

Delta-baryon mass in a covariant Faddeev approach

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We present a calculation of the three-quark core contribution to the mass of the Δ resonance in a Poincaré-covariant Faddeev framework. A consistent setup for the dressed-quark propagator, the quark-quark and quark-”diquark” interactions is used, where all the ingredients are solutions of their respective Dyson-Schwinger or Bethe-Salpeter equations in rainbow-ladder truncation. We discuss the evolution of the Δ mass with the current-quark mass and compare to the previously obtained mass of the nucleon.

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I. INTRODUCTION

Experimentally, the first excited state of the nucleon, the Δ resonance, is observed in scattering pions, photons, or electrons off nucleon targets. Recent precise experiments at LEGS, BATES, ELSA, MAMI, and JLAB do not only report on its mass ($M_\Delta = 1.232$ GeV) and width ($\Gamma_\Delta = 120$ MeV) but also measure the electromagnetic $N \rightarrow \Delta \gamma$ transition form factors [1–7]. The basic properties of the lightest resonance of the nucleon have been calculated within various approaches such as constituent-quark models [8–17], Skyrme models [18–20], chiral cloudy-bag models [21,22], chiral effective field theory methods [23–27], and lattice-regularized QCD [28–32].

Ultimately it is desirable to obtain a detailed understanding of the rich structure of N and Δ baryons and their properties in terms of QCD’s quark and gluon degrees of freedom in a quantum-field theoretical framework. The Dyson-Schwinger equation (DSE) point of view employed in this paper offers a nonperturbative continuum approach to QCD, reviewed recently in [33,34]. This fully self-consistent infinite set of coupled integral equations provides a tool to access both the perturbative and the non-perturbative regimes of QCD. The most prominent phenomena emerging in the latter are dynamical chiral symmetry breaking and confinement, which, in the same way as bound states, require a nonperturbative treatment.

Hadrons and their properties are studied in this approach via covariant bound-state equations. While mesons can be described by solutions of the $q\bar{q}$ -bound-state Bethe-Salpeter equation (BSE), the case of a baryon is more involved and has therefore not yet been addressed at the same level of sophistication. Based on the observation that the attractive nature of quark-antiquark correlations in a color-singlet meson is also attractive for $\bar{3}_C$ quark-quark correlations within a color-singlet baryon, a first step is to study a two-body problem by means of a quark-diquark

bound-state BSE. In a basic setup, parametrizations for the needed quark and diquark propagators and diquark amplitudes were used to calculate N and Δ masses [35,36] and nucleon electromagnetic form factors [37–41]. The next step on the way to the full covariant three-body bound-state equation is to replace the parametrizations by solutions of the corresponding DSE (quark propagator) and BSEs (diquark amplitudes) in analogy to sophisticated meson studies. The simplest consistent setup of this kind is the rainbow-ladder (RL) truncation, which was implemented in this fashion and discussed in full detail for the nucleon in [42]; for a short summary, see [43]. Recently, evidence has been provided that in Landau gauge QCD all components of the quark-gluon vertex are infrared divergent [44,45]. However, there are indications that for most hadronic observables the corresponding contributions beyond RL truncation are small. In this respect mesonic effects in the quark and quark-gluon-vertex DSEs (see, e.g., [46–48]) are more important. Because of the existence of these and other contributions beyond RL truncation, instead of aiming at actually reproducing light-quark baryon properties in RL truncation, a different approach was recently taken. It allows an identification of RL results with a hadron’s ’quark core’ via estimating all corrections that are expected beyond RL and subsequently tuning the RL contribution such that the corrected result would describe experimental data. The corresponding prescription was introduced in [49] and applied to the nucleon mass and electromagnetic form factors in [50]. In the present work we adopt this procedure and calculate the respective core contribution to the mass of the $\Delta(1232)$, a necessary step for a reliable analysis of the nontrivial $N \rightarrow \Delta \gamma$ transition.

The paper is organized as follows: first we briefly summarize the Poincaré-covariant Faddeev approach to baryons and its simplification to a quark-diquark picture. After discussing the ingredients of the covariant quark-diquark BSE and the implications of the interaction as well as the diquark concept, we present the tensor decomposition of the Δ amplitude. Finally we discuss the results for the mass

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of the Δ baryon and compare to the mass of the nucleon obtained in the same approach, together with a selection of results from lattice QCD.

Throughout this paper we work in Euclidean momentum space and use the isospin-symmetric limit $m_u = m_d$.

II. COVARIANT FADDEEV EQUATIONS

Baryonic bound states correspond to poles in the three-quark scattering matrix, i.e., the amputated and connected quark 6-point function. The residue at a pole associated to a baryon of mass M defines the respective bound-state amplitude. It satisfies a covariant homogeneous integral equation, which, upon neglecting irreducible three-body interactions, leads to a covariant equation of the Faddeev type [51].

The binding of baryons in this framework is dominated by quark-quark correlations. In particular, a color-singlet baryon emerges as a bound state of a color-triplet quark and a color-antitriplet diquark correlation [52]. The same mechanism that binds color-singlet mesons is suitable to account for an attraction in the corresponding diquark channels [52,53]. To implement this concept, one approximates the quark-quark scattering matrix that appears in the covariant Faddeev equations by a separable sum of pseudoparticle-pole contributions corresponding to various diquark correlations. This procedure leads to a quark-diquark BSE on the baryon's mass shell, where scalar 0^+ and axial-vector 1^+ diquarks have been used to describe the nucleon. Of those only the axial-vector diquark correlation contributes to the spin-3/2 and isospin-3/2 flavor symmetric Δ ; therefore its quark-diquark BSE reads [35,38]

$$\Phi_{\alpha\beta}^{\mu\nu}(p, P) = \int_k \{K^{\mu\rho}(p, k, P)S(p_q)\Phi^{\sigma\nu}(k, P)\}_{\alpha\beta} D^{\rho\sigma}(p_d), \quad (1)$$

where P is the total baryon momentum, p_q, p_d are quark and diquark momenta, p, k are the quark-diquark relative momenta, and \int_k denotes $\int \frac{d^4k}{(2\pi)^4}$. Greek superscripts represent Lorentz indices, Greek subscripts fermion indices. The amplitudes $\Phi^{\mu\nu}(p, P)$ are the matrix-valued remainders of the full quark-diquark amplitude $\Phi_{\alpha\beta}^{\mu\nu}(p, P)u_{\beta}^{\nu}(P)$ for the Δ , where $u_{\beta}^{\nu}(P)$ is a Rarita-Schwinger spinor describing a free spin-3/2 particle with momentum P . In addition one needs to specify the dressed-quark propagator S , the diquark propagator $D^{\rho\sigma}$, and the axial-vector diquark amplitude Γ^{ν} and its charge-conjugate $\bar{\Gamma}^{\nu}$ which appear in the quark-diquark kernel:

$$K_{\alpha\beta}^{\mu\nu}(p, k, P) = \{\Gamma^{\nu}(k_r, k_d)S^T(q)\bar{\Gamma}^{\mu}(p_r, -p_d)\}_{\alpha\beta}, \quad (2)$$

where subscripts r denote quark-quark relative momenta and d diquark momenta. By virtue of Eqs. (1) and (2) the mechanism which binds the Δ is an iterated exchange of roles between the single quark and any of the quarks

