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Radion clouds around evaporating black holes

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A Kaluza-Klein model, with a matter source associated with Hawking radiation from an evaporating black hole, is used to obtain a simple form for the radion effective potential. The environmental effect generally causes a matter-induced shift of the radion vacuum, resulting in the formation of a radion cloud around the hole. There is an albedo due to the radion cloud, with an energy-dependent reflection coefficient that depends upon the size of the extra dimensions and the temperature of the hole.

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I. INTRODUCTION

Kaluza-Klein–type models involving compactified extra dimensions produce effective four-dimensional (4D) theories containing moduli fields (radions) that are associated with scale factors of the compact dimensions. Particle masses typically exhibit a radion dependence, and local matter sources contribute to an effective potential for these scalar fields. If the compactification is *inhomogeneous*, particle masses and charges can have spatial and temporal variations. Solitonlike structures associated with these scalars may result (see, for example, [1–5]) with possibly observable consequences. An extreme condition is considered here, where the matter density from an evaporating microscopic black hole (MBH) may become large enough to give rise to an inhomogeneous compactification, resulting in a radionic cloud around the black hole. The reflective properties of this cloud endow the near-horizon region with a radion induced albedo.

Scenarios of this type, involving a radion coupled to the radiative field around an evaporating black hole, have been studied previously in Refs. [6,7]. In [6] rather general conditions were considered, and in [7] attention was focused on finite temperature, one-loop quantum corrections due to a partially thermalized medium surrounding an evaporating black hole. The resulting effective potential is quite complicated and difficult to represent in a simple closed form. Here, however, I take a more classical approach and consider a simple, but explicit, class of potentials $V(\varphi)$ for the radion field φ. This type of radion potential was studied first by Davidson and Guendelman (DG) in [1,2] in the context of a Freund-Rubin compactification and later by Carroll, Geddes, Hoffman, and Wald (CGHW) in [8], using an extra dimensional 2-form magnetic field. Both treatments result in the same radion potential for the case of two extra dimensions, but the former treatment also holds for an arbitrary number n of extra dimensions. In either case, the resulting potential has a rather simple form for an assumed set of parameter relations. For simplicity and concreteness, I specialize to the case of two extra dimensions, and refer to the radion potential $V(\varphi)$ as the DG-CGHW radion potential, although its generalization for arbitrary n, given by [1,2], is also given below. In addition to functional simplicity, the potential allows the extra dimensions to be stabilized classically, without explicit quantum corrections.

Using this simple DG-CGHW model, the full effective radion potential $U(\varphi)$ can be developed which includes a matter-sourced correction due to the Hawking radiation matter field. The matter source contribution to the radion effective potential $U(\varphi)$ depends, in a simple way, upon the local matter density $\rho(\mathbf{r}, t)$ and radion mass m_φ. A knowledge of these parameters then allows, in principle, a determination of the spatial and temporal variation of the radion field in the vicinity of the evaporating hole. The matter contribution can induce a shift, or complete destabilization, of the radion vacuum near the horizon, resulting in a "radion cloud" around the MBH. This cloud has an associated energy-dependent reflectivity, which can result in a distortion of the infrared portion of the Hawking radiation spectrum, as well as a partial reflection of low energy particles incident upon the MBH from the outside. By looking at the forms of the effective potential U near the hole's horizon and at asymptotic distances, it is suggested that the radion cloud has an evolving size $R \lesssim m_\varphi^{-1}$ and a maximal reflection coefficient $\mathcal{R}_{\max}$ that depends upon the matter density $\rho_{\rm hor}$ near the horizon through the ratio $\rho_{\rm hor}/(m_\varphi^2 M_0^2)$, where $M_0 = 1/\sqrt{8\pi G}$ is the reduced Planck mass.

II. RADION EFFECTIVE POTENTIAL

A. Effective 4D action

We start by considering a $D = (4 + n)$-dimensional spacetime, having n compact extra spatial dimensions endowed with a metric given by

$$\begin{aligned} ds_D^2 &= \tilde{g}_{MN} dx^M dx^N \\ &= \tilde{g}_{\mu\nu}(x) dx^\mu dx^\nu + b^2(x^\mu) \gamma_{mn}(y) dy^m dy^n, \end{aligned} \quad (2.1)$$

where $x^M = (x^\mu, y^m)$. Here $M, N = 0, 1, 2, 3, \ldots, D - 1$ label all the spacetime coordinates, while $\mu, \nu = 0, 1, 2,$

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3, label the 4D coordinates, and m, n label those of the compact extra space dimensions. The extra dimensional scale factor $b(x^\mu)$ is assumed to be independent of the y coordinates and takes the form of a scalar field in the 4D effective theory. The extra dimensional metric $\gamma_{mn}(y)$ depends upon the geometry of the extra dimensional space and is related to $\tilde{g}_{mn}(x, y)$ by $\tilde{g}_{mn} = b^2\gamma_{mn}$. As in Refs. [1,2,8], consideration is restricted to extra dimensional compact spaces with constant curvature, with a curvature parameter k defined by

$$k = \frac{\tilde{R}[\gamma_{mn}]}{n(n-1)}. \tag{2.2}$$

The action for the D-dimensional theory is

$$S_D = \int d^D x\sqrt{|\tilde{g}_D|}\left\{\frac{1}{2\kappa_D^2}[\tilde{R}_D[\tilde{g}_{MN}] - 2\Lambda] + \tilde{\mathcal{L}}_D\right\}, \tag{2.3}$$

where $\tilde{g}_D = \det\tilde{g}_{MN}$, $\tilde{R}_D$ is the Ricci scalar built from $\tilde{g}_{MN}$, Λ is a cosmological constant for the D-dimensional spacetime, $\tilde{\mathcal{L}}_D$ is a Lagrangian for the fields in the D dimensions, $\kappa_D^2 = 8\pi G_D = V_y\kappa^2 = V_y 8\pi G$, where $G(G_D)$ is the 4D (D)-dimensional gravitational constant, and $V_y = \int d^n y\sqrt{|\gamma|}$ is the coordinate "volume" of the extra dimensional space. A mostly negative metric signature $(+, -, -, \ldots, -)$ is used here.

