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Long-distance behavior of the quark-antiquark static potential. Application to light-quark mesons and heavy quarkonia

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Screening effects from sea pairs on the quark-antiquark static potential are analyzed phenomenologically from the light-quark to the heavy-quark meson spectra. From the high excited light-quark meson spectrum, a universal form for the screened static potential is proposed. This potential is then successfully applied to heavy quarkonia. Our results suggest the assignment of $X(4260)$ to the $4s$ state of charmonium and the possible existence of a $5s$ bottomonium resonance around 10748 MeV.

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I. INTRODUCTION

One remaining problem in our understanding of QCD has to do with quark confinement in hadrons. We expect confinement to be the dominant quark-antiquark ($q - \bar{q}$) or quark-quark interaction at large separation distances and therefore to be determinant to explain the properties of highly excited (large sized) hadrons. In recent years, there has been important progress in the knowledge of the spectrum of highly excited mesons in the light-quark (u, d) [1] as well as in the heavy-quark (c, b) [2,3] sectors. In particular, highly excited light-quark mesons show an intriguing hydrogenlike spectral pattern [4] which can be reproduced, within a nonrelativistic constituent quark model framework, by means of a static $q - \bar{q}$ interaction which becomes asymptotically Coulombic [5]. Here, we suggest that this asymptotic behavior has to do with confinement (nonperturbative gluonic effects) and not with the perturbative gluonic Coulomb interaction as suggested in [5]. We propose that string breaking gives rise, from a linear confining interaction, to an asymptotically Coulombic potential. When this screened confinement is complemented with a screened ‘‘gluonic’’ Coulomb interaction, an accurate description of the highly excited light-quark meson spectrum is achieved. The resulting static potential depends on four parameters. The two entering in the confinement term—the string tension and the string breaking distance—are considered universal in the sense of having the same values in all meson sectors. This is also the case for the orbital-angular-momentum parameter related to the onset for states of confinement. The remaining parameter, the effective gluonic Coulomb strength, obtains different values when going from light to heavier quarks. It should be pointed out that the potential does not contain an additive constant. The calculated meson masses, obtained by adding the mass of the quark and the mass of the antiquark to the eigenvalues of the Schrödinger equation, are directly compared to the experimental meson masses. In practice, for equal quark and antiquark masses, the quark mass and the effective gluonic Coulomb strength are fixed from two well-established experimental meson masses in the region of applicability of the static approach.

The successful spectral description obtained in the light-quark meson case can be extended to other meson sectors, in particular, to heavy quarkonia where an accurate description of the highly excited states may be very helpful for an unambiguous quantum numbers assignment. Our results suggest that $X(4260)$ could be the $4s$ state of charmonium and that a noncataloged bottomonium resonance around 10748 MeV might exist.

The contents of this article are organized as follows. In Sec. II we establish the general character of the constituent quark model approximation that we use and the general criterion of validity of the static potential in the different meson sectors. In Sec. III the asymptotically Coulombic potential inferred from the study of the highly excited light-quark meson spectrum is derived from a screened confinement potential ansatz. The implementation of an effective gluonic interaction and the consideration of an additional L-dependent correcting factor for it allow for a precise description of the known static spectrum. In Sec. IV the same potential is applied to heavy quarkonia—charmonium and bottomonium—where a distinct quantum numbers assignment for highly excited states comes out. Finally, in Sec. V we summarize our main results and conclusions.

II. QUARK MODEL APPROACH

In the study of the meson spectra, from light to heavy quarkonia, we shall rely on a nonrelativistic constituent quark model (NRCQM) framework. We will solve the Schrödinger equation for a static potential. Although the application of the NRCQM to heavy-quark systems, at least for bottomonium, can be taken for granted, its application to light-quark systems ($m_u = 340$ MeV) is always a matter of debate. In the spirit of NRCQM calculations, the effective values of the parameters take into account, at least to some extent, relativistic corrections in the kinetic and potential energies. Actually, it has been recently shown [5] that the known spectrum of highly excited light-quark mesons can be nicely reproduced within such a calculation framework despite its very relativistic character indicated by the calculated values of $p_u/m_u \gtrsim 1$.

The main distinctive feature of the effective potential employed in [5] is its asymptotically Coulombic tendency. Explicitly,

$$V_{\text{light-quark}}(r \to \infty) = \sigma_u r_s - \frac{k_u}{r} + C_u, \quad (1)$$

where $\sigma_u = 932.7$ MeV/fm stands for the string tension, $k_u = 2480$ MeV $\cdot$ fm for a Coulomb strength, and $C_u = 1070$ MeV for a constant to fix the origin of the potential. The distance r_s represents the onset for the screening of the interaction due to the presence of light quark-antiquark pairs popping out of the vacuum. The value chosen $r_s = 1.15$ fm is inferred from lattice calculations (see [6], and references therein). As for σ_u, the value used is extracted from the phenomenological analysis of the $(\rho, a_2, \ldots,)$ Regge trajectory (see also [6]). Concerning the value of k_u, one could tentatively try to ascribe it to the chromo-electric one gluon exchange (OGE) interaction as done in [5]. However, as we shall show in the next section, it may be rather giving to account for the long distance attenuation, due to string breaking, of the linear confining term.

It should be recalled that the accurate energy description of the meson states is linked to the correct long distance behavior of their wave functions. Given the relativistic character of the fitted spectrum, we should not trust much, at intermediate and short distances, the nonrelativistic wave functions obtained from the Schrödinger equation. Only very large sized light-quark mesons [in our scheme the higher the root mean square (rms) radius of the meson, the lower the p_q/m_q value], for which there are no available data yet, can be considered nonrelativistic systems. For them, the wave function coming out from the Schrödinger equation might also be accurate at short and intermediate distances.

For the sake of completeness, it is worthwhile to recall the criterion derived in [5] for the applicability of a static potential to a meson $(q\bar{q})$ sector within our NRCQM framework. It reads

$$\langle r^2 \rangle^{1/2} \gg \frac{1}{m_q}, \quad (2)$$

where $\langle r^2 \rangle^{1/2}$ stands for the rms radius of the meson state. So only for mesons with a large size, as compared to $1/m_q$, the static approach makes sense. For u and d quarks ($m_q = 340$ MeV) this means $\langle r^2 \rangle^{1/2} \gg 0.6$ fm. In fact, the light-quark meson spectrum has been well reproduced for states with rms radii greater than or equal to three and a half times this limit: $\langle r^2 \rangle^{1/2} \gtrsim 2.1$ fm.