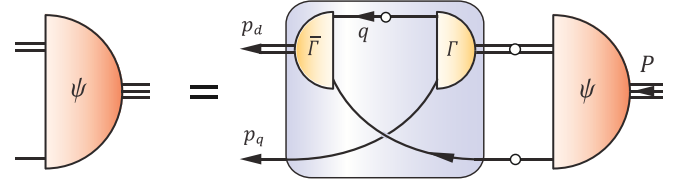


FIG. 1 (color online). The quark-diquark BSE, Eq. (1).

contained in the diquark. This exchange is depicted in the quark-diquark BSE in Fig. 1. We proceed by detailing the ingredients of Eq. (1).

III. QUARK PROPAGATOR AND EFFECTIVE COUPLING

We begin with the renormalized dressed-quark propagator $S(p)$. It is expressed in terms of two scalar functions,

$$S^{-1}(p) = A(p^2)(i\not{p} + M(p^2)), \quad (3)$$

where $1/A(p^2)$ is the quark wave-function renormalization and $M(p^2)$ the renormalization-point independent quark mass function. The quark propagator satisfies the quark DSE (also referred to as the QCD gap equation),

$$S_{\alpha\beta}^{-1}(p) = Z_2(i\not{p} + m_{\text{bare}})_{\alpha\beta} - \int_q^{\Lambda} \mathcal{K}_{\alpha\gamma, \delta\beta}(p, q)S_{\gamma\delta}(q), \quad (4)$$

where Z_2 is the quark renormalization constant and Λ a regularization mass-scale. The bare current-quark mass m_{bare} serves as an input of the equation. For the renormalization-independent current-quark mass \hat{m} , we use the one-loop evolution relation $m_{\text{bare}} = \hat{m}/[\ln(\Lambda/\Lambda_{\text{QCD}})]^{\gamma_m}$, where the anomalous dimension of the quark mass function is $\gamma_m = 12/(11N_c - 2N_f)$. We use $N_f = 4$ and $\Lambda_{\text{QCD}} = 0.234$ GeV. In the chiral limit, $\hat{m} = 0$.

The kernel \mathcal{K} of the quark DSE includes the dressed gluon propagator as well as one bare and one dressed quark-gluon vertex. The fully dressed gluon propagator and quark-gluon vertex could in principle be obtained as solutions of the infinite coupled tower of QCD's DSEs together with all other Green functions of the theory; cf. [45] and references therein. In practical numerical studies one employs a truncation of the infinite system of equations by solving only a subset explicitly. Green functions appearing in the subset but not solved are represented by substantiated *Ansätze*. In connection with the simultaneous solution of a meson BSE, it is imperative to employ a truncation that preserves the axial-vector Ward-Takahashi identity if one wants to correctly implement chiral symmetry and its dynamical breaking. This ensures a massless pion in the chiral limit—the Goldstone boson related to dynamical chiral symmetry breaking—and leads to a generalized Gell-Mann-Oakes-Renner relation for all pseudo-scalar mesons and all current-quark masses [54–56]. The

lowest order in such a symmetry-preserving truncation scheme [57,58], the RL truncation, has been extensively used in DSE studies of mesons, e.g., [59,60] (spectroscopy), [61–63] (electromagnetic properties), and references therein.

Out of the 12 general covariants of the quark-gluon vertex, the RL truncation retains only its vector part $\sim \gamma^\mu$. The nonperturbative dressing of the gluon propagator and the quark-gluon vertex are absorbed into an effective coupling $\alpha(k^2)$ which is given by an *Ansatz* invoked by the truncation. The kernel \mathcal{K} of the quark DSE then reads

$$\mathcal{K}_{\alpha\gamma,\beta\delta}(p, q) = Z_2^2 \frac{4\pi\alpha(k^2)}{k^2} T_k^{\mu\nu} \left(i\gamma^\mu \frac{\lambda^a}{2} \right)_{\alpha\gamma} \left(i\gamma^\nu \frac{\lambda^a}{2} \right)_{\beta\delta}, \quad (5)$$

where λ^a are the $SU(3)_C$ Gell-Mann matrices. $T_k^{\mu\nu} = \delta^{\mu\nu} - \hat{k}^\mu \hat{k}^\nu$ is a transverse projector with respect to the gluon momentum $k = p - q$, and $\hat{k}^\mu = k^\mu / \sqrt{k^2}$ denotes a normalized 4-vector. A convenient form of $\alpha(k^2)$ is given by [64]

$$\alpha(k^2) = \frac{c\pi}{\omega^7} \left(\frac{k^2}{\Lambda_0^2} \right)^2 e^{-k^2/\omega^2\Lambda_0^2} + \frac{\pi\gamma_m(1 - e^{-k^2/\Lambda_0^2})}{\ln\sqrt{e^2 - 1} + (1 + k^2/\Lambda_{\text{QCD}}^2)^2} \quad (6)$$

with a scale $\Lambda_0 = 1$ GeV. The second term reproduces QCD's perturbative running coupling and decreases logarithmically for large gluon-momenta. The first term of the interaction accounts for the nonperturbative enhancement at small and intermediate gluon momenta: it provides the necessary strength to allow for dynamical chiral symmetry breaking and the dynamical generation of a constituent-quark mass scale. With the UV part fixed by perturbative QCD, the coupling is characterized by two parameters, the strength c and width ω of the interaction in the infrared. The interaction of Eq. (6) provides a reasonable description of masses, decay constants, and electromagnetic properties of ground-state pseudoscalar and vector-mesons with equal-mass constituents up to bottomonium if the coupling strength is kept fixed for all values of the quark mass (see [60] and references therein). In these investigations a value $c = 0.37$ has commonly been used, which reproduces the phenomenological quark condensate and the experimental decay constant $f_\pi = 131$ MeV at a pion mass of $m_\pi = 140$ MeV. Furthermore, pseudoscalar- and vector-meson masses and decay constants have turned out to be insensitive upon variation of the coupling-width parameter ω in the range $\omega \approx 0.4 \pm 0.1$ [64].

