$B^- \to \pi^- \pi^0/\rho^- \rho^0$ to next-to-next-to-leading order in QCD factorization

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The approximate tree decays $B^- \to \pi^- \pi^0/\rho^- \rho^0$ may serve as benchmark channels for testing the various theoretical descriptions of the strong interaction dynamics in hadronic B meson decays. The ratios of hadronic and differential semileptonic $B \to \pi \ell \nu/\rho \ell \nu$ decay rates at maximum recoil provide particularly clean probes of the QCD dynamics. We confront the recent next-to-next-to-leading order calculation in the QCD factorization framework with experimental data and find support for the factorization assumption. A detailed analysis of all tree-dominated $B \to \pi\pi/\pi\rho/\rho\rho$ decay modes seems to favor somewhat enhanced color-suppressed amplitudes, which may be accommodated in QCD factorization by a small value of the first inverse moment of the B meson light-cone distribution amplitude, $\lambda_B \simeq 250$ MeV. Precise measurements of the semileptonic $B \to \rho \ell \nu$ spectrum could help to clarify this point.

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I. INTRODUCTION

A wealth of observables at current and future B physics experiments is related to exclusive hadronic decay modes. B decays into a pair of light (charmless) mesons are of particular phenomenological interest as they are mediated by rare flavor-changing $b \to q$ $(q = u, d, s)$ quark transitions, and the interference of several weak decay amplitudes may induce sizeable CP-violating effects.

The complicated strong interaction dynamics in hadronic decays pose a serious challenge for accurate theoretical predictions. In recent years systematic methods have been developed, which are based on the factorization of short- and long-distance effects in the heavy quark limit $m_b \gg \Lambda_{\rm QCD}$. The theoretical concepts are known as QCD factorization (QCDF) [1], soft-collinear effective theory [2] and the perturbative QCD approach [3].

In this article we consider the decays $B^- \to \pi^- \pi^0/\rho^- \rho^0$ within the QCDF framework, which is based on the statement that the hadronic matrix elements of the operators in the effective weak Hamiltonian simplify in the heavy quark limit according to [1]

$$\langle M_1 M_2|Q_i|\bar{B}\rangle \simeq F^{BM_1}(0) f_{M_2} \int du T_i^I(u) \phi_{M_2}(u) + \hat{f}_B f_{M_1} f_{M_2} \int d\omega dv du T_i^{II}(\omega, v, u) \times \phi_B(\omega) \phi_{M_1}(v) \phi_{M_2}(u). \quad (1)$$

The factorization formula implies, on the one hand, that the theoretical prediction requires nontrivial hadronic input parameters, such as decay constants f, moments of light-cone distribution amplitudes ϕ, and form factors F, which encode all long-distance effects in the limit $m_b \to \infty$. The power of the decomposition in (1) lies, on the other hand, in the fact that it provides the path to a systematic implementation of radiative corrections. The short-distance hard-scattering kernels $T_i^{I,II}$ are perturbatively calculable and currently being worked out to next-to-next-to-leading order (NNLO) [4–6].

The NNLO calculation is to date incomplete, but a subset of hard-scattering kernels, which specify the so-called topological tree amplitudes, has recently been determined to NNLO [4,6]. This allows us to present the first complete NNLO prediction within the QCDF framework for the decays $B^- \to \pi^- \pi^0/\rho^- \rho^0$, which are pure tree decays in the excellent approximation that small electroweak penguin amplitudes are neglected [7].

As the considered decays are likely to be dominated by their standard model contribution, they may serve as benchmark channels for testing the various theoretical descriptions of the strong interaction dynamics in hadronic B decays. By normalizing the hadronic decay rates to their semileptonic counterparts $B \to \pi \ell \nu/\rho \ell \nu$ at maximum recoil, most of the theoretical uncertainties from hadronic input parameters and $|V_{ub}|$ drop out, and one obtains precision observables for testing the QCD dynamics of the topological tree amplitudes. We confront the NNLO prediction in QCDF with experimental data and find support for the factorization assumption. We also take a look at the other tree-dominated $B \to \pi\pi/\pi\rho/\rho\rho$ decay modes and conclude that the color-suppressed tree amplitudes seem in general to be somewhat enhanced, which may hint at a smaller value of the first inverse moment of the B meson light-cone distribution amplitude, $\lambda_B \simeq 250$ MeV. We conclude our analysis with a comment on the so-called $B \to \pi\pi$ puzzle.

II. TREE AMPLITUDES

The decay amplitudes for hadronic B meson decays are conveniently parametrized by a set of topological amplitudes, which contain short-distance QCD and some electroweak effects. In the notation of [8] they read

$$\mathcal{A}(B^- \to \pi^- \pi^0) = \left[\lambda_u \left(\alpha_1 + \alpha_2 + \frac{3}{2}\alpha^u_{3,\mathrm{EW}} + \frac{3}{2}\alpha^u_{4,\mathrm{EW}} \right) + \lambda_c \left(\frac{3}{2}\alpha^c_{3,\mathrm{EW}} + \frac{3}{2}\alpha^c_{4,\mathrm{EW}} \right) \right] \frac{A_{\pi\pi}}{\sqrt{2}}, \quad (2)$$

with $\lambda_p = V_{pb}V^*_{pd}$ and $A_{\pi\pi} = iG_F/\sqrt{2}m_B^2 f_\pi F_+^{B\pi}(0)$ and similarly for $B^- \to \rho^-\rho^0$ with $f_\pi \to f_\rho$, $F_+^{B\pi} \to A_0^{B\rho}$ and $\alpha_i(\pi\pi) \to \alpha_i(\rho\rho)$. Whereas the electroweak penguin amplitudes $\alpha^p_{3/4,\mathrm{EW}}$ are currently known for $B \to \pi\pi/\pi\rho/\rho\rho$ to next-to-leading order (NLO) [1,8,9], the tree amplitudes $\alpha_{1,2}$ have recently been determined for $B \to \pi\pi$ to NNLO [4,6]. From the projection properties of the leading-twist π and ρ wave functions, we find that the respective expressions for $B \to \pi\rho/\rho_L\rho_L$ are identical (L refers to the longitudinal polarization). We, in particular, do not consider decays into transverse ρ mesons, which cannot be described model independently as they do not factorize.

