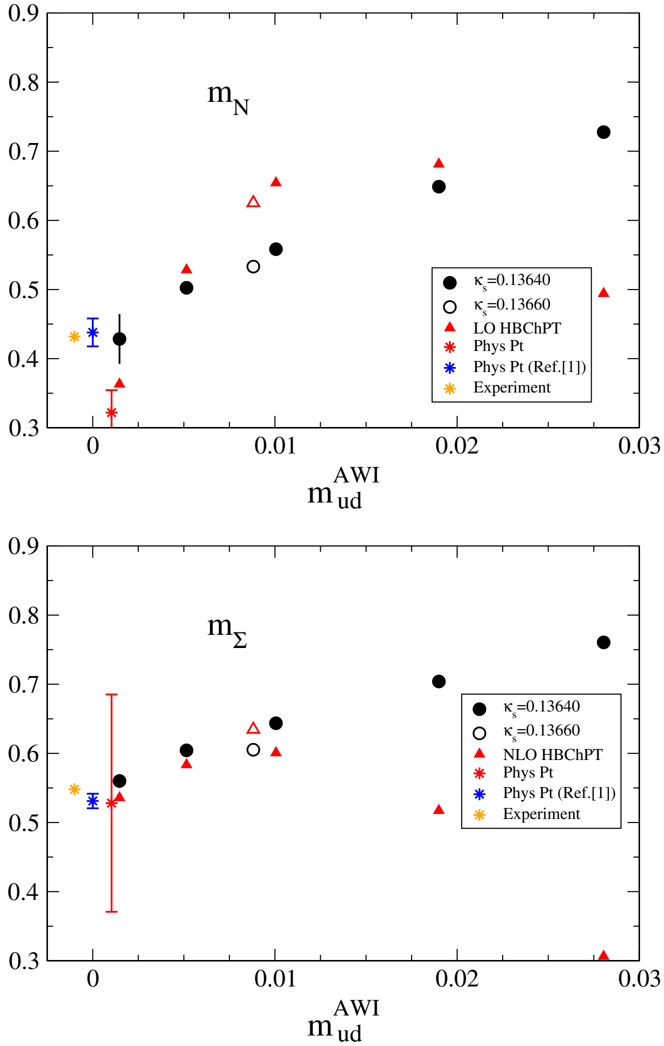

FIG. 8 (color online). Fit results with the SU(3) HBChPT up to NLO for the octet baryon masses. D, F, and $\mathcal{C}$ are fixed at the phenomenological estimates. Experimental values are given in lattice units with $a^{-1} = 2.176$ GeV in Ref. [1].

$$m_{T_i} = m_T + m_{T_i}^{(1)} + m_{T_i}^{(3/2)} + \cdots \tag{15}$$

with m_T the decuplet baryon mass in the SU(3) chiral limit. The LO and NLO corrections are given by

$$m_{\Delta}^{(1)} = \tfrac{2}{3}\gamma_M(3m_{\rm ud}) - 2\bar{\sigma}_M(2m_{\rm ud} + m_s), \tag{16}$$

$$m_{\Sigma^*}^{(1)} = \tfrac{2}{3}\gamma_M(2m_{\rm ud} + m_s) - 2\bar{\sigma}_M(2m_{\rm ud} + m_s), \tag{17}$$

$$m_{\Xi^*}^{(1)} = \tfrac{2}{3}\gamma_M(m_{\rm ud} + 2m_s) - 2\bar{\sigma}_M(2m_{\rm ud} + m_s), \tag{18}$$

$$m_{\Omega}^{(1)} = \tfrac{2}{3}\gamma_M(3m_s) - 2\bar{\sigma}_M(2m_{\rm ud} + m_s) \tag{19}$$

and

$$m_{T_i}^{(3/2)} = -\frac{1}{(4\pi f_0)^2}\sum_{\phi=\pi,K,\eta}\left[\frac{10}{9}\mathcal{H}^2 A_\phi^{T_i}\mathcal{F}(m_\phi, 0, \mu) + \mathcal{C}^2 B_\phi^{T_i}\mathcal{F}(m_\phi, -\Delta, \mu)\right], \tag{20}$$

where γ_M, $\bar{\sigma}_M$, and $\mathcal{H}$ are additional LECs, and $\mathcal{H}^2 A_\phi^{T_i}$ and $\mathcal{C}^2 B_\phi^{T_i}$ denote the contributions coming from the decuplet-decuplet- and the decuplet-octet-axial couplings, respectively, whose complete list is given in Table VII. Note that the function $\mathcal{F}(m_\phi, -\Delta, \mu)$ yields an imaginary part if $\Delta > m_\phi$ [5]. Our analyses are made by taking account of the real part only. In the region of $\Delta > m_\phi$ it might be problematic to apply the continuum chiral expansion for infinite spatial volume to lattice data. We leave the proper treatment to future work.

The LO fit results with the formula $m_{T_i} = m_T + m_{T_i}^{(1)}$ are presented in Figs. 10 and 11 and Tables II and VIII. The situation is similar to the case of the octet baryon masses.