The action can be expressed in terms of an effective 4D action (see, for example, [8,9] for details), which takes the form

$$S = \int d^4x\sqrt{-\tilde{g}}\left\{\frac{1}{2\kappa^2}[b^n\tilde{R}[\tilde{g}_{\mu\nu}] - 2nb^{n-1}\tilde{g}^{\mu\nu}\tilde{\nabla}_\mu\tilde{\nabla}_\nu b - n(n-1)b^{n-2}\tilde{g}^{\mu\nu}(\tilde{\nabla}_\mu b)(\tilde{\nabla}_\nu b) + n(n-1)kb^{n-2}] + b^n\left[\mathcal{L}_D - \frac{\Lambda}{\kappa^2}\right]\right\} \tag{2.4}$$

in the 4D Jordan frame (with metric $\tilde{g}_{\mu\nu}$), and I have defined a normalized field Lagrangian, $\mathcal{L}_D = V_y\tilde{\mathcal{L}}_D$. A 4D Einstein frame metric $g_{\mu\nu}$ can be defined as

$$\tilde{g}_{\mu\nu} = b^{-n}g_{\mu\nu}, \qquad \tilde{g}^{\mu\nu} = b^n g^{\mu\nu}, \qquad \sqrt{-\tilde{g}} = b^{-2n}\sqrt{-g}. \tag{2.5}$$

The action S in (2.4), in terms of the 4D Einstein metric, takes the form

$$S = \int d^4x\sqrt{-g}\left\{\frac{1}{2\kappa^2}\left[R[g_{\mu\nu}] + \frac{n(n+2)}{2}b^{-2}g^{\mu\nu}(\nabla_\mu b) \times (\nabla_\nu b) + n(n-1)kb^{-(n+2)}\right] + b^{-n}\left[\mathcal{L}_D - \frac{\Lambda}{\kappa^2}\right]\right\}, \tag{2.6}$$

where the total derivative terms have been dropped. Furthermore, an effective 4D source, or matter, Lagrangian $\mathcal{L}_m$ can be defined in terms of the D-dimensional source Lagrangian $\mathcal{L}_D = V_y\tilde{\mathcal{L}}_D$ and the scale factor b:

$$\mathcal{L}_m = b^{-n}\mathcal{L}_D. \tag{2.7}$$

A scalar radion field φ with a canonical kinetic term is now defined by

$$\sqrt{\frac{n(n+2)}{2\kappa^2}}\ln\frac{b}{b_0} = \varphi, \qquad b = b_0\exp\left(\sqrt{\frac{2}{n(n+2)}}\kappa\varphi\right), \tag{2.8}$$

where b_0 is some constant, which will be set equal to unity, and $\kappa = \sqrt{8\pi G} = \sqrt{8\pi}/M_P = M_0^{-1}$ is the inverse of the reduced Planck mass.

B. Radion potential V

For a simple and concrete example, I use the radion potential studied in Refs. [1,2,8] for the case of $n = 2$ extra dimensions having a constant positive curvature parameter k. I also adopt the same choices of parameter relations to obtain a simple functional form. The potential V has contributions from the curvature term in (2.6), the cosmological constant term Λ, and either (1) an extra dimensional magnetic field due to $F_{45} = \sqrt{|\gamma|}F_0$, where F_0 is a constant, in $\mathcal{L}_D$ (see [8]), or (2) a Freund-Rubin term (see [1,2]) of the form

$$\tilde{\mathcal{L}}_D = -\frac{1}{48}F^2, \qquad F_{MNPQ} = \partial_{[M}A_{NPQ]}, \qquad F_{\mu\nu\lambda\sigma} = \sqrt{\lambda}\frac{\sqrt{|\tilde{g}|}}{3b^n}\varepsilon_{\mu\nu\lambda\sigma}. \tag{2.9}$$

The potential V obtained by CGHW for the case of $n = 2$ extra dimensions and an extra dimensional magnetic field F_{45} leads to a simple potential (written here in terms of the scale factor b, rather than in terms of the radion field φ) given by

$$V(b) = \lambda(b^{-6} - 2b^{-4} + b^{-2}) \qquad \text{(CGHW potential)} \tag{2.10}$$

with the following relations and parameter choices: (see Eqs. (36) and (37) in Ref. [8])

$$\varphi = \frac{2}{\kappa}\ln b, \qquad b = e^{(1/2)\kappa\varphi}, \qquad \lambda = \frac{k}{2\kappa^2} = \frac{m_\varphi^2 M_P^2}{16\pi} = \frac{1}{2}m_\varphi^2 M_0^2, \tag{2.11}$$

where $k = \frac{\partial^2 V(\varphi)}{\partial\varphi^2}|_{\varphi=0} = m_\varphi^2$ is the mass^2 of the radion field [8], M_P is the Planck mass, and $M_0 = 1/\kappa = M_P/\sqrt{8\pi}$ is the reduced Planck mass. The form of V (see Fig. 1; V is also sketched in Refs. [1,8]) has one local minimum, which for the case $n = 2$ is located at $b = 1$ (or $\varphi = 0$) with $V(b = 1) = 0$, followed by a barrier for $b > 1$, then an asymptotic decrease with $V \to 0$ as $b \to \infty$.

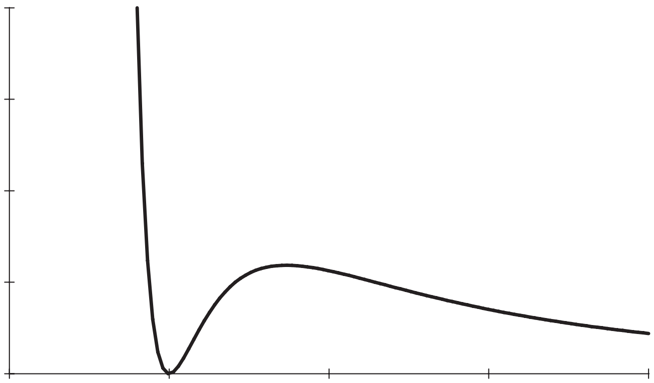

FIG. 1. A plot is shown of $V(b)/\lambda$ vs b. The local minimum occurs at $b = 1$ ($\varphi = 0$) where $V = 0$. The local maximum occurs at $b \approx 1.75$ where $V/\lambda \approx 0.15$. As $b \to \infty$, $V \to 0$.