Naturally, any description making use of a truncation is *a priori* incomplete. On the one hand, corrections come from pseudoscalar meson-cloud contributions which were introduced by the cloudy-bag model [22,65,66] and systematically incorporated into chiral perturbation theory

[67,68]. These corrections provide a substantial attractive contribution to the quark core of dynamically generated hadron observables in the chiral regime, whereas they vanish with increasing current-quark mass. Their impact on the chiral structure of the quark mass function and condensate, f_π , m_ρ , and nucleon and Δ observables has been demonstrated in the Nambu-Jona-Lasinio (NJL) model [69,70], DSE studies [36,46–48], and chiral extrapolations of lattice results [71]. On the other hand, a resummation of nonresonant diagrams beyond RL provides further attraction in the pseudoscalar and vector-meson channels [72,73]. The above corrections are in agreement with the quark-gluon-vertex DSE and the infrared properties of its solution [45]. In order to anticipate such corrections, a mere RL result should systematically overestimate the experimental masses. More concretely, in [49,50] a parametrization for the relation $m_\rho(m_\pi)$ was used to characterize the quark-core contribution to the meson masses:

$$x_\rho^2 = 1 + x_\pi^4/(0.6 + x_\pi^2), \quad x_\rho = m_\rho/m_\rho^0, \quad (7)$$

$$x_\pi = m_\pi/m_\rho^0,$$

with the chiral-limit value $m_\rho^0 = 0.99$ GeV. Starting from this value, the sum of chiral corrections would reduce m_ρ in the chiral limit by $\sim 25\%$ whereas above the s -quark mass the quark core approaches corresponding lattice results for m_ρ . In order to reproduce Eq. (7) by solving the ρ -meson BSE, the coupling strength c of Eq. (6) has to depend on the current-quark mass thereby reflecting the properties discussed above. The following fit yields Eq. (7):

$$c(\omega, \hat{m}) = 0.11 + \frac{0.86b(\Delta\omega)}{1 + 0.885x_q + (0.474x_q)^2}, \quad (8)$$

where $x_q = \hat{m}/(0.12$ GeV). At each value of the current-quark mass \hat{m} , the parametrization

$$b(\Delta\omega) = 1 - 0.15\Delta\omega + (1.50\Delta\omega)^2 + (2.95\Delta\omega)^3 \quad (9)$$

fixes the result for m_ρ to the same value in the range $\omega = \bar{\omega}(\hat{m}) \pm |\Delta\omega|$, with the central value given by $\bar{\omega}(\hat{m}) = 0.38 + 0.17/(1 + x_q)$. As demonstrated in Refs. [49,50], this procedure induces consistent (overestimated) values for a range of π , ρ , and nucleon observables as obtained from their respective meson and quark-diquark BSEs. These results are approximately ω -independent for $|\Delta\omega| \lesssim 0.1$, i.e., in the region where $b(\Delta\omega) \approx 1$.

In the present work we employ this ‘‘core model’’ of Eqs. (6)–(9) to directly compare the obtained Δ mass with the previously obtained result for the nucleon. We note here that all parameters of the interaction were fixed using information from the π and ρ masses.

IV. DIQUARKS

It follows from the separable diquark-pole *Ansatz* for the two-quark scattering matrix that the axial-vector diquark amplitude in Eq. (2) satisfies a diquark BSE with the on-shell condition $P^2 = -M_{\text{av}}^2$ for the total diquark momentum P^ν , which reads

$$\Gamma_{\alpha\beta}^\mu(p, P) = \int_q^\Lambda \mathcal{K}_{\alpha\gamma, \beta\delta}(p, q) \{S(q_+) \Gamma^\mu(q, P) S^T(q_-)\}_{\gamma\delta}, \quad (10)$$

where p is the relative momentum between the two quarks in the diquark bound state and $q_\pm = \pm q + P/2$ are the quark momenta. Greek subscripts represent Dirac and color indices of the involved quarks. The form of the corresponding scalar-diquark BSE is identical to Eq. (10) if the Lorentz superscript μ is dropped.

Consistency with the meson sector implies the identification of the irreducible two-quark kernel \mathcal{K} in the diquark BSE with the RL-truncated kernel of Eq. (5). This setup (self-consistently) yields real diquark masses. While this feature does not explicitly contradict diquark confinement (see, e.g., [74,75]), we note here that diquark bound-state poles are likely to be removed from the real timelike P^2 axis by large repulsive corrections beyond RL truncation [58,72,76,77]. Assuming that also a two-quark scattering matrix free of real timelike poles contains scales characterizing the diquark correlations, it is nonetheless meaningful to make use of the diquark concept. Evidence in this direction also comes from Coulomb-gauge QCD, where diquark correlations have recently been shown to retain their size while being removed from the physical spectrum [78].

The axial-vector diquark amplitude can be decomposed into a sum of 12 Lorentz-invariant dressing functions (a hat on a four-momentum indicates a normalized vector),

$$\Gamma_{\alpha\beta}^\mu(q, P) = \sum_{k=1}^{12} f_k^{\text{av}}(q^2, \hat{q} \cdot \hat{P}, P^2) \{i\tau_k^\mu(q, P) C\}_{\alpha\beta}, \quad (11)$$

of which, due to transversality at the axial-vector diquark pole, only 8 contribute on-shell. A suitable orthonormal basis for this on-shell part of the axial-vector amplitude is given by (cf. Appendix B.2 of Ref. [42])

$$\begin{aligned} \tau_1^\mu &= \gamma^\mu & \tau_2^\mu &= \gamma^\mu \hat{\not{P}} & \tau_3^\mu &= i\hat{q}^\mu \\ \tau_4^\mu &= \hat{q} \cdot \hat{P} \hat{q}^\mu \hat{\not{P}} & \tau_5^\mu &= \hat{q} \cdot \hat{P} (\gamma^\mu \hat{\not{q}}_T - \hat{q}^\mu) \\ \tau_6^\mu &= \frac{i\gamma^\mu}{2} [\hat{\not{P}}, \hat{\not{q}}] + i\hat{q}^\mu \hat{\not{P}} & \tau_7^\mu &= \hat{q}^\mu \hat{\not{q}}_T - \frac{\hat{q}_T^2}{3} \gamma^\mu \\ \tau_8^\mu &= \frac{\hat{q}^\mu}{2} [\hat{\not{P}}, \hat{\not{q}}] + \frac{\hat{q}_T^2}{3} \gamma^\mu \hat{\not{P}}, \end{aligned} \quad (12)$$

where the subscript T denotes transverse projection.

