String entanglement and D-branes as pure states

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We study the entanglement of closed strings degrees of freedom in order to investigate the microscopic structure and statistics of objects as D-branes. By considering the macroscopic pure state limit, whenever the entanglement entropy goes to zero (in such a way that the macroscopic properties of the state are preserved), we show that boundary states may be recovered in this limit and, furthermore, the description through closed string (perturbative) degrees of freedom collapses. We also show how the thermal properties of branes and closed strings could be described by this model, and it requires that dissipative effects be taken into account. Extensions of the macroscopic pure state analysis to more general systems at finite temperature are finally emphasized.

I. INTRODUCTION

Since the first microscopical description of the black hole entropy [1], one of the most interesting problems concerning D-branes is the development of a model where their thermodynamical properties and microscopical structure may be clarified. One may study features of D-branes at finite temperature by using dualities [2], however a detailed statistical model should start from the understanding of the true microscopical degrees of freedom that describe the brane.

Closed strings are believed to describe the perturbative degrees of freedom of a quantum theory that includes gravity, and D-branes are macroscopical objects which absorb or emit closed strings [3]; on the other hand, these ones are identified with classical gravitational solitons that can work as backgrounds in a perturbative scheme [4]. At present, there is no known theory in which the D-branes are described by vacuumlike states or by states created from a vacuum. Apart from this, any satisfactory description in this direction should include thermal effects, since generic nontrivial background spacetimes carry thermodynamic properties, such as temperature and entropy. A recent approach realizes these properties [5], but it is based on an open string description whose precise relation with the graviton excitations is unknown. Other models to describe D-branes at finite temperature, like boundary states of thermal closed strings came up in Refs. [6], these states have, however, nothing to do with vacuum configurations in any sense.

This work is devoted to a twofold purpose: to shed light on all these subjects by studying the general entanglement of the right/left sectors of closed strings; and, by doing that, to show how the boundary state that describes the D-brane may be obtained from statistical analysis of the pure macroscopic states without using any link to string boundary conditions. We actually show that boundary states may be recovered as macroscopic pure states in a vanishing entropy limit, and we see furthermore that novel (nonperturbative) degrees of freedom are needed to describe the dynamics of these objects.

There are examples in which the entanglement of the degrees of freedom of single closed strings is dynamically generated in backgrounds with gravitational fields [7–9]. This is the main motivation to study more deeply this behavior and to highlight the appearance of structures like the D-branes.

The paper is organized as follows. In Sec. II, we briefly review the boundary state formalism. In Sec. III, we study the general entanglement of the internal closed string modes and, by using the canonical symmetries, define canonical variables adapted to the $p + 1$ dimension of the brane. The main results are found in Sec. IV, where (coherent) macroscopical states, localized in the brane surface, are built up as fundamental states in a closed string Hilbert space. Furthermore, these states are proposed to describe the statistical properties of the brane, the entropy operator is canonically defined, and boundary states are recovered as pure states. The finite temperature behavior of branes (and closed strings being created in them) is also discussed. In Sec. V, the pure state limit is analyzed and its generalization to other thermodynamic systems is pointed out. Finally, in Sec. VI, we summarize the main conclusions of this approach.

II. CLOSED STRINGS AND BOUNDARY STATES

Let us consider a closed bosonic string in the Minkowski space-time ($\mathbb{R}^{26}$, $\eta_{\mu\nu}$). The general solution with periodical boundary conditions reads:

$$X^\mu = x_0^\mu - i\alpha' p^\mu t + \sqrt{\frac{\alpha'}{2}} \sum_{n>0} \frac{1}{\sqrt{n}} [(\alpha_n^\mu e^{-in(t-\sigma)}$$
$$+ \alpha_n^{\dagger\mu} e^{in(t-\sigma)}) + (\beta_n^\mu e^{-in(t+\sigma)} + \beta_n^{\dagger\mu} e^{in(t+\sigma)})],$$

$$(1)$$

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After its light-cone gauge quantization (LCG), Fourier coefficients α_n^I and β_n^I ($I, J, K = 1, \ldots 24$ denote the coordinates transverse to $X^{\pm} = X^0 \pm X^{25}$), for the left- and right-moving modes, respectively, can be redefined in order to obtain the physical creation and annihilation operators for each mode n in the different sectors. Namely,

$$A_n^I = \frac{1}{\sqrt{n}} \alpha_n^I, \qquad A_n^{I\dagger} = \frac{1}{\sqrt{n}} \alpha_{-n}^i, \qquad \forall \ n > 0, \quad (2)$$

$$B_n^I = \frac{1}{\sqrt{n}} \beta_n^I, \qquad B_n^{I\dagger} = \frac{1}{\sqrt{n}} \beta_{-n}^I, \qquad \forall \ n > 0. \quad (3)$$