When going to heavier quarkonia, we have $\langle r^2 \rangle^{1/2} \gg 0.4$ fm for $s\bar{s}$ ($m_s \sim 500$ MeV), $\langle r^2 \rangle^{1/2} \gg 0.14$ fm for $c\bar{c}$ ($m_c \sim 1400$ MeV), and $\langle r^2 \rangle^{1/2} \gg 0.04$ fm for $b\bar{b}$ ($m_b \sim 4800$ MeV), where typical values for the constituent quark masses have been chosen. Then, by using the same validity factor of 3.5 as in the light-quark case, we expect the static approximation to be valid for $(\langle r^2 \rangle^{1/2})_{s\bar{s}} \gtrsim 1.6$ fm, $(\langle r^2 \rangle^{1/2})_{c\bar{c}} \gtrsim 0.6$ fm, and $(\langle r^2 \rangle^{1/2})_{b\bar{b}} \gtrsim 0.16$ fm.

Notice though that the constituent quark mass in a meson sector is a parameter to be fixed from data in our model and that the values of $\langle r^2 \rangle^{1/2}$ result from the solution of the Schrödinger equation. Therefore the established criterion has to be checked *a posteriori*. Nonetheless, its consideration is essential to adequately select the specific data to be used to fix the free parameters. Thus, for light-quark mesons, data corresponding to $L = 4$ and $L = 5$ states were used, since these states are expected to have large rms radii due to the presence of the centrifugal barrier.

III. LIGHT-QUARK MESONS

A. String breaking

The static $q - \bar{q}$ potential can be derived from lattice QCD [6]. In the quenched approximation, only valence quark q_v and antiquark $\bar{q}_v$, it has the funnel form

$$\bar{V}(r) = \sigma r - \frac{\zeta}{r}, \quad (3)$$

where σ is the string tension and ζ is the strength of the Coulomb interaction. This potential has to be corrected at short distances so that ζ becomes a function of r [7]. When including sea quarks, an unquenched potential results from the screening of the static sources q_v and $\bar{q}_v$ by light $q\bar{q}$ pairs created in the hadronic vacuum. A parametrization of this effect was proposed 20 years ago [8]. Unquenched lattice results for the potential between two heavy static quarks separated by a distance r: $0 \to 1$ fm were described by the potential

$$\bar{V}_{\text{scr}}(r) = \left(\sigma r - \frac{\zeta}{r}\right)\left(\frac{1 - e^{-\mu r}}{\mu r}\right), \quad (4)$$

where μ^{-1} represented a screening length and ζ was related to the quark-quark-gluon coupling α_s through $\zeta = (4/3)\alpha_s$. The screening factor $H(r) \equiv (\frac{1-e^{-\mu r}}{\mu r})$ was constructed so that $\bar{V}_{\text{scr}}(r)$ has a Coulombic behavior at small distances while approaching a constant at large distances. When applied to heavy quarkonia, this potential form with effective values of its parameters provided a precise description of the spectrum of $b\bar{b}$ states with rms radii smaller than 1.1 fm [9,10]. However, up until now, lattice calculations did not allow extraction of the precise form of the QCD static potential at large distances [6,11]. So the asymptotic constant behavior should be considered as an educated guess.

Alternatively, an attenuated linear form of confinement has been implemented for the asymptotic potential in the framework of the QCD string approach (QCDSA) that has been successfully applied to light-quark [12,13] and heavy-quark [14,15] mesons. In this physical picture, the string tension σ is attenuated at separations $r \gtrsim R_1 \simeq 1.2$ fm, becoming a function of r so that for $r \gtrsim R_2 = 2.5$ fm, string breaking occurs with large probability. This attenuation plays a key role to correctly obtain the masses of the radial excitations of light-quark mesons within this approach. More explicitly, the confinement potential reads

$$V_{SA}(r) = \sigma(r)r = \sigma r\left(1 - \gamma \frac{\exp(\sqrt{\sigma}(r - R_1)}{B + \exp(\sqrt{\sigma}(r - R_1))}\right), \quad (5)$$

with $\sigma = 0.185\ \mathrm{GeV}^2 = 937.5$ MeV/fm, $\gamma = 0.4$, $R_1 = 6\,\mathrm{GeV}^{-1} = 1.18$ fm, and $B = 20$. The screening factor between parenthesis, which will be called $G(r)$ henceforth, varies from $\simeq 1$ for $r = 0$ to a value of $\simeq (1 - \gamma)$ for $r > R_2$.

Following the same philosophy as in [13], we shall attempt to extract information over the variation of the confining potential with r from a systematic analysis of the meson spectrum within our NRCQM framework. As mentioned above, the main distinct feature resulting from the application of the NRCQM to the light-quark meson spectrum is the Coulombic asymptotic behavior of the potential as given by Eq. (1). It is then interesting to examine the possibility that it may come from confinement as a result of string breaking. Indeed, the form of the potential in Eq. (1) can be derived from the screened confinement potential ansatz

$$V_{\mathrm{conf}}(r) = \sigma r(1 - e^{-(\nu/r)}) \equiv \sigma r F(r), \quad (6)$$

as can be easily checked by using $F(r) \to \frac{\nu}{r} - \frac{\nu^2}{2r^2}$ and making the identifications (up to order $1/r^2$)

$$\sigma\nu = \sigma_u r_s + C_u, \quad (7)$$

$$\frac{\sigma\nu^2}{2} = k_u. \quad (8)$$

Then from the numerical values of σ_u, C_u, and k_u previously quoted, we get

$$\nu = 2.3\ \mathrm{fm}, \quad (9)$$

$$\sigma = 925.5\ \mathrm{MeV/fm}. \quad (10)$$

Let us realize that the value of σ stays within the uncertainty interval of the phenomenological string tension extracted from the $\rho, a_2, \ldots,$ Regge trajectory as it should. Regarding $\nu \simeq 2r_s$, note its similarity to $R_2 \simeq 2R_1$ in the QCDSA. In the same manner, ν can be interpreted as the onset for string breaking to occur with large probability. We should realize though, that for $r > R_2 = 2.5$ fm, $G(r)$ keeps an almost constant value; whereas $F(r)$ varies in a Coulombic way.