For the more general case of n extra dimensions given by [1,2], the Davidson-Guendelman potential arising from the Freund-Rubin term above is given by (with factors of κ reinstated here)

$$V(b) = \frac{\lambda}{b^{3n}} - \frac{1}{2}\frac{k}{\kappa^2} n(n-1)\frac{1}{b^{n+2}} + \frac{\Lambda/\kappa^2}{b^n} \quad \text{(DG potential)}. \tag{2.12}$$

For the case of $n = 2$, requiring the potential to vanish at its minimum leads to the condition

$$\Lambda = \frac{k^2/\kappa^2}{4\lambda}. \tag{2.13}$$

Imposing the parameter choice $\lambda = k/2\kappa^2$ as in (2.11), then gives the same potential as in (2.10) with a local minimum at $b = 1$, where $V = 0$. For simplicity and concreteness, attention is restricted here to this simple form of the potential V for the particular case of $n = 2$ extra dimensions. For our case of $n = 2$, the potential V, given by (2.10) and (2.11), has a local minimum at $b = 1$ ($\varphi = 0$), where $V = 0$, a local maximum located at $b > 1$, and the potential falls off exponentially, $V \to 0$ as $b \to \infty$.

An equation of motion (EOM) is obtained from (2.6) for the radion field φ,

$$\Box\varphi + \frac{\partial V}{\partial\varphi} - \left\langle\frac{\partial \mathcal{L}_m}{\partial\varphi}\right\rangle = 0, \tag{2.14}$$

where $\mathcal{L}_m$ is the matter Lagrangian, which depends upon scalar, spinor, and vector matter fields, as well as the radion field. Using field redefinitions, the φ dependence of $\mathcal{L}_m$ appears in particle masses $m_A(\varphi)$ and in gauge coupling constants (see, e.g., [10]). The matter Lagrangian will therefore contribute to an effective potential $U(\varphi)$ for the radion.

C. Matter contribution–Hawking radiation

The matter contribution to the EOM for φ (or b) comes from the Hawking radiation [11] from an evaporating black hole with surface temperature T (as seen asymptotically). Denote this matter contribution to (2.14) by $\sigma = \langle\frac{\partial \mathcal{L}_m}{\partial\varphi}\rangle$. The Lagrangian $\mathcal{L}_m$ contains the matter and gauge fields, such as fermionic terms like [9,10] $\mathcal{L}_\psi = \bar\psi(i\gamma\cdot\partial - m(\varphi))\psi$ with $m(\varphi) = b^{-(n/2)}(\varphi)m_0$, ($m_0 = \text{const}$) and gauge field terms, such as that for the photon, $\mathcal{L}_F = -\frac{1}{4}b^n(\varphi)F_{\mu\nu}F^{\mu\nu}$. For a freely propagating electromagnetic field with $F_{\mu\nu}F^{\mu\nu} = 0$, we have no contribution to σ from $\mathcal{L}_F$. However, particle modes with nonzero rest mass do contribute to σ through terms like $-\alpha\langle m\bar\psi\psi\rangle$, where

$$\alpha_A = \frac{\partial \ln m_A(\varphi)}{\partial\varphi} \tag{2.15}$$

with A labeling the particle species. (In the nonrelativistic flat space limit, this σ term is proportional to the fermionic energy density, $-\alpha\langle m\bar\psi\psi\rangle \sim -\alpha\rho$. However, we want to consider σ terms beyond the nonrelativistic flat space limit. We will conclude that $\sigma \sim -\alpha g_{\mu\nu}\mathcal{T}^{\mu\nu}_{cl} = -\alpha\mathcal{T}_{cl}$ for the more general case, where $\mathcal{T}^{\mu\nu}_{cl}$ is a classical stress-energy tensor, and that, classically, $\alpha_A = \alpha = \text{const}$ is independent of particle species.)

Now, for simplicity, rather than using the field theoretic version of the matter Lagrangian $\mathcal{L}_m$, let us follow the approach used by Damour and Polyakov [10] and treat the matter with a classical description, replacing the field theoretic action with a classical particle action S_{cl}. We consider particle modes having a nonzero rest mass $m_A(\varphi)$ with $\partial_\varphi\mathcal{L}_A \neq 0$ and write a classical action

$$\begin{aligned} S_{cl} &= -\sum_A \int ds_A m_A \\ &= -\sum_A \int m_A[g_{\mu\nu}(x_A)dx^\mu_A dx^\nu_A]^{1/2} \\ &= -\sum_A \int d^4x \int m_A[g_{\mu\nu}(x_A)dx^\mu_A dx^\nu_A]^{1/2} \\ &\quad \times \delta^{(4)}(x - x_A) = \int d^4x\sqrt{-g}\,\mathcal{L}_{cl} \end{aligned} \tag{2.16}$$

and identify $\sqrt{-g}\,\mathcal{L}_{cl} = -\sum_A \int m_A[g_{\mu\nu}(x_A)dx^\mu_A dx^\nu_A]^{1/2} \times \delta^{(4)}(x - x_A)$. A field theoretic energy-momentum tensor for the matter fields, defined by $\mathcal{T}^{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\partial(\sqrt{-g}\mathcal{L}_m)}{\partial g_{\mu\nu}}$, is replaced by an energy-momentum tensor $\mathcal{T}^{\mu\nu}_{cl}$ for the classical particles, with[1]

$$\begin{aligned} \mathcal{T}^{\mu\nu}_{cl} &= -\frac{2}{\sqrt{-g}}\frac{\partial(\sqrt{-g}\,\mathcal{L}_{cl})}{\partial g_{\mu\nu}} \\ &= \frac{1}{\sqrt{-g}}\sum_A \int m_A u^\mu_A u^\nu_A \delta^{(4)}(x - x_A)d\tau_A, \end{aligned} \tag{2.17}$$