These redefined operators satisfy the oscillatorlike canonical commutation relations (CCR):

$$[A_n^I, A_m^{J\dagger}] = [B_n^{\mu I}, B_m^{\nu J\dagger}] = \delta_{nm} \eta^{IJ},$$
$$[A_n^I, B_m^J] = [A_n^I, B_m^{J\dagger}] = \ldots = 0. \quad (4)$$

The fundamental state of the closed bosonic string is defined by

$$A_n^I|0\rangle = B_n^I|0\rangle = 0, \quad (5)$$

where $|0\rangle = |p\rangle|0\rangle_A|0\rangle_B$, as usual. The momentum of the zero mode is often defined $p \equiv 0$, however it is actually arbitrary by virtue of the relativistic invariance of the string vacuum. The D_p-brane states are given by the following operatorial equations on states of the one-closed string Hilbert space, built from the usual boundary state conditions,

$$(A_n^a + B_n^{\dagger a})|B_p\rangle = 0, \qquad a = 0, 1, \ldots, p \quad (6)$$

$$(A_n^i - B_n^{\dagger i})|B_p\rangle = 0, \qquad i = p + 1, \ldots 25 \quad (7)$$

and for the zero mode we get

$$p^a|B_p\rangle = 0, \quad (8)$$

$$(x_0^i - x^i)|B_p\rangle = 0, \quad (9)$$

where x^i are the coordinates of the D_p-brane hyperplane. The solution reads as

$$|B_p\rangle = C_p \delta(x_0^i - x^i) \prod_{I,n>0} e^{-(S_p)_{IJ} A_n^{\dagger I} B_n^{\dagger J}} |0\rangle, \quad (10)$$

where we have defined $(S_p)_J^I \equiv (\delta_b^a, -\delta_j^i)$, C_p is a normalizing factor related to the brane tension by $T_p = 2C_p$.

The Hamiltonian operator writes

$$H = \sum_{n>0}^{\infty} n(A_n^{\dagger} \cdot A_n + B_n^{\dagger} \cdot B_n + \mathrm{tr}\eta^{IJ}) \equiv \sum_{n>0}^{\infty} nH_n, \quad (11)$$

where the dot represents an Euclidean scalar product in the transverse space, $a \cdot b \equiv \eta_{IJ} a^I b^J$. The negative quantity

$$V \equiv \sum_{n>0}^{\infty} n \, \mathrm{tr}\eta^{IJ} \quad (12)$$

is the energy of the tachyon in the rest frame.

To construct the physical Fock space it is necessary to fix the residual gauge symmetry generated by the world sheet canonical momentum Π. This imposes the level matching condition (LMC) on a physical state $|\Psi\rangle$:

$$\Pi|\Psi\rangle = \sum_{n=1}^{\infty} (nA_n^{\dagger} \cdot A_n - nB_n^{\dagger} \cdot B_n)|\Psi\rangle$$

$$= \sum_{n=1}^{\infty} n(N_n^A - N_n^B)|\Psi\rangle = 0, \quad (13)$$

where we have defined the number operators $N_n^A \equiv A_n^{\dagger} \cdot A_n$, $N_n^B \equiv B_n^{\dagger} \cdot B_n$.

III. STRING ENTANGLEMENT AND GENERALIZED VACUA

In what follows we study the internal quantum entanglement of closed strings, which may be reduced to entanglement between the right/left sectors. According to previous evidence [7–9], this behavior is generally induced in nontrivial backgrounds. Here, we will assume this generically, in order to study the consistency with expected statistic properties in gravitational backgrounds, so as other implications in the theory of D-branes.

The entanglement between two independent parts of a system is often described by a Bogoliubov transformation acting on the Hilbert space, which must be a tensor product of the two states space of each subsystem [10]. In this case we have

$$e^{iG(\theta)}: \mathcal{H}_A \otimes \mathcal{H}_B \mapsto \mathcal{H}_A \otimes \mathcal{H}_B, \quad (14)$$

where $G(\theta)$ is the generator of the transformation, called *the Bogoliubov operator*, and θ represents the set of parameters of the transformation, which shall depend on external conditions that induce the entanglement [10]. The pure closed string vacuum $|0\rangle = |p\rangle|0\rangle_A|0\rangle_B$, is transformed into a coherent mixed state

$$|0(\theta)\rangle = e^{iG(\theta)}|0\rangle, \quad (15)$$

which is annihilated by the transformed operators $A_n(\theta)$ and $B_n(\theta)$. The zero mode $|p\rangle$ is independent of the entangled sector.

In this framework one canonically quantizes the fields as operators and the statistical average of an operator Q is defined as its expectation value in the entangled vacuum state (15). So this state encodes the quantum and statistical information of the system on behalf of a density operator.