We evaluate the tree amplitudes with three-loop running coupling constant and next-to-next-to-leading logarithmic Wilson coefficients [10] of the operators in the weak effective Hamiltonian (we use the operator basis from [11]). The spectator scattering part (T_i^{II}) receives contributions from two perturbative scales, $\mu_h \sim m_b$ and $\mu_{hc} \sim (\Lambda_{\mathrm{QCD}} m_b)^{1/2}$, which gives rise to logarithms $\ln m_b/\Lambda_{\mathrm{QCD}}$ that we resum via renormalization group equations in soft-collinear effective theory to leading logarithmic approximation. Other scale dependent quantities are treated as described in [6], except for the parameters of the B meson wave function, which we evolve with fixed order relations as their evolution from their input scale does not induce parametrically large logarithms.

We also include certain power corrections to the tree amplitudes that are related to subleading-twist wave functions of the light mesons. As these chirally enhanced contributions do not factorize, we use the model proposed in [1] to estimate their size.

Our theoretical input parameters are listed in Table I. We deduced our default values for the hadronic parameters from recent lattice and sum rule calculations (where available). In general the parameters related to the pion (f_π, a_2^π [12,13], $F_+^{B\pi}$ [14]) are better determined than the ones related to the rho meson (f_ρ [15], a_2^ρ [13,16], $A_0^{B\rho}$ [17]). While there exists a large number of calculations for the B meson decay constant f_B [18], less is known about the moments of the B meson wave function ($\lambda_B, \sigma_{1,2}$) [19]. Our value for λ_B is based on a QCD sum rule calculation and on estimates from the operator product expansion, accounting for recent claims that higher dimensional operators lower the value of λ_B (last paper of [19]).

TABLE I. List of input parameters (in units of GeV or dimensionless). Scale dependent quantities refer to $\mu = 1$ GeV.

Parameter	Value	Parameter	Value
f_π	0.131	$\Lambda^{(n_f=5)}_{\overline{\mathrm{MS}}}$	0.204
f_ρ	0.216 ± 0.005	$\Lambda^{(n_f=4)}_{\overline{\mathrm{MS}}}$	0.283
f_B	0.200 ± 0.020	$m_{b,\mathrm{pole}}$	4.8
$F_+^{B\pi}(0)$	0.26 ± 0.04	$m_{c,\mathrm{pole}}$	1.4 ± 0.2
$A_0^{B\rho}(0)$	0.30 ± 0.05	$\lvert V_{cd}\rvert$	0.230 ± 0.011
a_2^π	0.25 ± 0.15	$10^3\lvert V_{cb}\rvert$	41.2 ± 1.1
a_2^ρ	0.15 ± 0.15	$10^3\lvert V_{ub}\rvert$	3.95 ± 0.35
λ_B	0.400 ± 0.150	γ	$(70 \pm 20)^\circ$
σ_1	1.5 ± 1.0	μ_h	$4.8^{+4.8}_{-2.4}$
σ_2	3 ± 2	μ_{hc}	$1.5^{+1.5}_{-0.7}$

We estimate the size of higher order perturbative corrections by varying the factorization scales μ_h and μ_{hc} independently within the ranges specified in Table I. On the other hand, we evaluate the nonfactorizable power corrections at a fixed scale $\mu_0 = 1.5$ GeV. The latter introduces certain model parameters (ρ_H, ϕ_H) and some additional hadronic parameters. We use $(\bar{m}_u + \bar{m}_d)(2\ \mathrm{GeV}) = 8$ MeV, $\bar{m}_b(\bar{m}_b) = 4.2$ GeV, and $f_\rho^\perp(1\mathrm{GeV}) = 165$ MeV.

This brings us to our NNLO prediction of the color-allowed (α_1) and color-suppressed (α_2) tree amplitudes. In the $B \to \pi\pi/\rho_L\rho_L$ channels we obtain

$$\begin{aligned}
\alpha_1(\pi\pi) &= 1.013^{+0.017+0.008+0.014}_{-0.031-0.011-0.014} + (+0.027^{+0.006+0.020+0.014}_{-0.010-0.013-0.014})i = 1.013^{+0.023}_{-0.036} + (+0.027^{+0.025}_{-0.022})i,\\
\alpha_2(\pi\pi) &= 0.195^{+0.119+0.025+0.055}_{-0.066-0.025-0.055} + (-0.101^{+0.017+0.021+0.055}_{-0.010-0.029-0.055})i = 0.195^{+0.134}_{-0.089} + (-0.101^{+0.061}_{-0.063})i,\\
\alpha_1(\rho_L\rho_L) &= 1.017^{+0.017+0.010+0.014}_{-0.029-0.011-0.014} + (+0.025^{+0.007+0.019+0.014}_{-0.013-0.013-0.014})i = 1.017^{+0.024}_{-0.034} + (+0.025^{+0.025}_{-0.023})i,\\
\alpha_2(\rho_L\rho_L) &= 0.177^{+0.110+0.025+0.055}_{-0.063-0.029-0.055} + (-0.097^{+0.021+0.021+0.055}_{-0.012-0.029-0.055})i = 0.177^{+0.126}_{-0.089} + (-0.097^{+0.062}_{-0.063})i,
\end{aligned} \quad (3)$$

where the uncertainties in the intermediate results stem from the variation of hadronic input parameters, higher order perturbative corrections, and the considered model for power corrections, respectively, which have been added in quadrature for our final error estimate.