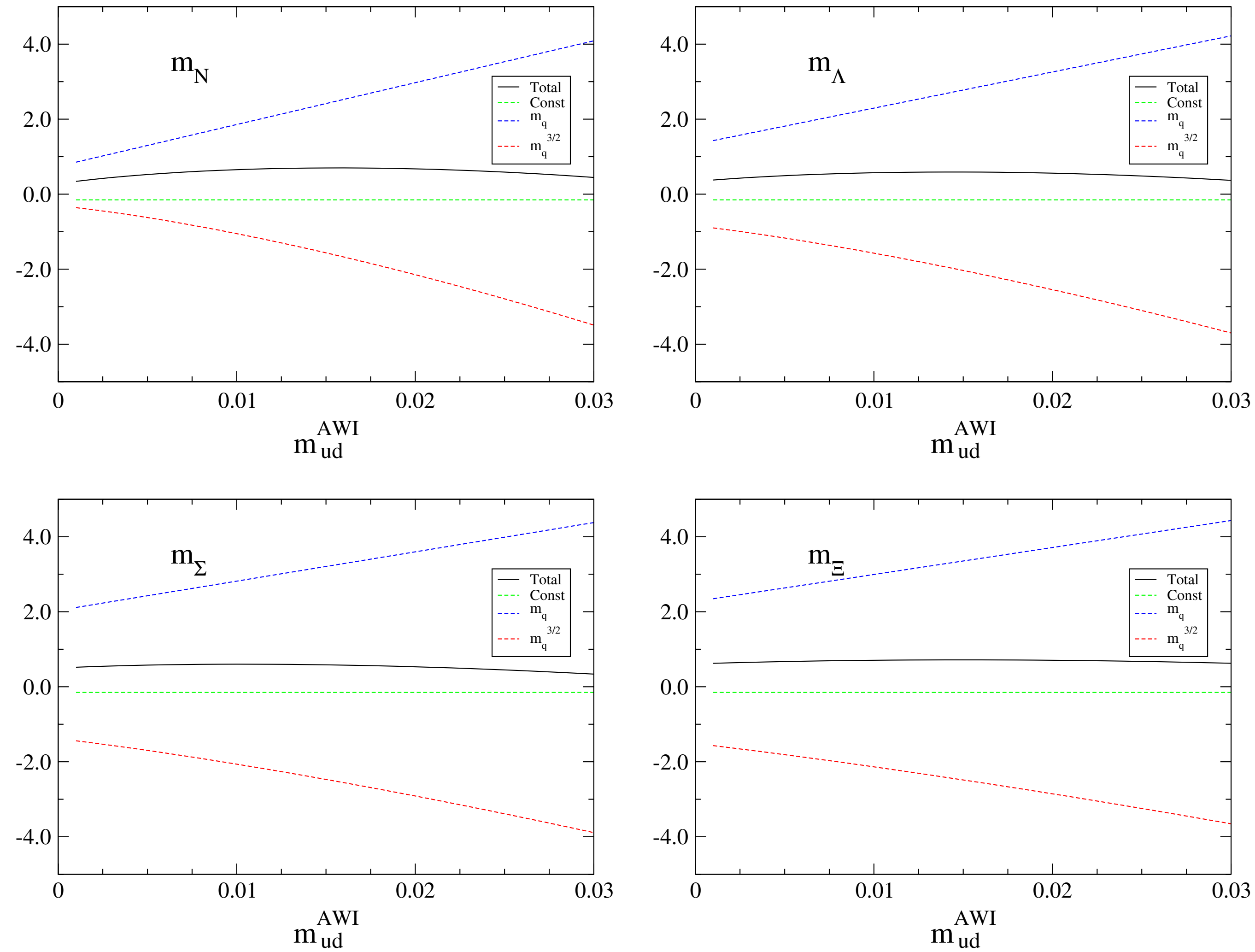

FIG. 9 (color online). Convergence behavior for the octet masses with the SU(3) HBChPT up to NLO. D, F, and $\mathcal{C}$ are fixed at the phenomenological estimates. m_{s} is fixed with the measured value at $(\kappa_{\mathrm{ud}}, \kappa_{\mathrm{s}}) = (0.137\,54, 0.136\,40)$.

The LO formula successfully describes the quark mass dependence of the decuplet baryon masses. The extrapolated values at the physical point do not show any sizable deviations beyond error bars from those of the SU(2) linear fit [Eq. (1)]. The contributions of the $O(m_{\mathrm{s}})$ term, however, are rather large compared to the magnitude of m_T. The SU(2) linear formula shows a better convergence behavior than the $SU(3)$-symmetric linear formula.

The NLO fit is carried out with and without fixing $\mathcal{C}$ at the phenomenological estimate. The results for the latter case are given in Fig. 12 and Tables II and VIII. We find a reasonable value for χ^2/dof, though the vanishing result for $\mathcal{C}$ shows inconsistency with the phenomenological estimate. Figure 13 illustrates the situation when $\mathcal{C}$ is fixed to the phenomenological value: The NLO fit hardly describes the quark mass dependence, and suffers from an unacceptable value of χ^2/dof. In Fig. 14 we find that the convergence is worsened by the NLO contributions. The mixed action case in Ref. [11] also shows that the fit results for the LECs disagree with the phenomenological values.

TABLE VII. Coefficients for the decuplet baryons $A_\phi^{T_i}$ and $B_\phi^{T_i}$.

	$A_\phi^{T_i}$			$B_\phi^{T_i}$		
ϕ	π	K	η	π	K	η
Δ	$\frac{5}{6}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{2}{3}$	0
Σ^*	$\frac{4}{9}$	$\frac{8}{9}$	0	$\frac{5}{9}$	$\frac{4}{9}$	$\frac{1}{3}$
Ξ^*	$\frac{1}{6}$	1	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{3}$
Ω	0	$\frac{2}{3}$	$\frac{2}{3}$	0	$\frac{4}{3}$	0

IV. SU(2) HBCHPT ANALYSES ON NUCLEON MASS

In the framework of SU(2) HBChPT the nucleon mass up to $O(m_{\mathrm{ud}}^{(3/2)})$ is given by

