

One-loop results for kink and domain wall profiles at zero and finite temperatureAnton Rebhan,^{1,*} Andreas Schmitt,^{1,†} and Peter van Nieuwenhuizen^{2,‡}¹*Institut für Theoretische Physik, Technische Universität Wien, Wiedner Hauptstrasse 8-10, A-1040 Vienna, Austria*²*C.N. Yang Institute for Theoretical Physics, State University of New York, Stony Brook, New York 11794-3840, USA*

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Using dimensional regularization, we compute the one-loop quantum and thermal corrections to the profile of the bosonic (1 + 1)-dimensional φ^4 kink, the sine-Gordon kink, and the CP^1 kink, and higher-dimensional φ^4 kink domain walls. Starting from the Heisenberg field equation in the presence of the nontrivial kink background we derive analytically results for the temperature-dependent mean field which display the onset of the melting of kinks as the system is heated towards a symmetry-restoring phase transition. The result is shown to simplify significantly when expressed in terms of a self-consistently defined thermal screening mass. In the case of domain walls, we find infrared singularities in the kink profile, which corresponds to interface roughening depending on the system size. Finally we calculate the energy density profile of φ^4 kink domain walls and find that in contrast to the total surface tension the local distribution requires composite operator renormalization in 3 + 1 dimensions.

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I. INTRODUCTION

Solitons [1] in scalar field theories in 1 + 1 dimensions, which we call generically kinks, have played an important role in increasing our understanding of various nontrivial aspects of quantum field theories, ranging from exactly solvable examples of strong-weak coupling dualities [2,3] to the theory of topological defects generated at phase transitions with applications in condensed matter physics [4] as well as cosmology [5].

In a number of models one knows in closed form the spectrum of fluctuations about the kink background which allows one to perform complete calculations of one-loop corrections to the mass of the kinks. However, these calculations turn out to be full of subtleties, in particular (but not only), in the presence of fermions. For example, for the minimally supersymmetric kink a number of authors have concluded from explicit calculations that there was a cancellation of the one-loop effects on mass and central charge in a certain minimal renormalization scheme [6–8], a result widely accepted since the mid 1980s. Only in 1997 two of the present authors have reopened this issue by demonstrating an incompatibility of the methods employed for the supersymmetric kinks with known exact results for the nonsupersymmetric sine-Gordon model [9], with correct results for the mass eventually being established in Refs. [10–12] and a resulting puzzle concerning BPS saturation solved in Ref. [12] by the discovery of a new anomaly in the central charge.¹ More recently this has led to a similar revision also in the case of four-

dimensional supersymmetric monopoles [14,15], where a long-standing (20 years) but unnoticed discrepancy of direct calculations [16,17] with newer developments (notably Seiberg-Witten theory [18]) was eventually cleared up.²

In the present paper we shall consider bosonic kinks at finite temperature,³ applying and slightly generalizing the method used in Refs. [12,21] to calculate the profile of kink to one-loop order. Following Ref. [21] we start from the Heisenberg field equation. Other authors have used the method of the effective action with x -dependent background fields (see [4] and references therein). Both methods are of course completely equivalent but we found the former to be simpler to implement.

Quantum and thermal corrections to kinks have been considered in various approximations of self-consistent, nonperturbative frameworks e.g. in Refs. [22–24]. Here we shall restrict ourselves to one-loop effects in a regime where perturbation theory is still reliable, refraining therefore from attempts to cover the physics of the symmetry restoring phase transition itself and the corresponding actual melting of kinks. However, we can reliably cover the onset of the melting of kinks and in this way provide benchmarks for more daring approximations and approaches. Systematic calculations of one-loop corrections to kink profiles and domain walls at zero and finite temperature have been carried out before by several authors

²For more extensive reviews see e.g. [19,20].³Finite temperature breaks supersymmetry so that supersymmetric kinks no longer display features that are not also found in the bosonic case. Moreover, thermal contributions from fermions provide no special difficulty and are easily added on to what follows.

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¹Actually the conformal central charge [13].

[25–29]. Our results are in agreement with those once the differences in the renormalization schemes are taken into account.⁴ However, we find significant simplifications in the corrections to the kink profile provided renormalization conditions are formulated in terms of a self-consistent thermal screening mass, which may be of practical importance in applications at noninfinitesimal coupling.

We shall in turn cover three different models in $1 + 1$ dimensions: the well-known, exactly solvable, sine-Gordon model with $\mathbb{Z} \times \mathbb{Z}_2$ symmetry, a closely related massive CP^1 model with $U(1) \times \mathbb{Z}_2$ symmetry, and the familiar φ^4 model with \mathbb{Z}_2 symmetry. All the above models have a discrete symmetry which is spontaneously broken at zero temperature and which leads to topological solitons (kinks). Since the φ^4 model is renormalizable in higher dimensions as well, we shall use it also to discuss domain walls in $2 + 1$ and $3 + 1$ dimensions.

At sufficiently high temperature T one expects symmetry restoration and the disappearance (“melting”) of the kinks. In $3 + 1$ dimensions it is well known that perturbation theory allows one to derive the leading order result for the phase transition temperature T_c , but in simple scalar field theories next-to-leading order results for the transition temperature as well as the order of the phase transition are beyond perturbation theory [32,33].

Therefore, let us explain the validity and limitations of perturbative methods in the various dimensions. In $3 + 1$ dimensions, symmetry restoration in φ^4 theory with coupling constant λ is brought about by a thermal mass term $\Delta m^2 \varphi^2 \sim \lambda T^2 \varphi^2$ which outweighs the wrong-sign mass term $\propto -\mu^2 \varphi^2$ in the classical potential for sufficiently high temperature $T > T_c \sim \mu/\sqrt{\lambda}$. After a resummation of the leading-order thermal mass, perturbation theory around the minimum of the effective potential has a loop expansion parameter $\sim \lambda T/m$, where m is the (thermally corrected) mass of the fluctuations around the minimum, with $m \sim \sqrt{2}\mu$ at low T in the broken phase, and $m \sim \Delta m$ for large T in the restored phase. With $\lambda \ll 1$, perturbation theory works for all temperatures except very close to T_c where m gets parametrically small. (As long as $m \sim \mu$, the expansion parameter $\lambda T/m \lesssim \sqrt{\lambda}$ up to temperatures of the order of T_c ; for $T \gg T_c$, one has $m \sim \sqrt{\lambda}T$ and the expansion parameter is of order $\sqrt{\lambda}$ throughout the symmetry-restored phase.)

⁴In Ref. [30] two of us have previously stated that we disagreed with Ref. [25] regarding results on the surface tension at zero temperature in a zero-momentum renormalization scheme, but agreed with Ref. [31]. As has been shown in Ref. [29], this apparent discrepancy was due to having compared two versions of a zero-momentum renormalization scheme. The zero-momentum scheme considered in Refs. [30,31] renormalizes the derivative of the two-point function with respect to momentum to one, while the scheme in Refs. [25,29] does not introduce nontrivial wave-function renormalization, which is possible at one-loop order. Taking this difference into account resolves this issue.

In lower dimensions, the situation is much more dire. In $1 + 1$ dimensions, the coupling constant of φ^4 theory as well as of the sine-Gordon model has scaling dimension mass squared, and we have to assume that the loop expansion parameter at zero temperature $\lambda/m^2 \ll 1$. (For the CP^1 kink we have a dimensionless coupling constant replacing λ/m^2 .) At finite temperature the expansion parameter is $(\lambda/m^2) \times (T/m)$. High-temperature thermal mass terms are however only linear in T , $\Delta m^2 \sim \lambda T/m$. For symmetry restoration we would need $|\Delta m^2| \gtrsim m^2$, but this contradicts the requirement $(\lambda/m^2) \times (T/m) \ll 1$. Hence, in perturbation theory we can reliably study the high-temperature limit $T/m \gg 1$ only as long as $T/m \ll m^2/\lambda$, i.e. in the broken phase where $|\Delta m^2| \ll m^2$. It is therefore not mandatory to resum the thermal mass Δm , although we shall find that it will be natural to do so.

An important difference to the $3 + 1$ -dimensional case is, however, that $\Delta m^2 \sim \lambda T/m$ in the $1 + 1$ -dimensional theory is generated only by Matsubara zero modes, whereas in the $3 + 1$ -dimensional case the leading terms in Δm^2 are generated by “hard,” short-distance modes with wavelength $\sim T^{-1}$, which for $T/m \gg 1$ are insensitive to the presence of a nontrivial kink background with inherent length scale $\sim m^{-1}$.