[1]With this set of definitions, we have $\mathcal{T}_{00} > 0$ for both fields and classical particles.

where $u^\mu = dx^\mu/d\tau$ satisfies an "on shell" constraint $g_{\mu\nu}u^\mu u^\nu = 1$. Taking the trace gives

$$\mathcal{T}_{cl} = g_{\mu\nu}\mathcal{T}^{\mu\nu}_{cl} = \frac{1}{\sqrt{-g}}\sum_A \int m_A \delta^{(4)}(x - x_A)d\tau_A$$
$$= -\mathcal{L}_{cl}. \tag{2.18}$$

We, therefore, find

$$\sigma = \frac{\partial \mathcal{L}_{cl}}{\partial \varphi} = \sum_A \alpha_A \mathcal{L}_{cl,A} = -\sum_A \alpha_A \mathcal{T}_{cl,A} \tag{2.19}$$

for $\alpha_A =$ const. The constant α_A takes a value $\alpha_A = \alpha = -\sqrt{\frac{n}{2(n+2)}}\kappa$, which for our $n = 2$ model becomes $\alpha = -\kappa/2$. This can be seen [12] by considering the matter action

$$S = -\sum_A \int m_{0,A}\, d\tilde{s}_A, \tag{2.20}$$

where $m_{0,A}$ is the constant Jordan frame particle mass for species A, and $d\tilde{s} = \sqrt{\tilde{g}_{\mu\nu}dx^\mu dx^\nu}$ is the Jordan frame line element, which by (2.5), is related to the Einstein frame line element $ds = \sqrt{g_{\mu\nu}dx^\mu dx^\nu}$ by $d\tilde{s} = b^{-(n/2)}ds$. The matter action rewritten in the Einstein frame is

$$S = -\sum_A \int m_{0,A}\,(b^{-(n/2)}ds_A) = -\sum_A \int m_A ds_A, \tag{2.21}$$

where the Einstein frame mass is

$$m_A = b^{-(n/2)}m_{0,A} = \exp\left(-\sqrt{\frac{n}{2(n+2)}}\kappa\varphi\right)m_{0,A}. \tag{2.22}$$

Equations (2.15) and (2.22) then give

$$\alpha_A = \frac{\partial \ln m_A(\varphi)}{\partial \varphi} = -\sqrt{\frac{n}{2(n+2)}}\kappa \rightarrow -\frac{\kappa}{2} \quad \text{for } n = 2. \tag{2.23}$$

From (2.19) we therefore have $\sigma = -\alpha\mathcal{T}_{cl} = \frac{\kappa}{2}\mathcal{T}_{cl}$ for our $n = 2$ model.

The Hawking radiation (for a neutral nonrotating black hole) is assumed to be a fluid with energy density $\rho = \mathcal{T}^0_0$ and a normal radial pressure component $p_r = -\mathcal{T}^r_r$, and tangential pressure[2] $p_T = -\mathcal{T}^\theta_\theta = -\mathcal{T}^\phi_\phi$ in the center of momentum frame, i.e., the rest frame of the black hole. We also assume the energy density and pressures to be related by the equations of state

$$p_r = w_r\rho, \qquad p_T = w_T\rho,$$
$$0 \le w_r \le 1, \qquad 0 \le w_T \le 1, \tag{2.24}$$

where $w_{r,T}$ are constants. For an isotropic perfect fluid $p_T = p_r = p$ and $w_r = w_T = w$ with $0 \le w \le 1$. Let us define an effective pressure p and a parameter w by

$$p = \tfrac{1}{3}(p_r + 2p_T), \qquad w = \tfrac{1}{3}(w_r + 2w_T), \qquad 0 \le w \le 1 \tag{2.25}$$

so that an effective equation of state can be written in the form $p = w\rho$, as assumed by Zurek and Page [13] and by 't Hooft [14], where the Hawking radiation is regarded as a perfect fluid with the constant $w = p/\rho \in [0, 1]$.

The trace of the stress-energy tensor becomes

$$\mathcal{T}_{cl} = \rho - (p_r + 2p_T) = [1 - (w_r + 2w_T)]\rho$$
$$= (1 - 3w)\rho. \tag{2.26}$$

For an ideal gas of noninteracting massless particles in thermal equilibrium, $w = 1/3$. However, we proceed by leaving w as a free parameter, subject to $0 \le w \le 1$ as assumed in [13,14]. This allows for a value of $\mathcal{T}_{cl}$ that can be positive, negative, or zero. With (2.19) and (2.26), we obtain our approximate result

$$\sigma = \frac{\partial \mathcal{L}_{cl}}{\partial \varphi} = -\alpha\mathcal{T}_{cl} = -\alpha(1 - 3w)\rho, \tag{2.27}$$

with the energy density ρ being dominated by relativistic particle modes.

A couple of remarks are in order here. First, we note that $\mathcal{T}_{cl}$ and therefore ρ in (2.27) are generated by the particle modes with nonzero rest mass and do not include the energy density ρ_0 and pressure p_0 that is due to the massless (e.g. photon) components of the radiation. The total energy density and pressure of the entire fluid would be $\rho_{\rm tot} = \rho + \rho_0$ and $p_{\rm tot} = p + p_0$, respectively. Second, it may be that there are multiple components of the fluid corresponding to various particle species, with $\rho = \sum_A \rho_A$, and each species may have an equation of state $p_A = p_A(\rho)$, which, in principle, could be complicated. However, in order to study the effects of the radiation, these difficulties are avoided here by our simple assumption that $p/\rho = w =$ const, the same assumption made in the Hawking radiation fluid models of Refs. [13,14]. This seems palatable for a case where there are very few relativistic massive modes, at least on time scales sufficiently small compared to the black hole evaporation time scale $M/\dot{M}$.