$$m_N = m_0 - 4c_1 m_\pi^2 - \frac{3g_A^2}{16\pi f^2} m_\pi^3, \tag{21}$$

where m_0 and f are defined in the SU(2) chiral limit, c_1 is a

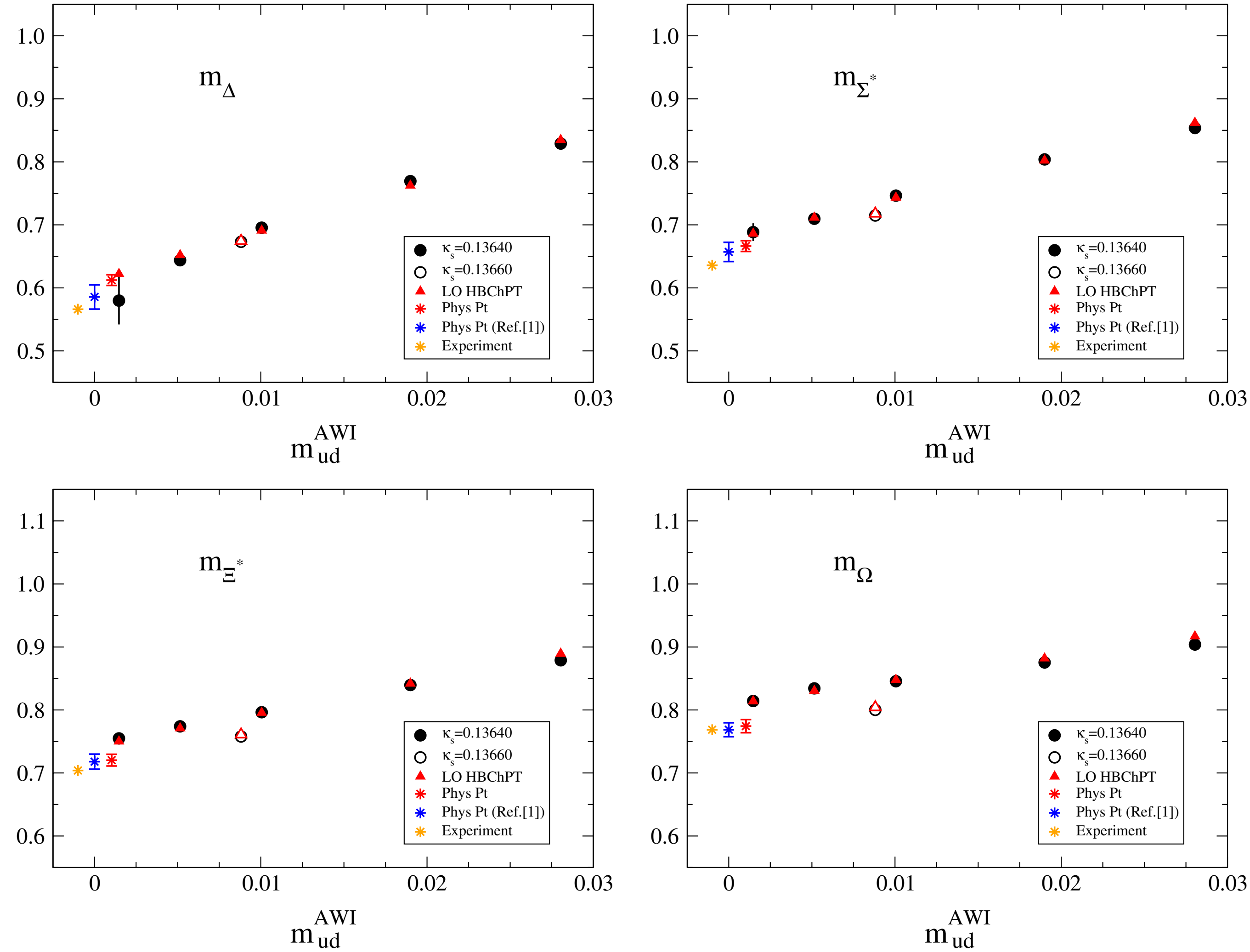

FIG. 10 (color online). Fit results with the SU(3) HBChPT up to LO for the decuplet baryon masses. Experimental values are given in lattice units with $a^{-1} = 2.176$ GeV in Ref. [1].

low energy constant, and g_A denotes the axial vector coupling of the nucleon. We further expand m_0 around the physical strange quark mass:

$$m_0 = \bar{m}_0 + m_0'(m_s - m_{s,\mathrm{ph}}). \tag{22}$$

Since $|m_s - m_{s,\mathrm{ph}}|$ is comparable to m_{ud} in our simulations, the analytic expansion of c_1, g_A, and f in terms of $m_s - m_{s,\mathrm{ph}}$ yields higher order corrections beyond $O(m_{\mathrm{ud}}^{(3/2)})$.

We apply the formula (21) to the four data points with $am_{\mathrm{ud}} \lesssim 0.01$, employing the experimental axial coupling $g_A = 1.267$, and the value of f already determined from the SU(2) ChPT fit for m_π, f_π, and f_K in Ref. [1]. The results are given in Fig. 15 and Table IX. The value of χ^2/dof is sufficiently small. However, we should remark two points. First, Fig. 15 shows that the fit results fail to predict the quark mass dependence of the data beyond $m_{\mathrm{ud}}^{\mathrm{AWI}} = 0.01$. The extrapolated value at the physical point, for which we obtain $m_N = 0.382(25)$, also undershoots the experimental one sizably. Second, the LO and NLO contributions quickly increase as the quark mass increases so that the good convergence region is restricted near the chiral limit.

It is worthwhile to make a comparison of our fit results with the two flavor twisted mass case given in Ref. [7] where the authors apply the NLO formula (21) to the nucleon mass choosing $g_A = 1.2695(29)$ and $f_\pi = 0.092\,419(7)(25)$ GeV [12]. A particular interest exists in the parameter c_1 which is responsible for the pion-nucleon sigma term: $\sigma(t = 0) = -4c_1 m_\pi^2$ with t the squared momentum transfer to the nucleon. Our result is $c_1 = -1.44(31)$ GeV^{-1} with $a^{-1} = 2.176$ GeV, which is consistent with $c_1 = -1.19(1)$ GeV^{-1} in the continuum limit of the two flavor twisted mass case. These values, however, are inconsistent with the $c_1 \approx -0.4$ GeV^{-1} obtained with the NLO fit in the 2 + 1 flavor mixed action [11]. We