In $2 + 1$ -dimensional theory, the situation is not better than in $1 + 1$ dimensions. The loop expansion parameter at zero temperature is $\lambda/m \ll 1$. The thermal expansion parameter $\lambda T/m^2 \ll 1$ equally implies that the thermal mass squared $\Delta m^2 \sim \lambda T \ll m^2$, precluding a perturbative analysis of a symmetry-restoring phase transition. When going to high temperatures $T/m \gg 1$, perturbation theory is reliable only as long as $T/m \ll m/\lambda$, so again only the broken phase is accessible.⁵

In order to study the actual melting of kinks, nonperturbative methods are needed. (In the lower dimensional cases, also the symmetry-restored phase with $T \gg T_c$ is nonperturbative.) In some cases, other systematic expansions like large- N expansions [36,37] can be put to work, but often (self-consistent) approximation schemes are employed which lack an expansion parameter controlling the approximations. In order to have credibility, such approximations should be able to reproduce our perturbative results as limiting case.

In Sec. II we consider the sine-Gordon model and the massive CP^1 model, which have closely related fluctuation spectra, and we calculate the one-loop corrections to the field profile of the kinks in these models. In Sec. III we turn to the familiar φ^4 model, which has a more complicated fluctuation spectrum and correspondingly more complicated one-loop corrections. Since the latter model is re-

⁵The critical temperature of the $2 + 1$ -dimensional φ^4 model has been estimated by renormalization-group methods in Ref. [34] to be given by a relation of the form $T_c/\mu \propto \mu/\lambda \log(c\lambda/T_c)$ and the constant c subsequently measured on the lattice in Ref. [35].

normalizable in higher dimensions, we shall use it to discuss one-loop corrections to domain walls in $2 + 1$ and $3 + 1$ dimensions, with a detailed discussion of our dimensional regularization and renormalization scheme, and its appropriate generalization at finite temperature. In particular, with the help of a remarkable identity for integrals involving the Bose-Einstein distribution function, we find that a self-consistent definition of the thermal screening mass removes certain artifacts in the one-loop kink profile.

In the calculation of the profile of the higher-dimensional domain walls, we encounter infrared singularities associated with the massless modes that correspond to the translational zero mode of the $1 + 1$ -dimensional kink. In accordance with Refs. [26,27,29], these singularities are interpreted as the field theoretic equivalent of system-size dependent interfacial roughening [38–40]. Finally we calculate the energy profile of the φ^4 kink and the corresponding domain walls and show that in contrast to the total mass (surface tension) the local energy density profile is ambiguous, depending on improvement terms to the stress tensor, and that in $3 + 1$ dimensions composite operator renormalization through improvement terms is required.

II. SINE-GORDON AND CP^1 KINKS

We begin by calculating the one-loop quantum and thermal corrections to the field profile of the $1 + 1$ -dimensional sine-Gordon model and a massive version of the CP^1 model, which both happen to have a simpler fluctuation spectrum than the φ^4 model (in particular, no bound states). A full-fledged discussion of our method of dimensional regularization will be introduced in Sec. III, where we turn to the φ^4 model in $1 + 1$ and higher dimensions, which also turns out to involve less trivial renormalization conditions.

A. Sine-Gordon kink

The Lagrangian of the sine-Gordon model is

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi + \frac{m^4}{\lambda}\left[\cos\left(\frac{\sqrt{\lambda}}{m}\varphi\right) - 1\right], \quad (1)$$

with a real scalar field φ . We have a discrete $\mathbb{Z}_2 \times \mathbb{Z}$ symmetry, given by $\varphi \rightarrow -\varphi$ and $\varphi \rightarrow \varphi + 2\pi n \frac{m}{\sqrt{\lambda}}$ with $n \in \mathbb{Z}$. The field equation in $1 + 1$ dimensions is

$$\partial_\mu\partial^\mu\varphi \equiv (-\partial_t^2 + \partial_x^2)\varphi = \frac{m^3}{\sqrt{\lambda}}\sin\left(\frac{\sqrt{\lambda}}{m}\varphi\right). \quad (2)$$

The constant solution of this equation yields the classical vacua

$$\varphi = 2\pi n \frac{m}{\sqrt{\lambda}}, \quad (3)$$

which break the discrete symmetry spontaneously. The

kink solution interpolating between the vacua with $n = 0$ ($x = -\infty$) and $n = 1$ ($x = +\infty$) is

$$\varphi_K(x) = \frac{4m}{\sqrt{\lambda}}\arctan e^{m(x-x_0)}. \quad (4)$$

The energy of this configuration is $M = 8m^3/\lambda$. The equation for the fluctuations $\eta(x, t)$ around the kink solution,

$$\partial_\mu\partial^\mu\eta(x, t) - m^2\eta(x, t)\cos\left[\frac{\sqrt{\lambda}}{m}\varphi_K(x)\right] = 0, \quad (5)$$

becomes, using $\eta(x, t) = e^{-i\omega t}\phi(x)$ and the kink profile (4),

$$\left[-\partial_x^2 + m^2\left(1 - \frac{2}{\cosh^2 mx}\right)\right]\phi(x) = \omega^2\phi(x), \quad (6)$$

where we have set $x_0 = 0$. From this equation we obtain the zero-mode with energy $\omega = 0$,

$$\phi_0(x) = \sqrt{\frac{m}{2}}\frac{1}{\cosh mx}, \quad (7)$$

and the continuum with $\omega_k = \sqrt{k^2 + m^2}$,

$$\phi_k(x) = \frac{m}{\omega_k}e^{ikx}\left(\tanh mx - i\frac{k}{m}\right). \quad (8)$$

We note the useful relation between the continuum and the zero-mode [Ref. [21], Eq. (12)]

$$|\phi_k(x)|^2 = 1 - \frac{2m}{\omega_k^2}\phi_0^2. \quad (9)$$

We can now compute the correction to the kink profile in the presence of the fluctuations. To this end, following Ref. [21], we interpret the equation of motion (2) as a Heisenberg field equation and write $\varphi(x, t) \rightarrow \varphi_K(x) + \varphi(x, t) = \varphi_K(x) + \phi_1(x) + \eta(x, t)$. Here, $\phi_1(x) \equiv \langle\varphi(x, t)\rangle$, and the quantum fluctuation field $\eta(x, t) \equiv \varphi(x, t) - \phi_1(x)$ obeys $\langle\eta\rangle = 0$.

The equation of motion for $\phi_1(x)$ becomes

$$\left[\partial_x^2 - m^2\cos\left(\frac{\sqrt{\lambda}}{m}\varphi_K\right)\right]\phi_1 = -\frac{m\sqrt{\lambda}}{2}\sin\left(\frac{\sqrt{\lambda}}{m}\varphi_K\right)\langle\eta^2\rangle_{\text{ren}}, \quad (10)$$

where

$$\langle\eta^2\rangle_{\text{ren}} = \langle\eta^2\rangle - \langle\eta^2\rangle|_{x\rightarrow\infty} \quad (11)$$

is the renormalized propagator. To obtain Eq. (10) from Eq. (2) we have employed the following renormalization. We have fixed δm^2 and $\delta\lambda$ in the renormalization of the mass $m^2 \rightarrow m_0^2 = m^2 + \delta m^2$ and the coupling constant $\lambda \rightarrow \lambda + \delta\lambda$ such that all one-loop (finite-temperature) graphs with one vertex cancel [41]. This condition implies that the ratio m^2/λ is unchanged under renormalization, and

$$\delta m^2 = m^2 \frac{\delta \lambda}{\lambda} = \frac{\lambda}{2} \langle \eta^2 \rangle|_{x \rightarrow \infty}. \quad (12)$$

The propagator at finite temperature is given by

$$\begin{aligned} \langle \eta^2 \rangle &= T \sum_n \int_{-\infty}^{\infty} \frac{dk}{2\pi} \frac{|\phi_k(x)|^2}{\omega_n^2 + \omega_k^2} \\ &= \int_{-\infty}^{\infty} \frac{dk}{2\pi} \frac{1 + 2n(\omega_k)}{2\omega_k} |\phi_k(x)|^2, \end{aligned} \quad (13)$$

where $\omega_n = 2n\pi T$ are the bosonic Matsubara frequencies and $n(\omega) = [\exp(\omega/T) - 1]^{-1}$ is the Bose-Einstein distribution. Here we have dropped the zero mode of Eq. (7), which corresponds to the translational degree of freedom of the kink and which can be taken care of by collective quantization [42]. This expression is UV divergent and thus requires regularization. In Sec. III we shall introduce our method of dimensional regularization adapted to solitons, but in this and the next section we shall suppress its details, since the finite results for the kink profile below do not depend on them. (If we were to calculate also local energy densities, as we shall do for the model of Sec. III, we would need to be more careful.) The treatment of the zero mode in our way of dimensional regularization will also be discussed more fully in Sec. III, where we cover the more general case of domain walls.