The EOM $\Box\varphi + \partial_\varphi V - \sigma = 0$ for the radion field φ becomes

$$\Box\varphi + \frac{\partial V}{\partial \varphi} + \alpha(1 - 3w)\rho = 0. \tag{2.28}$$

An effective potential $U(\varphi)$ is now defined by $U = V - \sigma\varphi = V + U_{\rm matter}$ or

[2]The spacetime is assumed to have spherical symmetry.

$$U(\varphi) = V(\varphi) + \alpha(1-3w)\rho\varphi$$
$$= V(\varphi) + \frac{2}{\kappa}\alpha(1-3w)\rho \ln b$$
$$\text{(effective potential)}, \tag{2.29}$$

where $\ln b = \frac{1}{2}\kappa\varphi$ for our model with two extra dimensions and we set $U(\varphi = 0) = 0$. The matter contribution to the radion effective potential is

$$U_{\text{matter}} = -\sigma\varphi = -\frac{2}{\kappa}\sigma \ln b = \frac{2\alpha}{\kappa}\mathcal{T}_{cl}\ln b$$
$$= -\mathcal{T}_{cl}\ln b = -(1-3w)\rho \ln b, \tag{2.30}$$

where the result $2\alpha/\kappa = -1$ from (2.23) has been used. The sign of this matter term is controlled by the parameter w, and in the special case $w = 1/3$ then $\sigma \to 0$ and the matter term does not contribute to the radion effective potential. We note that for $w < 1/3$ then U_{matter} is a decreasing function for $b > 1$, while for $w > 1/3$ we have that U_{matter} is an increasing function for $b > 1$.

D. Radion effective potential, U

With (2.10), (2.11), (2.29), and (2.30) we can now write an explicit, but simple, effective potential in the form $U(b) = V(b) - \mathcal{T}_{cl}\ln b$ or

$$\frac{1}{\lambda}U(b) = (b^{-6} - 2b^{-4} + b^{-2}) - \frac{\mathcal{T}_{cl}}{\lambda}\ln b, \tag{2.31a}$$
$$= (b^{-6} - 2b^{-4} + b^{-2}) - \zeta \ln b, \tag{2.31b}$$

where

$$\zeta \equiv \frac{\mathcal{T}_{cl}}{\lambda} = \frac{2\mathcal{T}_{cl}}{m_\varphi^2 M_0^2} = \frac{(1-3w)\rho}{\lambda}. \tag{2.32}$$

Here, the parameter ζ is dimensionless and $\zeta = \zeta(r, t)$ is a function of radial distance r from the black hole since $\rho = \rho(r, t)$. The assumed range of w allows a value of ζ in the range $-2\rho/\lambda \leq \zeta \leq \rho/\lambda$.

Figure 2 gives a representation of $U(b)/\lambda$ for various *positive* values of ζ ($p \leq \rho/3$). The asymptotic vacuum value of b occurs at $b_0 = 1$ for $\zeta = 0$, but for values of $0 < \zeta \lesssim .5$ the vacuum value of b is shifted to larger values, $b > 1$, and the minimum of U becomes more negative. For values $\zeta \gtrsim .5$, the local minimum disappears, the vacuum state is completely destabilized, and $U(\varphi)$ is a monotonically decreasing function whose slope depends on ζ.

Figure 3 gives a representation of $U(b)/\lambda$ for various *negative* values of ζ ($p \geq \rho/3$). The asymptotic vacuum value of b occurs at $b_0 = 1$ for $\zeta = 0$, but for larger values of $|\zeta|$ the vacuum value of b is shifted to smaller values, $b < 1$, and the minimum of U becomes more negative.

Thus, the vacuum values b_{vac} and φ_{vac} become r dependent in general. Far from the hole, $\zeta \to 0$ and $\varphi_{\text{vac}} \to 0$, $b_{\text{vac}} \to 1$. Near the hole, where $\zeta \neq 0$, then $\varphi \neq 0$ and $b \neq 1$. Therefore φ interpolates between a positive or negative value $\varphi \neq 0$ near the horizon to $\varphi = 0$ at asymptotic distances. As the hole evaporates and $|\zeta|$ increases, any vacuum state near the horizon gets further shifted to smaller or larger values, depending on the sign of ζ. For $\zeta > 0$ a stable vacuum eventually disappears and the radion rolls to larger values until the hole's explosive end.

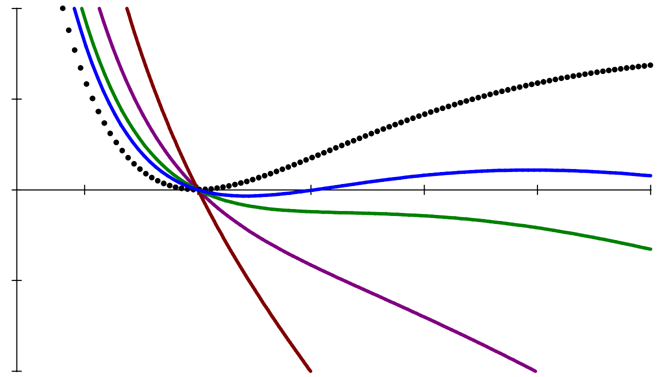

FIG. 2 (color online). Plots of $U(b)/\lambda$ vs b are shown for positive values of ζ. The dotted curve, with a minimum at $U = 0$ and $b = 1$, has $\zeta = 0$ and corresponds to the radion potential $V(b)/\lambda$. The solid curves have $\zeta = 0.3, 0.5, 1, 2$, with the more negatively sloped curves corresponding to bigger ζ. The vacuum value of b occurs at $b_0 = 1$ for $\zeta = 0$, but for $0 < \zeta \lesssim 0.5$ the vacuum value of b is shifted to larger values, $b_{\text{vac}} > 1$, and the minimum of U completely disappears for $\zeta \gtrsim 0.5$.

