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Properties of charmonia in a hot equilibrated medium

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We investigate the properties of charmonia in a thermal medium, showing that with increasing temperature the decay widths of these mesons behave in a nontrivial way. Our analysis is based on a potential model with interaction potential extracted from thermal lattice QCD calculations of the free-energy of a static quark-antiquark pair. We find that in the crossover region some decay widths are extremely enhanced. In particular, at temperatures $T \sim T_c$ the decay widths of the J/Ψ that depend on the value of the wave function at the origin are enhanced with respect to the values in vacuum by about a factor 2. In the same temperature range the decay width of the process $\chi_{cJ} \to J/\Psi + \gamma$ is enhanced by approximately a factor 6 with respect to the value in vacuum. At higher temperatures the charmonia states dissociate and the widths of both decay processes become vanishing small.

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I. INTRODUCTION

One of the central planks of the heavy-ion collision program is to identify and characterize the properties of deconfined quark matter. Many valuable probes of the properties of the medium produced in heavy-ion collisions are available, which include jets, electromagnetic signals, and heavy quarkonia states ($Q\bar{Q}$) [1]. In particular, much work has been devoted to understanding how the presence of deconfined quarks and gluons may affect the binding of quarkonia [2].

Whether or not hadrons survive in the quark gluon plasma and how their decay widths are modified is a long-standing issue [3]. Since the pioneering work by Matsui and Satz [4] it is known that the screening effect due to a thermal medium induces the dissociation of quarkonia states. Subsequent analyses of the binding energies of mesonic states as a function of the temperature have been done by various authors using potential models, with an interaction potential extracted from lattice QCD calculations [5–8]. These analyses show that quarkonia dissociate at temperatures close to the critical temperature of QCD, T_c. Analogous results have been obtained by studying the correlation functions of charmonia above deconfinement [9].

In the present paper we study the properties of heavy quarkonia states with increasing temperature. In particular, we are interested in determining how the decay widths for the leptonic, hadronic and radiative channels are influenced by the presence of the thermal medium. In order to achieve this result we first determine the radial wave function of some charmonia states and the corresponding binding energies. In our analysis we employ the nonrelativistic Schrödinger equation (and for comparison the relativistic Salpeter equation) with interaction potential extracted from thermal lattice QCD simulations. A first analysis of the decay widths of charmonia have been carried out in [10] by studying the quark correlators above the deconfinement temperature. Instead, we employ a wave equation and study the decay widths for any temperature up to the dissociation temperature of the pertinent quarkonium state.

The remarkable result of our analysis is that some decay widths change drastically close to T_c. In the transition region we find that the decay widths of the J/Ψ that depend on the value of the wave function at the origin change by approximately a factor 2 with respect to the corresponding values in vacuum. We also investigate the properties of the χ_c meson and, in particular, of the $\chi_{cJ} \to J/\Psi + \gamma$ transition. This radiative decay contributes to the total inclusive J/Ψ production by a fraction of about 0.3 as determined in proton proton and $\pi^+\pi^-$ data [11]. We find that this J/Ψ production mechanism is enhanced close to the critical temperature by approximately a factor 6 with respect to the value in vacuum.

Although it has been shown in perturbation theory that the potential has a sizable imaginary part [12], we shall employ a real potential as extracted from lattice QCD calculations of the free energy. Since a nonperturbative estimate of the imaginary part of the potential is not yet available, it is hard to estimate how large will be its contribution to the processes we are considering.

II. THE METHOD

Potential models have been quite successful in describing the properties of heavy quark bound states in vacuum (see e.g. [13]). Our key assumption is that at any temperature T, the interaction between heavy quarks can be approximated by an instantaneous potential, $V(r, T)$, where r is the radial coordinate.