We see, on the one hand, that the color-allowed tree amplitudes α_1 can be computed precisely in the factorization framework. The color-suppressed amplitudes α_2 suffer, on the other hand, from substantial theoretical uncertainties. The problem is related to certain cancellations between various perturbative contributions, which make the real parts particularly sensitive to the spectator scattering mechanism which is proportional to the hadronic ratio $f_{M_1}\hat{f}_B/\lambda_B F^{BM_1}(0)$. Our poor knowledge of the B

meson parameter λ_B, in particular, translates into the uncertainties $^{+0.107}_{-0.049}$ and $^{+0.096}_{-0.043}$ for the real parts of $\alpha_2(\pi\pi)$ and $\alpha_2(\rho_L\rho_L)$, respectively.

III. BRANCHING RATIOS

The branching ratios of $B^- \to \pi^-\pi^0/\rho^-\rho^0$ depend in addition on electroweak penguin amplitudes, cf. (2). These amplitudes have not yet been determined to NNLO [20], but their numerical values are small ($|\alpha^p_{3/4,\mathrm{EW}}| \lesssim 0.01$). As they are not Cabibbo-Kobayashi-Maskawa (CKM) enhanced in tree-dominated decays, it is consistent to treat these amplitudes in the NLO approximation. The explicit NLO results can be found in [1,8,9] (they are formulated in a different operator basis of the effective Hamiltonian). The *CP*-averaged branching ratios become

$$10^6 \mathrm{Br}(B^- \to \pi^-\pi^0) = 6.22^{+1.14+2.03+0.16+0.43}_{-1.05-1.65-0.18-0.42} = 6.22^{+2.37}_{-2.01},$$
$$10^6 \mathrm{Br}(B^- \to \rho_L^-\rho_L^0) = 21.0^{+3.9+7.4+0.5+1.5}_{-3.5-6.1-0.7-1.4} = 21.0^{+8.5}_{-7.3}, \quad (4)$$

where the uncertainties in the intermediate results are due to CKM parameters, hadronic parameters, higher order perturbative corrections, and nonfactorizable power corrections, respectively.

Our NNLO results are in good agreement with experimental data [21,22],

$$10^6 \mathrm{Br}(B^- \to \pi^-\pi^0)|_{\mathrm{exp}} = 5.59^{+0.41}_{-0.40},$$
$$10^6 \mathrm{Br}(B^- \to \rho_L^-\rho_L^0)|_{\mathrm{exp}} = 22.5^{+1.9}_{-1.9}, \quad (5)$$

i.e. the experimental values are reasonably well reproduced by the *central values* of our NNLO prediction, which is based on the input parameters from Table I. One should keep in mind, however, that we could also have obtained similar numbers for the branching ratios with rather different values of the tree amplitudes, the form factors, and $|V_{ub}|$. As we discuss in the following section, a much stronger test of the factorization assumption can be obtained by considering the ratios of hadronic and differential semileptonic decay rates, where the dependence on the form factors and $|V_{ub}|$ drops out to a large extent.

We may also take a look at the other tree-dominated $B \to \pi\pi/\pi\rho/\rho\rho$ decay modes. We emphasize that the NNLO calculation of these branching ratios is to date still incomplete, since the QCD penguin amplitudes have not yet been determined to NNLO (this is why we do not discuss *CP* asymmetries in this article). These modes also differ conceptually from $B^- \to \pi^-\pi^0/\rho^-\rho^0$ in the sense that they receive contributions from weak annihilation, which constitutes another class of nonfactorizable power corrections. We again use the model from [1] to estimate their size.

Our results for the *CP*-averaged branching ratios are shown in Table II. Apart from some exceptions ($\pi^+\pi^-$, $\pi^0\pi^0$, $\pi^+\rho^-$, $\pi^-\rho^+$), our default prediction (with central values) is again in reasonable agreement with the data. The agreement is, however, less pronounced than for the pure tree decays $B^- \to \pi^-\pi^0/\rho^-\rho^0$. Moreover, we point out that the color-suppressed modes ($\pi^0\pi^0$, $\pi^0\rho^0$, $\rho^0\rho^0$) are subject to sizeable theoretical uncertainties. This is partly related to the problem mentioned at the end of the previous section (λ_B) and in addition to the fact that these modes are more likely to be affected by $1/m_b$ corrections.

In order to illustrate the correlation of the theoretical uncertainties, we show in Table II the central values of some extreme scenarios (in the spirit of [8]):

In Scenario A we study the dependence on the weak phase γ (we set $\gamma = 110°$). Modes that show a strong dependence on this scenario ($\pi^+\pi^-$, $\pi^0\pi^0$, $\rho^0\rho^0$) are not particularly suited for our purposes, as we focus on testing the QCD dynamics of the topological tree amplitudes in this work.

TABLE II. *CP*-averaged branching ratios (in units of 10^{-6}). Experimental values for $B \to \pi\pi/\pi\rho$ are taken from [21], whereas the ones for $B \to \rho_L\rho_L$ have been inferred from [22]. The different scenarios correspond to: large γ (A), large color-suppressed amplitude (B), large weak annihilation (C), and a combined scenario (D). Further details are given in the text.

