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Gluon-glueball duality and glueball searches

Shmuel Nussinov[1,2] and Robert Shrock[3]

[1]*School of Physics and Astronomy, Tel Aviv University, Tel Aviv, Israel*

[2]*Schmid College of Science, Chapman University, Orange, California 92866, USA*

[3]*C.N. Yang Institute for Theoretical Physics, Stony Brook University, Stony Brook, New York 11794, USA*

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We discuss a notion of gluon-glueball duality analogous to quark-hadron duality. We apply this idea to the radiative decay of heavy orthoquarkonium, $Q\bar{Q} \to \gamma gg$, which has been used to search for glueballs. The duality is first introduced in two simplified contexts: (i) a hypothetical version of QCD without any light quarks and (ii) QCD in the large-N_c limit. We then discuss how an approximate form of this duality could hold in real QCD, based on a hierarchy of time scales in the temporal evolution of the gg subsystem in radiative orthoquarkonium decay. We apply this notion of gluon-glueball duality to suggest a method that could be useful in experimental searches for glueballs.

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I. INTRODUCTION

Quantum chromodynamics (QCD) is a very successful theory describing quark and gluon interactions. There are ample observations of gluon jets in high-energy collider data, and lattice QCD calculations of the pure gluonic sector of the theory have yielded a detailed spectrum of (color-singlet) bound states of gluons, commonly called glueballs [1–3]. The lightest of these can be modeled as gg states, where g denotes a gluon; these include a state with $S = 0$, $L = 0$, and $J^{PC} = 0^{++}$, and a heavier state with $S = 2$, $L = 0$, and $J^{PC} = 2^{++}$. Radial excitations, states with angular momentum $L \geq 1$, and ggg states also appear in the spectrum. Over the years there have been numerous experimental searches for glueballs. It was pointed out early on that a promising method is to use the radiative decay of a heavy $Q\bar{Q}$ orthoquarkonium state [4,5]. At the level of elementary constituents, this decay is $Q\bar{Q} \to \gamma gg$, so that when the two gluons are emitted with an invariant mass close to that of a glueball, they have substantial probability to bind to form this state. Other production channels have also been used. At present, there are strong indications for hadrons with large gluonic components, although there is still no consensus concerning the details of the mixing of $q\bar{q}$ and gluonic components to form various physical mass eigenstates [1–3].

In this paper we examine the temporal evolution of glueball production in radiative orthoquarkonium decay. We use the fact that glueballs have a smaller density of states than $q\bar{q}$ mesons, as a function of mass, in conjunction with the Heisenberg uncertainty principle, to infer that one can generically measure the formation of a glueball sooner than the formation of a $q\bar{q}$ meson. On the basis of this observation, we propose a notion of gluon-glueball duality. We apply this to comment on current experimental searches for glueballs and to suggest a method that could be useful for these searches. An outline of the paper is as follows. In Sec. II we review quark-hadron duality. In Secs. III and IV we give some background on glueball properties and searches. In Sec. V we introduce the notion of gluon-glueball duality in two simplified contexts, and in Sec. VI we discuss it in full QCD. We point out that in studying the production and decay of glueballs, it is useful to analyze the temporal evolution of the gg subsystem as it is produced, binds to form a protoglueball, mixes with $q\bar{q}$ components, and finally decays. Section VII suggests some future lattice gauge measurements that are relevant to gluon-glueball duality, while in Sec. VIII we apply our observations to experimental searches for glueballs.

II. QUARK-HADRON DUALITY

We first give some background on ideas of duality in hadronic physics. The reader who is familiar with this material can skip this section and proceed directly to our new observations in Secs. V and VI. The idea of quark-hadron duality in several related forms [6,7] dates back to the early period in the development of the quark-parton model. In the Bloom-Gilman form [7], it states, roughly speaking, that in a reaction such as an electron scattering off a nucleon, the sum of the cross sections for the full set of exclusive hadronic final states X_h that are kinematically accessible at a given center-of-mass energy $E_{\rm c.m.} = \sqrt{s}$ is equivalent to the cross section for the elementary reaction $e + q \to e + q$ involving the quarks in the nucleon. A similar duality relation applies to charged-current neutrino reactions such as $\nu_\mu + N \to \mu + X_h$. Let us denote the four-momenta of the incident and scattered leptons as ℓ_1 and ℓ_2, with $\ell_1 - \ell_2 = q$, $(\ell_1^0)_{\rm lab} = E$, $(\ell_2^0)_{\rm lab} = E'$, and the four-momenta of the target nucleon and final hadronic state as p and p_X. We further recall the standard Bjorken variables $x = -q^2/(2q \cdot p)$ and $y = q \cdot p/\ell_1 \cdot p = (E - E')/E$. Then this duality is the statement that

$$\sum_{X_h} \sigma(\nu_\mu + N \to \mu + X_h) \sim \int_0^1 dx \int_0^1 dy \frac{d\sigma}{dxdy} \times (\nu_\mu + f \to \mu + f'), \quad (2.1)$$

