PHYSICAL REVIEW D **80,** 054022 (2009)

Extending soft collinear effective theory to describe hard jets in dense QCD media

Ahmad Idilbi and Abhijit Majumder*

Department of Physics, Duke University, Durham, North Carolina 27708, USA

(Received 18 August 2008; revised manuscript received 15 June 2009; published 25 September 2009)

An extension to the soft collinear effective theory description of hard jets is motivated to include the leading contributions between the propagating partons within the jet with partons radiated from a dense extended medium. The resulting effective Lagrangian, containing both a leading and a power suppressed (in the hard scale Q^2) contribution, arises primarily from interactions between the hard collinear modes in the jet with Glauber modes from the medium. In this first attempt, the interactions between the hard jet and soft and collinear partonic modes have been ignored, in an effort to focus solely on the interactions with the Glauber modes. While the effect of such modes on vacuum cross sections are suppressed by powers of the hard scale compared to the terms from the Lagrangian, such subleading contributions are enhanced by the extent of the medium and result in measurable corrections. The veracity of the derived Lagrangian is checked by direct comparison with known results from full QCD calculations of two physical observables: the transverse momentum broadening of hard jets in dense media and a reanalysis of the transverse momentum dependent parton distribution function.

DOI: 10.1103/PhysRevD.80.054022 PACS numbers: 13.60.Hb, 12.38.Bx, 13.87.-a

I. INTRODUCTION

The study of hard jets in QCD is now a considerably mature science. Experiments at e^+e^- annihilation, deep-inelastic scattering (DIS), and p-p machines have yielded a wide array of measurements on a variety of jet observables including single particle production, multiparticle correlations as well as event shapes. On the theory side, sophisticated factorization theorems have been written down which factorize the final-state "jet function" from the initial state and the hard cross section at leading twist [1]. In the derivation leading to such factorization theorems [2–6] the infinite class of Feynman diagrams are subjected to a Landau analysis. Regions of momentum space which yield pinch singularities are identified. These represent the leading contributions to such processes and may be decomposed into classes of Feynman diagrams which in turn allow for proofs of factorization. An alternate and equivalent approach has recently been afforded by the methods of effective field theories such as the soft collinear effective theory (SCET) [7–11]. While not specifically devised to rederive factorization, SCET presents a formalism where the analysis resulting in the identification of the leading contributions may be carried out within the QCD Lagrangian resulting in the derivation of an effective Lagrangian which is only applicable to processes within the prescribed kinematic regime. In such a formalism, factorization occurs at the level of the Lagrangian and at the level of operators [12,13]. The Feynman rules which arise from an expansion in a small parameter λ may then be used to systematically study hard processes.

Power corrections to hard processes in vacuum are suppressed in the presence of a hard scale $Q^2 \gg \Lambda^2_{\rm QCD}$. However, there exist scenarios where a specific set of power corrections (often arising from operators with higher twist) may be enhanced and become non-negligible compared to the leading process. One example of this is the case of single-spin asymmetry in DIS, where leading twist processes yield vanishing results [14]. Another example, where the leading twist term does not vanish but power corrections may become enhanced is in the presence of a medium is in the case of DIS on a large nucleus [15,16]. The inclusive cross section receives contributions from power suppressed operators which are enhanced by a factor $A^{1/3}$ arising from the length of a large nucleus with mass number A [17,18].

In semi-inclusive processes such as single hadron inclusive events in DIS on a large nucleus, a hard jet is formed in the collision of the virtual photon with a hard quark. This jet then begins to shower and lose virtuality on its way to hadronization. Some part of this space-time evolution occurs within the nuclear medium. Multiple scattering of the jet in the medium modifies the final distribution of high momentum hadrons emanating from such a hard jet [19,20]. Experimental measurements of single and multi-hadron production from such modified jets and their comparison with jets produced in DIS on a proton or in p-p collisions allow one to quantify the gluon distribution in dense extended QCD media [21].

This modification depends on a class of higher twist operators evaluated in the nuclear medium. While there exists considerable information regarding the ground state (nucleon) structure of large nuclei which may be invoked in the modeling of these higher twist operators, there exists practically no such information regarding the bulk structure of the deconfined matter produced in high-energy

*Present address: Department of Physics, 191 W. Woodruff Avenue, The Ohio State University, Columbus, OH 43210, USA.

heavy-ion collisions. The modification of hard jets, in the deconfined matter produced in heavy-ion collisions, has assumed center stage in the experimental program as the primary probe of the structure of the produced matter [22]. This is due in part to the dramatically large effects seen in comparison with cold confined matter as well as the possibility for a first principles computation of this modification from perturbative QCD (pQCD). There are many approaches to this calculation, all involving a different set of approximations about the medium [23–27]. For a review of the different approaches see Ref. [28].

While the benefits of an effective field theory description of power corrections to hard processes in QCD cannot be overstated, to date there has not been a single attempt to incorporate the leading effects of medium enhanced higher twist within an effective theory formalism such as SCET. There exists an effective theory description of the dense deconfined matter in the limit of very high temperature. Here, a consistent effective theory of the medium without a jet is set up first, then the hard jet is assumed to have interactions with the soft field in the medium which are similar to those encountered by a hard thermal parton [29]. This article will take a different route, by trying to extend an existing effective theory of jets in vacuum to incorporate the effects of scattering in a medium. In what follows, we undertake the simplest extension to an SCET like formalism in the presence of an extended QCD medium.

In Sec. II, the emergent scales in the problem will be discussed and the presence of a new mode, called the Glauber mode, not present in the current vacuum implementation of SCET will be motivated. In Sec. III, an effective Lagrangian which includes the interaction of such Glauber modes with collinear quarks will be derived from the QCD Lagrangian. In this first attempt, we will ignore the further interactions between these Glauber modes and the soft and collinear gluon modes of the usual SCET Lagrangian. A new set of Feynman rules arising from such an effective Lagrangian will be outlined and their equivalence with the Feynman rules of full QCD demonstrated at an amplitude by amplitude level. In Secs. IV and V, the Feynman rules will be used to compute cross sections in physical processes; two examples will be dealt with: the transverse broadening of jets in DIS on large nuclei and the transverse momentum dependent parton distribution function (TMDPDF). It will be shown explicitly that, in light-cone gauge, the Glauber gluons give rise to the transverse gauge link that enters the definition of the fully gauge-invariant TMDPDF. The results obtained will be compared with published calculations in full QCD. Concluding discussions will be presented in Sec. VI.

II. THE ENERGY SCALES FROM A DENSE MEDIUM

Consider the DIS of a hard photon with momentum q and virtuality $q^2 = -Q^2$ on a nucleon with momentum p in vacuum or contained within a large nucleus. In the Breit frame, the photon has the momentum components

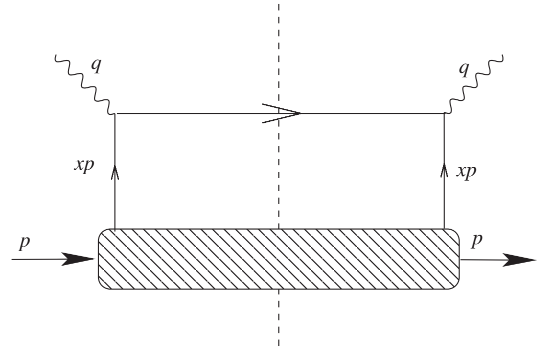

FIG. 1. Single-inclusive DIS on a nucleon or a large nucleus.

$$q \equiv [q^+, q^-, \vec{q}_\perp] = [Q^2/2q^-, q^-, 0] \sim Q(-1, 1, 0). \quad (1)$$

The off-shellness of the photon Q is taken as a representative of the hard scale in the process. This strikes a hard, almost on-shell quark with a large momentum in the $+z$ direction or a large (+) component of momentum,

$$p_i \equiv (x_B p^+, p_i^-, \vec{p}_{i,\perp}) \sim Q(1, \lambda^2, \lambda). \quad (2)$$

The number of partons in the infinite momentum frame which carry an $x_B \sim 1$ fraction of the nucleon momentum is rather small and λ is a small dimensionless variable ($\lambda \to 0$). We then trigger on events where an almost on-shell jet is produced in the final state (see Fig. 1). This partonic jet moves with large momentum in the (−) direction,

$$p_f \equiv (p_f^+, q^-, \vec{p}_{f,\perp}) \sim Q(\lambda^2, 1, \lambda), \quad (3)$$

where, at leading order, $\vec{p}_{f,\perp} = \vec{p}_{i,\perp}$.

In the case of DIS on a single nucleon, this partonic jet immediately escapes the medium and eventually after a time $\sim 1/(\lambda^2 Q)$[1] would have decayed into multiple partons of lower invariant mass and eventually turns into a jet of hadrons. The invariant mass of the final jet is $m \sim \lambda Q$ and its total forward energy from all produced hadrons which have arisen from this jet is

$$E = q^-/\sqrt{2} \sim Q. \quad (4)$$

In this article, we initiate the construction of the effective theory for the propagation of such jets through the dense matter within nucleons and large nuclei.

The first step in such an endeavor is the assignment of relations between all relevant dimensionfull quantities that appear in the problem. We assume that the scaling variable λ is so chosen that perturbation theory may be applied down to momentum transfer scales at or above $\lambda^2 Q$. The

[1]The formation time of the radiation may be estimated from the virtuality of the jet $\sim\lambda Q$ and the boost of the jet $\sim\lambda^{-1}$.

medium, also introduces its own set of scales, e.g., the mass of a nucleon $M_N \simeq 1$ GeV. This is assumed to scale with a new scaling variable μ such that it is comparable to $\Lambda_{\rm QCD}$ and in general much smaller than the soft perturbative scale, i.e.,

$$M_N \sim \Lambda_{\rm QCD} \sim \mu Q \lesssim \lambda^2 Q. \tag{5}$$

One may immediately surmise that in the Breit frame when $x_B \sim 1$, and $p^+ \sim Q$, the boost or γ factor is of order μ^{-1}. While the inverse size of the nucleon may be thought of as an even softer scale, in this effort, we will assume that the hard scales Q, λQ are much harder than the medium scale μQ and thus the inverse length will be assumed to be of the order of the mass of the nucleon,

$$l_N \sim \frac{1}{\mu Q}, \tag{6}$$

All our considerations will be carried out in the Breit frame. The scaling introduced at the beginning of this section regarding the momentum components of the incoming and outgoing partons were set up in the Breit frame. Thus the nucleon (or medium) will have to be boosted to this frame. The ensuing boost to the Breit frame will lead to the contracted length of the nucleon,

$$\frac{l_N}{\gamma} \sim \frac{1}{Q}. \tag{7}$$

The very introduction of an alternative soft scale such as μQ may lead the reader to imagine a much more complicated effective theory which will manifestly involve both λ and μ and will require a relation between these two scaling variables. However, as will be demonstrated in the next two sections, with specific examples, it is possible to construct an effective theory in dense matter involving only the hard scale Q and the vacuum scaling variable λ.

The construction of an in-medium effective theory which depends on only the hard scale and the scaling variables from the vacuum theory has one further requirement. In specific examples, such as in Sec. IV, certain in-medium matrix elements will be enhanced by media with sizes much larger than a nucleon, e.g., in the case of large nuclei or a deconfined quark gluon plasma (QGP). In the case of large nuclei (with mass number $A \gg 1$), the enhancement factor is usually the nuclear length in units of the nucleon length, i.e., $A^{1/3}$. These situations will require that the enhancement be expressed in powers of λ, i.e., $A^{1/3} \sim \lambda^{-n}$. The number n is so far unspecified and will turn out to be observable dependent.