The most basic feature that characterizes a Bogoliubov transformation is that it preserves the canonical commutation relations. This reflects the fact that the original nature of the degrees of freedom is preserved, even if the effective

dynamics is different. Thus, it shall be assumed that this map preserves both the CCR and the level matching condition in order to ensure that the transformed degrees of freedom are also closed strings; thus, in particular, the generator G commutes with the operator Π. We will also assume that this map is unitary in order to preserve the amplitudes and probability measures.

The general form of the Bogoliubov transformation that fix the form of the generator is given by the following relation [11,12]

$$
\begin{pmatrix} A' \\ B^{\dagger\prime} \end{pmatrix} = e^{-iG} \begin{pmatrix} A \\ B^{\dagger} \end{pmatrix} e^{iG} = \mathcal{B} \begin{pmatrix} A \\ B^{\dagger} \end{pmatrix},
$$
$$
\begin{pmatrix} A^{\dagger\prime} & -B' \end{pmatrix} = \begin{pmatrix} A^{\dagger} & -B \end{pmatrix} \mathcal{B}^{-1},
$$
(16)

where G is Hermitian, and then $\mathcal{B}$ is a 2×2 complex matrix

$$
\mathcal{B} = \begin{pmatrix} u & v \\ v^* & u^* \end{pmatrix}, \tag{17}
$$

such that

$$
|u|^2 - |v|^2 = 1, . \tag{18}
$$

The relation (18) encodes the preservation of the CCR and it will be important for our interpretation of the results. The operators that satisfy the relations (16) and (18) have the following form [12]

$$
G_{1_n} = \theta_{1_n}(A_n \cdot B_n + B_n^{\dagger} \cdot A_n^{\dagger}),
$$
$$
G_{2_n} = i\theta_{2_n}(A_n \cdot B_n - B_n^{\dagger} \cdot A_n^{\dagger}),
$$
$$
G_{3_n} = \theta_{3_n}(A_n^{\dagger} \cdot A_n + B_n^{\dagger} \cdot B_n + \delta_{nn}\mathrm{tr}\eta^{\mu\nu}) = \theta_{3_n}H_n,
$$
(19)

where H_n in the last line, generates the time evolution of the modes labeled by n. The θ's are the real parameters which, for convenience, have been included in the operators. It is easy to verify that the generators (19) satisfy the $SU(1, 1)$ algebra

$$
[G_{i_n}, G_{j_n}] = -i\Theta_{ijk}G_{k_n}, \tag{20}
$$

where we have defined

$$
\Theta_{ijk} \equiv 2\frac{\theta_{i_n}\theta_{j_n}}{\theta_{k_n}}. \tag{21}
$$

As we can see from (19), the most general entanglement generator $G = \sum_n(G)_n$ takes the following form

$$
G_n = \lambda_{1_n}B_n^{\dagger} \cdot A_n^{\dagger} - \lambda_{2_n}A_n \cdot B_n
$$
$$
+ \lambda_{3_n}(A_n^{\dagger} \cdot A_n + B_n^{\dagger} \cdot B_n + \delta_{nn}\mathrm{tr}\eta^{\mu\nu}) \quad (22)
$$

and the coefficients represent complex linear combinations of θ's

$$
\lambda_{1_n} = \theta_{1_n} - i\theta_{2_n}, \qquad \lambda_{2_n} = -\lambda_{1_n}^*, \qquad \lambda_{3_n} = \theta_{3_n}. \tag{23}
$$

These are called generalized Bogoliubov transformations or simply G-transformations [11,12] which form a $su(1, 1)$ algebra. At this point we wish to point out that other alternatives to unitary transformations may be considered, although in general they lead to the same expressions for the generators but a different relations between these parameters [11,12].

By applying the disentanglement theorem for $su(1, 1)$ [13], one can write the most general closed string vacuum (15) under the following form

$$
|0(\theta)\rangle = \prod_n e^{\Omega_{1_n}(B_n^{\dagger} \cdot A_n^{\dagger})} e^{\log(\Omega_{3_n})(A_n^{\dagger} \cdot A_n + B_n^{\dagger} \cdot B_n + \delta_{nn}\mathrm{tr}\eta^{\mu\nu})}
$$
$$
\times e^{\Omega_{2_n}(A_n \cdot B_n)}|0\rangle, \tag{24}
$$

where the coefficients of various generators are given by the relations

$$
\Omega_{1_n} = \frac{-\lambda_{1_n}\sinh(i\Lambda_n)}{\Lambda_n\cosh(i\Lambda_n) + \lambda_{3_n}\sinh(i\Lambda_n)},
$$
$$
\Omega_{2_n} = \frac{\lambda_{2_n}\sinh(i\Lambda_n)}{\Lambda_n\cosh(i\Lambda_n) + \lambda_{3_n}\sinh(i\Lambda_n)},
$$
(25)

$$
\Omega_{3_n} = \frac{\Lambda_n}{\Lambda_n\cosh(i\Lambda_n) + \lambda_{3_n}\sinh(i\Lambda_n)}, \tag{26}
$$

and

$$
\Lambda_n^2 \equiv (\lambda_{3_n}^2 + \lambda_{1_n}\lambda_{2_n}). \tag{27}
$$