The diquark amplitude is a solution of Eq. (10) only on the diquark's mass shell. However, within a baryon the

two-quark system moves at all possible values of the total diquark momentum. Therefore, to fully obtain the kernel of the quark-diquark BSE (1), one has to specify the off-shell behavior of the diquark amplitude and propagator as well. For completeness we recapitulate the off-shell extension introduced in Ref. [50]: we use an on-shell approximation for the dressing functions $f_k^{\text{av}}(q^2, \hat{q} \cdot \hat{P}, P^2) \approx f_k^{\text{av}}(q^2, 0, -M_{\text{av}}^2)$. Assuming that only the dominant amplitude remains relevant at large P^2 , each subleading amplitude $f_{k>1}^{\text{av}}$ is suppressed by a factor $g(x) = 1/(x+2)$, where $x = P^2/M_{\text{av}}^2$; and each occurrence of \hat{P} in the basis (12) is augmented by a factor $h(x) = -i\sqrt{x/(x+2)}$ to ensure a sensible analytic continuation for off-shell momenta which does not alter the power-law behavior at $P^2 \rightarrow \infty$. The off-shell *Ansätze* $g(x)$ and $h(x)$ leave the BSE solutions on the diquark's mass shell invariant, $g(-1) = h(-1) = 1$.

Reinsertion of the diquark pole *ansatz* into the Dyson series for the two-quark T -matrix yields the following expression for the diquark propagator [42,79]:

$$D_{\mu\nu}^{-1}(P) = M_{\text{av}}^2 \{ \lambda \delta_{\mu\nu} + \beta F_{\mu\nu}(x) + Q_{\mu\nu}(x) \}, \quad (13)$$

$$Q^{\mu\nu}(x) = \frac{1}{2M_{\text{av}}^2} \int_q \text{Tr}_D \{ \bar{\Gamma}^\mu(q, -P) S(q_+) \Gamma^\nu(q, P) S^T(q_-) \} \quad (14)$$

with $\lambda = -Q_T(-1)$ and $\beta = 1 - Q_T'(-1)$. $Q_T = Q_{\mu\nu} Q_{\mu\nu}/3$ denotes the transverse contribution to the quark-loop integral $Q_{\mu\nu}$. The two-loop integral $F_{\mu\nu}(x)$ emerges via inclusion of the subleading amplitudes $f_{k>1}^{\text{av}}$. The *ansatz*

$$F_{\mu\nu}(x) = \delta_{\mu\nu} (1 - 1/(x+2)^2)/2 \quad (15)$$

ensures the on-shell behavior $D_T^{-1}(P^2 \rightarrow -M_{\text{av}}^2) \rightarrow P^2 + M_{\text{av}}^2$ and a satisfactory approximation in the UV extracted from the result of the numerical two-loop calculation. We note that the systematic uncertainty connected to the diquark amplitude's off-shell *ansatz* is minimized through use of the consistent expression for the diquark propagator, Eq. (13). While a different choice for the off-shell functions $g(x)$ and $h(x)$ changes the values of $F(x)$ and $Q(x)$ accordingly and necessitates a modified *ansatz* (15), it has no material impact on calculated observables since only the product of two diquark amplitudes and the propagator (i.e., the quark-quark T -matrix) enters the covariant Faddeev equation's kernel.

V. QUARK-DIQUARK AMPLITUDES FOR THE Δ

Finally, to compute the amplitude and mass of the Δ baryon numerically, the structure of the on-shell quark-diquark amplitude Φ of Eq. (1) needs to be specified. Now denoting the total Δ momentum by P , with $P^2 = -M_\Delta^2$ and $\hat{P} = P/(iM_\Delta)$, it is decomposed into 8 covariant struc-

tures [35]:

$$\Phi_{\alpha\beta}^{\mu\nu}(p, P) = \sum_{k=1}^8 f_k^{\Delta}(p^2, \hat{p} \cdot \hat{P}) \times \{\tau_k^{\mu\rho}(p, P)\Lambda_+(P)\Lambda^{\rho\nu}(P)\}_{\alpha\beta}, \quad (16)$$

where the basis elements include the Rarita-Schwinger projector onto positive-energy and spin-3/2 spinors, constructed from $\Lambda_+(P) = (\mathbb{1} + \hat{\not{P}})/2$ and

$$\Lambda^{\mu\nu}(P) = \delta^{\mu\nu} + \frac{1}{3}(\hat{P}^\mu \gamma^\nu - \hat{P}^\nu \gamma^\mu - \gamma^\mu \gamma^\nu - 2\hat{P}^\mu \hat{P}^\nu). \quad (17)$$

The number of basis elements can be inferred from the Clifford algebra. A general Green function with two fermion legs, two vector legs, and two independent momenta p and P allows for 40 possible Dirac covariants $\tau_{k=1\dots 40}^{\mu\rho}$, where four come from $\delta^{\mu\rho}\{\mathbb{1}, \not{p}, \not{P}, \not{p}\not{P}\}$ and 36 from all possible combinations of the $3 \times 3 \times 4$ terms $\{p^\mu, P^\mu, \gamma^\mu\} \times \{p^\rho, P^\rho, \gamma^\rho\} \times \{\mathbb{1}, \not{p}, \not{P}, \not{p}\not{P}\}$. The elements $P^\rho, \gamma^\rho, \not{P}$, and $\not{p}\not{P}$ become redundant upon contraction with the Rarita-Schwinger projector: $P^\rho \Lambda_+ \Lambda^{\rho\nu} = \gamma^\rho \Lambda_+ \Lambda^{\rho\nu} = 0$, $\{\not{P}, \not{p}\not{P}\}\Lambda_+ = \{\mathbb{1}, \not{p}\}\Lambda_+$. This leaves 8 Dirac covariants for which a convenient orthogonal set is given by

$$\begin{aligned} \tau_1^{\mu\rho} &= \delta^{\mu\rho} & \tau_2^{\mu\rho} &= \frac{1}{\sqrt{5}}(2\gamma_T^\mu q^\rho - 3\delta^{\mu\rho} \not{q}) \\ \tau_3^{\mu\rho} &= -\sqrt{3}\hat{P}^\mu q^\rho \not{q} & \tau_4^{\mu\rho} &= \sqrt{3}\hat{P}^\mu q^\rho \\ \tau_5^{\mu\rho} &= -\gamma_T^\mu q^\rho \not{q} & \tau_6^{\mu\rho} &= \gamma_T^\mu q^\rho \not{q} - \delta^{\mu\rho} - 3q^\mu q^\rho \\ \tau_7^{\mu\rho} &= -\gamma_T^\mu q^\rho & \tau_8^{\mu\rho} &= \frac{1}{\sqrt{5}}(\delta^{\mu\rho} \not{q} + \gamma_T^\mu q^\rho + 5q^\mu q^\rho \not{q}), \end{aligned} \quad (18)$$

with $q^\mu := iT_p^{\mu\nu} \hat{p}^\nu / \sqrt{1 - (\hat{p} \cdot \hat{P})^2}$ and $T_p^{\mu\nu} = \delta^{\mu\nu} - \hat{P}^\mu \hat{P}^\nu$. The corresponding orthogonality relation reads

$$\text{Tr}\{\bar{\sigma}_k^{\mu\rho}(p, -P)\sigma_l^{\mu\rho}(p, P)\} = 4\delta_{kl}(-1)^{k+1}, \quad (19)$$

where $\sigma_k^{\mu\nu}(p, P) = \tau_k^{\mu\rho}(p, P)\Lambda_+(P)\Lambda^{\rho\nu}(P)$ and the conjugated amplitude is defined as (the superscript T denotes the Dirac transpose):

$$\bar{\sigma}_k^{\mu\rho}(p, -P) = C\sigma_k^{\mu\rho}(-p, -P)^T C^T. \quad (20)$$

The basis (18) corresponds to a partial-wave decomposition in terms of eigenfunctions of the quark-diquark total spin and orbital angular momentum in the Δ rest frame [35,80]. Since there is only one spherically symmetric s -wave component (f_1^Δ), the Δ 's deviation from sphericity = 1 can be explained by an admixture of p ($f_{2,4,7}^\Delta$), d ($f_{3,5,6}^\Delta$), and f (f_8^Δ) waves, which contribute a significant amount of orbital angular momentum to its amplitude.