The particular choice of scaling of nucleon size and momentum leads to certain obvious physical consequences: In the Breit frame, valence (large x) partons carrying order one fractions of the forward momentum of the nucleon, have momenta that may be expressed as (by simply boosting momenta of the order of M_N),

$$k \sim Q(1, \lambda^2, \lambda). \tag{8}$$

These partons will be found to be completely confined within nucleons. The off-shellness of these partons $\sim\lambda^2 Q$ is very small and, as a result, for processes which involve momentum transfers of the order of λQ or larger, these radiated partons may be considered as asymptotic in-states. In the following, we will often ignore discussion of the ($-$) components of the partonic momenta in the in-state; these play almost no role in the computation of jet modification and the TMDPDF.

Because of interactions, a variety of gluons may be radiated from these valence partons with momenta constrained by overall energy-momentum conservation and determined by the kinematics of the process being triggered on. In the case of transverse broadening, the type of radiated gluon which plays a leading role will be those with momenta which scale as

$$k \sim Q(\lambda^2, \lambda^2, \lambda). \tag{9}$$

Gluons with momenta which scale as in the above equation are referred to as Glauber gluons, or gluons in the Glauber region. The role of Glauber gluons in transverse broadening (as well as in transverse momentum dependent structure functions) may be easily understood in the case of DIS with a hard jet in the final state. The momenta of the produced quark jet scales as in Eq. (3). Imagine the multiple scattering of the struck quark off the remnants of the nucleon or nucleus. The diagrams under consideration are of the form of Fig. 2. In order for the produced jet to escape from the nucleon or nucleus without undergoing any induced radiation, the interactions with the nucleon have to be such that they do not induce a major change in the off-shellness of the quark. In order to see how this comes about we explicitly write out the expression for the q_2 propagator where $q_2 = q + p_i + k_1$ represents the quark momentum, which, after the hard scattering (with momentum $q + p_i$) scatters off one extra gluon with momentum k_1, (in order to simplify the expression we assume that $p^i_\perp = 0$ and $p_i^+ = x_B p^+ = -q^+$) in Fig. 2:

$$S(q_2) = \frac{\gamma^+ q^- + \gamma^\perp \vec{k}^1_\perp + \gamma^- k_1^+}{2q^-(k_1^+) - |\vec{k}^1_\perp|^2} \simeq \frac{\gamma^+ q^-}{2q^-(k_1^+) - |\vec{k}^1_\perp|^2}. \tag{10}$$

Since, $q^- \sim Q$ and $k^1_\perp \sim \lambda Q$, the forward momentum has

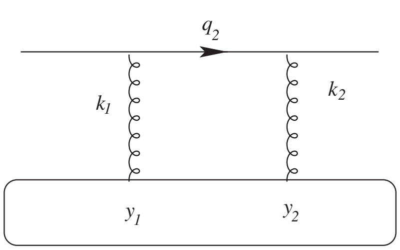

FIG. 2. The multiple scattering of a produced jet in deep-inelastic scattering.

to scale as $k_1^+ \sim \lambda^2 Q$ for the jet to remain off-shell by no more than $\lambda^2 Q^2$. If the forward momentum scales with a higher power, i.e., $k^+ \sim \lambda Q$, this will cause the jet to go off-shell by λQ^2 and lead to the radiation of momenta with large transverse momenta $l_\perp \sim \lambda^{1/2} Q$. This process will lead to the radiative energy loss of the propagating quark and will be discussed in a future effort. The reader will note that the absorption of gluons which are collinear to the outgoing quark, i.e., with a momentum that scales as $Q(\lambda^2, 1, \lambda)$, also do not raise the off-shellness of the quark beyond $\lambda^2 Q^2$. However, the number of such gluons emitted from a medium moving with a large collinear momentum in the (+) direction is vanishingly small. Hence the effect of such partons will be ignored.

In the current manuscript, the focus will remain on the production of a single jet with nonzero transverse momentum. Since the number of gluons with a forward momentum $p^+ \sim \lambda^2 Q$ far exceeds those with $p^+ \sim \lambda Q$ for the same transverse momentum, the jet will tend to encounter multiple interactions with gluons with a soft forward momentum. These will result in the transverse broadening of the hard jet. The neglect of gluons with a larger (+) component of momentum suppresses radiation from the hard parton.

With the power counting of the different momentum components identified, there remains the issue of determining the power counting (in terms of λ) of the 4-vector potential A_a^μ in this regime of momenta. In the case of effective theories of QCD in a vacuum, the power counting of the A_a^μ field is determined by an estimation of the powers of λ from the gluon propagator. In the case of Glauber gluons such a methodology will yield incorrect results. In the Glauber region of momenta, the gluon propagator obtained from the full QCD Lagrangian is never on-shell. With the transverse momenta being larger than the light-cone components, Glauber gluons are always spacelike off-shell. As demonstrated above, an on-shell collinear parton may interact with a Glauber gluon and have its transverse momentum changed by order λQ while still remaining on-shell. The simplest extension of an effective theory containing only collinear modes (the jet) to a medium with collinear modes travelling in the opposite direction (the target) will contain the interactions of the collinear jet parton with Glauber gluons radiated off the partons in the target which move in the opposite direction. The effective action has the simple form,

$$S = \int d^4x [\mathcal{L}_{\mathrm{SCET}} + j_\mu^a(x) A^{a\mu}{}_G(x)]. \qquad (11)$$

where, A_G^μ is the Glauber field radiated from the target and j_μ^a is the current of the collinear partons from the jet. The kinetic and interaction terms for the collinear fields which constitute j_μ^a are contained within $\mathcal{L}_{\mathrm{SCET}}$ [8] along with terms for the soft fields. In this effort, $\mathcal{L}_{\mathrm{SCET}}$ will not contain the collinear or soft modes from the target. These will be integrated out and included in effective Glauber field A_G^μ.

Note that there is no kinetic term for the Glauber gluon; thus, it does not obey a classical equation of motion. It admits no mode expansion and is not quantized as the SCET modes. However such exchanges are included in the full QCD Lagrangian and are prevalent in the interaction of collinear modes from the jet with those from the target. The Glauber field A_G^μ represents the effective classical field of the target partons. The power counting of the various components of the Glauber field may be obtained from a calculation of its production in full QCD.

As our goal here is simply to estimate the power counting of the various components of the Glauber field, we will ignore subtleties associated with nonlinear terms in an interacting non-Abelian theory. We will estimate the λ power of the various components in an Abelian theory. In a classical Abelian theory, the gauge field A^μ is obtained from a solution of the inhomogeneous Maxwell's equation. This is given as

$$A^\mu(x) = A_0^\mu(x) + \int d^4y \mathcal{D}^{\mu\nu}(x-y) J_\nu(y), \qquad (12)$$

where $A_0^\mu(x)$ is a solution of the homogeneous Maxwell's equation. By restricting the current to be collinear to the target direction, and insisting that the incoming partons in the target remain close to on-shell, we restrict the field $A^\mu(x)$ to only its Glauber component $A_G^\mu(x)$. In this region of momenta, there exists no solution of the homogeneous Maxwell's equation, i.e., $A_{0G}^\mu(x) = 0$. As a result, the Glauber field is obtained from the second term on the right hand side of Eq. (12). We now evaluate the power counting of the various components of the Glauber field in covariant and light-cone gauge.

A. Covariant gauge

At leading order in covariant gauge, the gauge propagator $\mathcal{D}^{\mu\nu}$ is given as

$$\mathcal{D}^{\mu\nu}(x-y) = \int \frac{d^4k}{(2\pi)^4} \frac{-ig^{\mu\nu} e^{-ik\cdot(x-y)}}{k^2 + i\epsilon}. \qquad (13)$$

In Eq. (12), $J^\nu(y) = \bar{\psi}(y)\gamma^\nu \psi(y)$ is the current of partons in the target which generates the gauge field. The fermionic operator may be decomposed as

$$\psi(y) = \int \frac{dp^+ d^2p_\perp}{(2\pi)^3 \sqrt{p^+ + \frac{p_\perp^2}{2p^+}}} \sum_s u^s(p) a_p^s e^{-ip\cdot y} + v^s(p) b_p^{s\dagger} e^{ip\cdot y}. \qquad (14)$$

The scaling of the fermionic operator depends on the range of momentum which are selected from the in-state by the annihilation operator. Note that this influences both the scaling of the annihilation operator a_p as well as the bispinor $u(p)$. The power counting of the annihilation operator may be surmised from the standard anticommutation relation,

$$\{a_p^r, a_{p'}^{s\,\dagger}\} = (2\pi)^3\delta^3(\vec{p} - \vec{p}')\delta^{rs}, \tag{15}$$

and the power counting of the bispinor from the normalization condition,

$$\sum_s u_p^s \bar{u}_p^s = \not{p} = \gamma^- p^+ + \gamma^+ p^- - \gamma_\perp \cdot p_\perp. \tag{16}$$

Substituting the equation for the current in Eq. (13), and integrating out y, we obtain the expression for the $(+)$ component of the gauge field:

$$A^+ \simeq \int \frac{d^3pd^3q}{(2\pi)^6\sqrt{p^+}\sqrt{q^+}} \frac{-ie^{-i(p-q)\cdot x}}{(p-q)^2} a_q^\dagger a_p \bar{u}(q)\gamma^+ u(q). \tag{17}$$

If the incoming and outgoing momenta p and q scale as collinear momenta in the $(+)$ direction, i.e., $p \sim Q(1, \lambda^2, \lambda)$, then we get, $\delta^3(\vec{p} - \vec{p}') \sim [\lambda^2 Q^3]^{-1}$, as one of the momenta will involve the large $(+)$ component and the remaining are the small transverse components. Thus the annihilation (and creation) operator scales as $\lambda^{-1}Q^{-3/2}$. Also in the spin sum $\not{p} \sim Q$ and thus $u(p) \sim u(q) \sim Q^{1/2}$. The γ^+ projects out the large $(\sim Q)$ components in u and $\bar{u}$ in the expression $\bar{u}(q)\gamma^+ u(p)$. We also institute the Glauber condition that $p^+ - q^+ \sim \lambda^2 Q$, $p^- - q^- \sim \lambda^2 Q$, and $p_\perp - q_\perp \sim \lambda Q$.

Using these scaling relations we correctly find that the bispinor scales as $\lambda Q^{3/2}$. However, to obtain the correct scaling of the gauge field A^+ one needs to institute the approximation that $q^+ = p^+ + k^+$ where $k^+ \sim \lambda^2 Q$. This condition is introduced by insisting that the $(+)$ momentum of the incoming and outgoing state, which control the scaling of $a_q^\dagger$ and a_p, are separated by $k^+ \sim \lambda^2 Q$. This is used to shift the $dq^+ \to dk^+$ and as a result we obtain the scaling of the A^+ field as $\lambda^2 Q$. Following a similar derivation, with the replacement $\gamma^+ \to \gamma^\perp(\gamma^-)$ we obtain the scaling of the transverse and $(-)$ component of the gauge field as $A^\perp \sim \lambda^3 Q$ and $A^- \sim \lambda^4 Q$.

B. Light-cone gauge

The power counting of the gauge field is gauge dependent. In this last subsection we surmise the power counting, in light-cone gauge, for the Glauber field. The primary difference with Eq. (17) is the gauge field propagator. In the positive light-cone gauge: $n \cdot A = n^- A^+ = A^+ = 0$, the only nonzero components are $A_\perp$ and A^-. Note that a Glauber field with transverse momentum $k_\perp \sim \lambda Q$ can only be radiated from a collinear parton without changing the direction of the collinear parton. For $A_\perp$, the dominant contribution to the power counting equation arises not from the $g^{\mu\nu}$ term in the numerator of the propagator, but rather from the $(k^\mu n^\nu + k^\nu n^\mu)/k^+$ term, i.e.,

$$A_\perp \simeq \int \frac{d^3pd^3q}{(2\pi)^6\sqrt{p^+}\sqrt{q^+}} \times \frac{i(\frac{(p_\perp - q_\perp)n^-}{p^+ - q^+})e^{-i(p-q)\cdot x}}{(p-q)^2} a_q^\dagger a_p \bar{u}(q)\gamma^+ u(q). \tag{18}$$

Comparing this with Eq. (17), we obtain that $A_\perp \sim \lambda Q$. Similarly we obtain $A^- \sim \lambda^2 Q$. As a result, in light-cone gauge, the $(\perp)$ component of the gauge field is much more dominant than in covariant gauge and we expect this to change the power counting of various terms in the effective Lagrangian. Similar power counting arguments may also be surmised from the explicit expressions presented in Ref. [30] and references therein. See also the discussion at the end of Sec. V.