Simply subtracting Eq. (13) according to Eq. (11), and making use of Eq. (9) yields the renormalized propagator

$$\langle \eta^2 \rangle_{\text{ren}} = -\phi_0^2(x) \left[\frac{1}{m\pi} + 2m \int_{-\infty}^{\infty} \frac{dk}{2\pi} \frac{n(\omega_k)}{\omega_k^3} \right]. \quad (14)$$

Inserting this result into the differential Eq. (10) yields the correction to the kink profile

$$\phi_1(x) = \frac{\sqrt{\lambda}}{4m} \left(\frac{1}{2\pi} + m^2 \int_{-\infty}^{\infty} \frac{dk}{2\pi} \frac{n(\omega_k)}{\omega_k^3} \right) \frac{\sinh mx}{\cosh^2 mx}. \quad (15)$$

The total kink is then given by $\varphi_K(x) + \phi_1(x)$ with φ_K from Eq. (4). We see that there is a zero-temperature and a finite temperature correction, both being proportional to the derivative of the zero mode. This is a consequence of the renormalization (12) which subtracts the complete one-loop (seagull) diagram of the trivial sector including thermal contributions. Had we left out the latter (or any finite part), this would have produced extra contributions to ϕ_1 proportional to $m \frac{\partial}{\partial m} \varphi_K = \varphi_K + \frac{2m}{\sqrt{\lambda}} mx / \cosh(mx)$.

With the large-temperature expansion

$$m^2 \int_{-\infty}^{\infty} \frac{dk}{2\pi} \frac{n(\omega_k)}{\omega_k^3} = \frac{T}{2m} - \frac{1}{4\pi} + \mathcal{O}\left(\frac{m}{T}\right), \quad (16)$$

we see that the finite-temperature part of ϕ_1 is suppressed compared to φ_K by one power of the expansion parameter $\lambda T/m^3$ (while the zero-temperature part is suppressed by λ/m^2). Since this expansion parameter cannot be small in the case of symmetry restoration (see discussion in the introduction), this perturbative result is only valid for

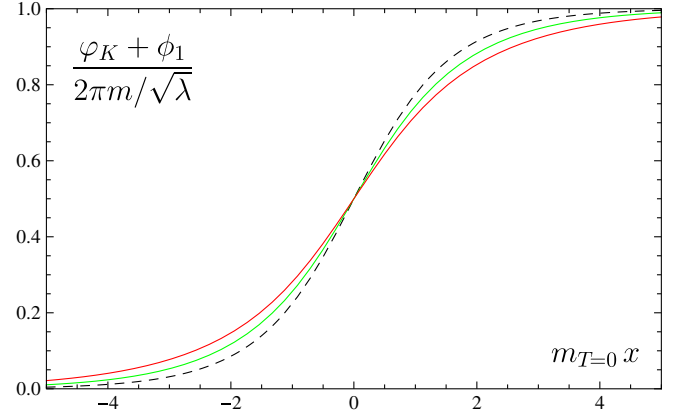


FIG. 1 (color online). Finite-temperature corrections to the sine-Gordon kink $\varphi_K(x) + \phi_1(x)$ according to the result (15) for $\lambda/m^2 = 0.2$ as a function of x times the zero-temperature mass $m_{T=0}$ and for three different temperatures, $T/m_{T=0} = 0, 10, 20$. The zero-temperature result is given by the dashed line, and with increasing temperature the kink becomes flatter.

temperatures much smaller than the critical temperature of symmetry restoration.

Identifying zero-temperature and thermal contributions of the mass counterterm $\delta m^2 \equiv \delta_{T=0} m^2 + \delta_T m^2$, we note that our renormalized mass $m^2 \equiv m_0^2 - \delta m^2 = (m_0^2 - \delta_{T=0} m^2) - \delta_T m^2 = m_{T=0}^2 + m_T^2$ differs from the renormalized mass at zero temperature by a negative thermal correction

$$\begin{aligned} m_T^2 &= -\delta_T m^2 = -\frac{\lambda}{2} \int_{-\infty}^{\infty} \frac{dk}{2\pi} \frac{n(\omega_k)}{\omega_k} \\ &= -\frac{\lambda}{4} \frac{T}{m} + \dots \quad \text{for } T \gg m. \end{aligned} \quad (17)$$

This means that thermal corrections tend to flatten out the potential by reducing the difference between maxima and minima of the potential, which is proportional to $m^2(m^2/\lambda)$ with m^2/λ invariant. However, the distance between the minima and thus the value of φ_K at $x = \pm\infty$ remains fixed. In Fig. 1 we plot the kink profile for three different temperatures as a function of $m_{T=0}x$, showing that with increasing temperature the kink profile becomes flatter. (Had we plotted the profile as a function of mx with m the temperature-dependent mass, the kink would have appeared to become steeper instead.)

B. CP^1 kink

The discussion of the finite-temperature kink in the mass-deformed CP^1 model is very similar to the sine-Gordon model of the previous subsection. The Lagrangian of the massive CP^1 model is

$$\mathcal{L} = -\frac{r}{(1 + \phi^\dagger \phi)^2} (\partial_\mu \phi^\dagger \partial^\mu \phi + m^2 \phi^\dagger \phi), \quad (18)$$

with a dimensionless coupling r and a complex field ϕ .

This model is renormalizable in 1 + 1 dimensions and requires a coupling constant counterterm $r \rightarrow r + \delta r$ which is equivalent to wave-function renormalization and a mass counterterm $m^2 \rightarrow m^2 + \delta m^2$.

The Lagrangian (18) has a $U(1)$ symmetry, $\phi \rightarrow e^{i\alpha}\phi$, and a discrete \mathbb{Z}_2 symmetry, $\phi \rightarrow 1/\phi^\dagger$. Since we work in 1 + 1 dimensions, the continuous symmetry cannot be spontaneously broken [43]. A vacuum expectation value of the field ϕ rather breaks the discrete \mathbb{Z}_2 symmetry spontaneously. The classical potential

$$V = \frac{rm^2\phi^\dagger\phi}{(1 + \phi^\dagger\phi)^2} \quad (19)$$

is minimized at $\phi = 0$ and $|\phi| = \infty$, i.e., by the south and north poles of the 2-sphere representing the compactified complex plane. [Both minima are invariant under $U(1)$ as required].

For a static solution, we can rewrite the Hamiltonian as

$$\mathcal{H} = \frac{r}{(1 + \phi^\dagger\phi)^2}(\partial_x\phi^\dagger - m\phi^\dagger)(\partial_x\phi - m\phi) - rm\partial_x(1 + \phi^\dagger\phi)^{-1} \quad (20)$$

and read off the Bogomolnyi equation $(\partial_x - m)\phi = 0$. Thus the classical solution is

$$\varphi_K(x) = e^{m(x-x_0)}, \quad (21)$$

and the classical energy is $M = rm$. We again set $x_0 = 0$.

Next we determine the spectrum of the fluctuations in the presence of the kink. From

$$\frac{\delta\mathcal{L}}{\delta\phi^\dagger} = \frac{r}{\rho^3}[\rho(\square - m^2)\phi - 2\phi^\dagger((\partial_\mu\phi)^2 - m^2\phi^2)],$$

$$\rho \equiv 1 + \phi^\dagger\phi, \quad (22)$$

it is clear that the field equation of the fluctuations is obtained by setting $\phi = \eta$, keeping ρ fixed. This yields

$$\left(\partial_0^2 - \partial_x^2 + \frac{4\varphi_K^2}{1 + \varphi_K^2}m\partial_x + m^2\frac{1 - 3\varphi_K^2}{1 + \varphi_K^2}\right)\eta(x, t) = 0. \quad (23)$$

With the ansatz

$$\eta(x, t) = r^{-1/2}(1 + \varphi_K^2)g(x)e^{-i\omega t} \quad (24)$$

the field equation for $g(x)$ becomes Eq. (6) of the sine-Gordon model. Consequently, as in the sine-Gordon model, we have a zero-mode

$$g_0(x) = \sqrt{\frac{m}{2}} \frac{1}{\cosh mx}, \quad (25)$$

and the continuous spectrum $\omega_k = \sqrt{k^2 + m^2}$ with

$$g_k(x) = \frac{m}{\omega_k} e^{ikx} \left(\tanh mx - i \frac{k}{m} \right). \quad (26)$$