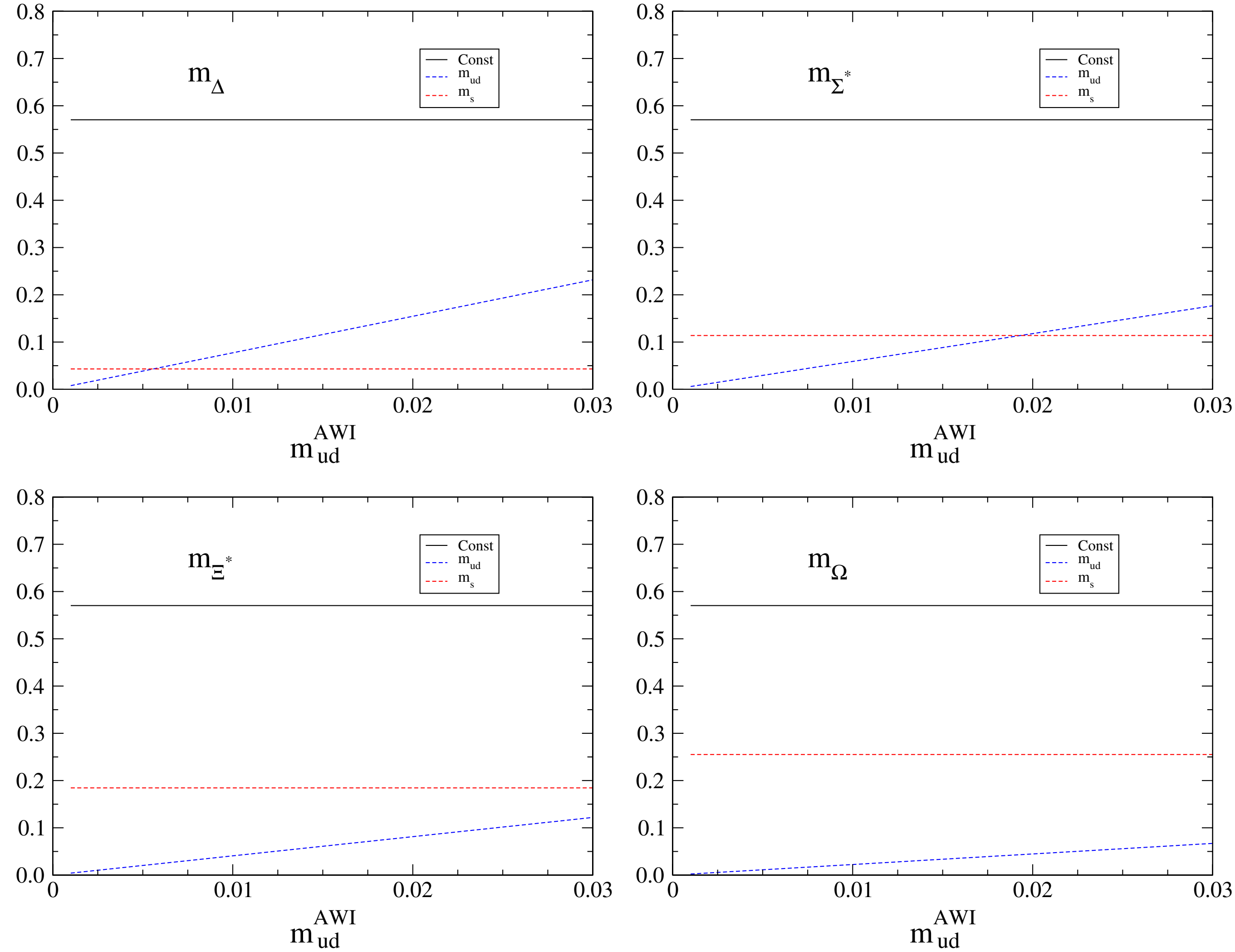

FIG. 11 (color online). Convergence behavior for the decuplet masses with the SU(3) HBChPT up to LO. m_s is fixed with the measured value at $(\kappa_{ud}, \kappa_s) = (0.137\,54, 0.136\,40)$.

obtain $\sigma(0) = 75(15)$ MeV for the sigma term, which prefers $\sigma(0) = 64 \pm 7$ MeV given by a recent analysis with the new experimental data [13] to the conventional phenomenological estimate $\sigma(0) \simeq 45$ MeV [14]. A reliable way for the precise determination of the sigma term, however, is the direct calculation of the forward matrix element of the nucleon $\langle N|\bar{q}q|N\rangle$ which requires both the connected and the disconnected diagrams [15–17].

TABLE VIII. Fit results with the SU(3) HBChPT for the decuplet baryon masses. NLO results are obtained with (case 2) and without (case 1) fixing D, F, and $\mathcal{C}$ at the phenomenological estimate.

		NLO		
	LO	case 1	case 2	Phenomenological
m_T	0.570(16)	0.550(43)	0.359(43)	
γ_M	2.745(80)	3.4(1.1)	3.88(25)	
$\bar{\sigma}_M$	−0.56(15)	−0.96(75)	−2.88(40)	
$\mathcal{C}$		$0.000(21) \times 10^{-8}$	1.5 fixed	1.5
$\mathcal{H}$		0.50(49)	$0.000(4) \times 10^{-8}$	
χ^2/dof	0.46(48)	0.50(60)	21.5(9.5)	