In the next section, the power counting arguments presented above will be used to derive the effective Lagrangian which describes the interaction of collinear modes with Glauber exchanges with the medium. While these power counting arguments have been derived for an Abelian theory, we expect them to remain true in a non-Abelian theory as well.

III. EFFECTIVE LAGRANGIAN FOR GLAUBER GLUONS

In the preceding section, the momentum scales associated with a new mode which arises in the presence of a medium was outlined. These so-called Glauber gluons present a mode that was absent in the derivation of the SCET Lagrangian. In what follows, we introduce these modes and construct a new additional effective Lagrangian called the Glauber Lagrangian. In this first attempt, the kinetic terms which represent the soft and collinear gluons of the SCET Lagrangian, along with their interactions with the collinear quarks, will be ignored. The two active fields will be the collinear quarks and the Glauber gluons. In principle there will be a similar contribution from Glauber interactions with a collinear gluon. While such interactions are not included in the derivations presented in the current paper, these represent a straightforward extension of the formalism presented in this section.

Before we start the derivation of the effective Lagrangian we comment on the off-shellness of the Glauber gluons. Since for Glauber gluons the product k^+k^- is much less than $|\vec{k}_\perp|^2$ then the Glauber modes are obviously off-shell degrees of freedom. In principle when one constructs an effective Lagrangian, the degrees of freedom involved have to be on-shell so that one can make use of the classical equations of motions. In our derivation below we do not make any use of the gluon equation of motion and it is only the Dirac equation for a collinear quark that is utilized. Thus the derivation below should be viewed merely as the limit of the contribution from gluons with arbitrary momentum taken to the Glauber region. This is similar to the well-known method of regions. In this sense we are actually deriving an effective ''vertex'' between a collinear quark and a Glauber gluon. This effective vertex could also be obtained on a case by case basis starting from the full QCD amplitudes and then taking the Glauber limit. This is illustrated below for a nontrivial case. The advantage of deriving an effective Lagrangian is mainly the consistent power counting (of the gluon fields and momenta) invoked in the derivation.

In DIS, in the Breit frame, the final state after the hard scattering consists of a hard outgoing quark in the $-z$ or simply the $(-)$ direction, which interacts with the remnants of the proton (or nucleus) which move in the $+z$ or simply the $(+)$ direction. These interactions are dominated by soft grazing scatterings with the fast-moving remnants. Some of these interactions may be hard as well, resulting in the hard quark going considerably off-shell and radiating a hard gluon. Such processes constitute the energy loss of the hard quark and will be ignored in this first attempt. The incorporation of such interactions will necessarily involve the reintroduction of both the soft and collinear modes as well as the inclusion of new interaction terms between the Glauber modes and these soft and collinear modes. In what follows we focus solely on the Glauber interactions of the hard outgoing quark with the glue field generated by the remnants of the struck proton (or nucleus).

Consider a fast-moving quark moving along the $\bar{n}$ direction where $\bar{n} = \frac{1}{\sqrt{2}}(1, 0, 0, -1)$. The $+z$ direction will be denoted by the unit vector, $n = \frac{1}{\sqrt{2}}(1, 0, 0, 1)$. This quark has large momentum in the $-z$ direction $p^- = n \cdot p \simeq Q$ where Q is the hard scale of the process considered. The interaction of this collinear quark with Glauber gluons leads to a change of the transver se momentum component of the collinear quark while the p^- component remains fixed up to $\mathcal{O}(\lambda^2)$. Thus the truly label-changing component is only the transverse one $p_\perp$. As such the starting point to describe the interaction of this collinear quark field with Glauber gluons would be, as in SCET [7,8], to extract the label momentum components from the full QCD field.

$$\psi(x) = e^{-ip^- x^+} \sum_{\vec{p}_\perp} e^{i\vec{p}_\perp \cdot \vec{x}_\perp} \psi_{\bar{n},\vec{p}_\perp}(x). \tag{19}$$

Again as in SCET, we decompose the $\psi_{\bar{n},\vec{p}_\perp}$ into a sum of two fields: $\xi_{\bar{n},\vec{p}_\perp} + \xi_{n,\vec{p}_\perp}$ where $\xi_{\bar{n},\vec{p}_\perp}$ carries the large momentum components in the $-z$ direction while $\xi_{n,\vec{p}_\perp}$ carries the small momentum components. The next step is to substitute this decomposition into Eq. (19) and then substitute the result into the interaction term in the full QCD Lagrangian. In order to maintain consistent power counting in the effective theory one has to specify the scalings (in terms of λ) of the relevant quantum fields. For the collinear quark the scaling is the same as in SCET, namely $\xi_{\bar{n},\vec{p}_\perp}$ scales as λ while $\xi_{n,\vec{p}_\perp}$ scales as λ^2. All derivatives acting on $\xi_{\bar{n},\vec{p}_\perp}$ or $\xi_{n,\vec{p}_\perp}$ will further suppress the power counting by λ^2. For the Glauber gluon gauge field the scaling was given in the previous section.

The starting point to obtain the effective Lagrangian is, as in SCET, the full QCD quark sector expressed in terms of the fields $\xi_{\bar{n}}$ and ξ_n

$$\begin{aligned}\mathcal{L}_g = {}& \bar{\xi}_{\bar{n},\vec{p}_\perp} \not{n}(i\bar{n} \cdot D)\xi_{\bar{n},\vec{p}_\perp} + \bar{\xi}_{n,\vec{p}_\perp} \not{\bar{n}}(n \cdot p + in \cdot D)\xi_{n,\vec{p}_\perp} \\ &+ \bar{\xi}_{\bar{n},\vec{p}'_\perp}(\not{p}_\perp + i\not{D}_\perp)\xi_{n,\vec{p}_\perp} + \bar{\xi}_{n,\vec{p}'_\perp}(\not{p}_\perp + i\not{D}_\perp)\xi_{\bar{n},\vec{p}_\perp}.\end{aligned} \tag{20}$$

We notice that in the last result there are terms that scale as $\mathcal{O}(\lambda^4)$, $\mathcal{O}(\lambda^5)$, and $\mathcal{O}(\lambda^6)$. We now eliminate the nondynamical field ξ_n by making use of the tree-level equation of motion

$$\xi_{n,\vec{p}_\perp} = \frac{\not{p}_\perp + i\not{D}_\perp}{2(n \cdot p + in \cdot D)} \not{n} \xi_{\bar{n},\vec{p}_\perp}. \tag{21}$$

We again have kept leading and subleading contributions in Eq. (21). It is useful to notice the difference between the case of Glauber gluons and collinear gluons. For collinear gluons, the gauge field component in the covariant derivative $(n \cdot A)$ in the denominator of Eq. (21) scales the same as $n \cdot p$. This component of the gauge field, eventually, leads to the presence of the collinear Wilson lines in SCET, as was demonstrated in Ref. [9]. However, for Glauber gluons this covariant derivative is suppressed compared to $n \cdot p$. Therefore, we expand the denominator and get

$$\begin{aligned}\mathcal{L}_g = {}& \sum_{\vec{p}_\perp,\vec{p}'_\perp} e^{i(\vec{p}_\perp - \vec{p}'_\perp)\cdot\vec{x}_\perp} \bar{\xi}_{\bar{n},\vec{p}'_\perp}(x) \bigg[\bar{n} \cdot iD + (\not{p}_\perp + i\not{D}_\perp) \\ &\times \frac{1}{2n \cdot p}\left(1 - \frac{in \cdot D}{n \cdot p}\right)(\not{p}_\perp + i\not{D}_\perp)\bigg] \not{n} \xi_{\bar{n},\vec{p}_\perp}(x),\end{aligned} \tag{22}$$

where higher orders in λ have been dropped out. In this article we will only consider Glauber gluons in covariant and light-cone $(A^+ = 0)$ gauge. In covariant gauge the leading order Lagrangian is given by

$$\mathcal{L}_g = \bar{\xi}_{\bar{n},\vec{p}_\perp}\left[i\bar{n} \cdot D + \frac{p_\perp^2}{2n \cdot p} \right] \not{n} \xi_{\bar{n},\vec{p}_\perp}, \tag{23}$$

where both terms in the square brackets are of order λ^2. This result was first obtained in Ref. [31]. In light-cone gauge the leading order interaction Lagrangian is given by

$$\mathcal{L}_g = \bar{\xi}_{\bar{n},\vec{p}'_\perp}\left[\frac{g_s(\not{p}'_\perp \not{A}_\perp + \not{A}_\perp \not{p}_\perp) + g_s^2 \not{A}_\perp^2}{2n \cdot p} \right] \not{n} \xi_{\bar{n},\vec{p}_\perp}. \tag{24}$$

The Feynman rules derived from the effective Lagrangians above are given in Fig. 3. As a first simple test of these rules we compute the amplitude for the case of two Glauber gluons attached to a collinear quark line and compare with the amplitude obtained from full QCD. We carry out both evaluations only in light-cone gauge as this is some what nontrivial. In covariant gauge, this same calculation can be straightforwardly carried out and will

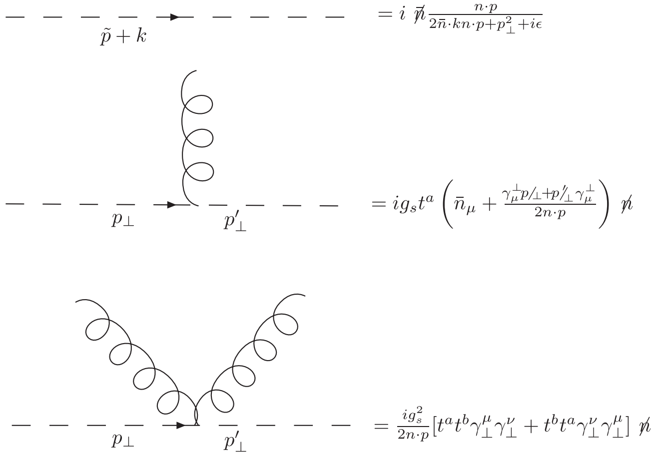

FIG. 3. Feynman rules for a collinear quark interacting with Glauber gluons. In principle, both terms in the parenthesis in the second diagram contribute in covariant as well as in light-cone gauge. However, the first term is leading (in terms of power counting in λ) in covariant gauge and is identically zero in light-cone gauge. The third diagram is only required for calculations in light-cone gauge.

be discussed in Sec. V. We show that the effective theory exactly reproduces the full QCD result at the level of the amplitudes of Feynman diagrams.