To obtain the corrections to the kink profile induced by the fluctuations we write, analogous to above, $\phi(x, t) = \varphi_K(x) + \phi_1(x) + \eta(x, t)$, and derive the equation for $\phi_1(x)$. The terms with $\langle \eta^* \eta \rangle$ are obtained by expanding the term $-2(\phi^*/\rho)[2\partial_x\varphi_K\partial_x\eta - 2m^2\varphi_K\eta]$ in Eq. (22) to first order in η^* . It is obvious from (22) that the propagator is diagonal: $\langle \eta\eta \rangle = 0$. Hence we may set $\phi = \varphi_K + \phi_1(x) + \eta_1(x, t)$ with real $\phi_1(x)$ and $\eta_1(x)$ and replace $\langle \eta^* \eta \rangle$ by $\langle \eta_1 \eta_1 \rangle$. Taking into account also the mass counterterm⁶ this yields

$$\left(\partial_x^2 - \frac{4\varphi_K^2}{1 + \varphi_K^2}m\partial_x - \frac{1 - 3\varphi_K^2}{1 + \varphi_K^2}m^2\right)\phi_1 = \frac{4\varphi_K m}{(1 + \varphi_K^2)^2} \langle \eta_1(\partial_x - m)\eta_1 \rangle + \delta m^2 \varphi_K \frac{1 - \varphi_K^2}{1 + \varphi_K^2}, \quad (27)$$

with

$$\langle \eta_1^2 \rangle = r^{-1}(1 + \varphi_K^2)^2 \int_{-\infty}^{\infty} \frac{dk}{2\pi} \frac{1 + 2n(\omega_k)}{2\omega_k} |g_k(x)|^2. \quad (28)$$

Using that $\langle \eta_1(\partial_x - m)\eta_1 \rangle = (\frac{1}{2}\partial_x - m)\langle \eta_1^2 \rangle$ and $(\frac{1}{2}\partial_x - m)(1 + \varphi_K^2)^2 = m(\varphi_K^2 - 1)(1 + \varphi_K^2)$ we find that a mass counterterm of the form

$$\delta m^2 = 4r^{-1}m^2 \int_{-\infty}^{\infty} \frac{dk}{2\pi} \frac{1 + 2n(\omega_k)}{2\omega_k} \quad (29)$$

corresponds to a UV-subtracted quantity

$$\langle \eta_1^2 \rangle_{\text{ren}} = r^{-1}(1 + \varphi_K^2)^2 \int_{-\infty}^{\infty} \frac{dk}{2\pi} \frac{1 + 2n(\omega_k)}{2\omega_k} [|g_k(x)|^2 - 1] = -r^{-1}(1 + \varphi_K^2)^2 g_0^2(x) \left(\frac{1}{m\pi} + 2m \int \frac{dk}{2\pi} \frac{n(\omega_k)}{\omega_k^3} \right), \quad (30)$$

since the functions $g_k(x)$ and $g_0(x)$ obey the same identity (9) as in the sine-Gordon case. With the explicit form of $g_0(x)$, Eq. (25), we finally obtain a vanishing result for the right-hand side of Eq. (27),

$$\langle \eta_1(\partial_x - m)\eta_1 \rangle_{\text{ren}} = \left(\frac{1}{2}\partial_x - m \right) \langle \eta_1^2 \rangle_{\text{ren}} = 0, \quad (31)$$

leading to the remarkably simple result

$$\phi_1(x) = 0. \quad (32)$$

This means that the kink profile remains unaltered at finite temperature, provided the mass is renormalized according to (29). The corresponding renormalization condition turns

⁶The coupling constant counterterm does not contribute because it amounts to wave-function renormalization and thus multiplies the classical field equation.

out to be an on-shell mass renormalization in the trivial sector including all thermal contributions.⁷ The thermal correction to the mass has the same form as in the sine-Gordon model, with the replacement $\lambda \rightarrow 8r^{-1}m^2$ in Eq. (17). Since it is negative, this means that the kink e^{mx} interpolating between the minima at $\phi = 0$ and $\phi = \infty$ becomes spread out with increasing temperature. For the height of the potential between these minima, also the coupling constant renormalization is required. Direct calculation shows that the one-loop self-energy equals $(p^2 + 3m^2)$ times a momentum independent expression, which implies that wave-function renormalization gives the counterterm $\delta r/r = \frac{1}{2}\delta m^2/m^2$. Hence, the thermal corrections of mass and wave function both work in the direction of diminishing the potential, given in Eq. (19), as the temperature is increased.

Once again we can only determine the onset of the melting of the kink. The above discussion for the sine-Gordon model applies with r^{-1} replacing the dimensionless coupling parameter λ/m^2 . The requirement of a small expansion parameter at finite temperature, $r^{-1}T/m \ll 1$, precludes the consideration of symmetry restoration as this would need $\delta m_T^2 \gtrsim m^2$.

III. φ^4 KINK AND DOMAIN WALLS

The Lagrangian for the kink with $\lambda\varphi^4$ interaction is

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{\lambda}{4}(\varphi^2 - v^2)^2, \quad v^2 \equiv \mu^2/\lambda, \quad (33)$$

with \mathbb{Z}_2 symmetry $\varphi \rightarrow -\varphi$.

To one-loop order we introduce counterterms $v^2 \rightarrow v^2 + \delta v^2$ and $\lambda \rightarrow \lambda + \delta\lambda$ and $\delta\mu^2 \equiv \delta(\lambda v^2) = \lambda\delta v^2 + v^2\delta\lambda$ (leaving out wave-function renormalization which would be needed only at two-loop order in 3 + 1 dimensions). δv^2 will be chosen such that in the topologically trivial sector tadpole contributions are subtracted completely (including thermal contributions). In the absence of wave-function renormalization, $\delta\lambda$ is fixed by a renormalization condition for the mass of fluctuations in the trivial sector, which at tree level is $m^2 = 2\mu^2$. (Note however that in the spontaneously broken model (33) the counterterms δv^2 and $\delta\lambda$ imply the mass counterterm $\delta m^2 = -\lambda\delta v^2 + 2v^2\delta\lambda \neq 2\delta\mu^2$.) At zero temperature, the various possibilities have been discussed in detail in Ref. [30]. Below we shall address this question at finite temperature and single out one particularly natural renormalization condition.

⁷In the CP^1 model the one-loop self-energy diagram is momentum-dependent, but it remains Lorentz-invariant at finite temperature, so that one does not have to distinguish between e.g. a screening mass or a plasmon mass.

The equation of motion in the φ^4 model reads

$$\partial_\mu\partial^\mu\varphi + \mu^2\varphi - \lambda\varphi^3 = 0. \quad (34)$$

A kink at rest at $x = x_0$ which interpolates between the two degenerate vacuum states

$$\varphi = \pm \frac{\mu}{\sqrt{\lambda}} \equiv \pm v \quad (35)$$

is classically given by [1]

$$\varphi_K(x) = v \tanh(\mu(x - x_0)/\sqrt{2}), \quad (36)$$

and its energy at tree level is $M = (\sqrt{2}\mu)^3/3\lambda$. From now on we set $x_0 = 0$ without loss of generality.

The kink can be trivially embedded in $d + 1$ spacetime dimensions where it represents a domain wall separating the two vacua of the model. In this case, M has the meaning of a surface tension, i.e. energy per unit transverse volume. In the following we shall make d continuous and use this for dimensional regularization in our renormalization program [30,44,45].

Fluctuations $\eta(\vec{x}, t)$ about the classical kink solution are simple plane waves in the transverse directions \vec{y} with transverse momentum $\vec{\ell}$, i.e., $\eta(\vec{x}, t) = \int e^{-i\omega_i t + i\vec{\ell}\cdot\vec{y}} \phi_i(x)$, with $\omega_{i\vec{\ell}}^2 \equiv \omega_i^2 + \ell^2$, where the x -dependent part $\phi_i(x)$ is then given by the 1 + 1-dimensional fluctuation equation

$$(-\partial_x^2 - \mu^2 + 3\lambda\phi_K^2)\phi_i(x) = \omega_i^2\phi_i(x). \quad (37)$$

The spectrum of the 1 + 1-dimensional fluctuation equation [1] has a zero-energy solution ($\omega_0 = 0$),

$$\phi_0(x) = \sqrt{\frac{3m}{8}} \frac{1}{\cosh^2 \frac{mx}{2}}, \quad (38)$$

a bound state with energy $\omega_B = \sqrt{3}m/2$,

$$\phi_B(x) = \sqrt{\frac{3m}{4}} \frac{\sinh \frac{mx}{2}}{\cosh^2 \frac{mx}{2}}, \quad (39)$$

and a continuous spectrum with energies $\omega_k = \sqrt{k^2 + m^2}$,

$$\phi_k(x) = \frac{m^2 e^{ikx}}{4\omega_k \sqrt{\omega_k^2 - \omega_B^2}} \left[-3 \tanh^2 \frac{mx}{2} + 1 + 4 \frac{k^2}{m^2} + 6i \frac{k}{m} \tanh \frac{mx}{2} \right]. \quad (40)$$