It is interesting to establish, from the comparison of the spectrum obtained from $V_{\mathrm{conf}}(r)$, Eq. (6), with data, whether the onset for the states of confinement in our model may have been experimentally reached or not. These states may be characterized for having a vanishing probability of presence for $r < r_c$ being r_c, a distance related to the confinement scale in QCD. From the quantum number standpoint, this means that meson states have orbital angular momentum L greater than or equal to a value L_c. A comparison of the light-quark meson spectrum obtained from $V_{\mathrm{conf}}(r)$ with data shows that for $L = 4$, for instance, the calculated mass is more than 100 MeV above the upper limit of the experimental interval. Although this difference between calculation and experiment increases when decreasing L, or equivalently decreases when increasing L, the significant discrepancy for $L = 4$ might suggest that we are still far from the pure confinement region, i.e. $L_c \gg 4$.

B. Phenomenological static potential

In order to accurately describe the known meson spectra, the unquenched confinement potential $V_{\mathrm{conf}}(r)$, Eq. (6), has to be complemented. The natural way to do it is through the incorporation of an effective gluonic Coulomb interaction so that one recovers at short distances, when the effect of $q\bar{q}$ pairs is negligible, the quenched (funnel) form of the potential. For the sake of simplicity, we shall assume the same screening factor used for confinement. Thus the potential reads

$$V_{sb}(r) = \left(\sigma r - \frac{\bar{\lambda}_q}{r}\right)(1 - e^{-(\nu/r)}), \quad (11)$$

where $\bar{\lambda}_q$ is the gluonic Coulomb strength. The subindex sb indicates that string breaking has been implemented in both terms of the potential. Certainly, $\bar{\lambda}_q$ keeps some relation with the quark-quark-gluon coupling α_s, since the chromoelectric OGE contribution should be contained in it. However, corrections to the kinetic and potential energies could also be taken into account through the effective value of $\bar{\lambda}_q$. These corrections may include, for instance, relativistic terms in the kinetic energy and in the OGE potential, nonperturbative contributions to the confinement term and to the quark-quark-gluon coupling, etc. Therefore $\bar{\lambda}_q$ has to be considered as a free parameter. To fix it from data, we calculate the high excited light-quark meson spectrum and require that the known states with a high orbital angular momentum, $L = 4$ for instance, for which we expect the static approximation works well, are reproduced. The results for $\bar{\lambda}_u = 1065\ \mathrm{MeV} \cdot \mathrm{fm}$ are presented in Table I. For the multiplets, we have used the quantum numbers notation (L, n_r), n_r: radial quantum number, as derived from the solution of the Schrödinger equation. The mass in a multiplet is denoted as M_{L,n_r}. Only states giving rise to $\langle r^2\rangle^{1/2} \gtrsim 2.1$ fm, for which the static approximation makes sense and for which there are well-established experimental candidates, are considered. Data are taken from the Cristal Barrel Collaboration (CBC), Ref. [1], and from the Particle Data Group (PDG) review, Ref. [2]. The ordering of the states has been chosen to

TABLE I. Calculated masses M_{L,n_r} and rms radii $\langle r^2\rangle^{1/2}$ for (L, n_r) multiplets from $V_{sb}(r)$ with $m_u = 340$ MeV, $\sigma = 925.5$ MeV/fm, $\nu = 2.3$ fm, and $\bar{\lambda}_u = 1065$ MeV · fm. Experimental average masses as in [5] from Ref. [1], $((M)_{L,n_r})_{\rm CBC}$, and Ref. [2], $((M)_{L,n_r})_{\rm PDG}$, are shown for comparison. The superindex † in the (4, 1) calculated mass indicates the average mass value chosen to fix $\bar{\lambda}_u$.

(L, n_r)	$\langle r^2\rangle^{1/2}$ fm	M_{L,n_r} MeV	$((M)_{L,n_r})_{\rm CBC}$ MeV	$((M)_{L,n_r})_{\rm PDG}$ MeV
(5, 1)	3.8	2432		$a_6(2450 \pm 130)$
(4, 1)	2.8	$2281^{\dagger}$	2262 ± 28	$\rho_5(2330 \pm 35)$
(3, 2)	3.4	2302	2258 ± 38	
(2, 3)	3.9	2329	2248 ± 37	$\rho_3(2250)$
(2, 2)	2.5	2089	1980 ± 23	$\rho_3(1990 \pm 20)$
(1, 4)	4.3	2359	2219 ± 43	
(1, 3)	2.9	2143	1947 ± 47	

make the bias of the results clear: the lower the L, the bigger the difference between calculated masses and data.

This deficiency can be corrected in an *ad hoc* manner by introducing an additional L-dependent factor in the gluonic Coulomb term so that

$$V(r) = \left(\sigma r - \frac{\lambda_q\left(1 - \sqrt{\frac{\vec{L}^2}{L_c(L_c+1)}}\right)}{r}\right)(1 - e^{-(\nu/r)}) \quad \text{if } L \leq L_c,$$
$$V(r) = V_{\rm conf}(r) \quad \text{if } L \geq L_c. \tag{12}$$

This form for $V(r)$ satisfies effectively the requirement that for $L \geq L_c$, the contributions to the energy from other terms in the potential different than the confining interaction $V_{\rm conf}(r)$, Eq. (6), are negligible. Moreover, the potential for states with L close to below L_c differs little from $V_{\rm conf}(r)$ as it should. Note also that the lower the L, the bigger the probability for short quark-antiquark separations and the bigger the relativistic corrections to the potential and kinetic energies. Therefore the value of λ_q (corresponding to the strength for $L = 0$) may be incorporating, at least to some extent, such corrections in an effective manner. Actually, the same discussion done for $\bar{\lambda}_u$ can be repeated here about the effective character of the gluonic Coulomb strength λ_u. Therefore any attempt to identify our λ_u with the coefficient of the chromoelectric Coulomb potential obtained from the OGE in QCD is risky.

TABLE II. Calculated masses and rms radii from $V(r)$ with $m_u = 340$ MeV, $\sigma = 925.5$ MeV/fm, $\nu = 2.3$ fm, $\lambda_u = 1600$ MeV · fm, and $L_c = 16$. Notation as in Table I. The experimental candidates that will be members of the multiplets are also indicated.