Let us consider the Feynman diagram given in Fig. 4 where two Glauber gluons are attached to the collinear quark field. In full QCD, the amplitude reads

$$I = -(ig_s)^2 \int \frac{d^4k_1}{(2\pi)^4} \frac{d^4k_2}{(2\pi)^4} \bar{u}(p) \times \frac{A\!\!\!/(k_1)[p\!\!\!/ - k\!\!\!/_1] A\!\!\!/(k_2)[p\!\!\!/ - k\!\!\!/_1 - k\!\!\!/_2]}{[(p-k_1)^2 + i\varepsilon][(p-k_1-k_2)^2 + i\varepsilon]}, \quad (25)$$

where $\bar{u}(p)$ represents the Dirac spinor for an outgoing quark and p has no transverse momentum. The scaling of

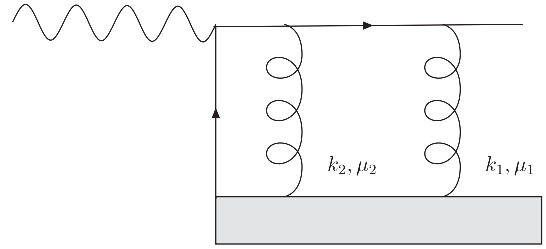

FIG. 4. Final-state interactions in DIS: two-gluon exchange.

the quark spinor will be ignored in the following, as it plays no role in the remaining discussion. When expanding the numerator in Eq. (25) one should invoke the same power counting for the gluon gauge fields and the momenta as the one used in deriving the effective theory. In light-cone gauge $A^+ = 0$ and by making use of the Dirac equation $[\bar{u}(p)p\!\!\!/ = p^- \bar{u}(p)\gamma^+ = 0]$, the leading contribution is

$$\mathcal{J} = -(ig_s)^2 \int \frac{d^4k_1}{(2\pi)^4} \frac{d^4k_2}{(2\pi)^4} \bar{u}(p) \times \frac{A\!\!\!/_\perp(k_1)[2k_1^+ p^- A\!\!\!/_\perp(k_2) + k\!\!\!/_{1\perp} A\!\!\!/_\perp(k_2)(k\!\!\!/_{1\perp} + k\!\!\!/_{2\perp})]}{[(p-k_1)^2 + i\varepsilon][(p-k_1-k_2)^2 + i\varepsilon]}. \quad (26)$$

We notice that each contribution in the square bracket scales as λ^4 and subleading terms have been dropped out.

In the effective theory (and again working in light-cone gauge) there are two Feynman diagrams that contribute. One [denoted below as $\mathcal{J}^{(1)}$] comes from two Glauber gluons attached at different points. The other contribution comes from the vertex of two Glauber gluons attached at the same point. It is denoted by $\mathcal{J}^{(2)}$. Using the Feynman rules given in Fig. 3 we obtain the first contribution from the effective theory as

$$\mathcal{J}^{(1)} = -(ig_s)^2 \int \frac{d^4k_1}{(2\pi)^4} \frac{d^4k_2}{(2\pi)^4} \bar{\xi}_{\bar{n}} \frac{A\!\!\!/_\perp(k_1) k\!\!\!/_{1\perp}[A\!\!\!/_\perp(k_2)(k\!\!\!/_{1\perp} + k\!\!\!/_{2\perp}) + k\!\!\!/_{1\perp} A\!\!\!/_\perp(k_2)]}{[2p^- k_1^+ + |\vec{k}_{1\perp}|^2 - i\varepsilon][2p^-(k_1^+ + k_2^+) + |\vec{k}_{1\perp} + \vec{k}_{2\perp}|^2 - i\varepsilon]}. \quad (27)$$

The second contribution from the effective theory is given as

$$\mathcal{J}^{(2)} = -(ig_s)^2 \int \frac{d^4k_1}{(2\pi)^4} \frac{d^4k_2}{(2\pi)^4} \bar{\xi}_{\bar{n}} \frac{A\!\!\!/_\perp(k_1) A\!\!\!/_\perp(k_2)}{[2p^-(k_1^+ + k_2^+) + |\vec{k}_{1\perp} + \vec{k}_{2\perp}|^2 - i\varepsilon]}. \quad (28)$$

In the above result we have considered only one contribution where the color and Lorentz indices are held fixed in the Feynman rule for the two-gluon vertex. The other contribution gives an identical result as $\mathcal{J}^{(2)}$ upon integrations over k_1 and k_2. It can be easily verified that $\mathcal{J} = \mathcal{J}^{(1)} + \mathcal{J}^{(2)}$ thus confirming that the effective theory reproduces the full QCD result at the level of the amplitudes of the relevant Feynman diagrams. The case of one-gluon attachment is trivial and one can easily verify that the effective Lagrangian also gives the same result as the one in full QCD.

In what follows, we investigate two physical applications of the derived effective Lagrangian: The transverse broadening experienced by hard jets in DIS on a large nucleus and the TMDPDF in a nucleon. In the first case, a final physical cross section will be computed and arguments on the enhancement of power corrections by large lengths in nuclei will be forwarded; hence, this application contains arguments beyond those used to derive the effective Lagrangian. However, the scaling of the momenta of the gluons will always lie within the strict boundary prescribed by the Glauber Lagrangian. It may not come as a surprise that the Glauber gluons which lead to the transverse broadening of hard jets, also play a principal role in the construction of the gauge-invariant TMDPDF.

IV. APPLICATION I: TRANSVERSE BROADENING IN LARGE NUCLEI

A straightforward application of the effective Lagrangian derived in Sec. III, is to the process of jet broadening in dense matter. As a specific example, we consider the process of jet broadening in deep-inelastic scattering in large nuclei. A virtual photon with momentum $q = [Q^2/2n \cdot q, n \cdot q, 0, 0]$ is incident on a large nucleus (A) with momentum $Ap \cdot \bar{n}$ where $p \cdot \bar{n}$ is the mean momentum of a nucleon. In the remaining section, we will refer to $q \cdot n$ as simply q^- and $p \cdot \bar{n}$ as simply p^+.

We compute the cross section for the semi-inclusive production of a hard jet in the final state with a net transverse momentum $\vec{l}_\perp$ with respect to the direction of the virtual photon, i.e.,

$$e(L_1) + A(p) \to e(L_2) + J(\vec{l}_\perp) + X. \tag{29}$$

In the frame chosen, the Bjorken variable is defined as $x_B = Q^2/(2p^+q^-)$. The differential cross section may be decomposed into a leptonic and a hadronic part as

$$\frac{E_{L_2} d\sigma}{d^3 L_2 d^2 l_\perp} = \frac{\alpha_{EM}^2}{2\pi s Q^4} L_{\mu\nu} \frac{dW^{\mu\nu}}{d^2 l_\perp}. \tag{30}$$

where $s = (p + L_1)^2$ is the total invariant mass of the lepton nucleon system. The leptonic tensor may be expressed as

$$L_{\mu\nu} = \tfrac{1}{2}\mathrm{Tr}[\not{L}_1 \gamma_\mu \not{L}_2 \gamma_\nu]. \tag{31}$$

The initial state of the incoming nucleus is defined as $|A; p\rangle$. The general final hadronic or partonic state is defined as $|X\rangle$. As a result, the semi-inclusive hadronic tensor may be defined as

$$\begin{aligned} W^{\mu\nu} &= \sum_X (2\pi^4)\delta^4(q + P_A - p_X)\langle A; p|J^\mu(0)|X\rangle \\ &\quad \times \langle X|J^\nu(0)|A; p\rangle \\ &= 2\,\mathrm{Im}\Big[\int d^4 y e^{iq\cdot y}\langle A; p|J^\mu(y)J^\nu(0)|A; p\rangle\Big], \end{aligned} \tag{32}$$

where the sum ($\sum_X$) runs over all possible hadronic states and J^μ is the hadronic electromagnetic current i.e., $J^\mu = Q_q \bar{\xi}_{\bar{n}} \gamma^\mu \xi_n$, where Q_q is the charge of a quark of flavor q in units of the positron charge e. It is understood that the factors of the electromagnetic coupling constant have already been extracted and included in Eq. (30). The leptonic tensor will not be discussed further. The focus in the remaining shall lie exclusively on the hadronic tensor.

In a full QCD calculation of Eq. (32), one computes the hadronic tensor, order by order, in the strong coupling. This leads to the introduction of a variety of processes leading to a modification of the structure of the jet. Such processes include radiative branchings, flavor changes of propagating partons, as well as transverse diffusion of the partons in the shower which ensues from the quark produced in the hard scattering. In this article, we will focus solely on the processes which lead to the transverse momentum diffusion or transverse broadening of the produced hard quark.

In Ref. [32], the leading contributions to transverse broadening without induced radiation, at all orders in coupling, were identified as those of Fig. 5. These diagrams depict processes where the propagating parton engenders multiple scattering off the glue field inside the various nucleons through which it propagates. However, scatterings do not change the small off-shellness of the propagating parton; as a result, large transverse momentum radiations do not occur. Using simple kinematics, the relation between the momentum components of the glue field k_i may be surmised by insisting that the off-shellness of the $i + 1$-th quark line be of the same order as the i-th line,

$$(p + k_i)^2 = p^2 + k_i^2 + 2p^+k_i^- + 2p^-k_i^+ - 2\vec{p}_\perp \cdot \vec{k}_\perp^i. \tag{33}$$

Insisting that $(p + k_i)^2 \sim p^2 \sim \lambda^2 Q^2$ and given the known scaling of the quark momenta (i.e., $p^+ \sim \lambda^2 Q$, $p^- \sim Q$, $\vec{p}_\perp \sim \lambda Q$), we obtain that $\vec{k}_\perp^i \sim \lambda Q$, $k_i^+ \sim \lambda^2 Q$, and k_i^- may scale with a range of different choices Q, λQ, $\lambda^2 Q$, etc. The first two cases for the scaling of k^- represent gluons which are emanated with large ($-$) momentum from a nucleon moving with large ($+$) momentum. The number of such gluons must be vanishingly small. The first nontrivial population of gluons emanating from a nucleon moving with a large ($+$) momentum, are those which scale as $k \sim [\lambda^2, \lambda^2, \lambda]$, which essentially constitute the Glauber sector.

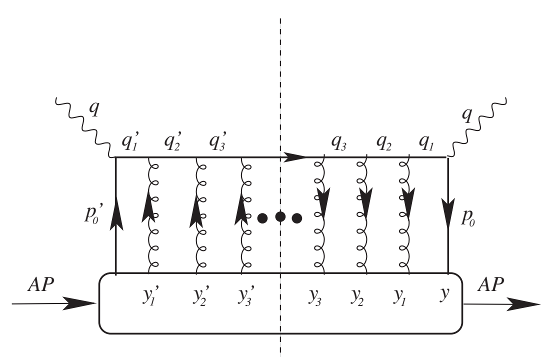

FIG. 5. An order n diagram which contributes solely to transverse broadening.

Using the Feynman rules derived for Glauber gluons in Sec. II, the leading component of n-th order diagrams such as those of Fig. 5 may be expressed as

$$W^{\mu\nu} = \int d^4y \frac{d^4l}{(2\pi)^4} \prod_{i=0}^{n-1} \prod_{j=0}^{n'-1} \left\{ d^4y_{i+1} d^4y'_{j+1} \frac{d^4k_i d^4k'_j}{(2\pi)^8} \right\} g^{n+n'} \langle A; p | \xi_n(0) \bar{\xi}_n(y) \gamma^\mu \times \prod_{i=1}^{n} \left[\frac{2q^-}{2q^- k_i^+ - |\vec{k}_\perp^i|^2 - i\epsilon} \right] 2l^- 2\pi \delta(2l^+ l^- - l_\perp^2) \prod_{j=n'}^{1} \left[\frac{2q^-}{2q^- k'^+_j - |\vec{k}'^j{}_\perp|^2 + i\epsilon} \right] t^{a_i} A^+_{a_i}(y_i) t^{a'_j} A^+_{a'_j}(y'_j) \gamma^\nu |A; p\rangle \times e^{-i \sum_{i=1}^{n-1} k_i \cdot y_i} e^{i \sum_{j=1}^{n'-1} k'_j \cdot y'_j} e^{[-iy_n \cdot \{l-(q+\sum_{i=0}^{n-1} k_i)\}]} e^{[iy'_{n'} \cdot \{l-(q+\sum_{j=0}^{n'-1} k'_j)\}]}, \quad (34)$$

where, it is understood that y'_0 is the origin and $y^0 \equiv y$. In the equation above, the gauge fields have been expressed in coordinate space. At this point an n-th momentum may be introduced, via

$$1 = \int d^4k_n \delta^4\left(l - \sum_{k=0}^{n} k_k - q\right). \quad (35)$$

This leads to a considerable simplification of the phase factors. The complete absence of the (−) components of the momentum, from all expressions except for the phase factors allows for the k^- and k'^- integrations to be done, resulting in the localization of the process on the negative light-cone.