The isospin matrices of the Δ quark-diquark amplitude are constructed via removal of the diquark contributions from the full Δ flavor-amplitude that is obtained by the Clebsch-Gordan prescription

$$\begin{aligned} \Delta^{++} &= (u, 0, 0), & \Delta^+ &= \left(\frac{1}{\sqrt{3}}d, \frac{2}{\sqrt{3}}u, 0\right), \\ \Delta^0 &= \left(0, \frac{2}{\sqrt{3}}d, \frac{1}{\sqrt{3}}u\right), & \Delta^- &= (0, 0, d). \end{aligned} \quad (21)$$

The first, second, and third entries in each vector correspond to the diquark's three symmetric isospin-1 states. Computing the flavor traces of the quark-diquark BSE leads to a global factor 1 in the equal-quark-mass case. The color matrix $\delta_{AB}/\sqrt{3}$ attached to each quark-diquark amplitude entails a global color factor -1 in the BSE.

The standard procedure to solve the quark-diquark BSE (1) is detailed in Ref. [81] for the analogous case of a nucleon. It involves a Chebyshev expansion in the angular variable $\hat{p} \cdot \hat{P}$ and leads to coupled one-dimensional eigenvalue equations for the Chebyshev moments of the dressing functions f_k^Δ . They match the BSE solution at $P^2 = -M_\Delta^2$, i.e., for an eigenvalue $\lambda(P^2) = 1$.

VI. RESULTS AND DISCUSSION

We solved Eqs. (1), (4), and (10) numerically using all ingredients as specified above and show the results in Fig. 2. The left panel depicts our calculated quark core values for m_ρ, M_N , and M_Δ , each together with a selection of lattice results and their chiral extrapolations (if available). As abscissa values we use the corresponding m_π^2 , in our calculation obtained from the pseudoscalar $q\bar{q}$ BSE. The solid curve for m_ρ is the input defined in Eqs. (6)–(9), which completely fixes the parameters in our interaction. The bands represent our results for M_N and M_Δ and explicitly show the sensitivity on the width parameter ω as described in Sec. III. At the physical pion mass we obtain $M_\Delta = 1.73(5)$ GeV; the corresponding mass for the nucleon $M_N = 1.26(2)$ GeV was reported in Ref. [50]. At larger quark masses the deviation from the lattice data diminishes. This result is in accordance with the assumption of Eq. (7), namely, that beyond-RL corrections to hadronic observables (except cases like highly excited states or low- x physics) become negligible in the limit of heavy quarks.

It is instructive to compare our results to core masses estimated via, e.g., the cloudy-bag model [82], NJL model [83], nucleon-pion Dyson-Schwinger studies [36,84], and a chiral analysis of lattice results [71,85]. These predictions lie in the range ~ 200 – 400 MeV for the correction to the nucleon mass induced by pseudoscalar meson loops; pseudoscalar-meson contributions to M_Δ are expected to be of a similar size or even smaller than those to the nucleon. In this respect, our value for M_N in the chiral region is roughly consistent with a pseudoscalar-meson

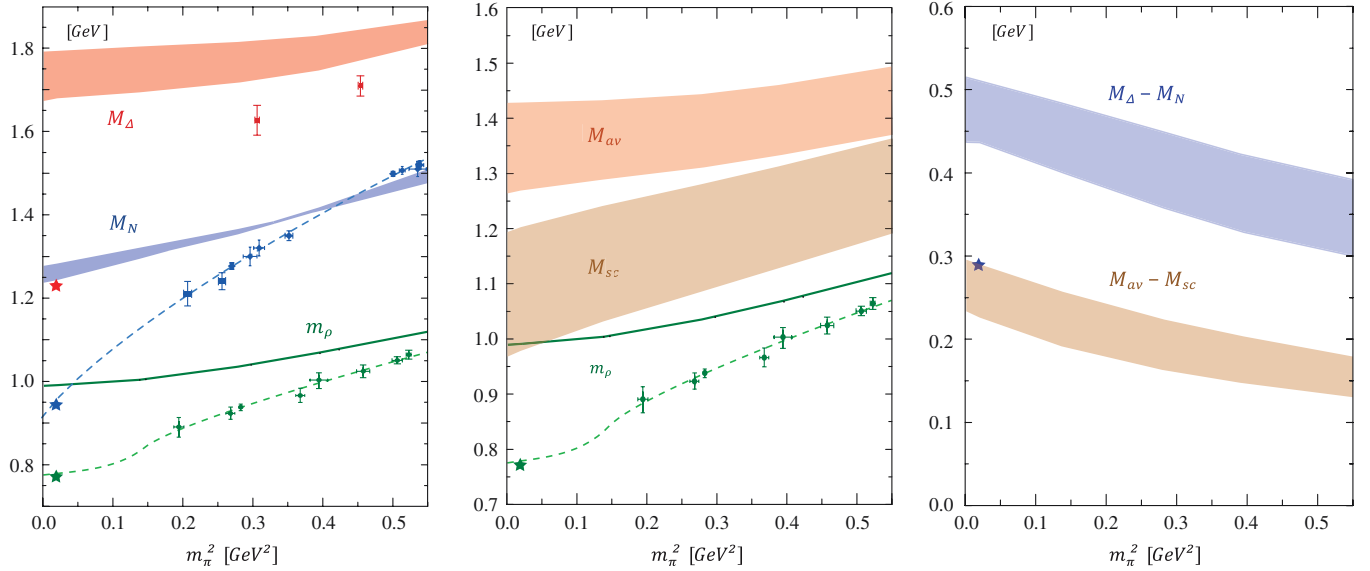


FIG. 2 (color online). Evolution of N and Δ masses (left panel), evolution of m_ρ , scalar and axial-vector diquark masses (center panel), and the mass splittings $M_\Delta - M_N$ and $M_{av} - M_{sc}$ (right panel) vs pion mass squared. The bands denote the sensitivity to a variation of $\omega = \bar{\omega}(\hat{m}) \pm 0.07$. The solid curve for m_ρ represents the chosen input of Eq. (7); nucleon and diquark masses were calculated in Ref. [50]. We compare to a selection of lattice data and their chiral extrapolations (dashed lines) for m_ρ [86,87], M_N [88–90], and M_Δ [28]. Stars denote the experimental values.