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where f denotes all of the charge $-1/3$ quarks (and charge $-2/3$ antiquarks) that can participate in this reaction. At a fundamental level, this duality is justified by the asymptotic freedom of QCD [8]. In the deep inelastic scattering of an electron or neutrino off a nucleon N, the hadronic part of the cross section involves the tensor

$$W_{\mu\nu}(q,p) = \frac{1}{2}\sum_X \langle N|J_\mu|X\rangle\langle X|J_\nu^\dagger|N\rangle(2\pi)^3\delta(p+q-p_X) \propto \int \frac{d^4z}{2\pi} e^{-iq\cdot z}\langle N|J_\mu(z)J_\nu(0)^\dagger|N\rangle, \tag{2.2}$$

where X denotes a hadronic final state and J_μ is the respective electromagnetic or weak (charged or neutral) current. One then uses the Wilson operator product expansion to express the bilocal product of currents in terms of a sum of local operators, applicable near to the light cone $z^2 \to 0$, as enforced by the kinematic conditions $-q^2 \gg \Lambda_{\rm QCD}^2$ and $q\cdot p \gg \Lambda_{\rm QCD}^2$, where $\Lambda_{\rm QCD} \simeq 300$ MeV is the scale where QCD confines and spontaneously breaks chiral symmetry. This enables one to express the deep inelastic scattering off the nucleon in terms of the scattering off quarks. The asymptotic freedom of QCD has the consequence that these quarks are quasifree when probed at short distances. Similarly, away from particle thresholds, one can calculate the total cross section for $e^+e^- \to$ hadrons at center-of-mass energy $\sqrt{s}$ in terms of the cross section for $e^+e^- \to q\bar{q}$, where $2m_q \lesssim \sqrt{s}$. One can consider the cross section for $e^+e^- \to$ hadrons, smeared over resonances, to be equivalent to the elementary reaction $e^+e^- \to q\bar{q}$, summed over the kinematically accessible quarks [9–11]

$$\sum_{X_h}\sigma(e^+e^- \to X_h) \sim \sum_q \sigma(e^+e^- \to q\bar{q}). \tag{2.3}$$

In the full QCD theory, the notion of quark-hadron duality is naturally generalized to parton-hadron duality, where the partons include both quarks and gluons, and the hadrons are understood to include not only qqq baryons and $q\bar{q}$ mesons, but also hadronic mass eigenstates that are linear combinations of $q\bar{q}$ and gg, ggg, etc. Possible exotic color-singlet hadrons such as, in the bosonic sector, $q\bar{q}q\bar{q}$ and $q\bar{q}g$ can, in principle, also be included in this set of physical states. In one sense, this duality amounts to the statements that (i) there is a complete orthonormal basis of perturbative quark and gluon states forming the Fock space of perturbative QCD, and there is a complete orthonormal basis of physical color-singlet hadronic mass eigenstates forming another Fock space; and (ii), given the asymptotic freedom of QCD, the cross section for an inclusive reaction involving the contributions of many exclusive physical channels with smearing over resonances as appropriate, can be expressed in terms of the corresponding cross section in terms of the elementary partonic degrees of freedom. In another sense, one can think of it as somewhat analogous to a Mittag-Leffler expansion, in which a function is written as a sum over its poles. In this context, one may recall that the Mittag-Leffler expansion of the Euler beta function forms part of the mathematical basis of the s-t duality in the Veneziano and Virasoro amplitudes in hadronic string theory [12–16].

A specific $\bar{q}q \leftrightarrow$ meson duality (and the analogous $gg \leftrightarrow$ glueball duality to be introduced next) is particularly useful. This is especially the case if one considers the large-N_c limit of QCD [17–19]. For large N_c, baryons become very heavy, and the kinematically accessible hadronic states X_h directly produced in e^+e^- annihilation are $\bar{q}q$ mesons. Since the decay rate of such a meson or glueball vanishes in the large-N_c limit, meson resonances are narrow in this limit. The energy integral in Eq. (2.3) then becomes essentially a summation over the contributions of these resonances.

Mesons and baryons are observed to lie on approximately linear Regge trajectories of the form

$$\alpha(m^2) = \alpha_0 + \alpha' m^2 \tag{2.4}$$

with respective intercepts α_0 and a common Regge slope $\alpha' = 0.9\ {\rm GeV}^{-1}$. Physical meson states occur where the angular momentum $\alpha(m^2)$ is equal to a non-negative integer. This behavior was originally motivated by analysis of potential scattering and was elegantly explained by hadronic string theory (the dual resonance model), according to which a meson is a mass eigenstate of an open string. It is believed (although it has not been proved) that the large-N_c limit of SU(N_c) QCD reproduces features of a hadronic string theory. In the hadronic string model, the string tension $\sigma = 1/(2\pi\alpha')$, so that $\sqrt{\sigma} \simeq 0.42$ GeV. Physically, this string tension represents the energy per unit length of the chromoelectric flux tube between the q and $\bar{q}$ forming the meson. An example of a Regge trajectory is that for the $S = 1$, $I = 1$ (isovector) mesons, which includes $\rho(770)$, $a_2(1320)$, $\rho_3(1690)$, and $a_4(2040)$, with increasing values of J indicated as subscripts (where $\vec{J} = \vec{L} + \vec{S}$). The radial excitations $\rho' = \rho(1450)$, $\rho'' = \rho(1700)$, etc. are on so-called daughter trajectories, forming a horizontal line in the plane with horizontal and vertical axes corresponding to $s = m^2$ and J, respectively.

A feature predicted by hadronic string theory (predating QCD) and consistent with data is that the density of $q\bar{q}$ meson states as a function of mass m grows rapidly with m. This is also the case for a specific flavor state such as $\bar{u}d$ and specific values of J, parity and charge conjugation quantum numbers, such as $J^{PC} = 1^{--}$. Let us denote the density of meson states, i.e., the number of states at a given mass m, counting those on the leading and daughter meson trajectories, as

$$n(m)_M \equiv \frac{dn_M(m)}{dm}, \tag{2.5}$$

where M stands for "meson." For the (bosonic) string in d

spacetime dimensions, the meson density of states $n(m)_M$ grows exponentially fast for $m^2 \gg (\alpha')^{-1}$ [20,21]:

$$n(m)_M \sim m^{-(d+1)/4} \exp[\pi m \sqrt{(2/3)(d-2)\alpha'}] \tag{2.6}$$

where d is the spacetime dimension. Hence, at sufficiently high mass, these resonances overlap. Indeed, even before one takes account of this asymptotic exponential growth in the density of states, the hadronic string model already implies that they will overlap, because on the leading Regge trajectory, Eq. (2.4) shows that two successive meson states with the same J^{PC}, that differ by two units of L and J, satisfy $\Delta J = 2 = \alpha'(m_{L+2}^2 - m_L^2)$, so that

$$m_{L+2} - m_L = \frac{2}{\alpha'(m_{L+2} + m_L)}. \tag{2.7}$$

Hence, as the masses of these states increase, their mass difference decreases, and eventually becomes less than their widths, so that they overlap. This happens when the mass difference $m_{L+2} - m_L$ becomes comparable to either of the widths Γ_L or Γ_{L+2}. For the present illustrative purposes, we approximate these as being roughly equal, and denote them as Γ, which we take to be $\Gamma \sim 0.25$ GeV. Setting $m_{L+2} - m_L = \Gamma$ and solving, we get

$$\frac{m_{L+2} + m_L}{2} \simeq \frac{1}{\alpha'\Gamma} \simeq 4.5 \text{ GeV}. \tag{2.8}$$

Thus, as (light-quark) meson masses increase beyond this scale, the states in their spectrum tend to merge. In the upper end of the mass region of interest here, from about 1.5 to 3 GeV, the asymptotic condition $m^2 \gg (\alpha')^{-1}$ begins to be satisfied, so the formula (2.6) is relevant. A hadronization model based on the chromoelectric flux tube between a q and $\bar{q}$ in conjunction with a Schwinger mechanism was given in Ref. [22]. The non-Abelian generalization, in which a constant chromoelectric field creates gluons, was analyzed in terms of relevant invariants in [23,24]. The flux-tube mechanism is incorporated in current hadronization computer programs such as PYTHIA [25]. Because of the increasing density of meson states for masses $m^2 \gg (\alpha')^{-1}$, the cross section for $e^-e^+ \to q\bar{q} \to$ hadrons then becomes a continuous curve which, according to the duality assumption, coincides with the continuous perturbative curve.

An important feature concerns the behavior in the mass region below approximately 3 GeV. The asymptotic freedom and precocious scaling properties of QCD make quark-hadron duality a property that is effectively local in mass already at masses that are only modestly greater than $\Lambda_{\rm QCD}$. Thus, the ρ and ρ' of masses 0.77 and 1.45 GeV largely account for the contributions in their mass region to finite-energy sum rules [12]. This is also manifest in Bloom-Gilman duality [7]. A difference is that Dolen-Horn-Schmid duality applies to $2 \to 2$ reactions involving on-shell hadrons, e.g., $\pi^+\pi^- \to \pi^+\pi^-$. Similarly, Bloom-Gilman duality applies to reactions such as exclusive electroproduction, e.g., $e + p \to e + p + \pi^0$.

III. REMARKS ON GLUEBALL PROPERTIES

In this section we note some properties of glueballs that we will use in our analysis. An especially important and relevant property that motivates our new suggestion is the density of states, but we begin with some basic facts. Since the gluons are bosons, Bose statistics implies that the total glueball wave function is symmetric under interchange of any two gluons. A difference between $q\bar{q}$ mesons and glueballs is that, although a confined quark picks up a (gauge-invariant) dynamical, constituent mass of order $\Lambda_{\rm QCD}$, one cannot ascribe a mass in the same manner to a bound gluon, since this would violate the color gauge invariance. This means that while a constituent quark model can provide a good description of baryons and $q\bar{q}$ mesons (see, e.g., [26] for a recent discussion and references to the literature), one cannot describe the glueball in quite so simple a manner. Furthermore, in the time evolution of an initial gluonic state, the splittings $g \to gg$ can occur in a manner that is leading in $1/N_c$, in the large-N_c limit. This is different from the time evolution of a $q\bar{q}$ state, for which the transition $q \to q + g$ is suppressed in the large-N_c limit. Thus, here a physical state denoted as gg strictly refers only to a state whose quantum numbers are most simply attainable via a (color-singlet) combination of two gluons. Keeping this caveat in mind, the lowest-lying glueballs can be modeled as gg bound states. For these, in the Clebsch-Gordon decomposition of the gg SU(3)c representations 8×8, the singlet appears as a symmetric combination. Hence, the product of the space and spin wave functions must be even under this interchange. The spin wave function involves the addition of two spin-1 angular momenta. If the resultant spin of the gg combination is $S = 0$ or $S = 2$, this spin wave function is even, so the relative angular momentum must also be even, and the ground state is $L = 0$. With $P = (-1)^L$ and $C = (-1)^{L+S}$ for this combination of two bosons, one thus expects that the lowest two glueball states have (i) $S = L = J = 0$, whence $J^{PC} = 0^{++}$ and (ii) $L = 0$, $S = J = 2$, whence $J^{PC} = 2^{++}$. The higher-lying glueball states can involve both nonzero internal angular momenta and radial excitations.

Estimates of glueball masses and widths have been made on the basis of a number of different methods [1,2,4,5,27–57]. Continuum approaches include the MIT bag model [36–48], flux-tube models, AdS/CFT approaches, and calculations based on the Bethe-Salpeter equation. Lattice calculations have achieved a rather high level of precision [27,49–57]. These naturally give the mass of a particular glueball in terms of the square root of the string tension, $\sqrt{\sigma} = 0.42$ GeV. For masses of glueballs in purely gluonic QCD, recent lattice calculations [51,55,57] yield

$$m(0^{++}) \simeq 1.7 \text{ GeV}, \tag{3.1}$$

$$m(2^{++}) \simeq 2.4 \text{ GeV}, \tag{3.2}$$

$$m(0^{-+}) \simeq m(0^{++\prime}) \simeq 2.6 \text{ GeV}, \tag{3.3}$$

and

$$m(1^{+-}) \simeq m(2^{-+}) \simeq 3.0 \text{ GeV}, \tag{3.4}$$

up to approximately 3 GeV. Here, the $0^{++\prime}$ glueball is a radial excitation of the 0^{++} glueball. Lattice measurements of higher-lying glueball masses have been made up to roughly 5 GeV [51,55,57]. Some unquenched calculations have also been reported [56].