The integrals over the momenta k_i^+, $k'^+{}_j$ may be reexpressed in terms of momentum fractions, i.e.,

$$Q^2 = 2x_B p^+ q^-, \qquad k_i^+ = x_i p^+, \qquad k'^+{}_j = x'_j p^+, \quad (36)$$

$$\sum_{k=0}^{i} 2\vec{k}_\perp^i \cdot \vec{k}_\perp^k + |\vec{k}_\perp^i|^2 = 2x_D^i p^+ q^-, \quad (37)$$

$$\sum_{l=0}^{j} 2\vec{k}'^j{}_\perp \cdot \vec{k}'^l{}_\perp + |\vec{k}'^j{}_\perp|^2 = 2x'^j{}_D p^+ q^-. \quad (38)$$

Integrating over all the x_i and x'_j momentum fractions by contour integration, we obtain the much simplified form of the hadronic tensor,

$$W^{\mu\nu} = g^{n+n'} \int \frac{d^2l_\perp}{(2\pi)^2} \prod_{i=0}^{n} dy_i^- d^2y_\perp^i \prod_{j=1}^{n'} dy'^-{}_j d^2y'^j{}_\perp \int \prod_{i=0}^{n} \frac{d^2k_\perp^i}{(2\pi)^2} \prod_{j=0}^{n'-1} \frac{d^2k'^j{}_\perp}{(2\pi)^2} (2\pi)^2 \delta^2(\vec{l}_\perp - \vec{K}_\perp) \frac{1}{2} \times (g^{\mu-} g^{\nu+} + g^{\mu+} g^{\nu-} - g^{\mu\nu}) e^{-ix_B p^+ y^-} \prod_{i=0}^{n} e^{-ix_D^i p^+ y_i^-} e^{i\vec{k}_\perp^i \cdot \vec{y}_\perp^i} \prod_{j=0}^{n'} e^{ix'^j{}_D p^+ y'^-{}_j} e^{-i\vec{k}'^j{}_\perp \cdot \vec{y}'^j{}_\perp} \prod_{i=n}^{1} \theta(y_i^- - y_{i-1}^-) \times \prod_{j=n'}^{1} \theta(y'^-{}_j - y'^-{}_{j-1}) \langle A; p | \bar{\xi}_n(y^-, y_\perp) \gamma^+ \xi_n(0) \,\mathrm{Tr}\left[\prod_{i=1}^{n} t^{a_i} A^+_{a_i}(y_i^-, \vec{y}_\perp^i) \prod_{j=n'}^{1} t^{a_j} A^+_{a_j}(y'^-{}_j, \vec{y}'^j{}_\perp) \right] |A; p\rangle. \quad (39)$$

The expression derived above has so far been a direct application of the Feynman rules derived in the preceding section. Hitherto, no assumption regarding the nature of the nuclear state has been made. As a result the nuclear or nucleon scale of μQ has also not appeared in any of the expressions. However, the hadronic tensor in Eq. (39) and any resulting transverse broadening will, ultimately, depend on the expectation of the $(n + n' + 2)$-parton operator as indicated in the last line of Eq. (39). To proceed further, approximations regarding the expectation of this partonic operator will have to be made. In these approximations, the in-medium scale μQ will appear. However, as we will show, the final transverse broadening will turn out to be independent of this scale under certain assumptions.

Following standard treatments, we approximate the nucleus as a weakly interacting homogeneous gas of nucleons. Such an approximation is only sensible at very high energy, where, due to time dilation, the nucleons appear to travel in straight lines almost independent of each other over the interval of the interaction of the hard probe. All forms of correlators between nucleons (spin, momentum, etc.) are assumed to be rather suppressed. As a result, the expectation value of the $n + n' + 2$ operators in the nuclear state may be decomposed as

$$\langle A; p | \bar{\xi}_{\bar{n}}(y^-, \vec{y}_\perp) \gamma^+ \xi_{\bar{n}}(0) \prod_{i=1}^{n+n'} A^+_{a_i}(y_i) |A; p\rangle = C^A_{p_0, p_2, \ldots p_n} \langle p_0 | \bar{\xi}_{\bar{n}}(y^-, \vec{y}_\perp) \gamma^+ \xi_{\bar{n}} | p_0 \rangle \times \prod_{i=1}^{(n+n')/2} \langle p_i | A^+_{a_i}(y_i) A^+_{a'_i}(y'_i) | p_i \rangle, \quad (40)$$

where, the factor $C^A_{p_0, p_1, \ldots, p_n}$ represents the correlations between the $(n + n')/2$ nucleons which interact with the propagating parton. In the decomposition above, we have restricted at most two parton operators per nucleon, insisting that any larger number of operators is suppressed. Note that this is only true outside the saturation regime [33,34]. The decomposition performed above, also restricts $n = n'$. The choice of gluon operators per nucleon is also special to the case of transverse broadening: The maximum broadening is obtained when one gluon operator from the amplitude is paired with one from the complex conjugate. This reason for this is immediately understood with further simplifications on each gluon pair (written with spin and color indices suppressed),

$$\int d^2y^i_\perp d^2y'^j{}_\perp \langle p|A^+(\vec{y}^i_\perp)A^+(\vec{y}'^j{}_\perp)|p\rangle \times \exp[-ix^i_D p^+ y^-_i + i\vec{k}^i_\perp\cdot\vec{y}^i_\perp + ix'^j{}_D p^+ y'^-{}_j - i\vec{k}'^j{}_\perp\cdot\vec{y}'^j{}_\perp] = (2\pi)^2\delta^2(\vec{k}^i_\perp - \vec{k}'^j{}_\perp)\int d^2y_\perp e^{-ix^i_D p^+(y^-_i - y'^-{}_j)} \times e^{i\vec{k}_\perp\cdot\vec{y}_\perp}\langle p|A^+(\vec{y}_\perp/2)A^+(-\vec{y}_\perp/2)|p\rangle, \quad (41)$$

where $\vec{y}_\perp$ is the transverse gap between the two gluon insertions and $\vec{k}_\perp = (\vec{k}^i_\perp + \vec{k}'^j{}_\perp)/2$. The physics of the above equation is essentially the transverse translation symmetry of the two-gluon correlator in a very large nucleus. This is then used to equate the transverse momenta emanating from the two gluon insertions. Thus, if the two operators were both chosen from the amplitude or the complex conjugate, then the momenta brought in by one gluon operator would be immediately taken out by the other and, as a result, the combination of the two operators will lead to no net transverse broadening. The integration above, also simplifies the longitudinal phase factors which now depend solely on the difference of the longitudinal positions of the two gluon insertions.

Further simplifications are introduced by Taylor expanding the transverse momentum dependent delta function, as

$$\prod_{i=1}^n \frac{\partial^2}{2!\partial^2 k^i_\perp}\delta^2(\vec{l}_\perp + \sum \vec{k}^i_\perp)\bigg|_{\vec{k}^i_\perp=0}\prod_{i=1}^n |\vec{k}^i_\perp|^2, \quad (42)$$

and combining the $|\vec{k}^i_\perp|^2$ with the expectation of the two-gluon operator $\langle p_i|A^+_{a_i}(\vec{y}^i_\perp/2)A^+_{a'_i}(-\vec{y}^i_\perp/2)|p_i\rangle$ to convert these into the expectation of field strengths in the nucleon $\langle p_i|F^{+\perp}_{a_i}(\vec{y}^i_\perp/2)F^{+\perp}_{a'_i}(-\vec{y}^i_\perp/2)|p_i\rangle$. The meaning of this decomposition of the transverse momentum delta function is the retention of solely the leading twist part of each of the two-point correlators in each nucleon. Higher powers of a given transverse momentum will necessarily lead to higher transverse moments of the two-gluon operator. One further assumption regarding the two-point function of Eq. (41), due to color confinement, leads to a constraint on the two longitudinal y^- integrations (ignoring color and spin indices),

$$\int dy^- dy'^- \langle p|F(y^-)F(y'^-)|p\rangle \simeq \int dy^- \langle FF\rangle y^-_{\rm conf}, \quad (43)$$

where, $\langle FF\rangle$ is the gluon expectation in a nucleon and $y^-_{\rm conf}$ is the confining distance.

Each such integral yields a factor of $L^- \sim A^{1/3} \sim 1/\lambda$ from the unconstrained y^- integration. The equating of the pairs of transverse momenta that appear in each two-gluon correlation, as well as the relation between the longitudinal momenta from the θ functions in Eq. (39), require that the largest transverse momentum broadening and largest length enhancement arises from the terms where the gluon correlations are built up in a mirror symmetric fashion, i.e., where the gluon insertion at y^i is contracted with that at y'^i.

Averaging over the spins and colors of the two-point functions in each nucleon, the remaining n longitudinal position integrals for the gluon insertions may be simplified as

$$\int \prod_{i=1}^n dy^-_i \theta(y^-_i - y^-_{i-1}) = \frac{1}{n!}\int\prod_{i=1}^n dy^-_i. \quad (44)$$

Invoking the above simplifications, the leading length enhanced contribution at order $2n$ to the differential hadronic tensor is obtained as

$$\frac{d^2W^{\mu\nu}_n}{d^2l_\perp} = C^A_{p_0,\ldots,p_n} W^{\mu\nu}_0 \frac{1}{n!}[\{\nabla^2_{l_\perp}\}^n\delta^2(\vec{l}_\perp)] \times\left[\frac{\pi^2\alpha_s}{2N_c}L^-\int\frac{dy^-}{2\pi}\langle p|F^{a+\alpha}F^a{}_{\alpha,}{}^+|p\rangle\right]^n, \quad (45)$$

where $W^{\mu\nu}_0$ is the leading order transverse momentum integrated hadronic tensor, given as

$$W^{\mu\nu}_0 = 2\pi[g^{\mu-}g^{\nu+} + g^{\mu+}g^{\nu-} - g^{\mu\nu}]\sum_q Q^2_q f_q(x_B), \quad (46)$$

where the expectation of the two quark operator in Eq. (40) leads to the quark structure function in the equation above.

There remains the overall coefficient $C^A_{p_0,\ldots,p_n}$ which contains the weak correlations between the various struck nucleons. A study of such correlations in Refs. [35,36] revealed that a simple factorized form such as $C^A_{p_0,\ldots,p_n} = C^A_{p_0}(\rho/2p^+)^n$, where ρ is the density of nucleons in a nucleus, is not completely inappropriate. Using this simple form one may sum over all n, i.e., over multiple scatterings of the quark in the nucleus, to obtain the resummed equation,

$$\frac{d^2W^{\mu\nu}}{d^2l_\perp} = e^{(DL^-)\nabla^2_{l_\perp}}\frac{d^2W^{\mu\nu}_0}{d^2l_\perp}, \quad (47)$$

where, $d^2W^{\mu\nu}_0/d^2l_\perp = W^{\mu\nu}_0\delta^2(\vec{l}_\perp)$, and the constant D is given as

$$D = \frac{\pi^2 \alpha_s}{2N_c} \rho \int \frac{d^3y d^2k_\perp}{(2\pi)^3 2p^+} \langle p | F^{a+\alpha}(y) F^a{}_{\alpha,}{}^+(0) | p \rangle \times \exp\left\{ -i \left(\frac{|\vec{k}_\perp|^2}{2q^-} y^- - \vec{k}_\perp \cdot \vec{y}_\perp \right) \right\}. \tag{48}$$

It is this constant D which controls the broadening experienced by the hard jet in the extended nucleus.