(L, n_r)	$\langle r^2\rangle^{1/2}$ fm	M_{L,n_r} MeV	$((M)_{L,n_r})_{\rm CBC}$ MeV	$((M)_{L,n_r})_{\rm PDG}$ MeV
(5, 1)	3.8	2432		2450 ± 130
				$a_6(2450)$
(1, 4)	3.7	2256	2219 ± 43	
			$b_1(2240)$	
			$a_1(2270)$, $a_2(2175)$	
(2, 3)	3.5	2252	2248 ± 37	
			$\pi_2(2245)$, $\rho(2265)$	
			$\rho_2(2225)$, $\rho_3(2260)$	$\rho_3(2250)$
(3, 2)	3.1	2250	2258 ± 38	
			$b_3(2245)$, $a_2(2255)$	
			$a_3(2275)$, $a_4(2255)$	
(4, 1)	2.7	2254	2262 ± 28	2330 ± 35
			$\pi_4(2250)$, $\rho_3(2260)$	
			$\rho_4(2230)$, $\rho_5(2300)$	$\rho_5(2350)$
(1, 3)	2.5	1967	1947 ± 47	
			$b_1(1960)$	
			$a_1(1930)$, $a_2(1950)$	
(2, 2)	2.2	1956	1980 ± 23	
			$\pi_2(2005)$, $\rho(2000)$	
			$\rho_2(1940)$, $\rho_3(1982)$	$\rho_3(1990)$

TABLE III. Predicted masses and rms radii from $V(r)$ for some (L, n_r) multiplets with $L + n_r \geq 6$. Parameters as in Table II.

$(L + n_r)$ (L, n_r)	$\langle r^2\rangle^{1/2}$ fm	M_{L,n_r} MeV
6	4.6 ± 0.8	2421 ± 11
(1, 5), (2, 4), (3, 3)		
(4, 2), (5, 1)		
7		
(1, 6), (2, 5), (3, 4)	6.4 ± 0.7	2524 ± 8
(4, 3), (5, 2), (6, 1)		
8		
(1, 7), (2, 6), (3, 5)	8.6 ± 0.6	2592 ± 6
(4, 4), (5, 3), (6, 2), (7, 1)		
9		
(1, 8), (2, 7), (3, 6), (4, 5)	11.0 ± 0.5	2638 ± 5
(5, 4), (6, 3), (7, 2), (8, 1)		

The calculated masses from $V(r)$, for $\lambda_u = 1600$ MeV $\cdot$ fm and $L_c = 16$, and their comparison to data are shown in Table II, where the states have now been ordered according to their sizes. As can be seen, the agreement is remarkable. We should not forget though that the values of the parameters ν, λ_u, and L_c have been fixed from the set of data in Table II. Regarding the other parameters of the model, let us recall that the value of σ comes from an external input: the phenomenological analysis of Regge trajectories involving lowly excited light-quark mesons. As for m_u, the value chosen corresponds to the average dynamic mass generated by spontaneous symmetry breaking in the energy region under consideration (see [5], and references therein).

Let us realize that in most multiplets, the difference in mass between members of the same multiplet is quite small due to what can be interpreted as the absence of significant spin-orbit and tensor contributions for the large sized states considered. On the other hand, the calculated meson states become less relativistic when increasing $(L + n_r)$. So p/m goes from $\simeq 1.5$ for $(L + n_r) = 4$ to $\simeq 1$ for $(L + n_r) = 6$. It is then interesting to give the model predictions for higher (L, n_r) multiplets for which the nonrelativistic treatment becomes less effective. The average values, from the calculated masses corresponding to the different (L, n_r) combinations giving the same $(L + n_r)$, are listed in Table III. A look at the Table shows the quite small difference with the predictions given in [5], as could be expected from the same asymptotic behavior of the potentials employed and the large meson radii involved. It should also be added that the limiting mass for the light-quark meson spectrum evaluated in [5] remains almost unaltered as it is given by

$$(M_{\mathrm{Lim}})_{u\bar{u}} = m_u + m_{\bar{u}} + \sigma\nu = 2809 \text{ MeV}. \tag{13}$$

IV. HEAVY QUARKONIA

The proposed form for the screened potential $V(r)$ should be tested in other meson sectors. Heavy quarkonia, in particular, the nonrelativistic bottomonium, constitute the ideal laboratory to test it, since the static approximation is expected to be valid for the whole spectrum. To apply $V(r)$ to different meson sectors, a criterion to fix the values of the parameters has to be established. As usual, we shall assume that the string tension σ is quite approximately flavour independent. Regarding ν, its value has to do with the screening effect caused dominantly by light sea quark-antiquark pairs. Consequently, it seems reasonable to take the same value for it in the different meson sectors. The universality can be tentatively extended to L_c given its connection to r_c, or equivalently to the confinement scale in QCD. Our results will justify this extension. As for the gluonic Coulomb strength λ_q and the quark mass m_q, they will be fixed to get the correct splitting and masses of two chosen states.

Let us remark that we are dealing with a spin independent potential. For s waves, we can assume that the experimental energy difference between spin singlet and spin triplet states [from $V(r)$, they are degenerate] comes mainly from the spin-spin interaction. Taking into account that the correction for the spin singlet is in absolute value 3 times bigger than for the spin triplet, we shall consider our calculated s-wave states to be describing spin-triplets. For p and d waves, spin-orbit and tensor interactions may give significant contributions to the mass. If we recall that for light-quark mesons these kind of contributions were suppressed for rms radii, as calculated in our model greater than 2 fm, we can expect a similar suppression in heavy quarkonia to take place perhaps at shorter distances, since the strength of the spin-orbit and tensor interactions decreases when increasing the mass of the quark.

A. Charmonium

In order to fix λ_c and m_c, we should rely on the highest well-established excitations with an unambiguous quantum numbers assignment. We only have $J/\psi(1s)$, $\psi(2s)$, and $\chi_{c0,c1,c2}(1p)$. As our model does not contain either spin-orbit or tensor interactions that can give account of the important mass splitting (140 MeV) in the $1p$ multiplet, we should choose $J/\psi(1s)$ and $\psi(2s)$. However, this choice is problematic, since the rms radius for J/ψ obtained from the fixed parameters $\langle r^2\rangle^{1/2} = 0.4$ fm would not satisfy the static condition $(\langle r^2\rangle^{1/2})_{c\bar{c}} \gtrsim 0.6$ fm, showing that the static approximation is invalid for J/ψ. Instead, we shall take for granted the conventional assignment of $\psi(4040)$ to $\psi(3s)$ and choose $\psi(2s)$ and $\psi(3s)$ as referents to fix the parameters. Notice that by attributing the mass difference $M[\psi(2s)] - M[\eta(2s)] \simeq 49$ MeV to the spin-spin interaction, the error in the determination of $M[\psi(2s)]$ due to the nonconsideration of such interaction is only of 12 MeV. For $M[\psi(3s)]$, we expect even a lower error.