Below we shall use the relation [21]

$$|\phi_k(x)|^2 = 1 - \frac{2m}{\omega_k^2} \phi_0^2(x) - \frac{m}{\omega_k^2 - \omega_B^2} \phi_B^2(x). \quad (41)$$

A. Kink profile

Interpreting (34) as a Heisenberg field equation and writing $\varphi(x, t) = \varphi_K(x) + \phi_1(x) + \eta(x, t)$ with the quantum fluctuation field obeying $\langle \eta \rangle = 0$, we have the follow-

ing equation for the one-loop correction to the kink profile,

$$\begin{aligned}
 & [\partial_x^2 + \mu^2 - 3\lambda\varphi_K^2(x)]\phi_1(x) \\
 &= 3\lambda\varphi_K(x) \left[\langle \eta^2 \rangle(x) - \frac{1}{3}\delta v^2 - \frac{\delta\lambda}{3\lambda}(v^2 - \varphi_K^2(x)) \right] \\
 &\equiv 3\lambda\varphi_K(x) \langle \eta^2 \rangle_{\text{ren}}(x). \tag{42}
 \end{aligned}$$

This equation is formally unchanged in the presence of a nonzero wave-function renormalization Z , although the counterterms δv^2 and $\delta\lambda$ will of course be different in different renormalization schemes. However, if two renormalization schemes, one with $Z = 1$ and one with $Z \neq 1$, have the same renormalization condition for the mass m (e.g. that it should be the pole mass in the trivial sector without a kink), the one with nontrivial Z differs from the other one only by an extra contribution⁸ $\Delta\phi_1 = -\frac{1}{2}(Z - 1)\varphi_K$.

Continuing with $Z = 1$, we have

$$\delta v^2 = 3\langle \eta^2 \rangle|_{\text{trivial sector}} = 3 \int \frac{d^{d-1}\ell dk}{(2\pi)^d} \frac{1 + 2n(\omega_{k\ell})}{2\omega_{k\ell}} \tag{43}$$

$$\begin{aligned}
 \langle \eta^2 \rangle_{\text{ren}}(x) &= \int \frac{d^{d-1}\ell dk}{(2\pi)^d} \frac{1 + 2n(\omega_{k\ell})}{2\omega_{k\ell}} \left[-\frac{3m^2/4}{k^2 + \frac{m^2}{4}} \frac{1}{\cosh^2(mx/2)} + \frac{(3m^2/4)^2}{(k^2 + m^2)(k^2 + \frac{m^2}{4})} \frac{1}{\cosh^4(mx/2)} \right] \\
 &+ \int \frac{d^{d-1}\ell}{(2\pi)^{d-1}} \frac{1 + 2n(\omega_{B\ell})}{2\omega_{B\ell}} \frac{3m}{4} \left(\frac{1}{\cosh^2(mx/2)} - \frac{1}{\cosh^4(mx/2)} \right) + \mathcal{N}_d(T) \frac{3m}{8} \frac{1}{\cosh^4(mx/2)} \\
 &- \delta\lambda \frac{m^2}{6\lambda^2} \frac{1}{\cosh^2(mx/2)}, \tag{44}
 \end{aligned}$$

where $\omega_{k\ell} = \sqrt{k^2 + \ell^2 + m^2}$ and $\omega_{B\ell} = \sqrt{\ell^2 + 3m^2/4}$. We have now also included the zero mode of the $1 + 1$ -dimensional fluctuation equation about the kink background, which in $d > 1$ spatial dimensions is a massless mode (the Goldstone mode associated with the spontaneous breaking of translational invariance),

$$\mathcal{N}_{d>1}(T) = \int \frac{d^{d-1}\ell}{(2\pi)^{d-1}} \frac{1 + 2n(\omega_{0\ell})}{2\omega_{0\ell}}, \quad \omega_{0\ell} = \sqrt{\ell^2}, \tag{45}$$

where the $T = 0$ part vanishes since in dimensional regularization scaleless integrals are identically zero. For $d = 1 + \epsilon$ we discard the thermal part as well, since the latter is finite and we can set $\epsilon = 0$ and $\omega_{0\ell} = 0$ there, assuming that this genuine zero mode is treated by collective quantization [42]. Thus we have

⁸As one can check (cf. Sec. 2.2.2 of Ref. [30]), the net difference of the right-hand side of (42) is then $+\lambda(Z - 1)\varphi_K^3$, which leads to the extra contribution $\Delta\phi_1 = -\frac{1}{2}(Z - 1)\varphi_K$.

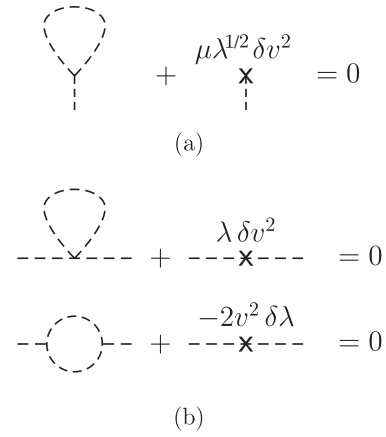


FIG. 2. Renormalization of the tadpole diagram (a) through the counterterm δv^2 and (b) of the self-energy diagrams through the counterterm $-\delta m^2 = \lambda\delta v^2 - 2v^2\delta\lambda$. Dashed lines correspond to the propagator in the trivial sector.

where we absorb the complete quantum and thermal contribution of the tadpole diagram in the renormalization of v [see Fig. 2(a)]. Writing everything out for $d + 1$ dimensions, this leaves us with

$$\mathcal{N}_{d=1+\epsilon}(T) \equiv 0; \quad \mathcal{N}_d(T = 0) = 0. \tag{46}$$

In keeping a nontrivial thermal contribution \mathcal{N}_d for $d \geq 2$ we differ from Ref. [46], and we shall later in Sec. III B provide evidence for the necessity of this.

As mentioned above, in $d = 3$, $\delta\lambda$ which appears in the last term of Eq. (44) has to absorb a UV divergence of the momentum integrals, whereas in lower dimensions one could also be content with a minimal renormalization scheme, where only tadpoles are renormalized. In Refs. [12,21] it was observed (for $d = 1$) that at zero temperature the requirement of an on-shell mass simplifies the result greatly—all terms proportional to $1/\cosh^2(mx/2)$ then cancel.

At nonzero temperature, there is actually not just one on-shell mass, but thermal masses are in general momentum dependent: at zero momentum, the thermal mass gives the plasma frequency above which propagating modes appear. Because of the absence of manifest Lorentz invariance, the effective mass of the propagating mode is generally not a constant. Thermal masses at frequencies below the plasma

frequency give inverse spatial screening lengths. As we shall show now, it is the static screening mass which has to be employed in the renormalization condition in order that the above mentioned simplification occurs.

Requiring that the renormalized parameter m be equal to the thermal static screening mass means that we have to subtract the self-energy diagram at zero frequency and imaginary spatial momentum⁹ $\vec{q}^2 = -m^2$. The mass counterterm follows from (33) after substituting $\varphi = v + \eta$, and reads $\delta m^2 = -\lambda \delta v^2 + 2v^2 \delta \lambda$. As indicated in Fig. 2(b), the seagull diagram is already canceled by the counterterm $-\lambda \delta v^2$, so the diagram with two propagators in the loop evaluated at $\vec{q}^2 = -m^2$ defines $2v^2 \delta \lambda$. Using Feynman parametrization (which is straightforward to use in finite-temperature integrals at zero frequency, albeit not otherwise [48]) we find

$$\delta \lambda = \frac{9\lambda^2}{2} \int_0^1 dt \int \frac{d^{d-1} \ell dk}{(2\pi)^d} \left[\frac{1 + 2n(\omega_t)}{2\omega_t^3} - \frac{n'(\omega_t)}{\omega_t^2} \right], \quad (47)$$

where

$$\omega_t \equiv \sqrt{k^2 + \ell^2 + m^2[1 - t(1 - t)]}. \quad (48)$$

In the $T = 0$ part of $\delta \lambda$, one can easily integrate over the Feynman parameter to find

$$\delta \lambda^{(T=0)} = \frac{9\lambda^2}{4} \int \frac{d^{d-1} \ell dk}{(2\pi)^d} \times \frac{1}{\sqrt{k^2 + \ell^2 + m^2}(k^2 + \ell^2 + 3m^2/4)}. \quad (49)$$

For later use we quote the closed-form result for this counterterm from Ref. [30],

$$\delta \lambda^{(T=0)} = 9\lambda^2 \frac{m^{d-3}}{(4\pi)^{(d+1)/2}} \Gamma\left(\frac{3-d}{2}\right) \times \left(\frac{3}{4}\right)^{(d-3)/2} {}_2F_1\left(\frac{3-d}{2}, \frac{1}{2}; \frac{3}{2}; -\frac{1}{3}\right). \quad (50)$$

However, keeping the integral over transverse momenta ℓ unevaluated and using

⁹In a non-Abelian gauge theory, one finds that only this self-consistent definition leads to a gauge-independent result for the (Debye) screening mass [47]. In the present scalar theory, it will be seen to remove certain artifacts in the kink profile.

$$\int_{-\infty}^{\infty} dk \frac{1}{\sqrt{k^2 + a^2}(k^2 + b^2)} = \frac{2}{b\sqrt{a^2 - b^2}} \times \arctan\left(\frac{\sqrt{a^2 - b^2}}{b}\right) \quad (51)$$

one can verify that all zero-temperature terms proportional to $1/\cosh^2(mx/2)$ in (44) cancel already under the integral over the transverse momentum ℓ . Thus the cancellations observed in Refs. [12,21] for zero temperature and $d = 1$ also take place for arbitrary d .