As shown in Ref. [32], Eq. (47) is a solution of the two dimensional transverse momentum diffusion equation, where the initial condition may be taken as a δ function in transverse momentum. Taking moments of the solution of the diffusion equation, we obtain the total transverse momentum squared acquired by the hard quark after traversing a length L^- in the nucleus as given by the simple relation,

$$\langle k_\perp^2 \rangle_{L^-} = 4DL^-. \tag{49}$$

The reader will note that we have used a two dimensional delta function as the input to the diffusion equation. This is an approximation to a very peaked distribution and one may use any other input distribution as well. The net extra broadening experienced by the input distribution is given by Eq. (49). Given that the initial parton is an SCET mode, the transverse momentum is of the order of $|\vec{k}_\perp|^2 \sim (\lambda Q)^2$. As a last step, we will demonstrate that the broadening obtained from multiple scattering in the large nucleus is of this order in power counting and thus one may continue to think of an SCET mode propagating in the extended medium.

The power counting of net transverse momentum squared may be easily estimated from counting powers of λ and μ in the expression for D in Eq. (48). As the expression is frame independent it will be evaluated in the Breit frame. In this frame, the z component of the length of the nucleon ($\sim 1/(\mu Q)$) is contracted to a length of order $1/Q$; hence the nucleon density ρ scales as

$$\rho = \frac{1}{V} \sim \mu^2 Q^3. \tag{50}$$

The dimension of the nucleon ket is obtained from the standard normalization of the on-shell nucleon state, given as

$$\langle p|q \rangle = (2\pi)^3 2p^+ \delta(p^+ - q^+) \delta^2(\vec{p}_\perp - \vec{q}_\perp) \Rightarrow |p\rangle \sim (\mu Q)^{-2}. \tag{51}$$

The $F^{+\perp}F^{+\perp}$ correlator scales as $(\lambda^3 Q^2)^2$ from the standard Glauber scaling rules for the transverse momentum and the vector potential. The enhanced length in the nucleus may be expressed in terms of the nuclear parameter $A^{1/3}$ and the length of a nucleon l_N, as

$$L^- = A^{1/3} l_N \sim \frac{A^{1/3}}{Q}. \tag{52}$$

Substituting the above relations in Eq. (48), and noting that the y^- and the $\vec{y}_\perp$ coordinates are conjugate to the $|k_\perp|^2/2q^-$ and $\vec{p}_\perp$ momenta, yields the λ power counting of the net transverse momentum squared picked up by the hard parton as

$$\langle p_\perp^2 \rangle_{L^-} \sim A^{1/3} \lambda^4 Q^2, \tag{53}$$

independent of the medium scaling parameter μ. The broadening is rather small in an object the size of a nucleon, but may get enhanced in large or dense media. As a result, for small nuclei where $A^{1/3} \ll \lambda^{-2}$ one may ignore this extra effect of final-state multiple scattering. For nuclei, where $A^{1/3} \sim \lambda^{-2}$, or the medium has a very large gluon density we may obtain a broadening which is comparable to the jets' inherent transverse momentum. In this case, we are in the "SCET-Glauber" region, where the derived effective Lagrangian in this paper may be used in combination with the SCET Lagrangian to understand the interaction of hard jets in dense media. For nuclei where $A^{1/3} \gg \lambda^{-2}$, the Glauber modes will broaden the propagating jets beyond the scaling assumed in the derivation of the SCET Lagrangian and a different set of effective theories will need to be constructed.

There is an unknown quantity that has been invoked a number of times in the discussion above: the inherent gluon density. This is the density of "small x" gluons that emanate from the current density in the medium and interact with the hard jet. The number of such gluons is a dimensionless quantity and thus difficult to estimate in a power counting calculation. The number of such gluons may in general also depend on the media in question. It is well known that the number of such gluons may become rather large at high energies and may thus lead to considerable broadening of hard jets. While the application in this section has focused on the broadening of jets in nuclei, the factorization properties afforded by SCET bode well for the applicability of this theory to jet broadening in dense QGPs created in heavy-ion collisions. While QGPs have been estimated to be from 10 to 100 times denser than nuclear matter, their lifetimes are rather short and the majority of jets propagate rather short distances in the densest part of such environments. Given these experimental considerations, we expect the derived effective theory in combination with SCET to have wide applicability.

V. APPLICATION II: TMDPDF

Inclusive hard scattering processes like DIS can be factorized into perturbatively calculable short-distance quantities convoluted with nonperturbative long-distance quantities [1]. The latter quantities are the familiar Feynman PDFs. For semi-inclusive processes where a single hadron is observed in the final state with a given transverse momentum, it is the TMDPDF that enters into the factorization formula for the cross section. More details can be found in [37,38]. The TMDPDF were introduced a long time ago in [4], as

$$f(x, \vec{k}_\perp) = \frac{1}{2}\int \frac{d\xi^- d^2\vec{\xi}_\perp}{(2\pi)^3} e^{-i(\xi^- k^+ - \vec{\xi}_\perp \cdot \vec{k}_\perp)} \times \langle P|\bar{\psi}(\xi^-, \vec{\xi}_\perp) L^\dagger_{\vec{\xi}_\perp}(\infty, \xi^-) \times \gamma^+ L_0(\infty, 0)\psi(0, \vec{0}_\perp)|P\rangle, \quad (54)$$

where L is a path-ordered gauge link along the light cone in the ($-$) direction, i.e.,

$$L_{\vec{\xi}_\perp}(\infty, \xi^-) = P\exp\left(ig_s \int_{\xi^-}^{\infty} d\xi^- A^+(\xi^-, \vec{\xi}_\perp)\right). \quad (55)$$

The Wilson line $L_{\vec{\xi}_\perp}$ has a well-known origin: It comes from the radiation of gluons which are collinear to the incoming parton. In SCET these Wilson lines are also the familiar collinear Wilson lines W [9]. In this work we will not discuss further the emergence of those Wilson lines as they are not related to Glauber gluons.

The TMDPDF is a physical quantity and thus has to be gauge invariant under arbitrary gauge transformation. However the above definition is gauge invariant only in the set of nonsingular gauges like covariant gauges where the gluon field vanishes at $\xi^- = \infty$. In singular gauges like light-cone gauge (with $A^+ = 0$) the gluon field (specifically the transverse components $A_\perp$) does not vanish at $\xi^- = \infty$ and a gauge transformation performed with $A_\perp(\xi^- = \infty)$ will generate a nonvanishing phase that is not compensated by any gauge link. Thus the above definition of the TMDPDF has to be modified by introducing an additional gauge link formed from the transverse components $A_\perp$:

$$L_{\xi^-=\infty}(\vec{\xi}_\perp, \vec{0}_\perp) = P\exp\left(ig_s \int_0^{\vec{\xi}_\perp} d\vec{\xi}'_\perp \cdot \vec{A}_\perp(\xi^- = \infty, \vec{\xi}'_\perp)\right), \quad (56)$$

where the line integral in the transverse plane can be performed in arbitrary direction. The above observations were first made in Ref. [30].

The important question that arises is what kind of interactions, say in DIS, build up this gauge link. This question was answered in the work of Belitsky, Ji, and Yuan (BJY) [39]. There it was shown that the final-state interactions between the struck quark and the remnants of the incoming proton are responsible for the appearance of this gauge link. Those final-state interactions are mediated by Coulomb gluons that carry mainly a momentum in the transverse direction. In the next section we will check whether these gluons are Glauber gluons or not. It is important to verify this issue as the final-state interactions are responsible for many physical effects like single-spin asymmetries and shadowing [40–42]. Any attempt to formulate an effective field theoretic approach to study such effects has to start from identifying the relevant momentum modes that mediate the interactions.

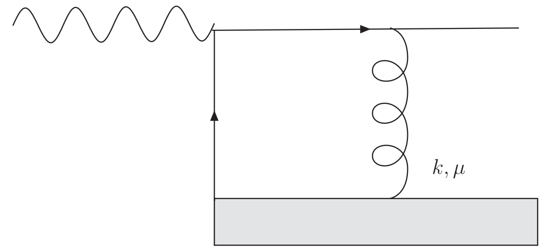

FIG. 6. Final-state interactions in DIS: one-gluon exchange.

We start by briefly reviewing the work of BJY by considering the Feynman diagram given in Fig. 6. A quark propagator with momentum $p - k$ has a denominator: $-2p^- k^+ + k^2$ where the quark has essentially large momentum in the $-z$ direction. The integral over the gluon momentum k gets contributions from a vanishing denominator. This could happen if (1) k is collinear to the outgoing quark. In this case both terms in the denominator scale as λ^2. Or, (2) k is soft which means $k^2 \ll k^+$ and k^+ scales as λ^2. Or, (3) k has Glauber scaling. Assuming the gluon is emitted from a fast-moving nucleon in the $+z$ direction then (1) is highly improbable. Case (2) does not lead to any transverse effects like transverse gauge link or transverse broadening as all componets of k scale similarly. Moreover soft gluons give rise to the familiar Eikonal soft Wilson lines. In SCET soft contributions can be handled by field redefinitions [12] and they get factorized from the collinear sector. The remaining contribution comes from (3) where $p^- k^+ \simeq |\vec{k}_\perp|^2 \simeq k^2$.

In BJY, the exact scaling of the gluons is not specified. This issue will be addressed in some detail below. For now, we continue the review of their work. By making use of the Chisholm's representation, one obtains the following form for the propagator:

$$\frac{1}{2p^- k^+ + k_\perp^2 - i\varepsilon} = i\int_0^\infty d\tau e^{-i\tau(2p^- k^+ + k_\perp^2 - i\varepsilon)}, \quad (57)$$

where the left-hand side is obtained from the limit $\tau = 0$. With the above representation one is then able to carry out the integrations over k^+ for the amplitude of Fig. 6. One then gets the gluon field $A^\mu(x)$ (in the mixed coordinate-momentum representation) evaluated at A^μ ($x^- = 2\tau p^-$, $x^+ = 0$, $\vec{k}_\perp$). Now take the scaling limit $p^- \to \infty$ before performing the τ integrations. This sets the argument of the gluon field in the light-cone direction at infinity and all the remaining τ dependence is now in the exponent. Then perform the τ integration. This will result in a $|\vec{k}_\perp|^2$ in the denominator. By repeating the above set of manipulations one gets the following result for a multigluon exchange:

$$\langle p^-, N|j_\nu(0)|P\rangle = (-1)^n(i)^n(ig_s)^n\bar{u}(p^-)\Pi_{i=1}^n\langle N|\int\frac{d^2\vec{k}_i}{(2\pi)^2} \times \not{A}(\infty,\vec{k}_i)\frac{\sum_{j=1}^i \vec{\not{k}}_{j\perp}}{|\sum_{j=1}^i\vec{k}_{j\perp}|^2 - i\varepsilon}\gamma_\nu\psi(0)|P\rangle. \quad (58)$$

In light-cone gauge it is Eq. (58) that gives the transverse gauge link. It is important to notice that the last result could be simply obtained by calculating, in full QCD, the amplitude for a Feynman diagram with arbitrary number of gluon attachments and then setting all the k^+ components of the gluon fields to 0 wherever they show up. The last equation is only valid if one takes the scaling limit first which amounts to setting all the k^+ to 0. This procedure clearly violates the Glauber scaling as one has to maintain the relative scalings of k^+ and $|\vec{k}|^2/(2p^-)$ intact.

The transverse gauge link that results from final-state interactions was also derived with a somewhat different set of manipulations in [43], however, the basic observation is still the same: it is gluons with vanishing k^+ that give rise to that gauge link. Another important issue related to the correct definition of the TMDPDF was raised in Refs. [43,44]. There it was claimed that a soft factor, built up of soft Wilson lines, needs to be subtracted from the standard definition of the TMDPDF so as to get the desired features of the anomalous dimension of the TMDPDF. Soft factor subtractions were also discussed in the traditional literature of pQCD [45–48] where this subtraction is aimed to avoid double counting among mainly the collinear and soft contributions. In the effective field approach the double counting issue was treated within the "zero-bin" subtraction [49] and later on a connection was made with the pQCD one [50–52]. It will be interesting to see whether the arguments of soft subtraction based on the anomalous dimension arguments are equivalent to the double counting issue. We will not discuss this issue further here and we leave it to a future work.