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We shall study bound states of heavy quarks by using the nonrelativistic Schrödinger equation

$$\left(2M_Q - \frac{\nabla^2}{M_Q} + V(r,T)\right)\psi_i = E_i\psi_i, \quad (1)$$

where M_Q is constituent mass of the heavy quark, ψ_i and E_i represent the wave function and the mass of the corresponding $Q\bar{Q}$ state, respectively. Comparing the results of this analysis with those obtained by the Salpeter equation for the S-wave states

$$(2\sqrt{M_Q^2 - \nabla^2} + V(r,T))\psi_i = M_i\psi_i, \quad (2)$$

we find excellent agreement between the outcomes of the two methods, meaning that for these mesonic states relativistic corrections are small.

There are some controversies about how to extract the potential $V(r,T)$ from the color-singlet free energy $F_1(r,T)$ (see [6,7,14]). In principle, in order to obtain the internal energy U_1 one has to subtract the entropy contribution from the free energy

$$U_1 = F_1 - T\frac{\partial F_1}{\partial T}. \quad (3)$$

However, this subtraction procedure has been put to question, one of the reasons being that around the critical temperature, the potential U_1 is more attractive than the potential at zero temperature. In [14] this effect has been interpreted as coming from an additional interaction between the constituents in the medium. Below T_c this effect should be due to hadrons and could be related to the string "flip-flop" interaction [15]; above T_c it should be related to the antiscreening properties of QCD. In both cases one can consider that at finite temperature the static sources Q and $\bar{Q}$ polarize the medium and a cloud of constituents forms around them. At short distances the main contribution to the potential energy comes from the $Q\bar{Q}$ direct interaction, which does not change with temperature. At intermediate distances the polarized clouds surrounding the two heavy quarks overlap and the interaction between the constituents of the clouds makes the interaction potential deeper than the potential at zero temperature. This indirect interaction leads to a change of the interaction strength at finite temperature and is extremely strong close to T_c. Some authors refer to this behavior of the internal energy as the "over-shooting" problem and propose to address it by defining $V(r,T) = xU_1(r,T) + (1-x)F_1(r,T)$, for $0 \le x \le 1$, see Ref. [2]. In agreement with Ref. [14] we shall use the internal energy in Eq. (3) as the interaction potential. As we will see, with this potential we obtain dissociation temperatures for charmonia consistent with lattice results obtained with the maximum entropy method [9]. Employing the free energy as interaction potential gives much smaller dissociation temperatures. Therefore, in the following we shall assume that $x = 1$; results obtained with $x = 0$ are shown for comparison.

Notice that at large distances the two heavy quarks are screened and they do not interact: in this limit the internal energy can be interpreted as a contribution to the constituent mass of the quark. For this reason it can be useful to define $\bar{U}_1(r,T) = U_1(r,T) - U_1(\infty,T)$.

At short distances the static quark-antiquark free energy is normalized in such a way that it reproduces the free energy at zero temperature

$$F(r, T=0) = -\frac{\alpha}{r} + \sigma r, \quad (4)$$

where σ is the string tension and α is the Coulomb coupling constant. The values of these two parameters can be phenomenologically fixed. Employing the numerical values $\sigma = 0.16\ \mathrm{GeV}^2$, $\alpha = 0.2$ and with $m_c = 1.28$ GeV one obtains $M_{J/\psi} \simeq 3.11$ GeV and $M_\chi \simeq 3.43$ GeV.

For nonvanishing temperatures we will consider the free energy proposed in [16] employing the formalism of the Debye-Hückel theory, with the parameterization of Ref. [17],

$$F_1(r,T) = \frac{\sigma}{\mu}\left[\frac{\Gamma(1/4)}{2^{3/2}\Gamma(3/4)} - \frac{\sqrt{x}}{2^{3/4}\Gamma(3/4)}K_{1/4}(x^2 + \kappa x^4)\right] - \frac{\alpha}{r}(\exp(-x) + x), \quad (5)$$

where $K_{1/4}$ is the modified Bessel function $x = \mu r$, while the functions $\mu \equiv \mu(T)$ and $\kappa \equiv \kappa(T)$ are determined by fitting the lattice data. Once these two functions are fixed, this parameterization of the free energy is in excellent agreement with lattice data for $T \le 2T_c$.