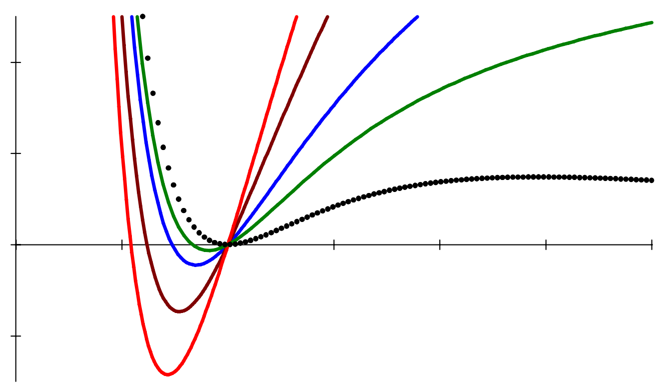

FIG. 3 (color online). Plots of $U(b)/\lambda$ vs b are shown for negative values of ζ. The dotted curve, with a minimum at $U = 0$ and $b = 1$, has $\zeta = 0$ and corresponds to the radion potential $V(b)/\lambda$. The solid curves have $\zeta = -0.5, -1, -2, -3$, with the lower minimum curves corresponding to bigger $|\zeta|$. The vacuum value of b occurs at $b_0 = 1$ for $\zeta = 0$, but for larger values of $|\zeta|$ the vacuum value of b is shifted to smaller values, $b_{\text{vac}} < 1$, and the minimum of U becomes more negative.

III. RADION CLOUD AND BLACK HOLE ALBEDO

A. Radion cloud

An energy-momentum tensor $\mathcal{T}_{\mu\nu}$ has been defined for the matter portion $\mathcal{L}_m$ of the effective 4D Lagrangian

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_\varphi(\varphi) + \mathcal{L}_m(\varphi, \sigma, \psi, \ldots) \\ &= \frac{1}{2}(\partial\varphi)^2 - V(\varphi) + \mathcal{L}_m(\varphi, \sigma, \psi, \ldots) \end{aligned} \quad (3.1)$$

and an energy-momentum tensor $S_{\mu\nu}$ can be written for the pure radion part $\mathcal{L}_\varphi$:

$$\begin{aligned}S_{\mu\nu} &= \partial_\mu\varphi\partial_\nu\varphi - g_{\mu\nu}\mathcal{L}_\varphi \\ &= \partial_\mu\varphi\partial_\nu\varphi - g_{\mu\nu}\Big[\frac{1}{2}\partial^\alpha\varphi\partial_\alpha\varphi - V(\varphi)\Big]. \end{aligned} \quad (3.2)$$

The energy density part for the radion field,

$$S_{00} = \tfrac{1}{2}(\partial_0\varphi)^2 - \tfrac{1}{2}g_{00}g^{rr}(\partial_r\varphi)^2 + g_{00}V(\varphi) \quad (3.3)$$

vanishes asymptotically, but becomes nonzero near the evaporating black hole where φ develops a nonzero vacuum value $\varphi_{\rm vac}$ due to the environmental effects of a nonzero ζ in the effective potential U.

For a nonradiating black hole (i.e., a matter-vacuum solution with $\mathcal{L}_m = 0$, $T_{\mu\nu} = 0$, $\sigma = 0$), we have the Einstein equation $R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -\kappa^2 S_{\mu\nu}$ along with the radion EOM $\Box\varphi + V'(\varphi) - \sigma = 0$. The minimal energy radion solution ($S_{\mu\nu} = 0$) is given by the trivial solution $\varphi = 0$ ($b = 1$). Therefore the gravitational field alone of the black hole has no effect on the radion and there is no radion cloud in this case. However, for a radiating black hole with nonzero values of σ and ζ outside the horizon, $\varphi = 0$ is not a solution of the radion EOM and the radion field $\varphi(r, t)$ must interpolate between a value of $\varphi_{\rm hor} \neq 0$ near the horizon and $\varphi = 0$ asymptotically. Since $\varphi \neq 0$ is not at the minimum of $V(\varphi)$, then $V(\varphi) > 0$, contributing a positive contribution to S_{00}. There are nonnegative gradient terms contributing to S_{00} as well. So near the horizon, $S_{00} > 0$, and asymptotically $S_{00} \to 0$.

The energy density of the scalar field φ is concentrated near the MBH, where the gradient terms are large, and the radion field forms a cloud around it. The exact structure of this cloud requires a knowledge of the solution $\varphi(r, t)$ to the EOM $\Box\varphi + U'(\varphi) = 0$. A crude estimate of the extent of this cloud of energy can be obtained by considering a thin shell of Hawking radiation at a radius $r \gg r_S$, where $r_S = 2GM$ is the radius of a Schwarzschild black hole. (Relativistic radiation with speed $v \sim 1$ is assumed, as a higher energy density is carried by relativistic modes.) At this asymptotic distance, the mass δM in the spherical shell is approximately constant as it propagates outward, so that $\delta M \approx 4\pi r^2\rho(r, t)\delta r \sim P(t)\delta t$, where $P(t) = -\dot{M}(t) > 0$ is the power output of the radiation from the evaporating MBH and for the sake of simplicity I have neglected any time retardation effects in $P(t)$. This gives a crude estimate for the matter density

$$\rho(r, t) \sim \frac{P(t)}{4\pi r^2}. \quad (3.4)$$

At a fixed instant t, the matter density drops off as r^{-2}, while at a fixed distance r the density increases with time as described by the power output $P(t)$. An outer "edge" of the evolving radion cloud, i.e., the cloud radius $R(t)$, can be defined as a radial distance where the density ρ assumes a sufficiently small constant value, i.e., $\eta \equiv \rho/\lambda \ll 1$ is a small constant, so that $\rho \approx 0$ outside this radius. From (3.4) this cloud radius, where η is a constant, is given by

$$R^2(t) \sim \frac{P(t)}{4\pi\rho} = \frac{P(t)}{4\pi\eta\lambda} = \frac{P(t)}{2\pi\eta m_\varphi^2 M_0^2}. \quad (3.5)$$