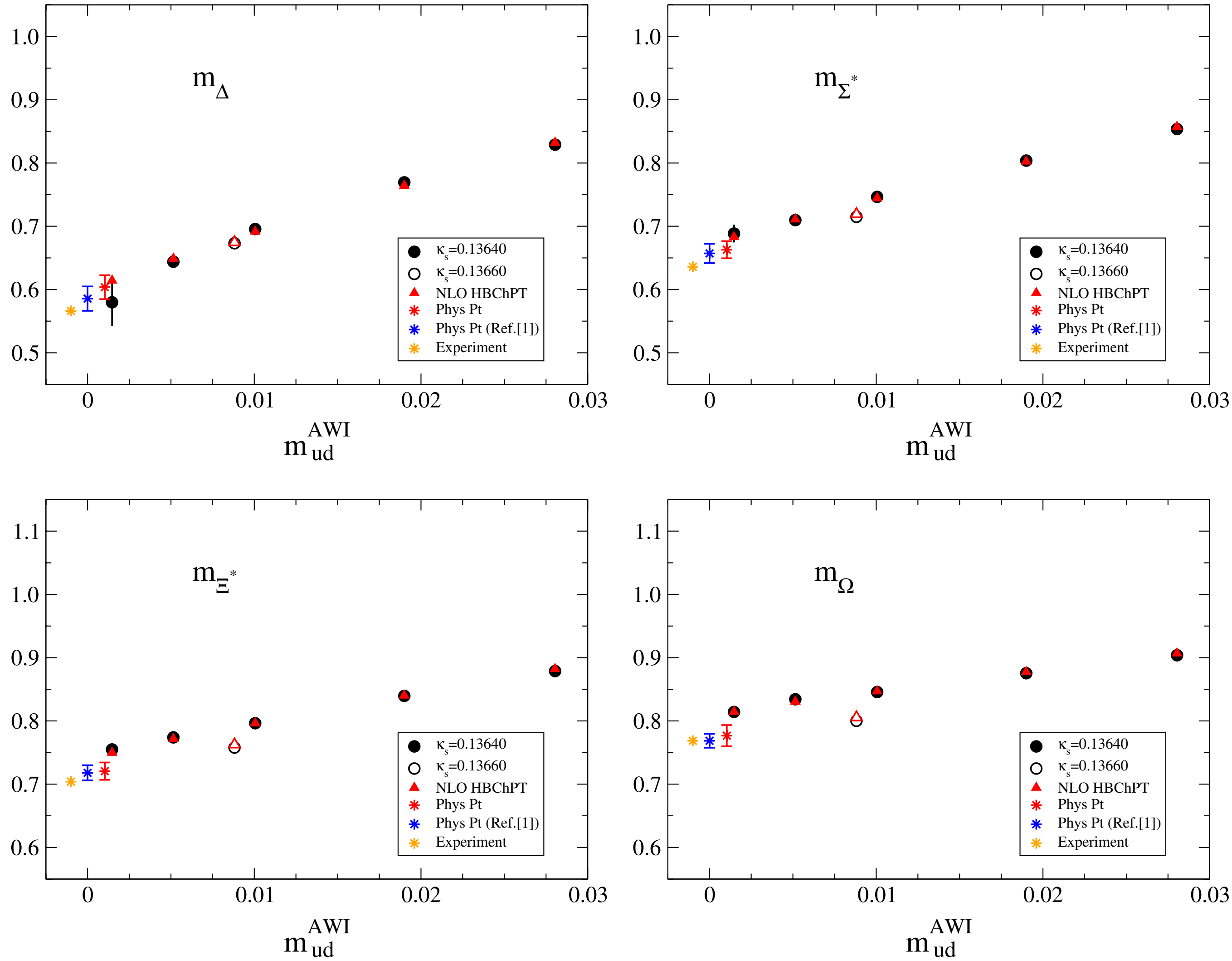

FIG. 12 (color online). Fit results with the SU(3) HBChPT up to NLO for the decuplet baryon masses. Experimental values are given in lattice units with $a^{-1} = 2.176$ GeV in Ref. [1].

V. FINITE SIZE EFFECTS

The one-loop correction for the baryon mass is evaluated by the following function:

$$\mathcal{F}^{(\infty)}(m_\phi, \Delta) = -8\pi^2 \int_0^\infty \frac{d^4 p}{(2\pi)^4} \times \frac{\vec{p}^2}{(ip_4 - \Delta)(p_4^2 + \vec{p}^2 + m_\phi^2)}, \quad (23)$$

where the integral is defined in the Euclidean space-time. This leads to Eq. (12) after the dimensional regularization and the renormalization in the $\overline{\text{MS}}$ scheme with the scale μ. In a finite spacial volume of linear size L, the integral over the spatial components of the loop momentum $\vec{p}$ is replaced by a sum over discrete momenta $\vec{p} = (2\pi/L)\vec{n}$,

$$\mathcal{F}^{(L)}(m_\phi, \Delta) = -8\pi^2 \int \frac{dp_4}{2\pi} \sum_{\vec{p}} \frac{1}{L^3} \times \frac{\vec{p}^2}{(ip_4 - \Delta)(p_4^2 + \vec{p}^2 + m_\phi^2)}, \quad (24)$$

where we assume that the time direction is infinite. We define the finite size correction for $\mathcal{F}$ as

$$\delta_L \mathcal{F}(m_\phi, \Delta) \equiv \mathcal{F}^{(L)}(m_\phi, \Delta) - \mathcal{F}^{(\infty)}(m_\phi, \Delta) \quad (25)$$

$$= 4\pi^2 \int_0^\infty d\lambda [\delta_L(\vec{p}^2 + \beta_\Delta^2)^{-(1/2)} - \beta_\Delta^2 \delta_L(\vec{p}^2 + \beta_\Delta^2)^{-(3/2)}], \quad (26)$$