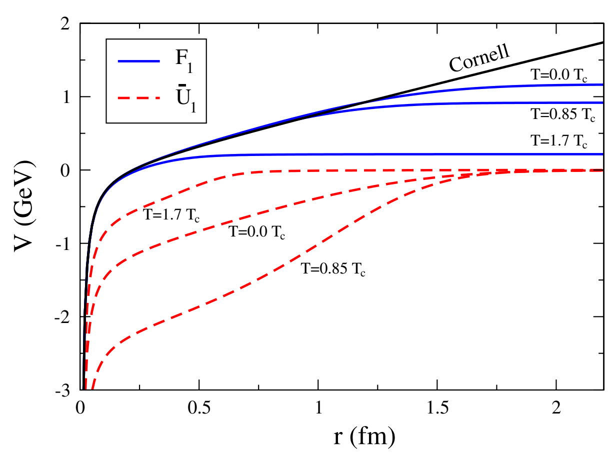

FIG. 1 (color online). Full black line corresponds to the Cornell potential, Eq. (4). Full blue lines correspond to F_1 and dashed red lines correspond to $\bar{U}_1$, obtained with the parametrization in Eq. (5), for temperatures $T = 0$, $T = 0.85T_c$ and $T = 1.7T_c$. Notice that F_1 and $\bar{U}_1$ have been normalized differently at large distances and that at vanishing temperature these two quantities only differ by a constant shift.

In Fig. 1 we compare the free energy with the internal energy. When the free energy is used as a potential, it results in an interaction that always decreases its strength with increasing temperature. Moreover, the only effect that is present is the screening of the interaction at large distances. On the other hand, $\bar{U}_1$ is more attractive close to T_c than at $T = 0$, the representing curve in Fig. 1 is for $T = 0.85T_c$. Around T_c the internal energy is extremely attractive; further increasing the temperature it eventually becomes weaker than in vacuum, as for the case of $T = 1.7T_c$ reported in Fig. 1.

III. MASSES AND DECAY WIDTHS

In order to determine the decay widths it is necessary to compute the wave function and the mass of the charmonia states. For $l = 0$ states, they are determined solving both the Schrödinger equation and the Salpeter equation through the Multhopp method [18], while for P-wave states we only solve the Schrödinger equation. For a potential with a singularity in $r = 0$, the wave function solution of the Salpeter equation is not well behaved at the origin and therefore we will cut off the short distance behavior of the potential. This procedure is analogous to the one employed in Ref. [19] in vacuum. In particular, we introduce a cutoff radius $r_{\min}$, such that for $r < r_{\min}$ the potential is constant and equal to $V(r_{\min})$. According to Ref. [20], we fix $r_{\min} = \frac{4\pi}{3M}$. In all cases we find that the typical radius of quarkonia states is much larger than $r_{\min}$, meaning that our results are not very sensitive to the value of the cutoff.

Various decay processes will be considered. Annihilation processes of charmonia can be viewed as two-stage factorized process. The first process consists in $c\bar{c}$ annihilation into gauge bosons. The annihilation takes place at the characteristic distance r of order $1/m_c$, so for a nonrelativistic pair $r \to 0$ and the annihilation amplitude is proportional to the wave function at the origin. Then, the produced gauge boson decays or fragments into leptons and hadrons. In vacuum it is assumed that the inclusive probability of the latter process is equal to one. At finite temperature we will assume that such a factorization persists. Therefore, the width of hadronic and leptonic decays will depend on temperature exclusively through the radial wave function at the origin and the mass of the meson [13]

$$\Gamma_A \propto \Gamma_{\ell^+\ell^-} \propto \Gamma(^3S_1 \to 3g) \propto \frac{|R_s(T,0)|^2}{M(T)^2}. \tag{6}$$

We also consider the radiative transitions $\chi_{cJ} \to J/\Psi + \gamma$ from the ^3P_J levels to the $^3S_1$ state with rate

$$\Gamma_B \propto \Gamma_{\chi_{cJ}\to J/\Psi+\gamma} \propto (2J+1)|I_{\rm PS}|^2, \tag{7}$$

where $I_{\rm PS}$ is the overlap integral between the radial wave functions of the corresponding states.