The ordinary Steffan-Boltzmann (SB) law for a perfect blackbody (no gravitational greybody effects assumed, for simplicity) with emitting surface area $4\pi r_S^2$ gives $P(t) = 4\pi r_S^2\sigma_{\rm SB}T^4(g/2)$ where $g = (N_B + \frac{7}{8}N_F)$ is the effective number of degrees of freedom of relativistic particles and $\sigma_{\rm SB} = \pi^2/60$ is the Steffan-Boltzmann constant. Using $r_S = 2GM = \frac{M}{4\pi M_0^2}$ and $T = \frac{1}{8\pi GM} = \frac{M_0^2}{M}$ (where $M_0 = 1/\kappa = 1/\sqrt{8\pi G}$ is the reduced Planck mass) leads to

$$P(t) = \frac{(g/2)\pi}{240}\frac{M_0^4}{M^2(t)}. \quad (3.6)$$

From this, the cloud radius is given by

$$R(t) \sim \sqrt{\frac{g/2}{480\eta}}\Big(\frac{M_0}{M(t)}\Big)m_\varphi^{-1} \sim \Big(\frac{M_0}{M(t)}\Big)m_\varphi^{-1}, \quad (3.7)$$

where, for simplicity, I have set $480\eta/(g/2) \sim 1$, and $M(t)$ is the black hole mass. The radion cloud grows in size as the hole shrinks, with $\dot{R}/R \sim -\dot{M}/M$. From (3.7), R approaches an upper limit $R_{\rm max} \sim m_\varphi^{-1}$ near the explosive end of the MBH as $M \to M_0$. The requirement that $r_S/R \ll 1$ implies that $m_\varphi \ll \frac{4\pi M_0^3}{M^2} \lesssim M_0$. Provided that the rather natural condition $m_\varphi \ll M_0$ is satisfied, then so is the requirement that $R \gg r_S$.

The energy-dependent reflection coefficient (see below) $\mathcal{R}(\omega)$ and transmission coefficient $\mathcal{T}(\omega) = 1 - \mathcal{R}(\omega)$ for a particle of energy ω will depend upon the variation in φ (or b) from the near-horizon region to the asymptotic region, along with the radius R of the cloud. The reflection coefficient approaches a maximal value $\mathcal{R}_{\rm max}$ as particle energy ω approaches a minimal value [9].

B. Radion reflectivity and black hole albedo

Basic features expected of particle reflectivity by the radion field—a radion induced black hole albedo—can be obtained from the (flat space) results of [9] and other studies of particle reflection from ordinary (nonradionic) scalar field domain walls (see, for example, [15–18]). In the case of photons, the radion cloud, treated as a scalar modulus domain wall of thickness $\sim R$, has a maximal reflection coefficient given by [9]

$$\mathcal{R}_{\max} = \frac{(b_1^2 - b_2^2)^2}{(b_1^2 + b_2^2)^2} = \frac{(b_{\rm hor}^2 - 1)^2}{(b_{\rm hor}^2 + 1)^2}, \tag{3.8}$$

where b_1, b_2 are the values of the scale factor b on the two different sides of a modulus wall. The two "sides" of our domain wall are the black hole horizon where $b = b_{\rm hor}$ and the asymptotic region approximately a distance R away where $b \to 1$. The transmission coefficient is $\mathcal{T}(\omega) = 1 - \mathcal{R}(\omega)$ and for photons of energy ω we have the thin wall limit with $\mathcal{R}(\omega) \to \mathcal{R}(0) = \mathcal{R}_{\max}$ in the infrared limit $\omega \to 0$, where the photon wavelength $\lambda_\gamma \gg R$. From [9], it was found from numerical calculations that $\mathcal{R}(\omega)/\mathcal{R}_{\max}$ typically begins to become significant for energies $\omega \lesssim 1/R$, i.e., the thin wall limit. Note that $\mathcal{R}_{\max} \sim 1$ when $b_{\rm hor} \gg 1$, or $b_{\rm hor} \ll 1$. We may have $b_{\rm hor} \gg 1$ for $\zeta_{\rm hor} \gtrsim .5$, at which point the near-horizon vacuum of $U(b)$ completely destabilizes. For a near-horizon value of $0 < \zeta_{\rm hor} < .5$, one expects $\mathcal{R}_{\max} \ll 1$, as there is a minimum of U that is not far removed from $b = 1$. On the other hand, for negative ζ with $|\zeta| \gg 1$ we have $b_{\rm hor} \to 0$ as $\zeta \to -\infty$, in which case $\mathcal{R}_{\max} \to 1$ again. Since $|\zeta|$ increases with black hole temperature T, one expects $\mathcal{R}_{\max}$ to increase with increasing T. These considerations lead us to expect a possible alteration of the infrared portion of the transmitted Hawking radiation, as well as a partial reflection from near-horizon regions of low energy particles incident upon the black hole from outside. (The details of the spectral distortion, however, will depend upon the structure of the radion cloud.) On the other hand, for high energy photons with $\omega \gg 1/R$, the cloud becomes transparent (thick-wall limit) [9] with $\mathcal{R} \to 0$. Similar qualitative statements are expected for massive particle modes. The black hole therefore has an energy-dependent albedo associated with the radion cloud, which, in turn, is due to an inhomogeneous compactification of the extra dimensions near the horizon.

The above deductions are based upon reflection and transmission characteristics in flat space. The effects of curved space would alter the gradient terms appearing in the $\Box\varphi$ portion of the radion EOM, and therefore the gradient nature of the solution $\varphi(r, t)$. The exact expressions for $\mathcal{R}(\omega)$ and $\mathcal{T}(\omega)$ would depend on the exact solution φ, but the basic qualitative features mentioned above for $\mathcal{R}(\omega)$ are not expected to be significantly affected.