Turning to the thermal contributions one finds that the individual integrals involving the Bose-Einstein factor n cannot be evaluated in closed form. By numerical integrations one can however verify without difficulty the rather abstruse identity

$$\begin{aligned} & \int_0^1 dt \int \frac{dk}{2\pi} \left[\frac{n(\omega_t)}{\omega_t^3} - \frac{n'(\omega_t)}{\omega_t^2} \right] \\ & + \int \frac{dk}{2\pi} \frac{n(\sqrt{k^2 + \ell^2 + m^2})}{\sqrt{k^2 + \ell^2 + m^2}} \frac{1}{k^2 + \frac{m^2}{4}} \\ & = \frac{1}{m} \frac{n(\sqrt{\ell^2 + \frac{3}{4}m^2})}{\sqrt{\ell^2 + \frac{3}{4}m^2}}, \end{aligned} \quad (52)$$

where ω_t was defined in Eq. (48). Using this to evaluate the Feynman parameter integral in $\delta \lambda$, Eq. (47), and inserting the result into Eq. (44) shows that at finite temperature it is the renormalization prescription of vanishing tadpoles and m being the finite-temperature screening mass which absorbs all terms proportional to $1/\cosh^2(mx/2)$ in (44). We then have

$$\langle \eta^2 \rangle_{\text{ren}}(x) = -m^{d-1} A_d(T/m) \frac{1}{\cosh^4(mx/2)} \quad (53)$$

with

$$\begin{aligned} A_d(T/m) &= \frac{3m^{2-d}}{4} \int \frac{d^{d-1} \ell}{(2\pi)^{d-1}} \frac{1 + 2n(\omega_{B\ell})}{2\omega_{B\ell}} \\ & - \frac{3m^{2-d}}{8} \mathcal{N}_d(T) - \frac{9m^{5-d}}{16} \int \frac{d^{d-1} \ell dk}{(2\pi)^d} \\ & \times \frac{1 + 2n(\omega_{k\ell})}{2\omega_{k\ell}} \frac{1}{(k^2 + m^2)(k^2 + \frac{m^2}{4})} \end{aligned} \quad (54)$$

which when inserted into Eq. (42) yields

$$\begin{aligned} \phi_1(x) &= v^{-2} m^{d-1} A_d(T/m) \frac{1}{\cosh^2(mx/2)} \varphi_K \\ & = v^{-1} m^{d-1} A_d(T/m) \frac{\sinh(mx/2)}{\cosh^3(mx/2)} \end{aligned} \quad (55)$$

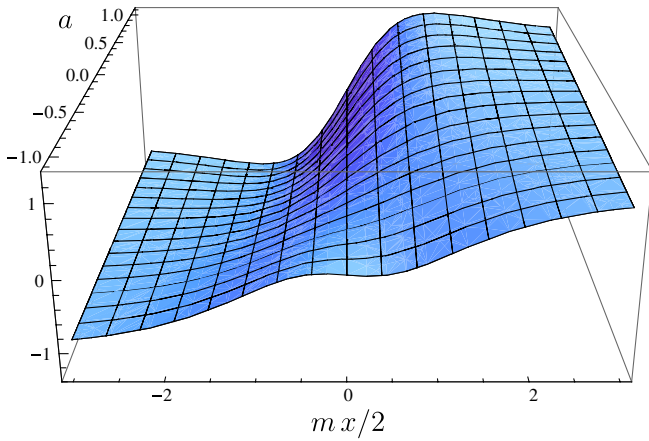


FIG. 3 (color online). The one-loop corrected φ^4 kink profile $\varphi_K(x) + \phi_1(x) \propto \tanh(z) + a \sinh(z)/\cosh^3(z)$ with $z = mx/2$ for $a = -1 \dots 1$, showing the effect of positive vs negative $a \propto A_d$ up to nonperturbatively large a . For $d = 1$, A_d is positive and increasing with temperature, as displayed in Fig. 4, seemingly leading to a steeper kink at higher temperatures. However, this plot does not yet take into account that at increasing temperature the asymptotic values of the kink are reduced by the thermal part of δv^2 [see Eq. (43)]. Including this effect, the kink profile does become flatter.

for the correction to the kink profile. In Fig. 3 the effect of such a correction to the classical kink profile is shown for (large) negative and positive coefficients.

In a generic renormalization scheme, where $\langle \eta^2 \rangle_{\text{ren}}(x)$ has either uncanceled $1/\cosh^2(mx/2)$ terms or uncanceled constant terms (by incomplete tadpole subtraction), one would find additional terms $\propto mx/\cosh^2(mx/2)$ in the kink profile correction ϕ_1 [aside from a different function $A_d(T/m)$]. *A posteriori*, the “bare” mx figuring in the latter term can be understood as an artifact of incomplete renormalization since the particular on-shell renormalization scheme considered above is evidently able to absorb these terms such that all mx appear only in exponentiated form.

So the simple result (55) depends on a renormalization scheme where m is the thermal static screening mass. This differs from the zero-temperature mass by the finite difference

$$m^2 - m_{T=0}^2 = 3\lambda \int \frac{d^{d-1}\ell dk}{(2\pi)^d} \frac{n(\omega_{k\ell})}{\omega_{k\ell}} \left(1 + \frac{3m^2/2}{k^2 + \frac{m^2}{4}}\right) - \frac{9\lambda m}{2} \int \frac{d^{d-1}\ell}{(2\pi)^{d-1}} \frac{n(\omega_{B\ell})}{\omega_{B\ell}}, \quad (56)$$

where we have again used the identity (52).

I. 1 + 1 dimensional kink

For $d = 1$, $\mathcal{N}_d(T)$ is absent and all integrals in (54) are individually finite and the ℓ integration can in fact be dropped. The $T = 0$ part is readily found to be

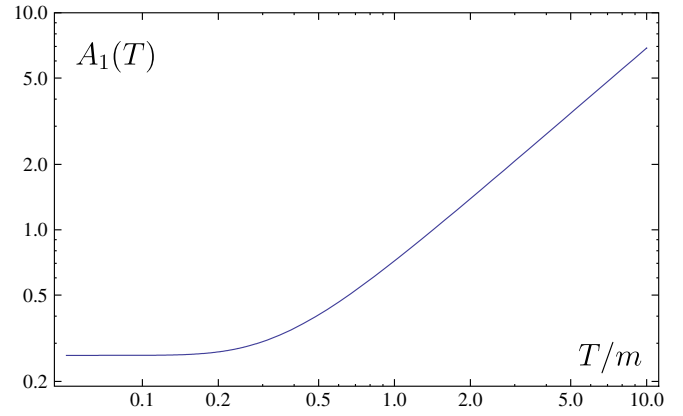


FIG. 4 (color online). The function $A_d(T/m)$ in a doubly logarithmic plot for $d = 1$. This function determines the temperature dependence of the kink profile [see Eq. (55)].

$$A_1(0) = \frac{1}{4\sqrt{3}} + \frac{3}{8\pi}, \quad (57)$$

in agreement with Refs. [12,21]. The thermal part of $A_1(T)$ turns out to be strictly positive and growing linearly with T/m for large T/m . The full function is plotted in Fig. 4. This corresponds to a positive parameter a in Fig. 3 which grows as the temperature is increased (with a having to remain sufficiently small so that perturbation theory is still valid). When plotted with fixed asymptotic values as in Fig. 3, the one-loop corrected kink profile which is slightly steeper than the classical one appears to become even steeper with increasing temperature. However, the asymptotic amplitude of the kink diminishes, because $v^2 \equiv v_0^2 - \delta v^2 \equiv (v_0^2 - \delta_{T=0}v^2) - \delta_T v^2$, and $\delta_T v^2$, the thermal part of Eq. (43), is positive and growing with temperature. Including this effect, the slope of the kink profile decreases at higher temperature. Moreover, also m has been renormalized such as to include thermal corrections and these reduce m as the phase transition is approached. When plotted in terms of a fixed zero-temperature mass $m_{T=0}$, the kink profile flattens even more quickly. At any rate, in perturbation theory we can only describe reliably the onset of the melting of the kink.