dressing providing the dominant correction to the quark-diquark core, whereas the result for M_Δ is not, which indicates a larger correction. As a concrete example, consider the reduction of N and Δ masses by intermediate $N\pi$ and $\Delta\pi$ states, estimated in the chiral limit by the expressions (e.g., [71])

$$M_N = M_N^{\text{core}} + \Sigma_N, \quad M_\Delta = M_\Delta^{\text{core}} + \Sigma_\Delta, \quad (22)$$

$$\Sigma_N = -\lambda \int_0^\infty dx x^2 u(x)^2 \left(1 + \frac{32}{25} \frac{x}{x+1}\right),$$

$$\Sigma_\Delta = -\lambda \int_0^\infty dx x^2 u(x)^2 \left(1 + \frac{8}{25} \frac{x}{x-1}\right). \quad (23)$$

$k = x\Delta M$ is the pion momentum, $\Delta M = 0.29$ GeV the physical $N - \Delta$ mass splitting, and $\lambda = 3g_A^2(\Delta M)^3/(8\pi^2 f_\pi^2)$ with $g_A = 1.26$ and $f_\pi = 131$ MeV. The $NN\pi$, $\Delta\Delta\pi$, and $N\Delta\pi$ vertex dressings $u(x)$ were assumed identical. Choosing a dipole form factor $u(x) = \{1 + (k/\Lambda)^2\}^{-2}$ with a regulator $\Lambda = 0.8$ GeV yields $\Sigma_N = -0.32$ GeV and $\Sigma_\Delta = -0.28$ GeV. Together with the experimental numbers for M_N and M_Δ , these values provide the simple estimates $M_N^{\text{core}} \sim 1.25$ GeV and $M_\Delta^{\text{core}} \sim 1.5$ GeV.

In this context one has to keep in mind that the identification of the baryonic quark core of Eqs. (22) and (23) with the quark-diquark “core” is more complicated than in the meson case. More precisely, Eq. (7) assumes that corrections to m_ρ are partly induced by a pseudoscalar-meson cloud, and to a lesser extent related to nonresonant corrections to RL truncation. In the baryon one has addi-

tional lines of improvement: inserting further diquark channels, abandoning the diquark pole *ansatz* in favor of the full qq scattering kernel, and including irreducible three-body interactions could affect N and Δ properties differently, but still describe a quark core in the sense of Eq. (7).

We also note that the solution for M_Δ exhibits a sizeable ω dependence, a feature not present in π , ρ , and nucleon observables [49,50]. The scalar and axial-vector diquark masses exhibit particularly large sensitivities to ω (see center panel of Fig. 2) which apparently cancel upon constituting the nucleon mass. In the M_Δ case, the same consideration could suggest that taking into account only an axial-vector diquark may not be sufficient for describing the Δ , and that a possible further isospin-1 (tensor) diquark component with a mass large enough to be irrelevant for the nucleon could diminish the Δ core mass.

A further remark concerns the coupling strength c of Eq. (6) at or close to the chiral limit. Specifically, a comparison of the core value induced by Eq. (8) and the input used in Ref. [64] provides valuable insight. The aim of [64] was to reproduce π and ρ properties; in addition, in our present setup the model also yields N and Δ masses that are close to the experimental values (set A in Table I; the core values are summarized as set B). While this result is perhaps incidental since a RL description is not likely to provide the complete underlying physical picture, it is still remarkable that the set A-to-set B ratios of all observables considered here are essentially identical. This is shown in Table I at the u/d -quark mass and can be understood as follows. Introducing $\eta = c^{1/3}/\omega$ and the scale

TABLE I. Comparison of several quantities calculated from the quark DSE ($\langle\bar{q}q\rangle$), meson BSE (m_ρ, f_π, f_ρ), diquark BSE (M_{sc}, M_{av}), and quark-diquark BSE (M_N, M_Δ). For the explicit calculation of the meson, diquark, and nucleon observables we refer the reader to Refs. [42,49,50]. The results correspond to a current mass $\hat{m} = 6.1$ MeV which is related to the physical pion mass $m_\pi = 138$ MeV. The first row quotes experimental or phenomenological values. Sets A and B are distinguished by different values for the input parameters $c = (\Lambda_{IR}/\Lambda_0)^3$ and ω ; set B represents the inflated quark core according to Eqs. (8) and (9). Only the central ω values are shown (note that these need not be identical to the ω -band averages quoted in the text.) The units of the first three rows (except c and ω) are GeV. The last row plots the ratios of sets A and B.

	c	ω	Λ_{IR}	$\langle\bar{q}q\rangle_{1\text{ GeV}}^{1/3}$	m_ρ	f_π	f_ρ	M_{sc}	M_{av}	M_N	M_Δ
Phen./Exp.				0.236	0.77	0.131	0.216			0.94	1.23
Set A	0.37	0.40	0.72	0.235	0.73	0.131	0.208	0.81	1.00	0.94	1.28
Set B (core)	0.93	0.54	0.98	0.319	0.99	0.176	0.280	1.08	1.35	1.25	1.72
(Set A)/(Set B)			0.73	0.74	0.74	0.74	0.74	0.75	0.74	0.75	0.74

$\Lambda_{IR} = c^{1/3}\Lambda_0$, the infrared contribution to the effective coupling $\alpha(k^2)$ can be rewritten as

$$\alpha_{IR}(k^2) = \pi\eta^7 x^2 e^{-\eta^2 x}, \quad x = k^2/\Lambda_{IR}^2. \quad (24)$$

An insensitivity of certain observables under variation of the width ω at a certain coupling strength c translates into an invariance with respect to η . Furthermore, the combined increase of c and ω according to Eqs. (8) and (9) to arrive at the quark core changes η by $\lesssim 5\%$; hence it can be viewed as a rescaling *only* of Λ_{IR} . This quantity defines the only relevant scale in the chiral-limit RL quark DSE and meson/diquark BSEs where no interference with a finite current-quark mass is possible. If the renormalization point is chosen large enough, a rescaling of Λ_{IR} equally affects the chiral-limit values of mass-dimensional observables; and it induces scale invariance of dimensionless quantities. The effect on dynamically generated observables is still approximately valid at the small u/d current-quark mass. It makes clear that any RL model which is able to reproduce experimental results for a given set of observables will, upon entering its core version, necessarily overestimate those results by the *same* percentage.