TABLE IV. Calculated $c\bar{c}$ masses and rms radii from $V(r)$. The superindex † indicates the masses used to fix $\lambda_c = 157$ MeV · fm and $m_c = 1448$ MeV. Masses for experimental candidates $(M_{L,n_r})_{\rm PDG}$ have been taken from [2] unless otherwise stated by means of a superindex: Be for Belle data [16], Ba for *BABAR* data [17]. For p waves, we quote the np_0, np_1, and np_2 states. Masses calculated in the QCD string approach [14], $(M_{L,n_r})_{\rm SA}$, are also shown for comparison.

n_rL	$\langle r^2\rangle^{1/2}$ fm	M_{L,n_r} MeV	$(M_{L,n_r})_{\rm PDG}$ MeV	$(M_{L,n_r})_{\rm SA}$ MeV
1s			3096.916 ± 0.011	3105
2s	0.9	$3686^{\dagger}$	3686.09 ± 0.04	3678
1d	1.0	3869	3772.92 ± 0.35	3800
3s	1.4	$4039^{\dagger}$	4039 ± 1	4078
2d	1.5	4148	4153 ± 3	4156
4s	2.0	4263	4263^{+8}_{-9}	4398
3d	2.1	4335	$4361 \pm 18^{\rm Be}$	4464
			$4324 \pm 24^{\rm Ba}$	
5s	2.6	4417	4421 ± 4	4642
4d	2.8	4468		4690
6s	3.3	4528		4804
5d	3.5	4565		
7s	4.1	4611		
6d	4.4	4639		
			$4664 \pm 16^{\rm Ba}$	
8s	5.0	4674		
1p	0.7	3574	$\chi_{c2}(3556.20 \pm 0.09)$	
			$\chi_{c1}(3510.66 \pm 0.07)$	
			$\chi_{c0}(3414.75 \pm 0.31)$	
2p	1.2	3965	$\chi_{c2}(3929 \pm 5 \pm 2)$	
3p	1.8	4212		
4p	2.4	4380		

The results for the static $c\bar{c}$ spectrum for $\lambda_c = 157$ MeV · fm and $m_c = 1448$ MeV are shown in Table IV as compared to data through a tentative quantum numbers assignment. QCDSA results for the n_rL states are also shown for comparison.

As expected, the lowest p and d states are not well reproduced. For 1p, with $\langle r^2\rangle^{1/2} = 0.7$ fm, the discrepancy goes from 20 MeV for χ_{c2} to 160 MeV for χ_{c0}. For 1d, with $\langle r^2\rangle^{1/2} = 1.0$ fm, the calculated mass differs about 100 MeV from the only known experimental candidate. The situation improves extraordinarily for $\langle r^2\rangle^{1/2} \gtrsim 1.5$ fm, since the calculated masses for the 2d, 4s, 3d, and 5s states can be put in perfect correspondence with experimental candidates [the resonances $Y(4360)$ from Belle and $Y(4324)$ from *BABAR* are assumed to correspond to the same state]. This is a very distinct feature of our model.

It is noteworthy that $X(4260)$ appears as a natural 4s state [instead in the QCDSA, the 4s state is identified with $\psi(4415)$]. Actually, the reluctancy to assign 4s quantum numbers to $X(4260)$ comes to some extent from the much higher mass predicted from conventional charmonium models [18], since experimental data might be accommodated by making such a choice [19]. Let us also point out that in our model, the $Y(4660)$ reported only by Belle could correspond to the overlap of the energetically close 7s and 8s states. Our 6s model state at 4528 MeV would be missed as well as other ns states with $n \geq 9$. These excitations would be very close in energy which could make their experimental disentanglement difficult despite the fact that the limiting mass of the spectrum is still quite far above

$$(M_{\rm Lim})_{c\bar{c}} \simeq m_c + m_{\bar{c}} + \sigma\nu = 5025 \text{ MeV}. \qquad (14)$$

It should also be remarked that the quite relativistic character of the fitted spectrum is indicated by the values $(p_c/m_c)^2 \sim 0.25$–0.16. Nonetheless, the values of the wave functions at the origin for ns states (n: 2, 3, 4, 5) differ at most 15% from the ones obtained from the solution of the Salpeter equation in the QCDSA [14]. Hence quite similar results (within a 20% of difference) would be obtained for the dielectron widths and the same conclusion inferred: the measured values for ns states [$nd(n$: 1, 2)] are systematically smaller (much bigger) than the calculated ones. This can be explained by the presence of $s - d$ states mixing as a consequence of their coupling to open channels. This mixing would significantly modify the values of the s- and d-wave functions at the origin. On the other hand, the very good fit obtained for the spectrum without mixing suggests that this should not have any significant effect on the calculated masses of the corresponding s and d states. Both features can be understood by realizing that dielectron widths are sensitive to the wave functions at the origin; whereas spectral masses are more related to their long distance behavior.

B. Bottomonium

An analysis that is parallel to the one just carried out for charmonium can be done for bottomonium. As ns states up to $n = 4$ have been experimentally identified, we choose $\Upsilon(3s)$ and $\Upsilon(4s)$ to fix the parameters λ_b and m_b. From $M[\Upsilon(3s)] = 10355$ MeV and $M[\Upsilon(4s)] = 10579$ MeV, we find $\lambda_b = 102.6$ MeV · fm and $m_b = 4795.5$ MeV. The results for the spectrum are shown in Table V and assigned to data. For comparison, results from the QCDSA are also listed. For the sake of completeness, it should be pointed out that the results for ns states with an "intermediate" model based on the asymptotically constant screened potential of Eq. (4) [10] lay in between ours and the QCDSA ones. On the other hand, quark potential models not incorporating screening [18] predict much larger energy splittings for high n_r.