A. A toy model for screening

We first introduce a toy model for studying the dependence of the decay widths on the screening length. It is based on the assumption that at short distance the interaction potential is given by the Cornell potential and is constant for $r > r_{\rm cut}$:

$$V(r, r_{\rm cut}) = \begin{cases} -\frac{\alpha}{r} + \sigma r & \text{for } r < r_{\rm cut} \\ -\frac{\alpha}{r_{\rm cut}} + \sigma r_{\rm cut} & \text{for } r > r_{\rm cut} \end{cases}. \tag{8}$$

This procedure roughly describes what happens for nonvanishing temperatures when one uses $F_1(r, T)$ as a potential, see Fig. 1. We find that the χ_c dissociates when $r_{\rm cut} \simeq 1.04$ fm, whereas the J/ψ dissociates at $r_{\rm cut} \simeq 0.64$ fm. The decay widths for the processes in Eqs. (6) and (7) are plotted in Fig. 2 as a function of $r_{\rm cut}$. The width of the process $\chi_{cJ} \to J/\psi + \gamma$ begins to decrease at $r_{\rm cut} \sim 1.2$ fm. At $r_{\rm cut} \simeq 1$ fm the width for this process is zero and the χ_c dissociates. The width for the processes in Eq. (6) is sensitive to smaller screening length, of less than 1 fm. These results are in agreement with the fact that these charmonia states have a radius of about 1 fm and that the J/Ψ is more compact than the χ_c. However, the potential in Eq. (8) does not take into account the contribution from the "cloud-cloud" interaction [14]. As we shall see, it is not the screening of the interaction that is important in characterizing these decay widths, but the antiscreening of the interaction.

B. Decay widths at different temperatures

Now we show the results for dissociation temperatures and decay widths obtained using as $Q\bar{Q}$ potential the internal energy in Eq. (3) and for comparison the free energy in Eq. (5).

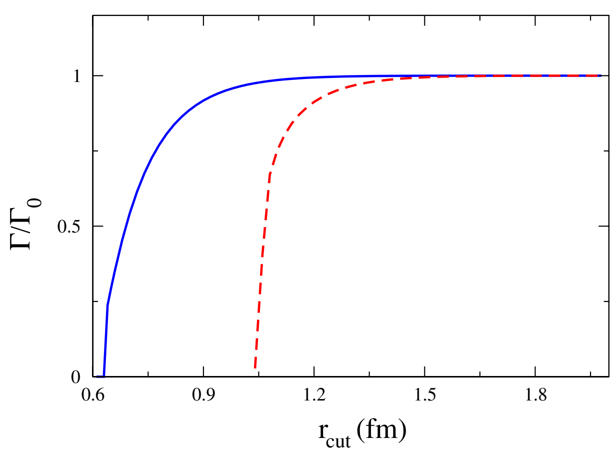

FIG. 2 (color online). Charmonia decay widths as a function of $r_{\rm cut}$, obtained feeding the potential of Eq. (8) in the Schrödinger Eq. (1). Full blue line, decay width of Eq. (6); dashed red line, radiative decay width of Eq. (7). The widths have been normalized to the values obtained for $r_{\rm cut} \to \infty$, corresponding to the widths in vacuum.