In the context of the Regge or hadronic string model, glueballs correspond to closed strings, which have a Regge slope equal to half of the Regge slope for open strings:

$$(\alpha')_{GB} = \frac{\alpha'}{2}. \tag{3.5}$$

It follows that for $m^2 \gg (\alpha')^{-1}$, the density of states for glueballs (closed strings), $n(m)_{GB} \equiv dn(m)_{GB}/dm$, is exponentially smaller than the density of states for $q\bar{q}$ mesons (open strings), $n(m)_M$. Quantitatively, from Eqs. (2.6) and (3.5), one finds that, for $m^2 \gg (\alpha')^{-1}$, the ratio of these densities of states is

$$\frac{n(m)_{GB}}{n(m)_M} \sim 2^{-(d+1)/4} \exp[\pi m(\sqrt{2}-1)\sqrt{(d-2)\alpha'/3}]. \tag{3.6}$$

With $\alpha' = 0.9 \text{ GeV}^{-1}$ (and $d = 4$),

$$\frac{n(m)_{GB}}{n(m)_M} \sim 0.3 \quad \text{for } m = 2 \text{ GeV} \tag{3.7}$$

and

$$\frac{n(m)_{GB}}{n(m)_M} \sim 0.1 \quad \text{for } m = 3 \text{ GeV}. \tag{3.8}$$

Thus, Eq. (3.6) indicates that the spectrum of low-lying glueball states is more sparse than that of the isoscalar $q\bar{q}$ mesons in the mass region from 1.5 to 3 GeV. To within the theoretical and experimental uncertainties, this is consistent with the data: for example, in the mass region 1.3 to 2 GeV, there are the following scalar 0^{++} states: $f_0(1370)$, $f_0(1500)$, $f_0(1710)$, and indications from recent BES data of an $f_0(1790)$ and $f_0(1810)$ [1]. The lattice estimates (to be discussed next) indicate that in this interval of masses, there is one 0^{++} glueball expected. In this channel, this gives a ratio of $n(m)_{GB}/n(m)_M \sim 0.25$.

Estimates have also been made of glueball widths. In the limit $N_c \to \infty$ with $g_s^2 N_c$ fixed and finite [17,18], where g_s is the $\text{SU}(3)_c$ gauge coupling, the width of a glueball vanishes like

$$\Gamma_{GB} = \frac{1}{\tau_{GB}} \propto \frac{\Lambda_{\text{QCD}}}{N_c^2}, \tag{3.9}$$

while the width of a $q\bar{q}$ meson M vanishes like

$$\Gamma_M = \frac{1}{\tau_M} \propto \frac{\Lambda_{\text{QCD}}}{N_c}. \tag{3.10}$$

The relations (3.9) and (3.10) follow from direct diagrammatic $1/N_c$ counting. As expected from the close correspondence between the large-N_c limit of QCD and the hadronic string picture, they can also easily be understood in a string picture. The decay of a $q\bar{q}$ meson resonance (an open string) takes place via a single cut in the string (flux tube), whereas the decay of a glueball requires a first cut to transform it from the initial closed string to an open string and then a second cut to produce the two-meson (e.g., $\pi\pi$) final state. With each cut being suppressed by a $1/N_c$ factor, the results on Γ_M and Γ_{GB} follow. Reverting from the large-N_c limit to real QCD, actual estimates of glueball widths have varied widely, ranging from a few MeV to $O(10^2)$ MeV [37,39,51].

IV. PREVIOUS SEARCHES FOR GLUEBALLS

Here we briefly review results of previous searches for glueballs. There is extended literature dealing with search criteria and analysis of data [1,2,4,5,58–87]. One signature is that glueballs would not fit into the standard set of $q\bar{q}$ states, including their angular momentum and radial excitations. Second, since the gluons carry no electric charge, one expects a small branching ratio of glueballs into photons. Third, since the gluons carry no flavor, it was originally expected that the decays of these states should be flavor independent, up to phase space considerations. On the other hand, however, it has been suggested that for $J = 0$ glueballs, there should be helicity suppression of decays to light-quark hadrons, at least if the decay amplitude element can be accurately modeled beginning with emission of a single $q\bar{q}$ pair [61]; if it involves higher initial multiplicity of (anti)quarks, then this helicity suppression would be reduced [62]. Fourth, some glueball states have exotic values of J^{PC} that cannot be obtained from $q\bar{q}$.

Experimental searches for glueballs have been carried out at many laboratories. Experiments using e^+e^- annihilation include Mark III and the Crystal Ball at SPEAR, the subsequent Crystal Ball experiment at DORIS, and experiments at other laboratories, including Orsay, CESR, Novosibirsk, BES, *BABAR*, and Belle [1,3,63–71]. We focus first on the isoscalar, $J^{PC} = 0^{++}$ channel, since the lightest pure glueball has these quantum numbers. There are three prominent isoscalar, Lorentz scalar 0^{++} meson resonances between about 1.0 and 1.7 GeV, namely, the $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$. The quark model is only expected to produce two such states, which would have $S = 1$, $L = 1$, $J = 0$ and be the analogues of the flavor SU(3) octet and singlet pseudoscalar mesons, η and η'.