Mode	Theory	CKM	had	μ	pow	A	B	C	D	Experiment
$B^- \to \pi^-\pi^0$	$6.22^{+2.37}_{-2.01}$	$^{+1.14}_{-1.05}$	$^{+2.03}_{-1.65}$	$^{+0.16}_{-0.18}$	$^{+0.43}_{-0.42}$	5.97	5.46	6.22	5.64	$5.59^{+0.41}_{-0.40}$
$B^- \to \rho_L^-\rho_L^0$	$21.0^{+8.5}_{-7.3}$	$^{+3.9}_{-3.5}$	$^{+7.4}_{-6.1}$	$^{+0.5}_{-0.7}$	$^{+1.5}_{-1.4}$	20.2	21.3	21.0	23.1	$22.5^{+1.9}_{-1.9}$
$B^- \to \pi^-\rho^0$	$9.34^{+4.00}_{-3.23}$	$^{+2.00}_{-1.81}$	$^{+3.22}_{-2.51}$	$^{+0.31}_{-0.34}$	$^{+1.24}_{-0.84}$	11.2	10.4	10.3	11.8	$8.3^{+1.2}_{-1.3}$
$B^- \to \pi^0\rho^-$	$15.1^{+5.7}_{-5.0}$	$^{+2.9}_{-2.8}$	$^{+4.8}_{-4.1}$	$^{+0.3}_{-0.4}$	$^{+1.0}_{-0.7}$	11.9	11.9	15.8	11.8	$10.9^{+1.4}_{-1.5}$
$\bar{B}^0 \to \pi^+\pi^-$	$8.96^{+3.78}_{-3.32}$	$^{+1.87}_{-1.91}$	$^{+3.02}_{-2.62}$	$^{+0.16}_{-0.20}$	$^{+1.28}_{-0.71}$	6.20	5.21	10.2	5.53	$5.16^{+0.22}_{-0.22}$
$\bar{B}^0 \to \pi^0\pi^0$	$0.35^{+0.37}_{-0.21}$	$^{+0.16}_{-0.14}$	$^{+0.20}_{-0.09}$	$^{+0.03}_{-0.03}$	$^{+0.26}_{-0.11}$	0.66	0.63	0.59	0.68	$1.55^{+0.19}_{-0.19}$
$\bar{B}^0 \to \pi^+\rho^-$	$22.8^{+9.1}_{-8.0}$	$^{+4.2}_{-4.0}$	$^{+7.8}_{-6.8}$	$^{+0.4}_{-0.5}$	$^{+1.9}_{-1.4}$	20.0	13.2	24.6	15.7	$15.7^{+1.8}_{-1.8}$
$\bar{B}^0 \to \pi^-\rho^+$	$11.5^{+5.1}_{-4.3}$	$^{+2.3}_{-2.1}$	$^{+4.2}_{-3.6}$	$^{+0.2}_{-0.2}$	$^{+1.8}_{-1.0}$	13.0	8.41	13.3	11.7	$7.3^{+1.2}_{-1.2}$
$\bar{B}^0 \to \pi^\pm\rho^\mp$	$34.3^{+11.5}_{-10.0}$	$^{+6.3}_{-5.7}$	$^{+8.9}_{-7.8}$	$^{+0.6}_{-0.7}$	$^{+3.7}_{-2.4}$	33.1	21.6	37.9	27.3	$23.0^{+2.3}_{-2.3}$
$\bar{B}^0 \to \pi^0\rho^0$	$0.52^{+0.76}_{-0.42}$	$^{+0.10}_{-0.09}$	$^{+0.62}_{-0.21}$	$^{+0.10}_{-0.10}$	$^{+0.41}_{-0.34}$	0.44	1.64	0.34	1.02	$2.0^{+0.5}_{-0.5}$
$\bar{B}^0 \to \rho_L^+\rho_L^-$	$30.3^{+12.9}_{-11.2}$	$^{+5.6}_{-5.3}$	$^{+11.2}_{-9.6}$	$^{+0.6}_{-0.7}$	$^{+2.9}_{-2.3}$	26.8	22.3	33.2	27.2	$23.6^{+3.2}_{-3.2}$
$\bar{B}^0 \to \rho_L^0\rho_L^0$	$0.44^{+0.66}_{-0.37}$	$^{+0.10}_{-0.09}$	$^{+0.50}_{-0.18}$	$^{+0.10}_{-0.09}$	$^{+0.40}_{-0.30}$	0.58	1.33	0.24	1.03	$0.69^{+0.30}_{-0.30}$

In Scenario B we pursue the question if the data are in accordance with a large color-suppressed amplitude, which may be realized in the factorization framework by a very low value of $\lambda_B = 200$ MeV (we moreover decrease the form factors to $F_+^{B\pi}(0) = 0.21$ and $A_0^{B\rho}(0) = 0.27$). This scenario shows a satisfactory description of the data, in particular, the ''problematic'' modes $\pi^+\pi^-$, $\pi^+\rho^-$ and $\pi^-\rho^+$ are—by construction—in much better agreement with the data.

It is tempting to understand the large experimentally observed $\pi^0\pi^0$ branching ratio as an indication for sizeable nonfactorizable power corrections. It is hard to address this issue in a model-independent way. We would like to emphasize, however, that some observables are indeed more likely to be affected by $1/m_b$ corrections than others (cf. the column labeled ''pow'' in Table II). We, in particular, expect the branching ratios of the tree decays $B^- \to \pi^-\pi^0/\rho^-\rho^0$ to be clean observables as they are free of weak annihilation contributions.