Since the pure vacuum is annihilated by a_n^{μ} and $\tilde{a}_n^{\mu}$, the only contribution to the mixed vacuum is given by

$$
|0(\theta)\rangle = \prod_n(\Omega_{3_n})^{2\delta_{nn}\mathrm{tr}\eta^{\mu\nu}}e^{\Omega_{1_n}(B_n^{\dagger} \cdot A_n^{\dagger})}|0\rangle. \tag{28}
$$

The string operators are mapped to entangled ones by the corresponding Bogoliubov generators

$$
A_n^{\mu}(\theta) = e^{-iG_n}A_n^{\mu}e^{iG_n}, \qquad B_n^{\mu}(\theta) = e^{-iG_n}B_n^{\mu}e^{iG_n}. \tag{29}
$$

Similar relations hold for the creation operators. The entangled operators satisfy the same canonical commutation relations as the pure operators at $\theta = 0$ by construction. Alternatively, one can organize the operators in doublets [11,12] and represent the Bogoliubov transformation as

$$
\begin{pmatrix} A_n^{\mu}(\theta) \\ B_n^{\mu\dagger}(\theta) \end{pmatrix} = \mathcal{B}_n \begin{pmatrix} A_n^{\mu} \\ B_n^{\mu\dagger} \end{pmatrix}, \tag{30}
$$

where the explicit form of the $\mathcal{B}_n$ matrices is given by

$$
\mathcal{B}_n = \cosh(i\Lambda_n)\mathbb{I} + \frac{\sinh(i\Lambda_n)}{(i\Lambda_n)}\begin{pmatrix} i\lambda_{3_n} & i\lambda_{1_n} \\ i\lambda_{2_n} & -i\lambda_{3_n}, \end{pmatrix} \tag{31}
$$

where $\mathbb{I}$ is the 2×2 identity matrix.

Symmetries and canonical *p*-coordinates

This general $su(1, 1)$ algebra contains the subalgebra of the canonical transformations, whose generators are those that commute with the total Hamiltonian (11):

$$G_{3_n} = \theta_{3_n} H_n. \tag{32}$$

These are the symmetries that may be thought of as the gauge freedoms in the present construction. Thus, we may use them to fix canonical coordinates encoding the information on the brane dimension that consist in the original fields multiplied by relative real phase factors. In particular, we may define this new set of annihilation operators as

$$A_n \rightarrow a_n, \qquad B_n \rightarrow -(S_p)_J^I B_n^J \equiv \tilde{a}_n^I, \tag{33}$$

and the corresponding creation operators are defined by their adjoints. One may easily verify that, in fact, these transformations may be generated by operators (32). This fixing will clarify the interpretation of our results on p branes.

The operators above annihilate the corresponding p-vacuum state,

$$a_n^I |0_p\rangle\rangle = \tilde{a}_n^I |0_p\rangle\rangle = 0, \tag{34}$$

for $n > 0$ and $|0_p\rangle\rangle = |0_p\rangle \otimes |\tilde{0}_p\rangle$ as usual, which is equivalent the direct product between the A and B vacua (up to a phase factor). Once more, the entangled fundamental state is obtained from this through a Bogoliubov transformation, e^{-iG}, which mixes the two independent right/left Hilbert spaces.

The normal modes redefined above, satisfy the canonical algebra:

$$[a_n^I, a_m^{\dagger J}] = [\tilde{a}_n^I, \tilde{a}_m^{\dagger J}] = \delta_{n,m} \delta^{I,J},$$
$$[a_n^{\dagger I}, \tilde{a}_m^J] = [a_n^{\dagger I}, \tilde{a}_m^{\dagger J}] = [a_n^I, \tilde{a}_m^J] = [a_n^I, \tilde{a}_m^{\dagger J}] = 0. \tag{35}$$

Let us notice finally that a transformation of this type does not generate any contribution to the entanglement entropy. In fact it does not mix between right- and left-modes, by virtue of the separated structure of the generators $G_{3_n} = G_{3_{n,A}} + G_{3_{n,B}}$ (the corresponding matrices $\mathcal{B}_{(3)n}$ are indeed diagonal).

IV. COHERENT STATES: ENTROPY AND BRANE THERMODYNAMICS

In order to keep the fixing above we shall consider a subgroup which led the p-coordinates choice invariant. In most of the systems, the thermal effects are described by standard one-parameter Bogoliubov transformations, generated by g_{2_n} [14,15]. In what follows, we focus on these type of transformations since one of our purposes is to describe those effects.