The fact that there are three f_0 states in this range is thus one of several pieces of evidence suggesting that the third may be primarily a glueball. The $f_0(1370)$ is quite broad, with $\Gamma \sim 300$ MeV, while the $f_0(1500)$ and $f_0(1710)$ have widths of roughly 100–140 MeV [3]. More recently, The Beijing e^+e^- collider BES has found evidence for an $f_0(1790)$ and $f_0(1810)$ [1]. Several theoretical fits to these data have been performed [1,2]. The authors of some of these fits concluded that the lightest glueball forms a primary component in the $f_0(1500)$ [79–82], while others concluded that this lightest glueball forms the primary component in the $f_0(1710)$ [51,83] and still others invoked important contributions from $q\bar{q}q\bar{q}$ states [84,85] (see also [34]). Further data and analyses should help to elucidate this situation [1,2,86].

V. GLUON-GLUEBALL DUALITY IN TWO SIMPLIFIED CONTEXTS

To explain our notion of gluon-glueball duality, we begin with two simplified forms of QCD, namely (i) without any light quarks, and (ii) in the large-N_c limit. Let us first consider the case of no light quarks. For definiteness, we imagine the standard model with one generation of fermions with quarks U and D having masses $m_U, m_D \gg \Lambda_{\rm QCD}$. We denote these quarks collectively as Q. We next consider the favored reaction for glueball production, namely the production, in e^+e^- annihilation, of the orthoquarkonium $Q\bar{Q}$ state, followed by its radiative decay $Q\bar{Q} \to \gamma gg$. An important feature of this world is that a number of the lowest-lying glueball states would be stable. Indeed, using the lattice estimates of low-lying glueball masses listed above, all six of the states listed would be stable; in order for a heavier glueball to be kinematically allowed to decay to two of the lightest glueballs, it would necessarily have a mass greater than about 3.4 GeV. Thus, the invariant mass distribution dN/dm_G for the mass of the gluonic states recoiling against the photon in the radiative orthoquarkonium decay $Q\bar{Q} \to \gamma gg$, i.e., at the physical level, $Q\bar{Q} \to \gamma + X_{GB}$, where X_{GB} denotes a glueball, would exhibit very sharp resonances for $m_{X_{GB}}$ equal to the mass of each of the stable glueballs, and then finite-width resonances for the higher-lying unstable glueballs, up to the kinematic limit allowed by the mass of the original orthoquarkonium state. The statement of gluon-glueball duality would be that, with appropriate smearing,

$$\int \left(\frac{dN}{dm}\right)_{GB} dm = \int \left(\frac{dN}{dm}\right)_{gg} dm, \tag{5.1}$$

where the first integral is over physical glueball final states and the second integral denotes the perturbative calculation of dN/dm, where m is the invariant mass of the gg subsystem in the decay $Q\bar{Q} \to \gamma gg$. In terms of the overall e^+e^- cross section, the gluon-glueball duality would be the relation, with appropriate smearing,

$$\sum_{GB} d\sigma(e^+e^- \to n^3S_1(Q\bar{Q}) \to \gamma + X_{GB}) \simeq d\sigma(e^+e^- \to n^3S_1(Q\bar{Q}) \to \gamma gg), \tag{5.2}$$

where again the second term represents the perturbative calculation of the production and decay.

In the $N_c \to \infty$ limit, $q\bar{q}$ mesons and glueballs become stable, as indicated by Eqs. (3.9) and (3.10). Furthermore, there is no mixing between glueballs and $q\bar{q}$ mesons. Here, gluon-glueball duality takes a particularly simple form. With N_c large but finite, so as to allow for the radiative decay of the heavy orthoquarkonium state, this duality would again be expressed via the relations (5.1) and (5.2). Quark-hadron duality also takes a particularly simple form in this large-N_c QCD. This type of connection between sums over resonances and properties of the underlying quarks and gluons was previously used with QCD sum rules to study correlators of various operators [28–34].

VI. GLUON-GLUEBALL DUALITY IN QCD

We next discuss our notion of gluon-glueball duality in real QCD. An important part of our discussion of this duality in the radiative decay of a heavy orthoquarkonium state $Q\bar{Q} \to \gamma gg$ is a careful treatment of the temporal evolution of the gg subsystem, as it is initially produced, as the gluons bind to form a protoglueball, as this glueball mixes with a $q\bar{q}$ component, and as it finally decays. To understand gluon-glueball duality, it is crucial to analyze the time evolution and hierarchy of time scales relevant to the $Q\bar{Q} \to \gamma gg$ decay, as compared with the production of mesons in a reaction such as $e^+e^- \to q\bar{q}$. A general statement concerns the time required for the formation of color-singlet states from the respective initial $q\bar{q}$ and gg states. Given the fact that QCD confines on a scale $\Lambda_{\rm QCD}$ and that hadrons have a corresponding size

$$r_{\rm had} \simeq \frac{1}{m_\pi} \simeq 1 \text{ fm}, \tag{6.1}$$

and given the causality condition that information cannot be communicated any faster than at the speed of light, it follows that a minimum time associated with the formation of color-singlet hadronic states is

$$t_{\rm had} = \frac{r_{\rm had}}{c} \simeq 0.3 \times 10^{-23} \text{ sec} \tag{6.2}$$