In order to quantify this question we study the influence of a large annihilation amplitude in Scenario C [within the Beneke-Buchalla-Neubert-Sachrajda (BBNS) model from [1]]. It turns out that it is almost impossible to enhance the $\pi^0\pi^0$ decay rate and to simultaneously decrease the $\pi^+\pi^-$ rate without fine-tuning the model parameters [23]. Moreover, the overall pattern of branching ratios and, in particular, the rates of the other color-suppressed modes seem to disfavour a generic scenario with large annihilation contributions. This is illustrated in Scenario C, where we double the default value of the BBNS model for universal weak annihilation, i.e. we set $\rho_A = 1$ and $\phi_A = 0$. We conclude that we do not see any clear pattern of abnormally large power corrections in the data and prefer to be guided by clean observables rather than by the color-suppressed and penguin-contaminated $\pi^0\pi^0$ branching ratio, which cannot be predicted precisely in the factorization framework anyway. We admit that our conclusion is a model-dependent statement, which is, however, supported by a light-cone sum rule analysis, which finds even smaller annihilation contributions than the BBNS model with default parameters [24].

Finally, Scenario D is motivated by our analysis of the following section. It combines elements from Scenario A and B, but it is based on a more moderate parameter choice $\gamma = 90°$, $\lambda_B = 250$ MeV, and $F_+^{B\pi}(0) = 0.23$, which are within the ranges of our default parameters from Table I. These values are inspired by a fit to a set of particularly clean observables that we discuss below. We refrain from presenting the details of our fit and prefer to simply illustrate the effects of such a combined scenario [25].

IV. PRECISION OBSERVABLES

Our predictions for the branching ratios from Table II typically have ~40% uncertainties, which are largely related to an overall normalization from $|V_{ub}|F_+^{B\pi}(0)$ and $|V_{ub}|A_0^{B\rho}(0)$. This particular source of uncertainties can be eliminated by normalizing the hadronic decay rates to the differential semileptonic rates at maximum recoil,

$$\frac{d\Gamma(\bar{B}^0 \to \pi^+\ell^-\bar{\nu}_l)}{dq^2}\bigg|_{q^2=0} = \frac{G_F^2(m_B^2 - m_\pi^2)^3}{192\pi^3 m_B^3}|V_{ub}|^2|F_+^{B\pi}(0)|^2 \tag{6}$$

and similarly for $\bar{B}^0 \to \rho^+\ell^-\bar{\nu}_l$ with $F_+^{B\pi} \to A_0^{B\rho}$ and $m_\pi \to m_\rho$. The situation is, however, different for the color-suppressed modes ($\pi^0\pi^0, \pi^0\rho^0, \rho^0\rho^0$), which are rather dominated by the uncertainties from λ_B and power corrections than by form factor uncertainties and $|V_{ub}|$. We therefore do not consider these modes in this section.

The BABAR collaboration has measured the semileptonic $B \to \pi\ell\nu$ decay spectrum to high accuracy [26]. The data has been investigated in detail under different types of form factor parametrizations in [27]. This analysis uses the world average for the absolute branching ratio and finds $|V_{ub}|F_+^{B\pi}(0) = (9.1 \pm 0.7) \times 10^{-4}$, which is to be compared with our default value 10.3×10^{-4} and 8.3×10^{-4} from Scenario B. The experimental value has been adopted in conjunction with our default value for $|V_{ub}|$ to fix the form factor $F_+^{B\pi}(0) = 0.23$ in Scenario D.

The analysis of the differential semileptonic $B \to \rho\ell\nu$ decay spectrum is more complicated as three different form factors contribute in this case (which confine to $|V_{ub}|A_0^{B\rho}(0)$ at maximum recoil). Recent measurements by BABAR, Belle, and CLEO provide data in 3–4 q^2 bins [28], which are not yet sufficient to extrapolate the decay spectrum in a model-independent way. In a recent analysis the data has been combined with (quenched) lattice calculations of the form factors in the high q^2 region and light-cone sum rule predictions for $q^2 = 0$ [29]. This analysis yields $|V_{ub}|A_0^{B\rho}(0) = (5.5 \pm 2.6) \times 10^{-4}$, which illustrates that the data is still premature. We therefore do not include this number in our analysis.

Our predictions for the ratios

$$\mathcal{R}_{M_3}(M_1M_2) \equiv \frac{\Gamma(\bar{B} \to M_1M_2)}{d\Gamma(\bar{B}^0 \to M_3^+\ell^-\bar{\nu}_l)/dq^2|_{q^2=0}} \tag{7}$$

are shown in Table III. For the $\pi\rho$ modes we chose the normalization such that the dependence on the form factor multiplying the color-allowed amplitude is most strongly eliminated. From Table III it can be seen that the theoretical uncertainties have been reduced considerably to the level of ~15%. Moreover, correlations among different sources of theoretical uncertainties have been resolved to a large extent.

The first two ratios in Table III provide particularly clean probes of the QCD dynamics of the topological tree amplitudes [8,30]. In the factorization framework we have

$$\mathcal{R}_\pi(\pi^-\pi^0) \simeq 3\pi^2 f_\pi^2|V_{ud}|^2|\alpha_1 + \alpha_2|^2, \tag{8}$$

where small electroweak penguin amplitudes have been

TABLE III. Ratios $\mathcal{R}_{M_3}(M_1M_2)$ of hadronic and differential semileptonic decay rates as defined in (7) (in units of GeV^2) and ratios $R(M_1M_2/M_3M_4)$ of hadronic decay rates from (11). The different scenarios A–D are described in the caption of Table II.