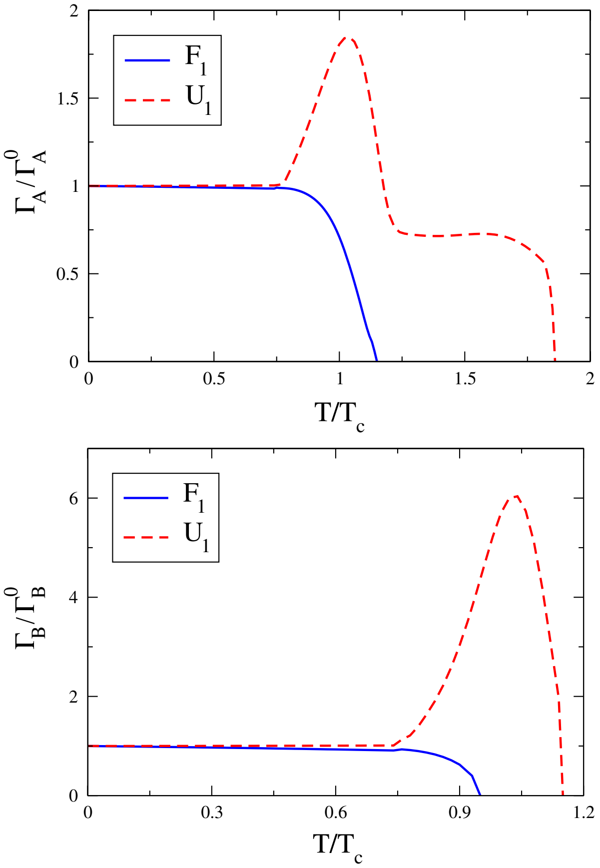

FIG. 3 (color online). Charmonia decay widths given in Eq. (6) [left panel] and in Eq. (7) [right panel]. We have used F_1, full blue lines, and U_1, dashed red lines, as potentials with the parameterization of Eq. (5), and normalized the widths to the vacuum value.

We find that using the free energy (5) the J/ψ dissociates at $1.2T_c$, while the χ_c dissociate at about $0.95T_c$. Using the internal energy as potential we find that the J/ψ dissociates at approximately $1.8T_c$, while the χ_c dissociates at about $1.15T_c$. We have estimated the dissociation temperatures from the binding energy of the pair. Using the internal energy in the wave equations, one obtains higher dissociation temperatures in agreement with the fact that U_1 is more attractive than F_1. Employing the Salpeter equation we find a difference in the dissociation temperatures of less than 5%, in qualitative agreement with results obtained in vacuum [21].

In Fig. 3 we report the values of the decay width for the process in Eq. (6), left panel, and for the process in Eq. (7), right panel, as a function of the temperature. Employing F_1 we find for both the decay processes considered here a behavior similar to the one reported in Fig. 2. Therefore, one can interpret the variation of these decay widths with the temperature as due to the screening of the potential. On the other hand, when using U_1 we find that across the transition region both decay widths become large. For the decays in Eq. (6) the width is about a factor 2 larger than in vacuum. For the radiative decay in Eq. (7), the ratio between the width in the thermal medium and in vacuum reaches a factor 6 across the transition region. This effect is not due to the screening of the potential, but is rather connected with the cloud-cloud interaction.

In order to understand whether this effect is observable one should consider the evolution of the medium in collider experiments. For a rough estimate, let us suppose that the χ_c is produced in the early stage of the heavy-ion collision and then travels for about 4 fm in the thermally equilibrated medium. If the temperature is sufficiently low, so that the decay width is approximately the same as in vacuum, one has that less than 1% of χ_c decay. On the other hand, for temperatures close to T_c, one should observe that about 5% of χ_c decay. Since for temperatures close to T_c one can neglect interactions of charmonia with in-medium hadrons [2,22] and gluons [2] the radiative decay process should give a sizable contribution to the total decay width of the χ_c. As regarding the inclusive width of the J/Ψ, one should consider that at temperatures below T_c, the process in Eq. (6) gives approximately the inclusive width. However, at larger temperatures the dominant contribution should be due to the interaction with in-medium partons [2].

The largest uncertainty in our calculations resides in the extraction of the potential from lattice QCD calculations and pertinent parameterizations to numerically evaluate the entropy. In order to test the robustness of our results one can employ a different parameterizations of the internal energy. Using the expression of $U_1(r, T)$ reported in Ref. [6] we find approximately the same results reported here. In particular, we find that in the crossover region the widths of the processes in Eqs. (6) and (7) are enhanced with respect to the vacuum values by approximately the same factors 2 and 6, respectively.

The analysis reported in the present paper can easily be extended to bottomia; moreover, it would be of some interest to include D mesons.

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