Again the 1p, 2p, and 1d states are not well described. Now for 1p(2p)with $\langle r^2\rangle^{1/2} = 0.4$ fm (0.7 fm), the discrepancy goes from 60 MeV (30 MeV) for χ_{b2} to 110 MeV (70 MeV) for χ_{b0}. For 1d, with $\langle r^2\rangle^{1/2} = 0.6$ fm, the calculated mass differs about 50 MeV from the only known experimental candidate. Unfortunately, we do not have

TABLE V. Calculated $b\bar{b}$ masses and rms radii from $V(r)$ with $\lambda_b = 102.6$ MeV · fm and $m_b = 4795.5$ MeV. Notation as in Table IV. The superindex Ba indicates now recent *BABAR* data [20]. Masses calculated in the QCD string approach are taken from [15].

n_rL	$\langle r^2\rangle^{1/2}$ fm	M_{L,n_r} MeV	$(M_{L,n_r})_{\rm PDG}$ MeV	$(M_{L,n_r})_{\rm SA}$ MeV
1s	0.2	9458	9460.30 ± 0.26	9453
2s	0.5	10037	10023.26 ± 0.31	10010
1d	0.6	10218	10161.1 ± 1.7	10144
3s	0.8	$10355^{\dagger}$	10355.2 ± 0.5	10356
2d	0.8	10471		10446
4s	1.1	$10579^{\dagger}$	10579.4 ± 1.2	10630
5s	1.4	10748		10862
6s	1.7	10880	10865 ± 8 $10876 \pm 2^{\rm Ba}$	11067
7s	2.0	10986	$10996 \pm 2^{\rm Ba}$ 11019 ± 8	11240
8s	2.4	11073		
9s	2.7	11144		
10s	3.1	11205		
11s	3.6	11256		
1p	0.4	9970	$\chi_{b2}(9912.21 \pm 0.57)$ $\chi_{b1}(9892.78 \pm 0.57)$ $\chi_{b0}(9859.44 \pm 0.57)$	9884
2p	0.7	10300	$\chi_{b2}(10268.65 \pm 0.72)$ $\chi_{b1}(10255.46 \pm 0.72)$ $\chi_{b0}(10232.5 \pm 0.9)$	10256
3p	1.0	10535		10541

data at our disposal for higher radial p or d excitations to fix a value for the rms radius beyond which spin-dependent contributions are negligible. If we assume a correct prediction for the $3p$ state, this radius would be of $\sim$1 fm.

A very good correspondence between calculated and experimental masses (a difference of 15 MeV at most) is found for $1s$, $2s$, $6s$, and $7s$ if the $\Upsilon(10860)$ is assigned to $\Upsilon(6s)$ [not to $\Upsilon(5s)$ as usually done] and $\Upsilon(11020)$ to $\Upsilon(7s)$. Notice that the recent measurements by *BABAR* [20] give 10876 MeV and 10996 MeV for the masses of these two resonances. Moreover, the $\Upsilon(11020)$ appears in [21] as a peak between 10990 MeV and 11060 MeV that is compatible with being the overlap of our $7s$ and $8s$ states.

It should be emphasized that the assignment of $\Upsilon(10860)$ to $\Upsilon(6s)$ implies the existence of a $\Upsilon(5s)$ resonance with a mass

$$M[\Upsilon(5s)] \sim 10748 \pm 15 \text{ MeV} \qquad (15)$$

that can be considered as a main prediction (the quoted error of 15 MeV has been estimated from Table V) and at the same time as a stringent test of our potential model. The presence of this resonance might have some relation with the experimental shoulder present on the tail of $\Upsilon(4s)$ with a mass of $10684 \pm 10 \pm 8$ MeV and a width of $131 \pm 27 \pm 23$ MeV in Ref. [22] and a mass between 10 670 and 10 730 MeV in [21] (see Table I of this reference). In the recent study by *BABAR* [20], there appears to be a small bump around 10700 MeV not identified as a resonance (see Fig. 1 of this reference) that might have to do with the predicted state. It should be added that the presence of the close $B_s\bar{B}_s$ threshold at 10732 MeV may complicate the experimental extraction of this resonance, if it exists.

An additional argument in favor of this resonance can be elaborated from the comparison of the experimental energy differences between contiguous s excitations in charmonium and bottomonium as done in Table VI.

The assumption that the $4s - 3s$ mass differences in bottomonium (224 MeV) and charmonium have close values as it is the case for $3s - 2s$, and $2s - 1s$ requires a $4s$ resonance around 4260 MeV for $c\bar{c}$, as our model predicts. Then, assuming that $X(4260)$ is the $4s$ state, the extension of the argument to the $5s - 4s$ mass differences (158 MeV in charmonium) implies the existence of $\Upsilon(5s)$ at about 10740 MeV. Alternatively, as is the case in the QCDSA, the $X(4260)$ could not be a $c\bar{c}$ state, and the $\Upsilon(10748)$ could not exist but then the energy difference pattern in charmonium and bottomonium would be very different (the $4s - 3s$ mass difference in charmonium would be 382 MeV against 224 MeV in bottomonium). It should be emphasized that this discrepancy in the interpretation of the experimental states is directly related to the different manner at which string breaking is implemented in both models. Then the experimental confirmation (refutation) of our results would serve to establish the Coulombic (non-Coulombic) character of the asymptotic quark-antiquark potential.

Concerning other ns states with $n \geq 8$, the small separation in energy between neighbors suggests important overlaps among them and difficulties for a separated identification. This may explain the nonidentification of any clear signal for a resonance in the region of 11 000–11 200 MeV recently explored by *BABAR* [20]. Let us realize that the limiting mass of the spectrum is still quite far above

$$(M_{\rm Lim})_{b\bar{b}} \simeq m_b + m_{\bar{b}} + \sigma\nu = 11\,720 \text{ MeV}. \qquad (16)$$

TABLE VI. Experimental mass differences (in MeV) between $(n_r + 1)s$ and n_rs states in charmonium and bottomonium. The superindex $*$ indicates that the corresponding difference has been calculated assuming that $X(4260)$ and $\psi(4415)$ are the $4s$ and $5s$ states of $c\bar{c}$.

$(n_r + 1)$	$(M_{0,n_r+1} - M_{0,n_r})$ $c\bar{c}$	$(M_{0,n_r+1} - M_{0,n_r})$ $b\bar{b}$
2	589	563
3	353	332
4	224^{*}	224
5	158^{*}	
6		

It is also worthwhile to emphasize the nonrelativistic character of the fitted static spectrum in bottomonium since $(p_b/m_b)^2 \sim 0.1$–0.06. Then we can tentatively identify $\lambda_b = 4(\alpha_s)_b/3$ where α_s stands for the quark-quark-gluon coupling at the bottomonium scale. This gives $(\alpha_s)_b = 0.39$ in agreement with the value derived from QCD in bottomonium for the $1p$ and $2p$ states [23].