where $\beta_\Delta^2 = \lambda^2 + 2\lambda\Delta + m_\phi^2$ and

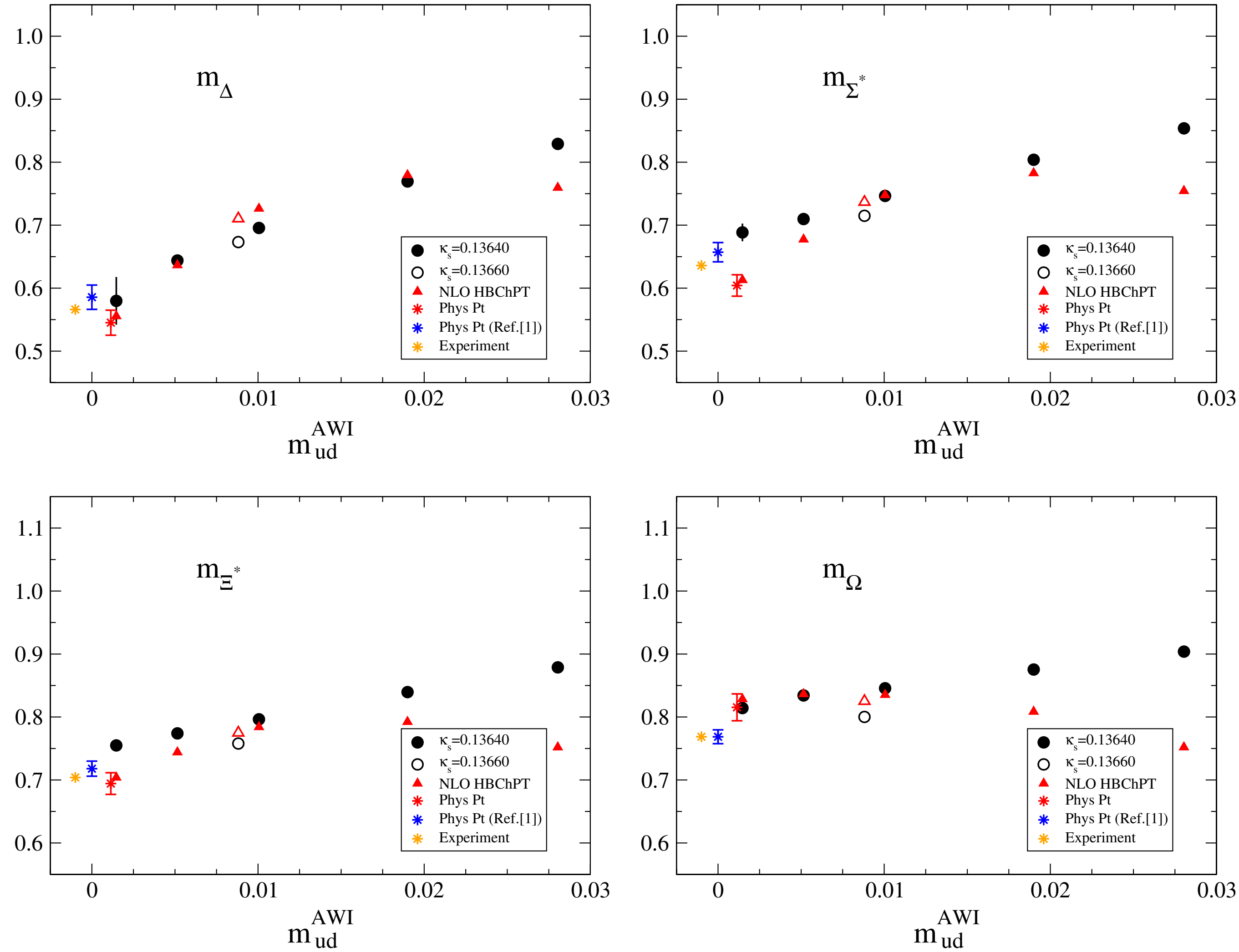

FIG. 13 (color online). Fit results with the SU(3) HBChPT up to NLO for the decuplet baryon masses. $\mathcal{C}$ is fixed at the phenomenological estimates. Experimental values are given in lattice units with $a^{-1} = 2.176$ GeV in Ref. [1].

$$\delta_L(\vec{p}^2+m^2)^{-r} \equiv \frac{1}{L^3}\sum_{\vec{p}}(\vec{p}^2+m^2)^{-r} - \int\frac{d^3p}{(2\pi)^3}(\vec{p}^2+m^2)^{-r}. \tag{27}$$

With the use of the master formula [18,19]

$$\delta_L(\vec{p}^2+m^2)^{-r} = \frac{1}{(4\pi)^{3/2}\Gamma(r)}\sum_{\vec{n}\neq 0}\left(\frac{L|\vec{n}|}{2m}\right)^{r-(3/2)} \times K_{r-(3/2)}(mL|\vec{n}|), \tag{28}$$

where $K_n(z)$ is a modified Bessel function of the second kind, one finds that the finite size corrections to the baryon masses at next-to-leading order are

$$\delta_L m_{B_i} = -\frac{2}{(4\pi f_0)^2}\sum_{\phi=\pi,K,\eta}[A_\phi^{B_i}\delta_L\mathcal{F}(m_\phi,0) + \mathcal{C}^2 B_\phi^{B_i}\delta_L\mathcal{F}(m_\phi,\Delta)], \tag{29}$$

$$\delta_L m_{T_i} = -\frac{1}{(4\pi f_0)^2}\sum_{\phi=\pi,K,\eta}\left[\frac{10}{9}\mathcal{H}^2 A_\phi^{T_i}\delta_L\mathcal{F}(m_\phi,0) + \mathcal{C}^2 B_\phi^{T_i}\delta_L\mathcal{F}(m_\phi,-\Delta)\right], \tag{30}$$

where $\delta_L m = m(L) - m(\infty)$ and $\delta_L\mathcal{F}(m_\phi,\Delta)$ is given by

$$\delta_L\mathcal{F}(m_\phi,\Delta) = 2\int_0^\infty d\lambda\beta_\Delta\sum_{\vec{n}\neq 0}\left[\frac{1}{L|\vec{n}|}K_1(L\beta_\Delta|\vec{n}|) - \beta_\Delta K_0(L\beta_\Delta|\vec{n}|)\right], \tag{31}$$

$$\delta_L\mathcal{F}(m_\phi,0) = -\pi m_\phi^2\sum_{\vec{n}\neq 0}\frac{1}{L|\vec{n}|}e^{-m_\phi L|\vec{n}|}. \tag{32}$$

In order to evaluate the magnitude of the finite size effects, let us consider the asymptotic form of $\delta_L\mathcal{F}$ [19]:

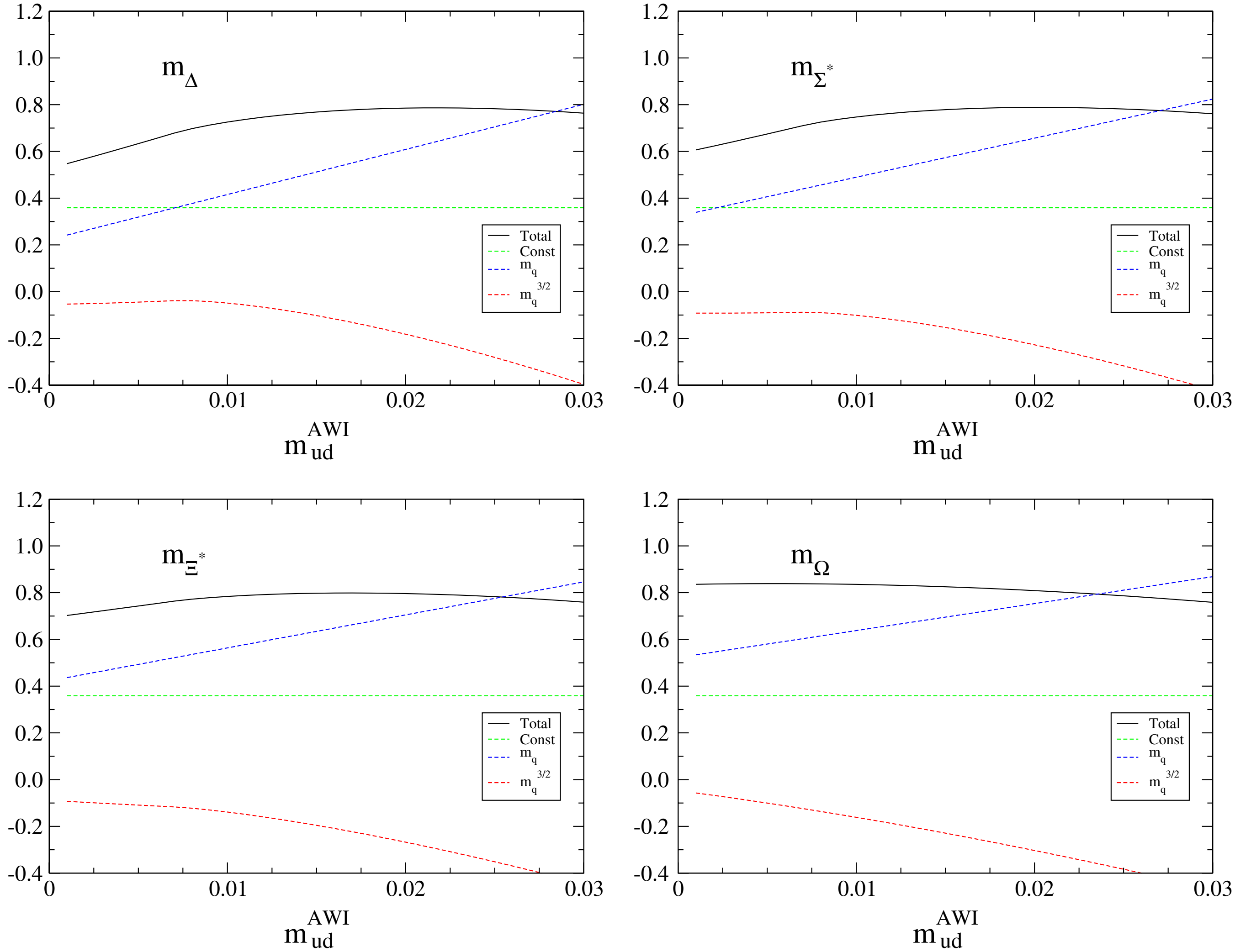

FIG. 14 (color online). Convergence behavior for the decuplet masses with the SU(3) HBChPT up to NLO. $\mathcal{C}$ is fixed at the phenomenological estimates. m_s is fixed with the measured value at $(\kappa_{ud}, \kappa_s) = (0.137\,54, 0.136\,40)$.

$$\delta_L\mathcal{F}(m_\phi, \Delta) = -6\sqrt{2\pi}m_\phi^{5/2}\frac{1}{L^{3/2}\Delta}\mathrm{e}^{-m_\phi L} + \cdots, \quad (33)$$

$$\delta_L\mathcal{F}(m_\phi, 0) = -6\pi m_\phi^2\frac{1}{L}\mathrm{e}^{-m_\phi L}. \quad (34)$$

The finite size effect for the baryon mass $\delta_L m = m(L) - m(\infty)$ is expressed as

$$\delta_L m = A\left\{\frac{3}{8\pi}\frac{m_\pi^3}{f_0^2}\frac{1}{(m_\pi L)}\mathrm{e}^{-m_\pi L}\right\} + B\left\{\frac{6\sqrt{2\pi}}{16\pi^2}\frac{m_\pi^4}{f_0^2\Delta} \times \frac{1}{(m_\pi L)^{3/2}}\mathrm{e}^{-m_\pi L}\right\} \equiv AE_1 + BE_2, \quad (35)$$

where we neglect the subleading contributions from m_K and m_η. An intriguing point is that E_1 and E_2 diminish as the pion mass decreases if the product of $m_\pi L$ is kept fixed. The values of E_1 and E_2 with $aL = 32$ at the physical point can be evaluated as

$$aE_1 = 6.61 \times 10^{-4}, \quad (36)$$

$$aE_2 = 1.43 \times 10^{-4}, \quad (37)$$

where we employ the following results in Ref. [1]:

$$m_{ud}^{ph}B_0 = 0.008\,59(11)\ [\mathrm{GeV}^2], \quad (38)$$

$$af_0 = 0.0546(39), \quad (39)$$

$$a^{-1} = 2.176(31)\ [\mathrm{GeV}]. \quad (40)$$

We also use $\Delta = m_T - m_B = 0.16$ obtained by the LO SU(3) HBChPT fit for the baryon masses. Table X summarizes the coefficients A, B and the relative finite size correction normalized by the baryon mass extrapolated at the physical point with the formula (1). To evaluate the numerical value of A and B we use the phenomenological