The φ^4 kink in 1 + 1 dimensions has been studied extensively in nonperturbative self-consistent approximations to the two-particle irreducible effective action [22–24]. Comparing our Fig. 3 with Figs. 2 and 3 of [24] we find a somewhat different behavior even when the corrections are still small. This can be traced to the fact that in Refs. [22,24] a variational ansatz for the dressed two-point function with coinciding arguments in the kink background, a nonperturbative generalization of our $\langle \eta^2 \rangle_{\text{ren}}(x)$, has been employed which is proportional to $1/\cosh^2(mx/2)$. Our result (53) however suggests that a more adequate ansatz would involve $1/\cosh^4(mx/2)$. In Ref. [22] the choice of the former was motivated by an analysis of the sine-Gordon model, where we have indeed

obtained a renormalized two-point function proportional to $1/\cosh^2(mx)$ [see Eq. (14)]. But, as we have demonstrated, the generalization to the φ^4 model is not justified. In this case, both the constant term in $\langle\eta^2\rangle$ as well as the term proportional to $1/\cosh^2(mx/2)$ are removable by renormalization, whereas in the sine-Gordon model only the constant term is. Note that in both cases the on-shell renormalized $\langle\eta^2\rangle_{\text{ren}}$ to one-loop order turns out to be

$$\begin{aligned} A_d(0) &= \frac{9}{16} \frac{1}{(4\pi)^{d/2}} \Gamma\left(\frac{2-d}{2}\right) \left[\left(\frac{3}{4}\right)^{(d-4)/2} - \frac{1}{\pi} \int_0^\infty dx \frac{(x^2+1)^{(d-4)/2}}{x^2+\frac{1}{4}} \right] \\ &= \frac{9}{16} \frac{1}{(4\pi)^{d/2}} \Gamma\left(\frac{2-d}{2}\right) \left[\left(\frac{3}{4}\right)^{(d-4)/2} - \frac{1}{\sqrt{\pi}} \frac{\Gamma(\frac{5-d}{2})}{\Gamma(\frac{6-d}{2})^2} F_1\left(\frac{4-d}{2}, \frac{1}{2}; \frac{6-d}{2}; \frac{3}{4}\right) \right]. \end{aligned} \quad (58)$$

In the limit $d \rightarrow 1$ this of course reproduces Eq. (57), but for $d \rightarrow 2$ we encounter a singularity, because the square bracket in Eq. (58) does not vanish for $d \rightarrow 2$. However, this divergence should not be a UV divergence, since, as we have shown above, mass, coupling constant, as well as wave-function renormalization can only modify constant terms and terms proportional to $1/\cosh^2(mx/2)$ in $\langle\eta^2\rangle$ and therefore not A_d . Indeed, by taking into account $\mathcal{N}_2(0)$, which is zero in dimensional regularization, but whose integral representation is both UV and IR divergent, one finds that a small IR regulator mass $\ell^2 \rightarrow \ell^2 + \nu^2$ in (45) changes (58) to

$$\begin{aligned} A_d(0; \nu/m) &= \frac{9}{16} \frac{1}{(4\pi)^{d/2}} \Gamma\left(\frac{2-d}{2}\right) \left[\left(\frac{3}{4}\right)^{(d-4)/2} - \frac{2}{3} \left(\frac{\nu^2}{m^2}\right)^{(d-2)/2} - \frac{1}{\pi} \int_0^\infty dx \frac{(x^2+1)^{(d-4)/2}}{x^2+\frac{1}{4}} \right] \\ &= \frac{9}{16} \frac{1}{(4\pi)^{d/2}} \Gamma\left(\frac{2-d}{2}\right) \left[\frac{4}{3} - \frac{2}{3} - \frac{2}{3} + \mathcal{O}(d-2) \right], \end{aligned} \quad (59)$$

which is finite for $d \rightarrow 2$ but now contains a term involving $\ln(\nu/m)$,

$$A_2(0) \sim -\frac{3}{16\pi} \ln \frac{m}{\nu}, \quad (60)$$

so that the IR regulator cannot be removed. [The finite- T contributions in $\mathcal{N}_d(T)$ have even more severe IR divergences.] Such IR divergences have in fact been discussed in the literature as being associated with the roughening of interfaces, leading to a logarithmic sensitivity of the width of interfacial profiles to the linear system size L . This phenomenon has been described in terms of the so-called capillary wave model [38–40], with the capillary waves corresponding to the massless modes associated with the kink zero mode [26]. In a field theoretic treatment similar to ours, but with the interpretation of the Euclidean action of the φ^4 model as the Landau-Ginzburg Hamiltonian of statistical field theory, interface roughening has been discussed using cutoff regularization in Ref. [29], and recently in $d = 3 + \epsilon$ dimensions in Ref. [27]. Our results for the one-loop profile, Eq. (55) and (60), agree with Refs. [27,29], the latter upon identifying $\nu \sim 1/L$. (Ref. [27] used a finite system with quadratic interface as IR regularization and also determined the resulting sub-logarithmic contributions.)

The results for the profile given in Refs. [26,27] also involve IR-finite contributions of the form

proportional to the zero mode of the respective model squared.

2. Domain wall profiles

For $d \geq 2$, the zero-temperature contributions in Eq. (54) are individually UV-divergent. Carrying out first the ℓ integral in dimensional regularization we obtain [using formula (3.259.3) of Ref. [49]]

$mx/\cosh^2(mx/2)$, which are due to the fact that there the results are expressed in terms of a static correlation length defined by a renormalization point $q^2 = 0$ instead of $q^2 = -m^2$. As we have shown, the latter definition, which is required to give m the meaning of an inverse exponential screening length, quite generally eliminates such contributions.

For $d = 3$, i.e., in the $3 + 1$ -dimensional case, we find that the $T = 0$ part is finite, yielding the (to our knowledge new) result

$$A_3(0) = -\frac{\sqrt{3}}{32\pi}. \quad (61)$$

Now the thermal contributions are logarithmically IR divergent, because of $\mathcal{N}_3(T)$, which leads to

$$A_3(T) \sim -\frac{3}{16\pi} \frac{T}{m} \ln \frac{m}{\nu}, \quad (62)$$

for an IR momentum cutoff $\nu \sim 1/L$, which indicates interfacial roughening effects also in the context of domain walls of relativistic field theories at finite temperature.

B. Local energy density

We now turn to the local energy density $\epsilon(x)$, which, following Ref. [21], we decompose as

$$\epsilon(x) = \epsilon_{\text{Cas}}(x) + \Delta\epsilon_{\text{Cas}}(x) + \Delta\epsilon_{(\phi_1)}(x). \quad (63)$$

Here $\epsilon_{\text{Cas}}(x)$ represents the local energy density of a domain wall in $d + 1$ dimensions (assuming dimensional regularization) due to the sum over zero-point energies, whereas the contributions $\Delta\epsilon_{\text{Cas}}(x)$ and $\Delta\epsilon_{(\phi_1)}(x)$ are total derivatives which do not contribute to the integrated total

energy (or surface tension) and which have been identified in Ref. [21].