(where we have explicitly indicated the speed of light, c). This is a rough estimate, accurate to a factor of order unity. For example, given that a glueball is represented by a closed string, one could consider a special case in which the closed string forms a circle, and one might argue that it is the circumference of this circle rather than the radius that is of order 1 fm. In this case, the radius would be $1/(2\pi)$ fm and the time taken for the formation, involving motion of the gluons outward from the center of the circle would be smaller than the value given in Eq. (6.2) by the

factor 2π. Because of the asymptotic freedom of QCD, for both (i) $e^+e^- \to \bar{q}q$ at center-of-mass energy $\sqrt{s} \gg \Lambda_{\rm QCD}$ and (ii) the radiative decay of heavy orthoquarkonium $\bar{Q}Q \to \gamma gg$, there exists a sufficiently short time $t_{\rm pert}$ such that for times $t < t_{\rm pert}$ the physics can be described using perturbative QCD. This satisfies the inequality

$$t_{\rm pert} < \frac{1}{\Lambda_{\rm QCD}} \sim t_{\rm had}. \tag{6.3}$$

Given the precocious scaling behavior of QCD, it is not necessary that $t_{\rm pert} \ll t_{\rm had}$. For the two specific cases under discussion, one could take $t_{\rm pert} \sim 1/\sqrt{s}$ for the reaction $e^+e^- \to q\bar{q}$ and $t_{\rm pert} \sim 1/(2m_Q)$ for the decay $Q\bar{Q} \to \gamma gg$. A typical value would be $t_{\rm pert} \sim 1/(3\ {\rm GeV}) \simeq 10^{-25}$ sec. Thus, in the reaction $e^+e^- \to \bar{q}q$, during the time interval $0 < t < t_{\rm pert}$, the q and $\bar{q}$ recede from each other in an approximately perturbative manner, with the first modification being the emission of a gluon, leading to a $q\bar{q}$ subsystem in a color octet state together with the emitted gluon g. In the radiative decay of the heavy orthoquarkonium state $Q\bar{Q} \to \gamma gg$, during the time interval $0 < t < t_{\rm pert}$, the gg final-state subsystem mainly evolves into more gluons via $g \to gg$ splittings. As noted above, this gluon splitting occurs at leading order in the large-N_c limit, in contrast to the $q \to q + g$ or $g \to (q\bar{q})_8$ processes, which start to mix $q\bar{q}$ with the initially purely gluonic gg subsystem.

After a time t_{MF}, where MF stands for "meson formation," the initial $\bar{q}q$ system will bind to form a meson, and after a corresponding time t_{GBF}, where GBF stands for "glueball formation," the initial gg system will bind to form a glueball. From the causality argument above, one has the general inequalities

$$t_{MF}, t_{GBF} \geq t_{\rm had} \tag{6.4}$$

and hence also the obvious inequalities $t_{MF}, t_{GBF} \geq t_{\rm pert}$. The q and $\bar{q}$ in the meson, and the gluons in the glueball, have minimum bound-state momenta $k_{\rm min} \sim \Lambda_{\rm QCD}$ because of confinement [35]. Several factors are relevant for the hadronic formation times t_{MF} and t_{GBF}, including (i) the intrinsic QCD hadronization time scale $t_{\rm had}$, (ii) the mixing of $q\bar{q}$ and gluonic states to form mass eigenstates, (iii) the decay widths Γ_i of various mesons and glueballs, and (iv) especially importantly for our current discussion, the density of meson and glueball states, $n(m)_M$ and $n(m)_{GB}$. The quantum-mechanical uncertainty relation $\Delta E \Delta t \gtrsim \hbar/2$ implies that the observation time interval Δt needed for an observer to measure the spectrum of states with a resolution in mass Δm is bounded below by $\Delta t \geq (\hbar/2)/\Delta m$. Here Δm is set by a combination of the density of states with the same quantum numbers (isospin and J^{PC}) and by the widths of these states. Let us consider a glueball search conducted in the range of masses $m_{GB} =$ 1.5–3 GeV. Given the inequality in the density of glueball versus $q\bar{q}$ meson states in Eq. (3.6), it follows that the time needed to experimentally measure and resolve glueball states is shorter than that needed for $q\bar{q}$ meson states. Using the hadronic string model as a theoretical guide, which is consistent with the observed states in the relevant mass region, one has, roughly,

$$t_{GBF} \simeq \frac{t_{MF}}{4}. \tag{6.5}$$

This leads us to suggest a different picture of glueball production than the one that is often used in analyses of experimental data on glueball searches. Conventional analyses use meson mass eigenstates that are linear combinations of $q\bar{q}$ states and gluonic states. Our new point is that it is crucial to take into account the actual temporal formation of the glueball states. Given that the glueball formation time is shorter than the meson formation time, with $t_{GBF} \simeq t_{MF}/4$ being a reasonable estimate, the glueball forms before significant mixing with the $q\bar{q}$ sector takes place. A concrete realization of both the $\bar{q}q \leftrightarrow$ meson and $gg \leftrightarrow$ glueball dualities can be obtained as follows. Starting with an initial pure $\bar{q}q$ entrance state, we implement the duality by letting the unitary QCD evolution operator, formally expressed as $U(t) = e^{-iHt}$, operate on this state, where here H denotes the QCD Hamiltonian, yielding

$$U(t_{MF})|\bar{q}q(t=0)\rangle = |M,\ {\rm meson}\rangle. \tag{6.6}$$

That is, the evolution over this time interval will yield a physical $\bar{q}q$ meson resonance. In a similar manner, in a purely gluonic sector

$$U(t_{GBF})|gg(t=0)\rangle = |GB,\ {\rm glueball}\rangle. \tag{6.7}$$

The two gluons in the gg subsystem produced in the radiative orthoquarkonium decay $\bar{Q}Q \to \gamma gg$ emerge from spacetime points that are separated by a small distance $\Delta r \sim 1/m_Q$, where $Q = c$ or b is a heavy quark. This is not precisely the same as the production of a scalar glueball by the action of the local operator

$$S(x) = G_{\mu\nu}(x)G^{\mu\nu}(x) \tag{6.8}$$

on the vacuum. However, a semiclassical argument leads to the conclusion that the gg usually bind with $L = 0$ relative orbital angular momentum. For example, in the case $Q = b$, the spatial separation of the points where the two gluons are emitted is $\Delta r \sim 1/m_b \sim 0.2\ {\rm GeV}^{-1}$. The three-momenta of the gluons in the gg rest frame are $|\vec{k}_g| \sim m_{\rm GB}/2 \sim O(1)$ GeV. The resultant average value of the relative orbital angular momentum is

$$\langle L \rangle \sim |\vec{k}_g|\Delta r \lesssim 0.2. \tag{6.9}$$

Hence, one expects that this production mechanism will yield mainly glueball states with $L = 0$, namely, the 0^{++} and 2^{++} mentioned before.