One Glauber exchange

We start our analysis of the TMDPDF in light-cone gauge $\bar{n}\cdot A = 0$ by considering the relatively simple case of only one Glauber gluon interacting with a collinear quark. The kinematics are that of DIS: the incoming quark is collinear in the $+z$ direction with p^+ large and of order of Q. This quark carries a longitudinal momentum fraction x and carries transverse momentum $\vec{k}_\perp$. The virtual photon moves in the $-z$ direction with momentum $q = (0, 0, 0, -Q)$. Thus after the hard interaction the quark has momentum with $p^- = Q/\sqrt{2}$ and transverse momentum $\vec{k}_\perp$. The Glauber gluon has also $\vec{k}_\perp$ so the outgoing quark has only p^-. Following the notation of BJY we consider the contribution of Fig. 6 to the matrix element $\langle p^-, N|j_\nu(0)|P\rangle$.

For Glauber gluons we now use the Feynman rules given in Fig. 3. We get

$$I_1 = g_s\bar{\xi}_{\bar{n}}\frac{\gamma^{\mu\perp}}{2p^-}\int\frac{dk^+}{2\pi}\int\frac{d^2\vec{k}_\perp}{(2\pi)^2} \times \frac{\not{k}_\perp}{k^+ + \frac{|\vec{k}_\perp|^2}{2p^-} - i\varepsilon}A_{\mu\perp}(k^+,\vec{k}_\perp), \quad (59)$$

where we used $\bar{u}\frac{\not{n}\not{\bar{n}}}{2} = \bar{u}$. We now invoke the Fourier transform of $A(k^+,\vec{k}_\perp)$:

$$A_{\mu\perp}(k^+,\vec{k}_\perp) = \int dx^-\int d^2\vec{x}_\perp\tilde{A}_{\mu\perp}(x^-,\vec{x}_\perp)e^{i(k^+x^- - \vec{k}_\perp\cdot\vec{x}_\perp)}, \quad (60)$$

and substitute for $A(k^+,\vec{k}_\perp)$ into Eq. (59) and carry the k^+ integral by contour integration picking up the pole from $k^+ = -\frac{|\vec{k}_\perp|^2}{2p^-} + i\varepsilon$. The result is (from now on we drop the tilde on $\tilde{A}$),

$$I_1 = ig_s\bar{\xi}_{\bar{n}}\frac{\gamma^{\mu\perp}}{2p^-}\int dx^-\theta(x^-)\int d^2\vec{x}_\perp A_{\mu\perp}(x^-,\vec{x}_\perp) \times\int\frac{d^2\vec{k}_\perp}{(2\pi)^2}\not{k}_\perp e^{-i([|\vec{k}_\perp|^2x^-/2p^-]-\vec{k}_\perp\cdot\vec{x}_\perp)}. \quad (61)$$

In order to carry out the integral over $d^2\vec{k}_\perp$ we complete the square in the exponent and shift the integration variable to $\vec{k}'_\perp = \vec{k}_\perp + \frac{p^-}{x^-}\vec{x}_\perp$. The resulting integral proportional to $\vec{k}'_\perp$ vanishes by symmetry. The remaining $d^2\vec{k}_\perp$ integral is obtained as

$$\begin{aligned} &= \int_{-\infty}^{\infty}\frac{d^2\vec{k}_\perp}{(2\pi)^2}e^{-i(|\vec{k}_\perp|^2x^-/2p^-)} \\ &= \frac{2p^-}{x^-}\frac{1}{(2\pi)^2}\left(\int_{-\infty}^{\infty}dk_x e^{-ik_x^2x^-/2p^-}\right)^2 \\ &= \frac{2p^-}{x^-}\frac{1}{(2\pi)^2}\left((1-i)\sqrt{\frac{\pi}{2}}\right)^2 = \frac{-i}{2\pi}\frac{p^-}{x^-}. \end{aligned} \quad (62)$$

Introducing the above mentioned simplifications in Eq. (61), the result for I_1 reads

$$I_1 = g_s\frac{1}{2\pi}\frac{\gamma^{\mu\perp}}{2p^-}\int dx^-\theta(x^-)\int d^2\vec{x}_\perp\vec{\not{x}}_\perp A_{\mu\perp}(x^-,\vec{x}_\perp) \times\left(\frac{p^-}{x^-}\right)^2 e^{-ip^-\vec{x}_\perp|^2/2x^-}. \quad (63)$$

Let us consider the x^- integral:

$$\tilde{I} = \int dx^-\theta(x^-)A_{\mu\perp}(x^-,\vec{x}_\perp)\left(\frac{p^-}{x^-}\right)^2 e^{-ip^-|\vec{x}_\perp|^2/2x^-}, \quad (64)$$

and perform the integration by parts. To do so we notice that in light-cone gauge the x^- dependence of $A_{\mu\perp}$ on x^- is just a $\theta(x^-)$ [53–55]. [See Eq. (68) below.] Moreover the contribution from a highly oscillating phase (obtained from the lower limit of the integral) would give a vanishing contribution upon integration over the transverse coordinates. The result of the integral over x^- is then given as

$$\tilde{I} = \frac{2ip^-}{|\vec{x}_\perp|^2} \times A_{\mu\perp}(\infty, \vec{x}_\perp). \tag{65}$$

Substituting the above result in the expression for I_1, we obtain

$$I_1 = ig_s \frac{1}{2\pi} \gamma^{\mu\perp} \int d^2\vec{x}_\perp \frac{\vec{\not{x}}_\perp}{|\vec{x}_\perp|^2} A_{\mu\perp}(\infty, \vec{x}_\perp). \tag{66}$$

In what follows, we decompose, $\vec{x}_\perp$ as $\vec{x}_\perp = |\vec{x}_\perp|\hat{n}_\theta$ where $\hat{n}_\theta \equiv (\cos\theta, \sin\theta)$. The integration measure is given as $d^2\vec{x}_\perp = |\vec{x}_\perp| d|\vec{x}_\perp| d\theta$. Substitution in the expression for I_1 leads to the form,

$$I_1 = ig_s \frac{1}{2\pi} \gamma^{\mu\perp} \gamma^i \int_0^{2\pi} d\theta \int_0^\infty d|\vec{x}_\perp| (\hat{n}_\theta)_i A_{\mu\perp}(\infty, \vec{x}_\perp), \tag{67}$$

with $i = 1, 2$. Using the trivial relation: $\gamma^{\mu\perp}\gamma^i = \frac{1}{2} \times ([\gamma^{\mu\perp}, \gamma^i] + \{\gamma^{\mu\perp}, \gamma^i\})$ and the fact that in light-cone gauge the gluon field $A_{\mu\perp}$ at $x^- \to \infty$ is a pure gauge [39], we obtain

$$A^\perp(x^- \to \infty, \vec{x}_\perp) = \theta(x^-)\vec{\nabla}\phi(r), \tag{68}$$

where $r \equiv |\vec{x}_\perp|$ and ϕ is an arbitrary scalar function. Then we have

$$A^\perp(x^- \to \infty, \vec{x}_\perp) = \frac{d\phi}{dr}\hat{n}_\theta, \tag{69}$$

which shows that the gluon field $A_\perp$ is directed in the radial direction. With this, it is straightforward to show that the contribution from the commutator of the γ matrices vanishes by symmetry. Thus we obtain

$$\begin{aligned} I_1 &= ig_s \frac{1}{2\pi} \int_0^{2\pi} d\theta \int_0^\infty d|\vec{x}_\perp| (\hat{n}_\theta)_i A_i^\perp(\infty, \vec{x}_\perp) \\ &= ig_s \int_0^\infty d|\vec{x}_\perp| \hat{n} \cdot A(\infty, \vec{x}_\perp), \end{aligned} \tag{70}$$

where, the second equality in the equation above is derived from the use of Eq. (69).

The above analysis shows that it is indeed the Glauber gluons, arising from final-state interactions, that build the transverse gauge link. In this sense, any (gauge-invariant) effective field theory formulation, of the TMDPDF, in particular, (see e.g. Ref. [56]) or semi-inclusive hadronic processes in general, requires the introduction of the Glauber mode in addition to the soft and collinear modes.

Two remarks are in order. We first notice that the above treatment could also be carried out with a multiple of Glauber gluons attachments. The power counting of such Feynman diagrams would still be leading since those contributions arise from the leading order Lagrangian. The sum of all those contributions would give the transverse gauge link. Second, we notice that the power counting of the Glauber gluon field, in light-cone gauge could be read off from Eq. (69). In Feynman gauge, the power counting of A^+ could also be read-of from explicit expressions [see, e.g., Eq. (14) in [30]].

VI. CONCLUSIONS

Effective theories now constitute a mainstay in the collection of theoretical methods used to apply perturbative QCD to phenomenological questions. The soft collinear effective theory has been identified as a rigorous and systematic effective approach in the application to phenomena involving hard jets in vacuum. In this article, we have instituted the first extension of this ‘‘leading twist’’ effective theory to include power corrections from the medium. As a guide to understanding the effects of the medium on a jet, we have focused on a description of the rescattering encountered by a hard quark produced in deep-inelastic scattering on a nucleon in vacuum or within a large nucleus. While most of the results derived in this manuscript are immediately applicable to quark jets propagating through confined media, these may be straightforwardly extended to gluon jets as well as to propagation in deconfined media.

A jet in SCET is endowed with a very particular relation as regards the range of its different momentum components,

$$p^\mu \equiv [p^+, p^-, \vec{p}_\perp] \sim Q[\lambda^2, 1, \lambda], \tag{71}$$

where, we have specified the case for a jet moving in the ($-$) direction with Q, a hard scale and λ a small parameter. The virtuality of this jet allows it to resolve modes in the medium with transverse momentum $k_\perp \sim \lambda Q$. If the forward or ($+$) momentum components of these in-medium modes scale as Q (or even as λQ), this will result in an intermediate parton with large off-shellness of the order of Q^2 (or λQ^2) and almost immediate hard radiation with large transverse momentum. Such interactions will, no doubt, change the large momentum label of the propagating SCET mode and will be dealt with in more detail in a future effort. If the forward momenta scale as $\lambda^2 Q$, the off-shellness of the propagating mode remains within the scaling prescribed by SCET and as a result, the simplest extension to this vacuum theory is suggested in Sec. II: the interaction between hard collinear quarks (or gluons) with gluons in the medium which scale as

$$k^\mu \sim Q[\lambda^2, \lambda^2, \lambda]. \tag{72}$$

Such gluons are referred to as Glauber gluons and in Sec. III we have constructed the effective Lagrangian which describes their interactions with the hard collinear modes. Although we have denoted the scaling of the k^- momentum to be $\lambda^2 Q$, it could indeed have any scaling $k^- \lesssim Q$ i.e., not be a hard collinear mode traveling in the ($-$) direction. In the Breit frame with the medium moving with a large boost $\gamma \sim \lambda^{-2}$ in the $(+)$ direction, such modes are energetically disfavored.

Since Glauber modes carry a small fraction of the forward energy of the nucleon being struck by the hard jet moving in the $(-)$ direction, they are quite pervasive and thus the inclusion of such modes and their interactions with the hard collinear modes is rather important. Such interactions occur continuously on a hard jet propagating through a dense medium. Given their off-shellness, SCET modes may traverse distances of the order of $(\lambda^2 Q)^{-1}$ before decay. When hard jets traverse large distances in dense matter, their total transverse momentum distribution is broadened. As a first application of the effective Glauber Lagrangian, this transverse broadening is derived for the case of DIS on a large nucleus in Sec. IV. Multiple interactions with Glauber modes may eventually lead to the generation of off-shellness or transverse momenta beyond the range of applicability of SCET and a completely different effective theory will have to be constructed. The transverse broadening as a function of the distance travelled allows for an estimation of the range in size of media, within which the effective theory will remain applicable. This is estimated in Sec. IV with the aid of some phenomenological input. It is argued that the derived effective theory has a wide range of applicability which may easily encompass jet propagation in the cold confined matter in large nuclei to that in hot deconfined matter created in high-energy heavy-ion collisions.