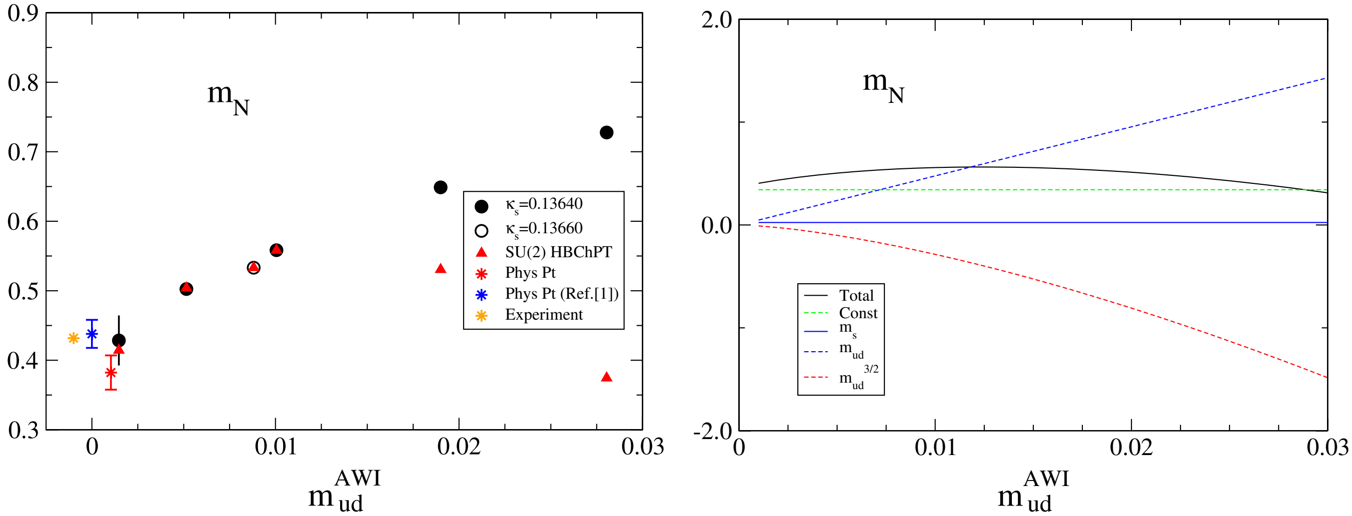

FIG. 15 (color online). Fit results with the SU(2) HBChPT up to NLO for the nucleon mass (left) and comparison of LO and NLO contributions (right). Experimental values are given in lattice units with $a^{-1} = 2.176$ GeV in Ref. [1].

TABLE IX. Fit results with SU(2) HBChPT up to NLO for the nucleon mass.

	NLO
$\bar{m}_0$	0.258(63)
m_0'	2.8(1.4)
c_1	−3.14(68)
χ^2/dof	0.2(9)

TABLE X. Relative finite size correction normalized by the baryon mass extrapolated at the physical point with the formula [Eq. (1)]. LECs are chosen to be $D = 0.80$, $F = 0.47$, $\mathcal{C} = 1.5$, and $\mathcal{H} = 1$.

	A	B	$R = \delta_L m/m[\%]$
m_N	$3(D+F)^2$	$\frac{8}{3}\mathcal{C}^2$	0.89
m_Λ	$4D^2$	$2\mathcal{C}^2$	0.45
m_Σ	$\frac{4}{3}(D^2+6F^2)$	$\frac{4}{9}\mathcal{C}^2$	0.34
m_Ξ	$3(D-F)^2$	$\frac{2}{3}\mathcal{C}^2$	0.07
m_Δ	$\frac{25}{27}\mathcal{H}^2$	$\frac{2}{3}\mathcal{C}^2$	0.13
m_{Σ^*}	$\frac{40}{81}\mathcal{H}^2$	$\frac{5}{9}\mathcal{C}^2$	0.08
m_{Ξ^*}	$\frac{5}{27}\mathcal{H}^2$	$\frac{1}{3}\mathcal{C}^2$	0.03
m_Ω	0	0	0

estimates $D = 0.80$, $F = 0.47$, and $\mathcal{C} = 1.5$ and assume $\mathcal{H} = 1$ which is comparable to our fit results in Table VIII. We find that the HBChPT predicts fairly small finite size corrections even at the physical point: The magnitude is less than 1% for all the channels. We additionally remark that the Ω baryon mass is free from the leading contribution of the finite size effects. This is another fascinating feature to choose m_Ω as one of the physical inputs.

VI. CONCLUSION

We have investigated the chiral behavior of the octet and the decuplet baryon masses based on the SU(3) HBChPT. At LO we find reasonable fit results both for the octet and the decuplet baryon masses, though rather large strange quark contributions are observed. This point is contrary to the SU(2) linear chiral expansion where the LO strange quark contribution is well controlled around the physical m_s. Inclusion of the NLO contributions makes the situation worse: The fit results are incompatible with the phenomenological estimates for the low energy constants D, F, and $\mathcal{C}$ both for the octet and the decuplet cases. We have also applied the NLO SU(2) HBChPT to the nucleon mass. The quark mass dependence is reasonably described below $m_{ud}^{AWI} \lesssim 0.01$. The good convergence property, however, is observed only near the chiral limit. The finite size effects predicted by the SU(3) HBChPT turn out to be fairly small: at most less than 1%. However, we should be aware of the possibility that the value $m_\pi L \approx 2$ at the physical point for the $aL = 32$ lattice may be beyond the applicability range of Eq. (35). Comparisons with lattice data on larger lattices should settle the issue here.

In order to avoid the difficulties associated with the chiral extrapolation of the baryonic quantities, we are now carrying out a simulation directly on the physical point. This simulation is being made on a larger lattice size so that a direct study of finite size effects is possible.

ACKNOWLEDGMENTS

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