In the radiative $Q\bar{Q} \to \gamma gg$ decay of orthoquarkonium, the gg subsystem is manifestly purely gluonic to start with, and mixing with $q\bar{q}$ components occurs subsequently. In the large-N_c limit, this mixing is suppressed by $1/N_c$, which has led to the common expectation that there could be hadrons that are primarily gluonic, with only a small $q\bar{q}$ component. Our estimate that $t_{GBF} \simeq t_M/4$, in conjunction with the suggestion from large-N_c arguments that mixing of gluonic and $q\bar{q}$ components may be rather small, leads us to the important inference that the gluon-glueball duality could hold reasonably well in full QCD as well as in the simplified contexts which we initially used to introduce it. It is understood that there will be some corrections due to the mixing of the gluons in a primarily gluonic hadron with $q\bar{q}$ states.

Let us next consider the longer times required for the $\bar{q}q$ meson, or the glueball, to decay into hadrons that are stable with respect to the strong interactions. The formation and decay times for the meson resonances are comparable, although $\tau_M \gtrsim t_{MF} > t_{\rm pert}$, and similarly for the glueballs, one has $\tau_{GB} \gtrsim t_{GBF} > t_{\rm pert}$. This is to be contrasted with the situation for a very heavy quark, namely, the top quark, which decays weakly before it can form color-singlet hadronic $t\bar{t}$ or $t\bar{q}$ states. To the extent that the large-N_c limit is applicable to QCD, one expects that, other factors such as phase space being equal, the lifetime τ_{GB} might be somewhat longer than τ_M, i.e., the glueball width might be somewhat smaller than that for a $q\bar{q}$ meson of comparable mass. However, in actual QCD, glueball widths may not be suppressed, and may, indeed, be of order 100–300 MeV. This would be somewhat analogous to the situation with the η' meson; in the $N_c \to \infty$ limit (with $\lambda \equiv g^2 N_c$ fixed), instanton effects are exponentially suppressed by the factor $\exp(-8\pi^2/g^2) = \exp(-8\pi^2 N_c/\lambda)$, so that $\mathrm{U}(1)_A$ is a good global symmetry and the isoscalar pseudoscalar meson η' is an approximate Nambu-Goldstone boson. However, in real QCD the η' is rather heavy, with a mass of 958 MeV. An important point is that, with the hierarchy of time scales that we have noted, the glueball decays by popping two pairs of light $q\bar{q}$ quarks out of the vacuum to produce the two final-state mesons (π's, K's, etc.). This process is essentially equivalent to the process by which the initially purely gluonic state acquires a $q\bar{q}$ component.

VII. FURTHER POSSIBLE INSIGHT FROM LATTICE QCD

Lattice calculations have the appeal of providing a fully nonperturbative tool for studying the properties of QCD, and the advantage of being able to be continually improved with the use of larger lattices, longer running times, improved lattice actions, and careful analysis of statistical and systematic uncertainties. Most lattice QCD calculations of glueball masses have been performed using the quenched approximation. Some unquenched calculations have also been reported [56]. Both the necessity of evaluating the fermion determinant and the related presence of disconnected flavor loops appearing in unquenched calculations make these calculations more difficult than computations in quenched QCD. We suggest that it would be worthwhile for lattice gauge simulations to address some of the issues that we have raised in this paper. We are interested not just in minor shifts of the glueball spectrum, but rather in finding the time t^* by which the admixture of the initial glue state with the $\bar{q}q$ and multiquark states becomes significant. For this purpose it could be useful to study the correlator $C(t) = \langle S(0)S(t)\rangle$ of the above-mentioned scalar glueball operator and examine how its Euclidean time dependence might differ from a simple exponential of the form $\exp(-m(0^{++})t)$. (Here, it is understood that one would ideally have removed the effects of higher-lying glueball states with the same $J^{PC} = 0^{++}$ and also that one would have taken account of effects due to periodic lattice boundary conditions.) For long, asymptotic times t such that $t \gg 1/(2m_\pi)$, the behavior of this scalar correlator $C(t)$ is controlled by the lowest-mass s-channel threshold, namely, that for the 2π final state, but we are interested in shorter times. Similar calculations could be performed for the 2^{++} glueball state by using an appropriate color-singlet tensor correlator. Assessing the full lifetime until the glueball decays into final hadrons that are stable with respect to the strong interactions is challenging, but is not essential for our purposes here.