A. Vacuum/boundary states

In terms of the new canonical variables, standard unitary Bogoliubov transformations are generated by

$$G(\theta) = -i\delta_{IJ}\sum_n \theta_n (a_n^I \tilde{a}_n^J - \tilde{a}_n^{\dagger I} a_n^{\dagger J}), \tag{36}$$

for finite volume systems. The creation and annihilation are transformed according to

$$a_n^I(\theta_n) = e^{-iG} a_n^I e^{iG} = \cosh(\theta_n) a_n^I - \sinh(\theta_n)\tilde{a}_n^{\dagger I} \tag{37}$$

$$\tilde{a}_n^I(\theta_n) = e^{-iG}\tilde{a}_n^I e^{iG} = \cosh(\theta_n) a_n^I - \sinh(\theta_n)\tilde{a}_n^{\dagger I}. \tag{38}$$

These operators annihilate the states

$$a_n^I(\theta_n)|0(\theta)\rangle = \tilde{a}_n^I(\theta_n)|0(\theta)\rangle = 0; \tag{39}$$

that, by virtue of this, must be referred to as (entangled) vacuum states. By using the Bogoliubov transformation, these relations give rise to the vacuum state conditions:

$$[a_n^I - \tanh(\theta_n)\tilde{a}_n^{\dagger I}]|0(\theta)\rangle = 0, \tag{40}$$

$$[\tilde{a}_n^I - \tanh(\theta_n)a_n^{\dagger I}]|0(\theta)\rangle = 0, \tag{41}$$

which are also generalizations of the boundary conditions (8) and (9).[1] So the present framework handles a consistent twofold interpretation for these states: as ground states, and also as (deformed) boundary states.

The solution of (40) and (41) is

$$|0_p(\theta)\rangle\rangle = e^{-iG}|0_p\rangle\rangle$$
$$= \delta_p \prod_{n=1}\left[\left(\frac{1}{\cosh(\theta_n)}\right)^{D-2} e^{\tanh(\theta_n)\delta_{IJ}a_n^{\dagger I}\tilde{a}_n^{\dagger J}}\right]|0_p\rangle\rangle; \tag{42}$$

and in terms of the A/B modes, it expresses in the suggestive form:

$$|0_p(\theta)\rangle = \delta_p \prod_{n>0}\left(\frac{1}{\cosh(\theta_n)}\right)^{D-2} e^{-\tanh(\theta_n)(S_p)_{IJ}A_n^{\dagger I}B_n^{\dagger J}}|0\rangle. \tag{43}$$

By considering an ensemble of closed strings in their fundamental state ($p^i = 0$, $p^a = 0$) one may construct coherent states localized (so as a wave packet) in a p-dimensional surface $x^i = $ const. These vacua are all solutions of (40) and (41), and are determined by zero mode conditions (8) and (9). This is expressed by the prefactor $\delta_p \equiv \delta(x_0^i - x^i)$ in the expressions above [(42) and (43)]. Notice then that these generalized p-vacua are coherent states with macroscopic properties localized in the brane hypersurface.

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[1]These ones may naturally be associated to (θ)-deformed closed string solutions $X_{(\theta)}^I(t, \sigma)$.

Then, the Fock space of entangled string states is constructed by applying the mixed creation operators $a_n^{\dagger I}(\theta)$, $\tilde{a}_n^{\dagger I}(\theta)$, to the vacuum (42) that may properly be identified with a brane state (because of the condensate structure in itself) absorbing and emitting entangled closed strings. In fact, if the modes corresponding to the (θ)-string leaving the $D_{(\theta)}$-brane are created from this ground state (42), then in absence of such excitations, the $D_{(\theta)}$-brane on its own must be associated to the fundamental state.

Remarkably enough, (40) and (41) are a generalization of the boundary conditions (6) and (7) (to be recovered in the proper limit), and in this sense, this state is a natural candidate to describe a statistical ensemble associated to the D_p-brane, which handles its thermal properties. The following part is devoted to the study of this issue, and it is addressed to recover D_p-branes as ground states.

B. Statistic analysis

The entanglement entropy operator is defined such that its average value be proportional to the thermodynamical entropy of any free bosonic field divided by the Boltzmann's constant [14]. For a bosonic field, it can be computed as the expectation value of the entropy operator in the state that describes the system

$$
\begin{aligned}
S &\equiv \frac{1}{k_B}\langle K\rangle \\
&= -\left\{\sum_k [\mathcal{N}_k \log\mathcal{N}_k - (1 + \mathcal{N}_k)\log(1 + \mathcal{N}_k)]\right\},
\end{aligned}
$$