In Sec. V, as a second example of the role of Glauber gluons in hard processes, we have considered the treatment, in full QCD, of Belitsky, Ji and Yuan for the TMDPDF. In their calculation, they have shown that gluons with solely transverse momentum components build up a transverse gauge link which should be an integral part of the gauge-invariant definition of the TMDPDF. Their analysis seemed to depend on taking the scaling limit first. In the current effort, we have demonstrated that the transverse gauge link may also be derived by keeping the Glauber scaling of gluon momenta between k^+ and $|\vec{k}_\perp|^2/2p^-$ explicit throughout the calculation. The full link structure, in any gauge, has been shown to arise from a combination of collinear and Glauber gluons.

As a final remark we address certain situations where the Glauber gluons do not contribute. A standard example would be DIS on a nucleon with its related physical quantities: The quark form factor and the PDF. It has been demonstrated that the only relevant modes that produce the infrared behavior of QCD for DIS are the soft and collinear (see, e.g, [57] and references therein). Also for the factorization of the PDF itself (in the large x limit) similar arguments and conclusions have been given in Ref. [58]. The fact that Glauber gluons do not contribute to the factorization of DIS on a nucleon could, in principle, be shown (in the effective field theory approach) when one combines the soft, collinear, and Glauber in one framework and then certain cancellations of the Glauber contributions should become manifest. This is a somewhat more involved topic, which we leave for a future effort.

One may also compute the corrections to the single hadron inclusive cross section in DIS on a nucleon from the Glauber sector. In large nuclei, the produced jets tend to have a distribution in transverse momentum which is much wider than the case of DIS on a nucleon. It is well established that such corrections are not leading and are power suppressed, hence are unimportant in the case of DIS on a nucleon. This result is also consistent with the Glauber Lagrangian derived in the current manuscript. The magnitude of the correction from Glauber scattering may be estimated from Eq. (53) by setting $A = 1$. It is clear that the $\langle p_\perp^2 \rangle$ generated is suppressed by λ^2 compared to that in a purely SCET process. Thus in the computation of the single hadron inclusive cross section from DIS on a nucleon, the contribution of the Glauber Lagrangian is power suppressed. In the case of the Drell-Yan process, the relevance of Glauber gluons is a more complicated issue. We will not discuss it further and refer the reader instead to Refs. [5,59–61].

In future efforts, the interaction between the Glauber modes emanating from the medium and the soft and collinear gluons of the SCET Lagrangian will have to be derived. This will represent the first complete theoretical description of jets with off-shellness in the range of $(\lambda Q)^2$ propagating through dense media. The setup of such a formalism will allow for the first systematic approach to such difficult problems such as factorization in hard jet production and modification in heavy-ion collisions. Embellishments of the heavy quark effective theory with Glauber modes will lead to more rigorous formulations of heavy-quark propagation in dense matter. Such extensions of SCET and heavy quark effective theory will lead to important advancements in our understanding of parton propagation and energy loss in dense matter and will no doubt play a leading role in the detailed theory and experimental comparison currently underway in DIS and heavy-ion collisions.

ACKNOWLEDGMENTS

The authors wish to thank X. Ji, C. Kim, and X. N. Wang for helpful discussions. The authors also thank T. Mehen and B. Müller for a careful reading of an earlier version of the manuscript and for discussions. The research of A. I. was supported in part by the U.S. Department of Energy under Grants No. DE-FG02-05ER41368, No. DE-FG02-05ER41376, and No. DE-AC05-84ER40150. The research of A. M. was supported in part by the U.S. Department of Energy under Grants No. DE-FG02-05ER41367 and No. DE-FG02-01ER41190.

[1] J.C. Collins, D.E. Soper, and G. Sterman, *Perturbative Quantum Chromodynamics*, edited by A.H. Mueller (World Scientific, Singapore, 1989), p. 1.
[2] R.K. Ellis, H. Georgi, M. Machacek, H.D. Politzer, and G.G. Ross, Phys. Lett. **78B**, 281 (1978); Nucl. Phys. **B152**, 285 (1979).
[3] S.B. Libby and G. Sterman, Phys. Rev. D **18**, 3252 (1978); **18**, 4737 (1978).
[4] J.C. Collins and D.E. Soper, Nucl. Phys. **B194**, 445 (1982).
[5] J.C. Collins, D.E. Soper, and G. Sterman, Phys. Lett. **134B**, 263 (1984).
[6] J.C. Collins, D.E. Soper, and G. Sterman, Nucl. Phys. **B261**, 104 (1985).
[7] C.W. Bauer, S. Fleming, and M.E. Luke, Phys. Rev. D **63**, 014006 (2000).
[8] C.W. Bauer, S. Fleming, D. Pirjol, and I.W. Stewart, Phys. Rev. D **63**, 114020 (2001).
[9] C.W. Bauer and I.W. Stewart, Phys. Lett. B **516**, 134 (2001).
[10] M. Beneke, A.P. Chapovsky, M. Diehl, and T. Feldmann, Nucl. Phys. **B643**, 431 (2002).
[11] J. Chay and C. Kim, Phys. Rev. D **65**, 114016 (2002).
[12] C.W. Bauer, D. Pirjol, and I.W. Stewart, Phys. Rev. D **65**, 054022 (2002).
[13] C.W. Bauer, S. Fleming, D. Pirjol, I.Z. Rothstein, and I.W. Stewart, Phys. Rev. D **66**, 014017 (2002).
[14] X. Ji, J.W. Qiu, W. Vogelsang, and F. Yuan, Phys. Lett. B **638**, 178 (2006).
[15] J.w. Qiu and G. Sterman, Nucl. Phys. **B353**, 105 (1991).
[16] J.w. Qiu and G. Sterman, Nucl. Phys. **B353**, 137 (1991).
[17] M. Luo, J.w. Qiu, and G. Sterman, Phys. Lett. B **279**, 377 (1992).
[18] M. Luo, J.w. Qiu, and G. Sterman, Phys. Rev. D **50**, 1951 (1994).
[19] X.F. Guo and X.N. Wang, Phys. Rev. Lett. **85**, 3591 (2000); X.N. Wang and X.F. Guo, Nucl. Phys. **A696**, 788 (2001).
[20] A. Majumder, E. Wang, and X.N. Wang, Phys. Rev. Lett. **99**, 152301 (2007); A. Majumder and X.N. Wang, arXiv:0806.2653.
[21] A. Airapetian *et al.* (HERMES Collaboration), Eur. Phys. J. C **20**, 479 (2001); A. Airapetian *et al.* (HERMES Collaboration), Phys. Lett. B **577**, 37 (2003).
[22] K. Adcox *et al.* (PHENIX Collaboration), Nucl. Phys. **A757**, 184 (2005); J. Adams *et al.* (STAR Collaboration), Nucl. Phys. **A757**, 102 (2005).
[23] E. Wang and X.N. Wang, Phys. Rev. Lett. **89**, 162301 (2002).
[24] A. Majumder, C. Nonaka, and S.A. Bass, Phys. Rev. C **76**, 041902 (2007).
[25] R. Baier, Y.L. Dokshitzer, A.H. Mueller, S. Peigne, and D. Schiff, Nucl. Phys. **B483**, 291 (1997); **B484**, 265 (1997).
[26] M. Gyulassy, P. Levai, and I. Vitev, Phys. Rev. Lett. **85**, 5535 (2000); Nucl. Phys. **B594**, 371 (2001).
[27] U.A. Wiedemann, Nucl. Phys. **B588**, 303 (2000).
[28] A. Majumder, J. Phys. G **34**, S377 (2007).
[29] P. Arnold, G.D. Moore, and L.G. Yaffe, J. High Energy Phys. 06 (2002) 030; S. Jeon and G.D. Moore, Phys. Rev. C **71**, 034901 (2005); S. Turbide, C. Gale, S. Jeon, and G.D. Moore, Phys. Rev. C **72**, 014906 (2005).
[30] X. Ji and F. Yuan, Phys. Lett. B **543**, 66 (2002).
[31] F. Liu and J.P. Ma, arXiv:0802.2973.
[32] A. Majumder and B. Muller, Phys. Rev. C **77**, 054903 (2008).
[33] A.H. Mueller, Nucl. Phys. **B335**, 115 (1990); **B415**, 373 (1994).
[34] L.D. McLerran and R. Venugopalan, Phys. Rev. D **49**, 2233 (1994); **49**, 3352 (1994).
[35] J. Osborne and X.N. Wang, Nucl. Phys. **A710**, 281 (2002).
[36] A. Majumder and X.N. Wang, arXiv:0806.2653.
[37] J.C. Collins, Nucl. Phys. **B396**, 161 (1993).
[38] X. Ji, J.P. Ma, and F. Yuan, Phys. Rev. D **71**, 034005 (2005).
[39] A.V. Belitsky, X. Ji, and F. Yuan, Nucl. Phys. **B656**, 165 (2003).
[40] S.J. Brodsky, P. Hoyer, N. Marchal, S. Peinge, and F. Sannino, Phys. Rev. D **65**, 114025 (2002).
[41] S.J. Brodsky, D.S. Hwang, and I. Schmidt, Phys. Lett. B **530**, 99 (2002).
[42] S.J. Brodsky, D.S. Hwang, and I. Schmidt, Nucl. Phys. **B642**, 344 (2002).
[43] I.O. Cherednikov and N.G. Stefanis, Nucl. Phys. **B802**, 146 (2008).
[44] I.O. Cherednikov and N.G. Stefanis, Phys. Rev. D **77**, 094001 (2008).
[45] R. Akhoury, M.G. Sotiropoulos, and G. Sterman, Phys. Rev. Lett. **81**, 3819 (1998).
[46] J.C. Collins and F. Hautmann, Phys. Lett. B **472**, 129 (2000).
[47] J.C. Collins and F. Hautmann, J. High Energy Phys. 03 (2001) 016.
[48] F. Hautmann, Phys. Lett. B **655**, 26 (2007).
[49] A.V. Manohar and I.W. Stewart, Phys. Rev. D **76**, 074002 (2007).
[50] A. Idilbi and T. Mehen, Phys. Rev. D **75**, 114017 (2007).
[51] A. Idilbi and T. Mehen, Phys. Rev. D **76**, 094015 (2007).
[52] C. Lee and G. Sterman, Phys. Rev. D **75**, 014022 (2007).
[53] R. Jackiw, D. Kabat, and M. Ortiz, Phys. Lett. B **277**, 148 (1992).
[54] I. Robinson and K. Rozga, J. Math. Phys. (N.Y.) **25**, 499 (1984).
[55] G. 't Hooft, Phys. Lett. B **198**, 61 (1987).
[56] J. Chay, arXiv:0711.4295.
[57] G. Sterman, arXiv:hep-ph/9606312.
[58] X. Ji, J.P. Ma, and F. Yuan, Phys. Lett. B **610**, 247 (2005).
[59] G.T. Bodwin, Phys. Rev. D **31**, 2616 (1985); **34**, 3932(E) (1986).
[60] R. Doria, J. Frenkel, and J.C. Taylor, Nucl. Phys. **B168**, 93 (1980); J. Frenkel, J.G.M. Gatheral, and J.C. Taylor, Nucl. Phys. **194**, 172 (1982).
[61] R.J. Fries, A. Schafer, E. Stein, and B. Muller, Nucl. Phys. **B582**, 537 (2000); Phys. Rev. Lett. **83**, 4261 (1999); R.J. Fries, Phys. Rev. D **68**, 074013 (2003).