VIII. APPLICATION TO EXPERIMENTAL SEARCHES FOR GLUEBALLS

In this section we apply our notion of gluon-glueball duality to suggest a method that could be useful in experimental searches for glueballs in radiative orthoquarkonium decays, in particular, those involving the $\Upsilon(1S)$, $\Upsilon(2S)$, and $\Upsilon(3S)$ states. There are very high-statistics data sets from radiative J/ψ decays, which have been used quite effectively for glueball searches. However, radiative Υ decays allow one to search in a wider mass range and reduce phase space suppression for decays into final states involving more massive glueballs. While a major purpose of the experiments at *BABAR* and Belle was to study B physics and CP violation, they accumulated of order 10^9 events from decays of $\Upsilon(1S)$, $\Upsilon(2S)$, and $\Upsilon(3S)$, as well as the $\Upsilon(4S)$ state that provided a copious source of B_d mesons [70,71]. These data extended the already impressive data sets collected by the CLEO experiment at CESR in its later years of high-intensity running [65,66]. For an average radiative decay branching ratio of 1.5% we expect of order 1.5×10^7 radiative decays in the *BABAR* and Belle data. In the radiative $\Upsilon(nS) \to \gamma gg$ (with $n = 1, 2, 3$), we label the four-momenta of the outgoing photon and gluons as k_γ, k_1, and k_2, and recall that it is necessary to symmetrize the amplitude under the interchange $k_1 \leftrightarrow k_2$ to take account of the two identical bosons in the (perturbative) final state. If one makes the approximation,

in the perturbative calculation of the amplitude, that the outgoing (massless) gluons interact only very weakly with each other, it follows that the three invariant mass combinations $(k_1 + k_2)^2$, $(k_\gamma + k_1)^2$, and $(k_\gamma + k_2)^2$ are uniformly distributed over the Dalitz plot, which becomes an equilateral triangle. In this Dalitz plot, the region of interest, which is assumed here to be dominated by the lowest-lying glueballs, is then a rectangular strip adjacent to the bottom of the triangle. The total area of this region is $2 \times (2.7)^2/100$, i.e. 15% of the total area of the Dalitz plot and hence includes approximately 2×10^6 events. The notion of gluon-glueball duality that we have discussed then leads us to the suggestion to analyze the inclusive mass distribution of these 2×10^6 events. This avoids any bias due to postselection by the final channel (which might prefer specific final $\bar{q}q$ or multiquark resonances). Our use of gluon-glueball duality is analogous to the use of quark-hadron duality in the sense that both of these dualities relate inclusive channels and sums of exclusive channels in the respective particle processes. For notational simplicity, we denote $X \equiv X_{GB}$. Clearly, only a crude resolution $\Delta M_X \simeq 0.5$ GeV is needed to resolve the two well-separated lowest-lying glueball states with $J^{PC} = 0^{++}$ and 2^{++} (or the excited 0^{++} state). Let us denote the invariant mass squared of the gg subsystem as $M_X^2 = (k_1 + k_2)^2$ and take particle energies to be measured in the rest frame of the decaying $Q\bar{Q}$ orthoquarkonium state. The elementary kinematic relation

$$M_{Q\bar{Q}}^2 = (k_\gamma + k_1 + k_2)^2 = 2E_\gamma(M_{Q\bar{Q}} - E_\gamma) + M_X^2 \quad (8.1)$$

implies that

$$\Delta M_X = \frac{(2E_\gamma - M_{Q\bar{Q}})\Delta E_\gamma}{M_X}. \quad (8.2)$$

As an illustration, we consider the *BABAR* detector [70]; similar numbers apply for the Belle detector [71]. The fractional resolution $(\Delta E_\gamma)/E_\gamma$ of the measurement of the photon energy by the electromagnetic calorimeter of this detector varies from about 2% to 3% over the range of E_γ from $\sim$8 GeV to 1 GeV [70]. Hence, the resultant resolution ΔM_X from Eq. (8.2), for the radiative decay of the $\Upsilon(1S)$, varies from approximately 0.55 GeV to 0.27 GeV as M_X varies from 1.7 GeV to 2.4 GeV. For the radiative decay of the $\Upsilon(2S)$ the resolution ΔM_X varies from about 0.65 GeV to 0.33 GeV as M_X varies from 1.7 GeV to 2.4 GeV. Considering the very high statistics of the data sets obtained by *BABAR* and Belle, this analysis could give useful information about glueballs via broad deviations from the phase space distribution that would occur in their absence. This analysis presumes that one takes careful account of pure quantum electrodynamic (QED) backgrounds and corrections. By insisting on some hadronic activity in the detector, one may reduce such QED backgrounds without excessive biasing such as would result if one were to fully reconstruct the final hadronic state. Obviously, the experimental procedure sketched here in the broadest terms is challenging. Nevertheless, one has observed how much useful new data *BABAR* and Belle have obtained concerning new hadronic states involving charm quarks, including $X(3782)$, new D_s states, and others. Provided that our analysis of the time evolution of the glueball production process discussed above is correct, then we believe that these facilities have the potential to considerably clarify the lingering puzzles in glueball physics.

IX. CONCLUSIONS

In this paper we have presented a different picture of glueball production than the one commonly used in current analyses of data. Using the hadronic string model, we have given a quantitative estimate of the smaller density of states of glueballs (closed strings) in the region of $\sim$2 GeV, as compared with $q\bar{q}$ mesons (open strings), and, from basic quantum mechanics, we have inferred a resultant hierarchy of formation times of observable (resolvable) glueballs, as compared with $q\bar{q}$ mesons, namely, Eq. (6.5). On the basis of this, together with the suggestion from the large-N_c expansion that mixing between glueball and $q\bar{q}$ states may be suppressed, we have argued that the glueballs produced in radiative orthoquarkonium decay could plausibly form without substantial mixing with $q\bar{q}$ states. This motivates a notion of gluon-glueball duality, which we have presented, namely that the summation over sufficiently many glueball states produced in radiative orthoquarkonium decay $Q\bar{Q} \to \gamma gg$, appropriately smeared, could be well fit with the perturbative calculation of this process. We have applied this notion of gluon-glueball duality to suggest a method that could be useful in experimental searches for glueballs using radiative decays of the $\Upsilon(1S)$, $\Upsilon(2S)$, and $\Upsilon(3S)$ states using the large data sets that are currently available on these decays.

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