Observable	Theory	CKM	had	μ	pow	A	B	C	D	Experiment
$\mathcal{R}_\pi(\pi^-\pi^0)$	$0.70^{+0.12}_{-0.08}$	$^{+0.01}_{-0.01}$	$^{+0.11}_{-0.06}$	$^{+0.02}_{-0.02}$	$^{+0.05}_{-0.05}$	0.68	0.95	0.70	0.82	$0.81^{+0.14}_{-0.14}$
$\mathcal{R}_\rho(\rho_L^-\rho_L^0)$	$1.91^{+0.32}_{-0.23}$	$^{+0.03}_{-0.04}$	$^{+0.28}_{-0.17}$	$^{+0.05}_{-0.07}$	$^{+0.13}_{-0.13}$	1.83	2.38	1.91	2.09	$\cdots$
$\mathcal{R}_\rho(\pi^-\rho^0)$	$0.85^{+0.22}_{-0.14}$	$^{+0.08}_{-0.07}$	$^{+0.17}_{-0.09}$	$^{+0.03}_{-0.03}$	$^{+0.11}_{-0.08}$	1.01	1.16	0.93	1.07	$\cdots$
$\mathcal{R}_\pi(\pi^0\rho^-)$	$1.71^{+0.27}_{-0.24}$	$^{+0.16}_{-0.18}$	$^{+0.18}_{-0.12}$	$^{+0.03}_{-0.05}$	$^{+0.11}_{-0.08}$	1.35	2.07	1.79	1.71	$1.57^{+0.32}_{-0.32}$
$\mathcal{R}_\pi(\pi^+\pi^-)$	$1.09^{+0.22}_{-0.20}$	$^{+0.15}_{-0.17}$	$^{+0.03}_{-0.06}$	$^{+0.02}_{-0.02}$	$^{+0.16}_{-0.09}$	0.75	0.97	1.24	0.86	$0.80^{+0.13}_{-0.13}$
$\mathcal{R}_\pi(\pi^+\rho^-)$	$2.77^{+0.32}_{-0.31}$	$^{+0.15}_{-0.17}$	$^{+0.15}_{-0.19}$	$^{+0.05}_{-0.06}$	$^{+0.23}_{-0.17}$	2.44	2.46	2.99	2.44	$2.43^{+0.47}_{-0.47}$
$\mathcal{R}_\rho(\pi^-\rho^+)$	$1.12^{+0.20}_{-0.14}$	$^{+0.07}_{-0.07}$	$^{+0.03}_{-0.06}$	$^{+0.02}_{-0.02}$	$^{+0.18}_{-0.10}$	1.27	1.01	1.29	1.13	$\cdots$
$\mathcal{R}_\rho(\rho_L^+\rho_L^-)$	$2.95^{+0.37}_{-0.35}$	$^{+0.15}_{-0.17}$	$^{+0.16}_{-0.21}$	$^{+0.06}_{-0.07}$	$^{+0.28}_{-0.22}$	2.61	2.68	3.22	2.64	$\cdots$
$R(\rho_L^-\rho_L^0/\rho_L^+\rho_L^-)$	$0.65^{+0.16}_{-0.11}$	$^{+0.03}_{-0.02}$	$^{+0.13}_{-0.07}$	$^{+0.03}_{-0.03}$	$^{+0.08}_{-0.08}$	0.70	0.89	0.59	0.79	$0.89^{+0.14}_{-0.14}$
$R(\rho_L^+\rho_L^-/\pi^-\rho^+)$	$2.64^{+0.34}_{-0.36}$	$^{+0.31}_{-0.31}$	$^{+0.13}_{-0.13}$	$^{+0.00}_{-0.00}$	$^{+0.06}_{-0.14}$	2.06	2.65	2.49	2.33	$3.23^{+0.69}_{-0.69}$
$R(\pi^+\pi^-/\pi^+\rho^-)$	$0.39^{+0.04}_{-0.05}$	$^{+0.03}_{-0.04}$	$^{+0.02}_{-0.02}$	$^{+0.00}_{-0.00}$	$^{+0.02}_{-0.00}$	0.31	0.39	0.42	0.35	$0.33^{+0.04}_{-0.04}$
$R(\pi^-\pi^0/\pi^+\pi^-)$	$0.65^{+0.19}_{-0.14}$	$^{+0.10}_{-0.07}$	$^{+0.14}_{-0.07}$	$^{+0.03}_{-0.03}$	$^{+0.08}_{-0.10}$	0.90	0.98	0.57	0.95	$1.01^{+0.09}_{-0.09}$
$R(\pi^+\pi^-/\pi^0\pi^0)$	$25.7^{+26.0}_{-18.7}$	$^{+22.7}_{-10.8}$	$^{+7.0}_{-11.0}$	$^{+2.6}_{-2.3}$	$^{+10.2}_{-10.4}$	9.33	8.32	17.3	8.13	$3.33^{+0.43}_{-0.43}$

suppressed. Our NNLO prediction for this ratio

$$\mathcal{R}_\pi(\pi^-\pi^0) = (0.70^{+0.12}_{-0.08})\ \text{GeV}^2 \tag{9}$$

is in good agreement with experimental data

$$\mathcal{R}_\pi(\pi^-\pi^0)|_{\text{exp}} = (0.81^{+0.14}_{-0.14})\ \text{GeV}^2, \tag{10}$$

which strongly supports the factorization assumption. It is, however, interesting that the central experimental value is in between our default prediction and the value 0.95 GeV^2 from Scenario B, which may hint at a somewhat larger value of the color-suppressed amplitude and hence a lower value of the parameter $\lambda_B \simeq 250$ MeV (which we adopt in Scenario D). Experimental data for the ratio $\mathcal{R}_\rho(\rho_L^-\rho_L^0)$ could help to clarify this point.