(44)

where $\mathcal{N}_k$ is the number of particles in the state k. This general expression is nothing but the expectation value of the von Neumann entropy operator $K \equiv -\mathcal{N}\log\mathcal{N}$, where the second term has taken into account the counting of antiparticles states [10]. Consequently, one defines the entropy operator for the bosonic string through the number operators $N_n^{A/B}$ for general $SU(1, 1)$ transformations (22) according to this formula. The LMC on physical states [Eq. (13)] implies that $K_n^A = K_n^B$, so the relevant quantity for our analysis is the entropy associated with one of the two (right/left) sectors. In the particular case of standard transformations generated by (36), the number operator of left-handed modes is proportional to $\sinh^2\theta_n$; therefore, in terms of canonical p-coordinates, the associated entropy reduces to

$$
\begin{aligned}
K = -\sum_{n=1} &\{a_n^{\dagger I}a_n^J\delta_{IJ}\ln(\sinh^2(\theta_n)) \\
&- a_n^I a_n^{\dagger J}\delta_{IJ}\ln(\cosh^2(\theta_n))\}.
\end{aligned}
$$

(45)

A straightforward algebra leads to the following expression for the vacuum state

$$
\begin{aligned}
|0_p(\theta)\rangle\rangle &= \delta_p e^{-K(\theta)/2}\prod_{I,n>0} e^{a_n^{\dagger I}\tilde{a}_n^{\dagger I}}|0_p\rangle\rangle \\
&= \delta_p e^{-\tilde{K}(\theta)/2}\prod_{I,n>0} e^{a_n^{\dagger I}\tilde{a}_n^{\dagger I}}|0_p\rangle\rangle,
\end{aligned}
$$

(46)

in terms of the entropy operator.[2] Then, according to (10), it may be written as

$$
|0_p(\theta)\rangle\rangle = \frac{1}{C_p}e^{-K(\theta)/2}|B_p\rangle = \frac{1}{C_p}e^{-\tilde{K}(\theta)/2}|B_p\rangle.
$$

(47)

This expression shows how the ground state of a system of self-entangled strings is related to the D_p-brane state through the entropy operator, and they both coincide in the formal limit $K \to 0$. This limit will be analyzed in the next section, and may be seen as an alternative way to construct the boundary state (10).

A very important remark has to be done at this point. Recalling the generalized vacuum state conditions [(40) and (41)], the boundary state (10) may be recovered as a vacuum state or, more precisely, as a proper (vanishing entropy) limit point of a many-string ground state.

Therefore, the generalized p-vacua [expressed by (43)] may be seen as statistical or thermodynamical extensions (parametrized by θ) of the D_p-brane state, since this is a coherent state of entangled string modes localized on the D-brane surface. In this sense, the brane tension may be promoted to an operator that acts on the pure brane state, defined in terms of the brane entropy:

$$
\hat{T}_p(\theta) \equiv 2e^{-K(\theta)/2}
$$

(48)

then the state that represents the *statistical brane* expresses as

$$
|0_p(\theta)\rangle\rangle = \frac{1}{2}\delta_p\hat{T}_p(\theta)\prod_{I,n>0} e^{a_n^{\dagger I}\tilde{a}_n^{\dagger I}}|0_p\rangle\rangle.
$$

(49)

D-branes are believed to be solitons in (super)-gravity theories that work as the ground states for gravitons (carried by closed strings). In agreement with this fact, the calculation above confirm that D-branes configurations indeed appear under general conditions of entanglement of closed string fields; provided that these entanglement effects arise in gravitational backgrounds [7–10]. Then as a by-product, if one associates the entropy $K(\theta)$ with the background, states as (43) actually describe solitons of the geometry with thermodynamic properties like black branes.

C. Temperature and dissipative effects

It is important to stress that this entropy may or may not be associated to equilibrium states and temperature.

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[2]A demonstration of this expression may be found in Ref. [10], where the entanglement of quantum fields in the presence of event horizons is studied.

However, the construction above handles a model to describe the thermal properties of D_p-branes and closed string states being created on them. Let us consider the following potential,

$$F = \langle H^a \rangle - \frac{1}{\beta} S \sim (\tilde{F}), \qquad (50)$$

and minimize it with respect to the transformation's parameters θ's [14]. Here $\langle H^a \rangle$ is given by computing the matrix elements of the right-handed Hamiltonian operator $\langle H^a \rangle$ in the state (42). The temperature is defined as the positive definite parameter $T \equiv (k_B \beta)^{-1}$ considered independent of the variations of the parameters θ. The solution for the angular parameters θ_n is given by the Bose-Einstein distribution:

$$N_n(= \tilde{N}_n) = \sinh^2 \theta_n = (e^{\beta E_n} - 1)^{-1}. \qquad (51)$$

The vacuum state conditions (40) and (41), valued on these parameters $\theta_n(\beta)$, provide the generalization of the boundary conditions at finite temperature. According to this formalism, a physical one-string excitation at temperature T, say $n, \tilde{n}$, may be created from this vacuum as usual:

$$|1; n, \tilde{n}, (\beta) \rangle\rangle = \frac{1}{n!} a_n^\dagger(\beta) \tilde{a}_n^\dagger(\beta) |0_p(\beta)\rangle\rangle. \qquad (52)$$

When the system is in equilibrium, we see from (51) that entropy of the bosonic brane goes to zero in the limit $T \to 0$. This guarantees that the third principle of thermodynamics is satisfied. Notice, however, that there is an apparent paradox in this model for thermal branes since the state (42) *is not* an eigenstate of the closed string Hamiltonian. Provided that (11) indeed describes the dynamics of the system, the state (42) evolves and (51) cannot then be considered an equilibrium distribution. Therefore, as a result, the present model for thermodynamic effects is incomplete and deviations from equilibrium should be included.