Finally, in the right panel of Fig. 2 we plot the $N - \Delta$ mass splitting. According to Eqs. (22) and (23), the pseudoscalar meson contribution to the experimental value $M_\Delta - M_N = 0.29$ GeV is small and positive: $\Sigma_\Delta - \Sigma_N = 0.04$ GeV (cf. Ref. [71]). This is apparently not the case in our calculation, where at the u/d mass $(M_\Delta - M_N)^{\text{core}} = 0.48(4)$ GeV and therefore predicts a negative correction to the full splitting. We also compare $M_\Delta - M_N$ with the diquark mass splitting $M_{av} - M_{sc}$. Both decrease with increasing current-quark mass; nevertheless there is no direct relationship between the two quantities, since the axial-vector diquark contribution to the mass of the nucleon does not vanish.

VII. CONCLUSIONS

We calculated the mass of the Δ baryon in a covariant Faddeev approach, where the Δ quark core is pictured as a quark-diquark bound state. The kernel of the quark-diquark Bethe-Salpeter equation is fully specified by the solutions of the Dyson-Schwinger equation for the dressed-quark propagator and the diquark Bethe-Salpeter equation which are both solved in rainbow-ladder truncation. The physical input is specified by an effective interaction $\alpha(k^2)$ that is constructed to describe an overestimated quark core for the ρ meson such that the (overall attractive) corrections to rainbow-ladder truncation as, e.g. a pseudoscalar meson-cloud dressing, are anticipated in the chiral regime. The calculation does not rely on any baryonic input, i.e., all parameters in the interaction are fixed via $m_\rho(m_\pi)$. Our result for the evolution of M_Δ with m_π^2 is justified *a posteriori*: at the u/d current-quark mass, $M_\Delta = 1.73(5)$ GeV, whereas at larger quark masses our curve approaches lattice data for M_Δ . A relatively pronounced interaction dependence as well as a significant overestimation of the expected M_Δ core mass indicate that further diquark degrees of freedom besides the axial-vector diquark could play a role in the construction of the Δ quark core. The approach presented herein can be systematically improved by eliminating the diquark *ansatz* in favor of a sophisticated three-quark interaction kernel and by developing suitable methods to directly incorporate meson-cloud effects.

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- [1] H. Schmieden, *Eur. Phys. J. A* **28S1**, 91 (2006).
- [2] N.F. Sparveris *et al.* (OOPS Collaboration), *Phys. Rev. Lett.* **94**, 022003 (2005).
- [3] R. Beck *et al.*, *Phys. Rev. C* **61**, 035204 (2000).
- [4] G. Blanpied *et al.*, *Phys. Rev. C* **64**, 025203 (2001).
- [5] D. Elsner *et al.*, *Eur. Phys. J. A* **27**, 91 (2006).
- [6] T. Pospischil *et al.*, *Phys. Rev. Lett.* **86**, 2959 (2001).
- [7] S. Stave *et al.*, *Eur. Phys. J. A* **30**, 471 (2006).
- [8] N. Isgur and G. Karl, *Phys. Rev. D* **20**, 1191 (1979).
- [9] A.J. Buchmann, E. Hernandez, and A. Faessler, *Phys. Rev. C* **55**, 448 (1997).
- [10] A. Faessler *et al.*, *Phys. Rev. D* **74**, 074010 (2006).
- [11] T. Melde, W. Plessas, and R.F. Wagenbrunn, *Phys. Rev. C* **72**, 015207 (2005); **74**, 069901(E) (2006).
- [12] T. Melde and W. Plessas, *Eur. Phys. J. A* **35**, 329 (2008).
- [13] T. Melde, W. Plessas, and B. Sengl, *Phys. Rev. D* **77**, 114002 (2008).
- [14] T. Melde, L. Canton, and W. Plessas, *Phys. Rev. Lett.* **102**, 132002 (2009).
- [15] B. Metsch, U. Loring, D. Merten, and H. Petry, *Eur. Phys. J. A* **18**, 189 (2003).
- [16] S. Migura, D. Merten, B. Metsch, and H.-R. Petry, *Eur. Phys. J. A* **28**, 55 (2006).
- [17] B. Metsch, *Eur. Phys. J. A* **35**, 275 (2008).
- [18] A. Wirzba and W. Weise, *Phys. Lett. B* **188**, 6 (1987).
- [19] A. Abada, H. Weigel, and H. Reinhardt, *Phys. Lett. B* **366**, 26 (1996).
- [20] H. Walliser and G. Holzwarth, *Z. Phys. A* **357**, 317 (1997).
- [21] G. Kaelbermann and J.M. Eisenberg, *Phys. Rev. D* **28**, 71 (1983).
- [22] A.W. Thomas, *Adv. Nucl. Phys.* **13**, 1 (1984).
- [23] T.R. Hemmert, B.R. Holstein, and J. Kambor, *Phys. Lett. B* **395**, 89 (1997).
- [24] T.R. Hemmert, B.R. Holstein, and J. Kambor, *J. Phys. G* **24**, 1831 (1998).
- [25] V. Pascalutsa and M. Vanderhaeghen, *Phys. Rev. Lett.* **95**, 232001 (2005).
- [26] C. Hacker, N. Wies, J. Gegelia, and S. Scherer, *Phys. Rev. C* **72**, 055203 (2005).
- [27] V. Pascalutsa and M. Vanderhaeghen, *Phys. Lett. B* **636**, 31 (2006).
- [28] J.M. Zanotti *et al.* (CSSM Lattice Collaboration), *Phys. Rev. D* **68**, 054506 (2003).
- [29] M. Gockeler *et al.* (QCDSF Collaboration), *Proc. Sci., LAT2007* (2007) 129.
- [30] A. Walker-Loud *et al.*, *Phys. Rev. D* **79**, 054502 (2009).
- [31] C. Gattringer *et al.*, *Proc. Sci., Lattice2008* (2009) 093.
- [32] C. Alexandrou *et al.*, *Phys. Rev. D* **79**, 014507 (2009).
- [33] C.S. Fischer, *J. Phys. G* **32**, R253 (2006).
- [34] C.D. Roberts, M.S. Bhagwat, A. Holl, and S.V. Wright, *Eur. Phys. J. ST* **140**, 53 (2007).
- [35] M. Oettel, G. Hellstern, R. Alkofer, and H. Reinhardt, *Phys. Rev. C* **58**, 2459 (1998).
- [36] M.B. Hecht *et al.*, *Phys. Rev. C* **65**, 055204 (2002).
- [37] M. Oettel, M. Pichowsky, and L. von Smekal, *Eur. Phys. J. A* **8**, 251 (2000).
- [38] M. Oettel, R. Alkofer, and L. von Smekal, *Eur. Phys. J. A* **8**, 553 (2000).
- [39] R. Alkofer, A. Holl, M. Kloker, A. Krassnigg, and C.D. Roberts, *Few-Body Syst. Suppl.* **37**, 1 (2005).
- [40] A. Höll, R. Alkofer, M. Kloker, A. Krassnigg, C.D. Roberts, and S.V. Wright, *Nucl. Phys. A* **755**, 298 (2005).
- [41] I.C. Cloet *et al.*, *Few-Body Syst. Suppl.* **42**, 91 (2008).
- [42] G. Eichmann, A. Krassnigg, M. Schwinzerl, and R. Alkofer, *Ann. Phys. (N.Y.)* **323**, 2505 (2008).
- [43] G. Eichmann, A. Krassnigg, M. Schwinzerl, and R. Alkofer, *Prog. Part. Nucl. Phys.* **61**, 84 (2008).
- [44] R. Alkofer, C.S. Fischer, and F.J. Llanes-Estrada, *Mod. Phys. Lett. A* **23**, 1105 (2008).
- [45] R. Alkofer, C.S. Fischer, F.J. Llanes-Estrada, and K. Schwenzer, *Ann. Phys. (N.Y.)* **324**, 106 (2009).
- [46] M.A. Pichowsky, S. Walawalkar, and S. Capstick, *Phys. Rev. D* **60**, 054030 (1999).
- [47] C.S. Fischer, D. Nickel, and J. Wambach, *Phys. Rev. D* **76**, 094009 (2007).
- [48] C.S. Fischer and R. Williams, *Phys. Rev. D* **78**, 074006 (2008).
- [49] G. Eichmann, R. Alkofer, I.C. Cloet, A. Krassnigg, and C.D. Roberts, *Phys. Rev. C* **77**, 042202(R) (2008).
- [50] G. Eichmann, I.C. Cloet, R. Alkofer, A. Krassnigg, and C.D. Roberts, *Phys. Rev. C* **79**, 012202(R) (2009).
- [51] R.T. Cahill, C.D. Roberts, and J. Praschifka, *Aust. J. Phys.* **42**, 129 (1989).
- [52] R.T. Cahill, C.D. Roberts, and J. Praschifka, *Phys. Rev. D* **36**, 2804 (1987).
- [53] P. Maris, *Few-Body Syst. Suppl.* **32**, 41 (2002).
- [54] P. Maris, C.D. Roberts, and P.C. Tandy, *Phys. Lett. B* **420**, 267 (1998).
- [55] P. Maris and C.D. Roberts, *Phys. Rev. C* **56**, 3369 (1997).
- [56] A. Holl, A. Krassnigg, and C.D. Roberts, *Phys. Rev. C* **70**, 042203 (2004).
- [57] H.J. Munczek, *Phys. Rev. D* **52**, 4736 (1995).
- [58] A. Bender, C.D. Roberts, and L. von Smekal, *Phys. Lett. B* **380**, 7 (1996).
- [59] R. Alkofer, P. Watson, and H. Weigel, *Phys. Rev. D* **65**, 094026 (2002).
- [60] P. Maris, *AIP Conf. Proc.* **892**, 65 (2007).
- [61] A. Holl, A. Krassnigg, P. Maris, C.D. Roberts, and S.V. Wright, *Phys. Rev. C* **71**, 065204 (2005).
- [62] P. Maris and P.C. Tandy, *Nucl. Phys. B, Proc. Suppl.* **161**, 136 (2006).
- [63] M.S. Bhagwat and P. Maris, *Phys. Rev. C* **77**, 025203 (2008).
- [64] P. Maris and P.C. Tandy, *Phys. Rev. C* **60**, 055214 (1999).
- [65] S. Theberge, A.W. Thomas, and G.A. Miller, *Phys. Rev. D* **22**, 2838 (1980).
- [66] D.-H. Lu, A.W. Thomas, and A.G. Williams, *Phys. Rev. C* **57**, 2628 (1998).
- [67] J. Gasser and H. Leutwyler, *Ann. Phys. (N.Y.)* **158**, 142 (1984).
- [68] V. Bernard, N. Kaiser, and U.-G. Meissner, *Int. J. Mod. Phys. E* **4**, 193 (1995).
- [69] M. Oettel, M. Buballa, and J. Wambach, *Phys. At. Nucl.* **64**, 698 (2001).
- [70] I.C. Cloet and C.D. Roberts, *Proc. Sci., LC2008* (2008) 047.
- [71] R.D. Young, D.B. Leinweber, A.W. Thomas, and S.V. Wright, *Phys. Rev. D* **66**, 094507 (2002).
- [72] M.S. Bhagwat, A. Holl, A. Krassnigg, C.D. Roberts, and P.C. Tandy, *Phys. Rev. C* **70**, 035205 (2004).