We recall that all other ratios from Table III receive contributions from QCD penguin amplitudes that are not yet completely available to NNLO. Among these $\mathcal{R}_\pi(\pi^+\rho^-)$, $\mathcal{R}_\rho(\rho_L^+\rho_L^-)$, and $\mathcal{R}_\rho(\pi^-\rho^+)$ are particularly suited to test the dynamics of the color-allowed amplitudes. Our prediction for $\mathcal{R}_\pi(\pi^+\rho^-)$ compares again well to the experimental value.

The fourth color-allowed ratio $\mathcal{R}_\pi(\pi^+\pi^-)$ is special, since the interference of the color-allowed amplitude with the QCD penguin amplitude is not negligible in this case. This ratio is thus particularly sensitive to the choice of the weak phase γ. One should keep in mind, however, that the power corrections from weak annihilation represent another important source of uncertainties for this ratio. It is interesting to replace the BBNS model for weak annihilation by the light-cone sum rule prediction from [24], which strongly reduces the uncertainties from weak annihilation and hence enhances the sensitivity to γ (we then find $1.03^{+0.14+0.03+0.02+0.03}_{-0.16-0.06-0.02-0.03}$ GeV^2). The current experimental value may then be considered as a hint at a large value $\gamma \gtrsim 90°$. A smaller value of γ, on the other hand, may then imply the presence of an additional contribution to the QCD penguin amplitude or that power corrections, which are neither from weak annihilation nor from chirally enhanced wave functions, have been underestimated in our approach. The latter would be conceptually important, as it would increase the total uncertainty from power corrections in QCDF. We refrain, however, from drawing any conclusions concerning $\mathcal{R}_\pi(\pi^+\pi^-)$ and its implications for γ, as long as the penguin amplitudes have not been calculated to NNLO.

In Table III we also show some ratios of hadronic decay rates defined by

$$R(M_1M_2/M_3M_4) \equiv \frac{\Gamma(\bar{B} \to M_1M_2)}{\Gamma(\bar{B}' \to M_3M_4)}. \tag{11}$$

The ratio $R(\rho_L^-\rho_L^0/\rho_L^+\rho_L^-)$ yields complementary information on the tree amplitudes from the ρ sector, where the contamination from the QCD penguin amplitudes is known to be less important [9,31]. We consider the experimental value for this ratio as another important piece of evidence in favor of an enhanced color-suppressed amplitude (Scenario B or D).

The ratios $R(\rho_L^+\rho_L^-/\pi^-\rho^+)$ and $R(\pi^+\pi^-/\pi^+\rho^-)$ of color-allowed modes can be predicted precisely in the factorization framework. Whereas the second ratio is in nice agreement with the data, the first one seems to somewhat disfavor a scenario with a large weak phase γ.

The last two ratios from Table III finally refer to what is known as the $B \to \pi\pi$ puzzle. Whereas the ratio $R(\pi^-\pi^0/\pi^+\pi^-)$ is by construction in Scenarios A, B, and D in better agreement with experimental data than our default prediction, the ratio $R(\pi^+\pi^-/\pi^0\pi^0)$ illustrates what we mentioned at the beginning of this section, i.e. the bulk of theoretical uncertainties does *not* drop out in ratios that involve color-suppressed modes. The uncertainties of our default prediction

$$R(\pi^+\pi^-/\pi^0\pi^0) = 25.7^{+26.0}_{-18.7} \qquad (12)$$

are thus extremely large, and the central value is in vast disagreement with the data. This ratio may be brought down by a factor of $\sim$3 in Scenarios A, B, and D, which may be seen as independent evidence in favor of these scenarios. The fact, however, that these predictions still suffer from $\sim$60% uncertainties related mainly to the power corrections, a_2^π, μ_{hc}, and f_B (in decreasing order of importance), shows that we cannot expect to predict this ratio precisely. We would like to add that there is no such puzzle in the $\pi\rho/\rho\rho$ channels, i.e. there is no general failure of QCDF to describe color-suppressed modes.

V. CONCLUSIONS

We presented the NNLO QCDF prediction for the approximate tree decays $B^- \to \pi^-\pi^0/\rho^-\rho^0$ and updated the global analysis of the other tree-dominated $B \to \pi\pi/\pi\rho/\rho\rho$ decay modes. Our analysis from Sec. IV showed that QCDF yields precise theoretical predictions for particular *ratios* of decay rates. We found in general support for the factorization assumption and uncovered some hints for enhanced color-suppressed amplitudes, which translate in QCDF into a small value of the B meson parameter λ_B. Theoretical progress from nonperturbative methods on the hadronic ratio $f_{M_1}\hat{f}_B/\lambda_B F^{BM_1}(0)$, as well as experimental measurements of the semileptonic $B \to \rho\ell\nu$ decay spectrum, may shed further light on this issue.