C. ζ near the horizon

In the limit of a static, ideal fluid in thermodynamic equilibrium, the local total energy density[3] is [13] $\rho_{\rm tot} \sim T^{*(w+1)/w}$, where $T^*(r) = T/\sqrt{g_{00}(r)} = (\sqrt{g_{00}}8\pi GM)^{-1}$ is the blueshifted Hawking temperature. In this limit, $\zeta = (1 - 3w)\rho/\lambda \sim (1 - 3w)T^{*(w+1)/w}/\lambda$ can become quite large or divergent near the horizon (or the would-be horizon). (There can be significant backreactions on the metric, and the studies [13,14] suggest that the horizon could be removed by a (static) Hawking "atmosphere," with ρ remaining finite.) If the horizon is not removed by backreactions, the local energy density can diverge on the horizon, due to the diverging blueshifted local temperature [19]. Furthermore, quantum field effects, such as vacuum polarization, [20–22] are expected to play important roles and may contribute to the $\sigma = \langle \partial \mathcal{L}_m/\partial\varphi \rangle$ term in the effective potential. In any case, whether ρ diverges or remains finite near the black hole, the local value of $|\zeta|$ and σ may become extremely large in the near-horizon region, possibly leading to either $b_{\rm hor} \gg 1$ or $b_{\rm hor} \ll 1$. In either of these cases $\mathcal{R}_{\max} \to 1$, indicating an infrared radionic reflectivity.

[3]Recall that $\rho_{\rm tot} = \rho + \rho_0$, or $\rho = f\rho_{\rm tot}$, where $f = \rho/\rho_{\rm tot} = 1 - \rho_0/\rho_{\rm tot}$ is the fraction of energy carried by nonmassless particles.

IV. SUMMARY

A Kaluza-Klein model with two spherically compactified extra dimensions, studied previously by Davidson and Guendelmann [1,2] and Carroll, Geddes, Hoffman, and Wald [8], is examined here with attention focusing on the development of a radion cloud around an evaporating neutral, nonrotating MBH. The cloud owes its existence not to the gravitational field alone (a Schwarzschild solution is accompanied by a trivial radion solution $\varphi = 0$), but arises in response to the environmental effect of the Hawking radiation. The radiation is modeled as a fluid with an effective energy density $\rho(r, t)$, contributing to the radion equation of motion. An effective pressure p is assumed to be related to ρ through an equation of state $p/\rho = w = \text{const}$ with $0 \le w \le 1$, resembling the perfect fluid Hawking atmosphere models of Refs. [13,14]. For the particular case $w = 1/3$, as is expected for a fluid of noninteracting massless particles in thermal equilibrium, there is no environmental effect on φ. However, it is not assumed here that the fluid is in equilibrium, and the particle modes contributing to the energy density $\rho = \rho_{\rm tot} - \rho_0$ (where ρ_0 is due to massless particle modes) are those associated with particles of nonzero rest mass. The matter contribution to the radion effective potential is $U_{\rm matter} = -\mathcal{T}_{cl}\ln b = -\lambda\zeta\ln b$, which can be positive, negative, or zero, depending on the sign of $\zeta = (1 - 3w)\rho/\lambda$. A classical description has been used to estimate the $\sigma = \langle \partial\mathcal{L}_m/\partial\varphi \rangle$ term in the radion equation of motion, but near the horizon quantum field effects such as vacuum polarization [20–22] are expected to be important and may contribute to a shift in the radion vacuum.

The radion φ approaches a normal vacuum value $\varphi \to 0$ ($b \to 1$) asymptotically, where the Hawking radiation energy density vanishes, but near the MBH the radion is shifted to a value $\varphi \neq 0$ ($b \neq 1$) for any $\zeta \neq 0$.

Gradients of φ and a nonzero radion potential $V(\varphi)$ then give rise to a radion cloud with nonvanishing energy density around the MBH. This radion cloud has an estimated size $R(t) \sim (\frac{M_0}{M(t)}) m_\varphi^{-1}$ and an energy-dependent reflection coefficient $\mathcal{R}(\omega)$ as studied in [9]. This reflection coefficient has a maximum value $\mathcal{R}_{\max}$ (given by (3.8) for the case of electromagnetic radiation in the flat space limit), which depends upon the parameter ζ near the horizon. $\mathcal{R}(\omega)/\mathcal{R}_{\max}$ begins to become significant for particle energies $\omega \lesssim R^{-1}(t)$. Since the asymptotic compactification radius for the extra dimensions is $\sim m_\varphi^{-1}$, the size of the cloud compared to that of the extra dimensions in asymptotic space is $R(t)/m_\varphi^{-1} \sim M_0/M(t)$, which is initially small, but becomes of order unity at the end stages of the evaporation. An infrared portion of the Hawking spectrum detected by an external observer will be suppressed if $\mathcal{R}_{\max} \to 1$, and some low energy particles incident upon the MBH from the outside will be reflected back. The amount of reflectivity depends upon the temperature T of the MBH (and therefore the parameter ζ near the horizon) and particle energy ω. For high energy particles ($\omega \gg R^{-1}$), the radion cloud is transparent. The Hawking radiation contributes heavily to the effective potential $U(\varphi)$ for large $|\zeta|$, in which case $\mathcal{R}_{\max}$ may approach unity. For $\zeta < 0$ one may have a vacuum with $b_{\rm hor} \ll 1$, while for positive values $\zeta \gtrsim .5$, the effective potential $U(\varphi)$ is completely destabilized, i.e., a local minimum disappears. In this case, the slope of U is negative, and the radion rolls outward in time with $b(t)$ increasing. In either case, when $\zeta \neq 0$, a radion cloud must develop, since $\varphi = 0$ and $b = 1$ is not a solution of the radion EOM. For $\zeta \neq 0$, the radion infrared albedo effect increases as the MBH evaporates. Since transparency begins to set in at particle energies $\omega \gtrsim R^{-1} \sim (\frac{M}{M_0}) m_\varphi \gtrsim m_\varphi$, the energy range of observable albedo effects ($\omega \lesssim m_\varphi$) will be very sensitive to the radion mass m_φ and therefore the size of the extra dimensions.

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