- [73] H. H. Matevosyan, A. W. Thomas, and P. C. Tandy, *Phys. Rev. C* **75**, 045201 (2007).
- [74] R. Alkofer and L. von Smekal, *Phys. Rep.* **353**, 281 (2001).
- [75] C. D. Roberts and S. M. Schmidt, *Prog. Part. Nucl. Phys.* **45**, S1 (2000).
- [76] G. Hellstern, R. Alkofer, and H. Reinhardt, *Nucl. Phys. A* **625**, 697 (1997).
- [77] A. Bender, W. Detmold, C. D. Roberts, and A. W. Thomas, *Phys. Rev. C* **65**, 065203 (2002).
- [78] R. Alkofer, M. Kloker, A. Krassnigg, and R. F. Wagenbrunn, *Phys. Rev. Lett.* **96**, 022001 (2006).
- [79] M. Oettel and R. Alkofer, *Eur. Phys. J. A* **16**, 95 (2003).
- [80] M. Oettel, Ph.D. thesis, University of Tübingen, arXiv: nucl-th/0012067.
- [81] M. Oettel, L. von Smekal, and R. Alkofer, *Comput. Phys. Commun.* **144**, 63 (2002).
- [82] B. C. Pearce and I. R. Afnan, *Phys. Rev. C* **34**, 991 (1986).
- [83] N. Ishii, *Phys. Lett. B* **431**, 1 (1998).
- [84] M. Oettel and A. W. Thomas, *Phys. Rev. C* **66**, 065207 (2002).
- [85] R. D. Young, D. B. Leinweber, and A. W. Thomas, *Prog. Part. Nucl. Phys.* **50**, 399 (2003).
- [86] A. Ali Khan *et al.* (CP-PACS Collaboration), *Phys. Rev. D* **65**, 054505 (2002).
- [87] C. R. Allton, W. Armour, D. B. Leinweber, A. W. Thomas, and R. D. Young, *Phys. Lett. B* **628**, 125 (2005).
- [88] A. Ali Khan *et al.* (QCDSF-UKQCD Collaboration), *Nucl. Phys.* **B689**, 175 (2004).
- [89] R. Frigori *et al.*, *Proc. Sci.*, LAT2007 (2007) 114.
- [90] D. B. Leinweber, A. W. Thomas, and R. D. Young, *Phys. Rev. Lett.* **92**, 242002 (2004).