As we are dealing with a nonrelativistic system, we expect that the wave functions obtained from the Schrödinger equation may accurately account for other observables. In particular, s-wave splittings and leptonic (dielectron) widths depend directly on the values of the wave functions at the origin. Thus in first order perturbation theory, the splitting energy between the triplet $\Upsilon(ns)$ and the singlet $\eta_b(ns)$ spin states is given by

$$M[\Upsilon(ns)] - M[\eta_b(ns)] = \frac{4}{3}(\alpha_s)_b \frac{2}{3m_b^2}|R_{n_r s}(0)|^2, \quad (17)$$

where $R_{n_r s}(0)$ stands for the radial wave function at the origin for $\Upsilon(ns)$. The resulting splitting for $n_r = 1$ is

$$M[\Upsilon(1s)] - M[\eta_b(1s)] = 173 \text{ MeV} \quad (18)$$

in accord with the experimental value

$$M[\Upsilon(1s)]_{\text{ex}} - M[\eta_b(1s)]_{\text{ex}} = 160 \pm 40 \text{ MeV}. \quad (19)$$

For $n_r = 2$, the predicted value is

$$M[\Upsilon(2s)] - M[\eta_b(2s)] = 70 \text{ MeV}. \quad (20)$$

Regarding the leptonic widths $\Gamma_{e^+e^-}$ for $n_r s$ states, they can be evaluated as [24]

$$\Gamma_{e^+e^-}(n_r s) = \Gamma^{(0)}_{e^+e^-}(n_r s)\left[1 - \frac{16(\alpha_s)_b}{3\pi} + \Delta(n_r s)\right]. \quad (21)$$

The terms with $[-16(\alpha_s)_b/3\pi]$ and $[\Delta(n_r s)]$ account for the leading order radiative and higher order radiative + relativistic corrections to

$$\Gamma^{(0)}_{e^+e^-}(n_r s) \equiv \frac{4e_b^2\alpha^2}{M^2_{n_r s}}|R_{n_r s}(0)|^2, \quad (22)$$

where $e_b = -1/3$ is the quark electric charge, $\alpha = 1/137.036$ is the fine structure constant, and $M_{n_r s}$ is the mass of the $n_r s$ state for which we shall use the experimental value. The calculated leptonic widths are shown in Table VII. Although the correction Δ depends on the particular $n_r s$ state, we shall consider it, for the sake of simplicity, as an effective constant. We fix its value from $\Gamma_{e^+e^-}(\Upsilon(10\,580))$ since it corresponds to the highest excitation with well identified quantum numbers ($4s$), and we expect the nonrelativistic and static approaches to be more accurate for it than for lower excited states. Then we get $\Delta = 0.22$, one third of the value of the first order radiative correction $16(\alpha_s)_b/3\pi = 0.66$.

It should be pointed out that the measured $\Gamma_{e^+e^-}(10\,860)$ in [2] might be contaminated by the hidden $\Upsilon(10\,748)$. Instead for $n_r \geq 5$ data from [22], where a resonance of about 1700 MeV is taken into account, are used.

TABLE VII. Leptonic widths $\Gamma_{e^+e^-}$ (in keV) for $b\bar{b}$. The superindex † indicates the value used to fix Δ. Data from [2] except for $n_r s$ with $n_r > 3$ are taken from [22] and indicated by a superindex *. The experimental number between the $7s$ and $8s$ states indicates that the resonance $\Upsilon(11020)$ in [22], to which this number is assigned, could be a result of the overlap of the $7s$ and $8s$ states.

$n_r L$	$\Gamma_{e^+e^-}$	$(\Gamma_{e^+e^-})_{\text{exp}}$
$1s$	1.7	1.340 ± 0.018
$2s$	0.61	0.612 ± 0.011
$3s$	0.39	0.443 ± 0.008
$4s$	0.27†	0.272 ± 0.029
$5s$	0.21	$0.20 \pm 0.05 \pm 0.10$*
$6s$	0.16	$0.22 \pm 0.05 \pm 0.07$*
$7s$	0.13	
		$0.095 \pm 0.030 \pm 0.035$*
$8s$	0.11	

Table VII shows clearly that a good agreement with the data (10% difference at most) may be achieved except for $\Gamma_{e^+e^-}(1s)$. This might have to do with either the Δ dependence on the $n_r s$ state or a deficient description of the wave function at the origin for $\Upsilon(1s)$, the more relativistic state for bottomonium with the more important spin-spin correction. Indeed, a 13% decrease in the value of $|R_{1s}(0)|$ would fit the central experimental value of $\Gamma_{e^+e^-}(1s)$ (notice that the $1s$ spin splitting would be 130 MeV, still within the experimental uncertainty). It should also be kept in mind that a systematic deviation of the values of the wave functions at the origin might be hidden through the effective value of Δ. Actually, the values we get for $R_{1s}(0)$ and $R_{2s}(0)$ are significantly bigger than the ones obtained in the QCDSA.

Dielectron widths can be also calculated for nd states, but no data are available. Therefore we will only mention that the calculated values in our model from (see for instance [15])

$$\Gamma^{(0)}_{e^+e^-}(nd) = \frac{25e_b^2\alpha^2}{2m_b^4 M^2_{nd}}|R''_{nd}(0)|^2, \quad (23)$$

where $R''_{nd}(0)$ stands for the second derivative of the radial wave function at the origin, are four orders of magnitude smaller than for $n_r s$ states.

For the sake of completeness, $E1$ decay widths are also evaluated. By using a single quark operator approximation, the width can be written as [25]

$$\Gamma^{E1}_{if} = \tfrac{4}{27}e_b^2\alpha k_{if}^3(2J_f + 1)D^2_{if}, \quad (24)$$

where k_{if} is the photon energy or momentum, J_f is the total angular momentum of the final meson, and D_{if} is the transition matrix element

TABLE VIII. $E1$ decay widths for $b\bar{b}$ (in keV) as compared to data from [2].