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[1] M. Beneke, G. Buchalla, M. Neubert, and C. T. Sachrajda, Phys. Rev. Lett. **83**, 1914 (1999); Nucl. Phys. **B591**, 313 (2000); **B606**, 245 (2001).
[2] C. W. Bauer, S. Fleming, D. Pirjol, and I. W. Stewart, Phys. Rev. D **63**, 114020 (2001); C. W. Bauer, D. Pirjol, and I. W. Stewart, Phys. Rev. D **65**, 054022 (2002).
[3] Y. Y. Keum, H. N. Li, and A. I. Sanda, Phys. Rev. D **63**, 054008 (2001).
[4] M. Beneke and S. Jäger, Nucl. Phys. **B751**, 160 (2006); N. Kivel, J. High Energy Phys. 05 (2007) 019; V. Pilipp, Ph.D. thesis, Ludwig Maximilians Universität München, 2007, arXiv:0709.0497; Nucl. Phys. **B794**, 154 (2008).
[5] M. Beneke and S. Jäger, Nucl. Phys. **B768**, 51 (2007); A. Jain, I. Z. Rothstein, and I. W. Stewart, arXiv:0706.3399.
[6] G. Bell, Nucl. Phys. **B795**, 1 (2008); Ph.D. thesis, Ludwig Maximilians Universität München, 2006, arXiv:0705.3133; arXiv:0902.1915.
[7] In our numerical analysis the electroweak penguin amplitudes will be included in the next-to-leading order approximation.
[8] M. Beneke and M. Neubert, Nucl. Phys. **B675**, 333 (2003).
[9] M. Beneke, J. Rohrer, and D. Yang, Nucl. Phys. **B774**, 64 (2007); M. Bartsch, G. Buchalla, and C. Kraus, arXiv:0810.0249.
[10] C. Bobeth, M. Misiak, and J. Urban, Nucl. Phys. **B574**, 291 (2000); M. Gorbahn and U. Haisch, Nucl. Phys. **B713**, 291 (2005).
[11] K. G. Chetyrkin, M. Misiak, and M. Munz, Nucl. Phys. **B520**, 279 (1998).
[12] V. M. Braun *et al.*, Phys. Rev. D **74**, 074501 (2006); J. Bijnens and A. Khodjamirian, Eur. Phys. J. C **26**, 67 (2002); A. P. Bakulev, S. V. Mikhailov, and N. G. Stefanis, Phys. Lett. B **578**, 91 (2004); P. Ball and R. Zwicky, Phys. Lett. B **625**, 225 (2005); P. Ball, V. M. Braun, and A. Lenz, J. High Energy Phys. 05 (2006) 004.
[13] P. A. Boyle, D. Brommel, M. A. Donnellan, J. M. Flynn, A. Juttner, and C. T. Sachrajda (RBC Collaboration and UKQCD Collaboration), arXiv:0810.1669.
[14] M. Okamoto *et al.*, Nucl. Phys. B, Proc. Suppl. **140**, 461 (2005); E. Gulez, A. Gray, M. Wingate, C. T. H. Davies, G. P. Lepage, and J. Shigemitsu, Phys. Rev. D **73**, 074502 (2006); **75**, 119906(E) (2007); P. Ball and R. Zwicky, Phys. Rev. D **71**, 014015 (2005); G. Duplancic, A. Khodjamirian, T. Mannel, B. Melic, and N. Offen, J. High Energy Phys. 04 (2008) 014.
[15] P. Ball, G. W. Jones, and R. Zwicky, Phys. Rev. D **75**, 054004 (2007).
[16] P. Ball and R. Zwicky, J. High Energy Phys. 04 (2006) 046.
[17] P. Ball and R. Zwicky, Phys. Rev. D **71**, 014029 (2005).
[18] E. Gamiz, C. T. H. Davies, G. P. Lepage, J. Shigemitsu, and M. Wingate (HPQCD Collaboration), Phys. Rev. D **80**, 014503 (2009); C. Bernard *et al.*, Proc. Sci., LAT2008 (2008) 278; A. A. Penin and M. Steinhauser, Phys. Rev. D **65**, 054006 (2002); M. Jamin and B. O. Lange, Phys. Rev. D **65**, 056005 (2002).
[19] V. M. Braun, D. Y. Ivanov, and G. P. Korchemsky, Phys. Rev. D **69**, 034014 (2004); S. J. Lee and M. Neubert, Phys. Rev. D **72**, 094028 (2005); H. Kawamura and K. Tanaka, Phys. Lett. B **673**, 201 (2009).
[20] Partial next-to-next-to-leading order results of the penguin amplitudes from spectator scattering can be found in [5].

[21] E. Barberio *et al.* (Heavy Flavor Averaging Group), arXiv:0808.1297, and updates at http://www.slac.stanford.edu/xorg/hfag/index.html.
[22] B. Aubert *et al.* (BABAR Collaboration), Phys. Rev. Lett. **102**, 141802 (2009); Phys. Rev. D **76**, 052007 (2007); A. Somov *et al.*, Phys. Rev. Lett. **96**, 171801 (2006); B. Aubert *et al.* (BABAR Collaboration), Phys. Rev. D **78**, 071104 (2008).
[23] It should be noticed that these decay rates depend on the same combination of annihilation amplitudes.
[24] A. Khodjamirian, T. Mannel, M. Melcher, and B. Melic, Phys. Rev. D **72**, 094012 (2005).
[25] We think that a sophisticated fit to the observables from Table III should be considered only when the semileptonic $B \to \rho \ell \nu$ spectrum has been measured precisely.
[26] B. Aubert *et al.* (BABAR Collaboration), Phys. Rev. Lett. **98**, 091801 (2007).
[27] P. Ball, Phys. Lett. B **644**, 38 (2007).
[28] B. Aubert *et al.* (BABAR Collaboration), Phys. Rev. D **72**, 051102 (2005); T. Hokuue *et al.* (Belle Collaboration), Phys. Lett. B **648**, 139 (2007); N. E. Adam *et al.* (CLEO Collaboration), Phys. Rev. Lett. **99**, 041802 (2007).
[29] J. M. Flynn, Y. Nakagawa, J. Nieves, and H. Toki, arXiv:0812.2795.
[30] J. D. Bjorken, Nucl. Phys. B, Proc. Suppl. **11**, 325 (1989).
[31] R. Aleksan, F. Buccella, A. Le Yaouanc, L. Oliver, O. Pene, and J. C. Raynal, Phys. Lett. B **356**, 95 (1995).