This may be seen as a notable coincidence with recent perspectives on the hydrodynamic properties of thermal (black) branes [16–18]. According to these references, the infrared behavior of theories whose dual bulk-gravities contain a black brane is governed by hydrodynamics, and the main observation in this sense is the existence of a universal value for the ratio of shear viscosity to entropy density [19], which should be investigated in the context of an appropriate microscopical model.

Observe that the temperature in this model might be interpreted as due to the immersion of the brane in a thermal bath measured by accelerated observers, according to the Unruh effect [8]. So, by identifying the temperature parameter with the relative acceleration of certain class of observers $a \equiv (2\pi k_B \beta)^{-1}$, this approach may also be viewed as describing accelerated branes [20]. In particular, for inertial observers, such that $a = 0$, the standard string vacuum $|0\rangle$ in a Minkowski space-time is recovered, and

the coherent state (43) disappears, in agreement with the membrane paradigm [21].

Let us finally mention that by virtue of the distribution (51), the divergence associated with the Hagedorn temperature is also present in this description, since the tension operator (48) tends to zero for higher temperatures, and thus the state $|0_p(\theta(\beta \to 0))\rangle$ is outside of the original Hilbert space in this limit [22]; i.e., the unitarity of the Bogoliubov transformation breaks down for sufficiently high temperatures.[3]

V. MACROSCOPIC PURE STATES AND COLLAPSE OF CLOSED STRING DEGREES OF FREEDOM

The state (42) describes a statistical state or ensemble; however, one could be interested in a sort of *nonthermodynamic* limit in order to get a description of the D-brane as a macroscopic, but quantum-mechanical system.[4] This is a coherent state with vanishing entropy.

In order to recover a quantum-mechanical picture, one usually studies the zero temperature limit, but there is a certain ambiguity in doing this. In particular by taking $\beta \to \infty$ in (42) one obtains the *microscopic* vacuum, $|0\rangle$ of one-closed string. However more generally, if one takes the zero-entropy limit, the pure state that describes the system is recovered, and Nerst's theorem guarantees that $T \to 0$ as a particular way of taking such a limit (through a succession of equilibrium states). In this approach the entropy is an operator defined not only for equilibrium ensembles, and this limit may be realized preserving the macroscopic character expressed in the structure of coherent state. This is what we call the macroscopic pure state (MPS) limit.

As argued before, the state (42) may be written in terms of the von Neumann entropy operator as follows

$$|0_p(\theta)\rangle\rangle = \frac{1}{C_p} e^{-K(\theta)/2} |B_p\rangle, \qquad (53)$$

where

$$|B_p\rangle \equiv \mathcal{B}_p |0\rangle \equiv C_p \delta_p \prod_{I, n > 0} e^{a_n^{\dagger I} \tilde{a}_n^{\dagger I}} |0\rangle\rangle. \qquad (54)$$

This describes the macroscopic system as a branelike condensate, through the collective variables θ, whose fundamental state corresponds to the fundamental equilibrium configuration given by the Bose-Einstein distribution (51).

By projecting Eq. (46) in the number basis $|n, \tilde{n}\rangle$ we have

$$\langle n, \tilde{n} |0_p(\theta)\rangle\rangle = e^{-(S_n(\theta_n)/2k_B)} \langle n, \tilde{n} |B_p\rangle, \qquad (55)$$

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[3]A consistent interpretation of this behavior was presented in Ref. [8] by studying closed strings crossing event horizons.
[4]The simplest analogy is a *perfect* crystal, an entropyless macroscopic system.

where $S_n(\theta_n)$ is the entropy of the nth level for an arbitrary distribution (or collective state), $\{\theta_n\} \equiv |0_p(\theta)\rangle\rangle$. Thus, we may observe the state $|0_p(\theta)\rangle\rangle$ approaches $|B_p\rangle$ in the zero-entropy limit $S_n(\theta_n) \to 0$,[5] which may be identified with the coherent pure state that describes the D-brane as a macroscopic quantum-mechanical object, such as we aimed.

The microscopical (stringy) degrees of freedom described by the string canonical variables collapse in this limit. In fact, according to the expressions (42) and (46), this limit may be expressed by $\tanh\theta_n \to S_p(= \pm 1)$, or simply

$$|(\nu_n/u_n)|^2 = |\tanh\theta_n|^2 \to 1; \tag{56}$$

thus if the canonical transformation is demanded to be unitary, the limit point cannot be reached unless the relation (18) is violated. By virtue of this, the canonical structure of the variables characterized by their CCR breaks down in this limit. This is not surprising since the emerging macroscopic quantum-mechanical system must have proper (few) canonical degrees of freedom, instead of many stringy ones. Apart from this, as one also would expect for solitonic objects, despite the state of the system, may even be represented in the string Fock space in this case [Eq. (46)], its dynamics can no longer be described by perturbative degrees of freedom (strings).