$b\bar{b}$ Transition	Γ_{E1}	$\Gamma_{\rm exp}$
$\Upsilon(2s)\to\gamma\chi_{b_0}(1P)$	1.7	1.2 ± 0.2
$\Upsilon(2s)\to\gamma\chi_{b_1}(1P)$	2.5	2.2 ± 0.2
$\Upsilon(2s)\to\gamma\chi_{b_2}(1P)$	2.6	2.3 ± 0.3
$\Upsilon(3s)\to\gamma\chi_{b_0}(2P)$	1.9	1.2 ± 0.2
$\Upsilon(3s)\to\gamma\chi_{b_1}(2P)$	3.2	2.6 ± 0.5
$\Upsilon(3s)\to\gamma\chi_{b_2}(2P)$	3.5	2.7 ± 0.6

$$D_{if}=\int_0^\infty dr u_i(r)\frac{3}{k_{if}}\left[\frac{k_{if}r}{2}j_0\left(\frac{k_{if}r}{2}\right)-j_1\left(\frac{k_{if}r}{2}\right)\right]u_f(r), \tag{25}$$

being that $u_{i,f}(r)$ is the reduced radial wave functions of the initial and final mesons, and j_0, j_1 are spherical Bessel functions. The results obtained for $\Upsilon(2s)\to\gamma\chi_{bJ}(1p)$ and $\Upsilon(3s)\to\gamma\chi_{bJ}(2p)$ are compiled in Table VIII. As a reminder, that the three calculated $\chi_{bJ}(np)$ states are degenerate in our model. Hence the same wave function is employed for all of them. This can be justified by assuming that the experimental masses are explained by the effect of spin-dependent interactions calculated in perturbation theory to the first order. The differences in Table VIII among the three $\Upsilon(2s)\to\gamma\chi_{bJ}(1p)$ or the three $\Upsilon(3s)\to\gamma\chi_{bJ}(2p)$ decays come from the use of the nondegenerate experimental χ_{bJ} masses to evaluate k_{if}.

A clear bias of the results is observed: they are systematically higher than data. The discrepancy is more pronounced for $\chi_{b0}(np)$ final states. As $\chi_{b0}(np)$ states are the ones requiring a bigger spin-dependent mass contribution in our model, the systematics may suggest the need to implement $\chi_{bJ}(np)$ wave function corrections.

V. SUMMARY

To summarize, a universal form for the quark-antiquark static potential, incorporating the screening of the color charges by sea pairs, has been proposed within a nonrelativistic quark model framework. This potential, with a confining long-distance Coulombic behavior, reproduces the highly excited light-quark meson spectrum and provides a successful spectral description of charmonium and bottomonium, suggesting the assignment of $X(4260)$ to the $4s$ state of $c\bar{c}$ and the existence of a noncataloged $\Upsilon(10748)$ resonance corresponding to the $5s$ state of $b\bar{b}$. These very distinctive predictions of our model come from the way screening has been implemented within it. Therefore their experimental confirmation or refutation could allow us to establish the Coulombic or non-Coulombic character of the long distance quark-antiquark static potential.

It should be remarked that the only dependence of the potential on the particular meson sector comes from the value of an effective gluonic strength. As the light-quark mesons, and to a lesser extent the charmonium, are clearly relativistic systems, one can tentatively think that some relevant relativistic corrections could be effectively taken into account through the value of this parameter. The fact that the gluonic Coulomb strength obtains a systematically greater value than the gluonic chromoelectric strength in QCD seems to point in this direction. For the nonrelativistic bottomonium, this gluonic strength can correspond with the strength of the chromoelectric one gluon exchange interaction in QCD or, equivalently, with the quark-quark-gluon coupling α_s at the corresponding Q^2 scale.

In our nonrelativistic treatment, the quark and antiquark masses are parameters of the model. Their values should be added to the binding energies to obtain the meson masses. A peculiarity of our potential is the absence of any additive constant to obtain acceptable values of the constituent quark masses (in the sense of being able to account for other observables, such as hadronic magnetic moments) from the fitted meson masses.

All of these features make the effective nonrelativistic quark model proposed very useful to identify excited states from existing experimental candidates and for assigning quantum numbers to them. Furthermore, it can be used to advance predictions on highly excited states in all meson sectors.

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[1] D. V. Bugg, Phys. Rep. **397**, 257 (2004); A. V. Anisovich, V. V. Anisovich, and A. V. Sarantsev, Phys. Rev. D **62**, 051502 (2000).
[2] C. Amsler *et al.* (Particle Data Group), Phys. Lett. B **667**, 1 (2008).
[3] E. S. Swanson, Phys. Rep. **429**, 243 (2006).
[4] S. S. Afonin, Mod. Phys. Lett. A **22**, 1359 (2007); Int. J. Mod. Phys. A **23**, 4205 (2008); arXiv:hep-ph/0707.1291 [Mod. Phys. Lett. A (to be published)].
[5] El Houssine Mezoir and P. González, Phys. Rev. Lett. **101**,

232001 (2008).
[6] G. S. Bali, Phys. Rep. **343**, 1 (2001).
[7] A. M. Badalian and D. S. Kuzmenko, Phys. Rev. D **65**, 016004 (2001).
[8] K. D. Born *et al.*, Phys. Rev. D **40**, 1653 (1989).
[9] Y.-B. Ding, K.-T. Chao, and D.-H. Qin, Phys. Rev. D **51**, 5064 (1995).
[10] P. González, A. Valcarce, H. Garcilazo, and J. Vijande, Phys. Rev. D **68**, 034007 (2003).
[11] A. Duncan, E. Eichten, and H. Thacker, Phys. Rev. D **63**, 111501 (2001).
[12] A. M. Badalian and B. L. G. Bakker, Phys. Rev. D **66**, 034025 (2002).
[13] A. M. Badalian, B. L. G. Bakker, and Yu. A. Simonov, Phys. Rev. D **66**, 034026 (2002).
[14] A. M. Badalian, B. L. G. Bakker, and I. V. Danilkin, Phys. At. Nucl. **72**, 638 (2009).
[15] A. M. Badalian, B. L. G. Bakker, and I. V. Danilkin, arXiv:0903.3643.
[16] X. L. Wang *et al.* (Belle Collaboration), Phys. Rev. Lett. **99**, 142002 (2007).
[17] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. Lett. **98**, 212001 (2007).
[18] E. Eichten, K. Gottfried, T. Kinoshita, K. D. Lane, and T. M. Yan, Phys. Rev. D **21**, 203 (1980); S. Godfrey and N. Isgur, Phys. Rev. D **32**, 189 (1985).
[19] F. J. Llanes-Estrada, Phys. Rev. D **72**, 031503 (2005).
[20] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. Lett. **102**, 012001 (2009).
[21] D. M. J. Lovelock *et al.*, Phys. Rev. Lett. **54**, 377 (1985).
[22] D. Besson *et al.*, Phys. Rev. Lett. **54**, 381 (1985).
[23] S. Titard and F. J. Ynduráin, Phys. Lett. B **351**, 541 (1995); Phys. Rev. D **51**, 6348 (1995).
[24] W. Buchmüller and S.-H. H. Tye, Phys. Rev. D **24**, 132 (1981).
[25] D. P. Stanley and D. Robson, Phys. Rev. D **21**, 3180 (1980).