Let us finally notice that the above macroscopic pure states may be also recovered from equilibrium states, when the temperature is analytically continued to purely imaginary values. This may be directly verified from the Bose-Einstein distribution: $\tanh\theta_n = e^{n\beta/2}$; by taking $\beta \to i\tau$, the MPS condition $|\tanh\theta_n|^2 = 1$ is fulfilled, or in other words, purely phase factors $\tanh\theta_n$ may be absorbed in the creating operators of the state (42) through a canonical and unitary transformation. This remarkable property will be further explored elsewhere [23].

Canonical systems at finite temperature

To end this work, we wish to point out briefly the implications of the ideas discussed above to very general thermodynamic systems since the MPS limit may be a supplementary ingredient to study the emergence of macroscopic states like branes in more diverse contexts. In this sense the so-called thermofield dynamcs (TFD) approach is a privileged ground to do that.

Thermofield dynamics, developed by Takahashi and Umezawa [11,14,15,24,25], is a real time approach to quantum field theory at finite temperature [26,27] where an identical but fictitious copy of the system is introduced. In TFD the full statistical information of a quantum system is encoded in the (thermal) vacuum state instead of the

density operator or partition function:

$$|0(\beta)\rangle\rangle = Z^{-1/2}\sum_n e^{-\beta E_n/2}|n\rangle|\tilde{n}\rangle, \tag{57}$$

where $|n;\tilde{n}\rangle$ denotes the nth energy eigenvalue of the two systems, the physical one and its auxiliary copy denoted by $\tilde{\ }$. This may be alternatively expressed in terms of a Bogoliubov transformation, which maps the Fock space based on the initial vacuum $|0\rangle \otimes |\tilde{0}\rangle$ (annihilated by a_n and $\tilde{a}_n$) to a new thermal vacuum state:

$$|0(\theta)\rangle\rangle = e^{-iG(\theta)}|0\rangle|\tilde{0}\rangle. \tag{58}$$

The equilibrium state (57) corresponds to the particular distribution $\tanh\theta_n = e^{n\beta/2}$, where the expectation value of the free energy operator: $\mathcal{A} = H - \beta^{-1}K$ in the state (58), is stationary. The canonical entropy operator is given here by

$$K = -\sum_{n=1}[a_n^\dagger a_n \ln(\sinh^2\theta_n) - a_n a_n^\dagger \ln(\cosh^2\theta_n)], \tag{59}$$

where $a_n^\dagger$, $\tilde{a}_n^\dagger$ refer to the canonical creation operators corresponding to the physical system and its fictitious copy, respectively.

Consequently the concept of MPS limit may also be introduced in TFD, since one may write

$$|0(\theta)\rangle\rangle = e^{-K(\theta)/2}\prod_{n>0}e^{a_n^\dagger \tilde{a}_n^\dagger}|0\rangle\rangle. \tag{60}$$

Thus the macroscopic pure states are

$$|I\rangle\rangle \equiv \mathcal{I}|0\rangle\rangle \equiv \prod_{n>0}e^{a_n^\dagger \tilde{a}_n^\dagger}|0\rangle\rangle, \tag{61}$$

which may also be obtained from the equilibrium ones (57) through analytic continuation of the temperature values.

VI. FINAL REMARKS

We built a statistical approach to a bosonic D_p-branes which handles a model to describe their thermodynamic properties, such that in the vanishing entropy limit the boundary states are recovered. The picture is an ensemble of closed strings, which in the thermodynamic limit may be seen as a sort of medium extended on p spacial dimensions filled with string excitations. The model includes the evidence of dissipative behavior and the need for a hydro-dynamic description [16–18]. This radically differs from previous approaches to thermal D-branes using the TFD formalism [6], which supposes a duplication of the closed string degrees of freedom. In future works we will investigate that (in)stability of the thermal D-brane states and furthermore the possibility of studying black branes using these ideas.

A remarkable strength of this description is that the D_p-branes are constructed as the fundamental states of a

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[5]This limit may be taken by choosing some arbitrary series of states $\{\theta_n\}_M \equiv \theta_{n,M}$ with decreasing entropies, $0 \leq S_n(\theta_{n,M}) < S_n(\theta_{n,M-1})$, $\forall n, M$.

closed string Fock space, which is consistent with the view of these objects as solitonic gravitational backgrounds.

This work may be considered as a previous step addressed to formulate an open/closed dictionary based in the consistency with the description of thermal branes in terms of open strings [5].

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