Subtracting off the energy density of the topologically trivial sector (including thermal contributions), the local energy density from the sum over zero-point energies is given by

$$\begin{aligned} \epsilon_{\text{Cas}}(x) = & \int \frac{d^{d-1}\ell}{(2\pi)^{d-1}} \frac{\omega_{0\ell}}{2} [1 + 2n(\omega_{0\ell})] \phi_0^2(x) + \int \frac{d^{d-1}\ell}{(2\pi)^{d-1}} \frac{\omega_{B\ell}}{2} [1 + 2n(\omega_{B\ell})] \phi_B^2(x) \\ & - m \int \frac{d^{d-1}k}{(2\pi)^d} \frac{\omega_{k\ell}}{2} [1 + 2n(\omega_{k\ell})] \left[\frac{2\phi_0^2(x)}{k^2 + m^2} + \frac{\phi_B^2(x)}{k^2 + \frac{m^2}{4}} \right] - \frac{\lambda}{2} \delta v^2 [\varphi_K^2(x) - v^2] + \frac{\delta\lambda}{4} [\varphi_K^2(x) - v^2]^2, \end{aligned} \quad (64)$$

where in the second line Eq. (41) has been used again to rewrite $|\phi_k(x)|^2 - 1$ in terms of $\phi_0(x)$ and $\phi_B(x)$, and where we have included the contribution from the counterterms. (Recall that $\omega_{k\ell} = \sqrt{k^2 + \ell^2 + m^2}$, $\omega_{B\ell} = \sqrt{\ell^2 + 3m^2/4}$, and $\omega_{0\ell} = \sqrt{\ell^2}$.) The first term in Eq. (64) is the contribution from the massless modes corresponding to the zero mode of the $1 + 1$ -dimensional kink. Its zero-temperature part is eliminated by dimensional regularization, but its thermal contribution is that of blackbody radiation in $d - 1$ spatial dimensions. It is to be omitted for $d = 1$, but contributes nontrivially in the case of domain walls. If it was not included (as done e.g. in Ref. [46]), the total one-loop surface tension $\int dx \epsilon_{\text{Cas}}$ would not vanish in the limit $m \rightarrow 0$, i.e. at the second-order phase transition where the kink has melted completely. To see this, note that for fixed $\ell \gg k$, one has IR singular limits $m \rightarrow 0$ of $\int dk \left(\frac{2}{k^2 + m^2} + \frac{1}{k^2 + m^2/4} \right)$ in the second line of Eq. (64) which cancel the explicit factor of m there. This yields thermal contributions which indeed compensate the $m \rightarrow 0$ limits of the thermal contributions of the bound state modes and the massless modes.

In contrast to the case of the kink profile we observe that in ϵ_{Cas} the massless modes do not lead to IR problems in either $2 + 1$ or $3 + 1$ dimensions, but we shall see that the IR singular kink profile plays a role in the remaining two (total derivative) contributions to $\epsilon(x)$. The total energy (or surface tension) M is already determined by the above (IR-safe) expression,

$$M = \int dx \epsilon_{\text{Cas}}(x). \quad (65)$$

(The integration over x is evaluated readily using $\int dx \phi_B^2(x) = \int dx \phi_0^2(x) = 1$.)

For the local distribution of the energy, however, the total derivative terms $\Delta\epsilon_{\text{Cas}}(x)$ and $\Delta\epsilon_{(\phi_1)}(x)$ are relevant. The first of these comes from a surface term associated to a partial integration of the spatial gradients in the kinetic energy [21] and reads

$$\begin{aligned} \Delta\epsilon_{\text{Cas}} = & \frac{1}{4} \partial_x^2 \langle \eta^2 \rangle(x) \\ = & \frac{1}{4} \partial_x^2 \left[\langle \eta^2 \rangle_{\text{ren}}(x) + \delta\lambda \frac{m^2}{6\lambda^2} \frac{1}{\cosh^2(mx/2)} \right], \end{aligned} \quad (66)$$

with $\langle \eta^2 \rangle_{\text{ren}}$ defined in Eq. (42).

The second contribution to the local energy density comes from the correction to the kink profile, ϕ_1 , that we considered above. This also does not contribute to the total energy, because the classical kink corresponds to a stationary point of the classical energy, but it gives a local modification of the energy density according to

$$\Delta\epsilon_{(\phi_1)} = \partial_x(\phi_1 \partial_x \varphi_K). \quad (67)$$

Inserting the results of Eq. (53) and (55) into these two expressions, we find that the terms with the function A_d , which appears in both, add with equal magnitude, yielding

$$\begin{aligned} \Delta\epsilon_{\text{Cas}} + \Delta\epsilon_{(\phi_1)} = & \frac{1}{2} m^{d+1} A_d(T) \left(-\frac{4}{\cosh^4 \frac{mx}{2}} + \frac{5}{\cosh^6 \frac{mx}{2}} \right) \\ & - \delta\lambda \frac{m^3}{18\lambda^2} [\phi_0^2(x) - \phi_B^2(x)]. \end{aligned} \quad (68)$$

This localized contribution with zero total energy is finite in $1 + 1$ dimensions, where it has been evaluated at zero temperature in Ref. [21], but in $2 + 1$ and $3 + 1$ dimensions it inherits the IR divergences found in the prefactor A_d of the one-loop kink profile discussed in the previous section. In $3 + 1$ dimensions it is moreover UV divergent because of the appearance of $\delta\lambda$ in $\Delta\epsilon_{\text{Cas}}$.

Let us now check for UV divergences in Eq. (64), which was evaluated in [12,21] for the $1 + 1$ -dimensional kink, but only in integrated form for $d + 1$ -dimensional domain walls in Ref. [30]. We shall now show that in the $2 + 1$ -dimensional case also the local energy density is rendered finite by on-shell renormalization (defined in the trivial sector). However, we shall find that in $3 + 1$ dimensions there are uncanceled UV divergences in $\epsilon_{\text{Cas}}^{(T=0)}(x)$ which require additional composite operator renormalization.

Potential UV divergences can only come from the $T = 0$ part (terms without n) in Eq. (64). Using that $\varphi_K^2(x) - v^2 = -\frac{2m}{3\lambda}(\phi_B^2 + 2\phi_0^2)$ and $[\varphi_K^2(x) - v^2]^2 = \frac{2m^3}{3\lambda^2}\phi_0^2$ we obtain

$$\begin{aligned} \epsilon_{\text{Cas}}^{(T=0)}(x) &= \phi_B^2(x) \frac{1}{2} \int \frac{d^{d-1}\ell}{(2\pi)^{d-1}} \left[\omega_{B\ell} + m \int \frac{dk}{2\pi} \left(\frac{1}{\omega_{k\ell}} - \frac{\omega_{k\ell}}{k^2 + \frac{m^2}{4}} \right) \right] \\ &+ \phi_0^2(x) m \int \frac{d^{d-1}\ell dk}{(2\pi)^d} \left[\frac{1}{\omega_{k\ell}} - \frac{\omega_{k\ell}}{k^2 + m^2} + \frac{3}{8} \frac{m^2}{\omega_{k\ell}(\omega_{k\ell}^2 - \frac{m^2}{4})} \right]. \end{aligned} \quad (69)$$

Integration over ℓ yields after some rearrangements

$$\begin{aligned} \epsilon_{\text{Cas}}^{(T=0)}(x) &= \phi_B^2(x) \frac{\Gamma(-\frac{d}{2})}{(4\pi)^{d/2}} \frac{1}{2} \left[-\left(\frac{3}{4}m^2\right)^{d/2} + m(1-d) \int \frac{dk}{2\pi} (k^2 + m^2)^{d/2-1} + m \int \frac{dk}{2\pi} (k^2 + m^2)^{d/2-1} \frac{3m^2/4}{k^2 + m^2/4} \right] \\ &+ \phi_0^2(x) \left[\frac{\Gamma(-\frac{d}{2})}{(4\pi)^{d/2}} m(1-d) \int \frac{dk}{2\pi} (k^2 + m^2)^{d/2-1} + \delta\lambda^{(T=0)} \frac{m^3}{6\lambda^2} \right], \end{aligned} \quad (70)$$

with $\delta\lambda^{(T=0)}$ given in (50). This expression is finite for $d \rightarrow 1$, and includes an ‘‘anomalous’’ term proportional to $(1-d) \int dk (k^2 + m^2)^{d/2-1}$, which would be missed in a naive momentum cutoff regularization. With it one recovers Eq. (18) of Ref. [21], except for the last term, which is due to $\delta\lambda$ and which is finite by itself.¹⁰ For $d \rightarrow 2$ the result is also finite [the divergent $\Gamma(-\frac{d}{2})$ which is present in all terms except $\delta\lambda$ is canceled after combining the UV finite expressions and using that $\int dk (k^2 + m^2)^{d/2-1}$ is proportional to $1/\Gamma(1-d/2)$], but for $d = 3 - 2\epsilon$ one finds divergent terms $1/\epsilon$. All remaining finite integrals over k can be evaluated separately for $d = 1, 2, 3$ by using the substitution $k/m = \tan t$, but using the formulas given in Ref. [30], the following comparatively compact result can be derived,

$$\begin{aligned} \epsilon_{\text{Cas}}^{(T=0)}(x) &= \frac{m^d}{(4\pi)^{(d+1)/2}} \frac{2\Gamma(\frac{3-d}{2})}{d} \\ &\times \{ \phi_B^2(x) [(3/4)^{(d-1)/2} f(d) - 1] \\ &+ \phi_0^2(x) [d(3/4)^{(d-1)/2} f(d) - 2] \} \end{aligned} \quad (71)$$

where

$$f(d) = {}_2F_1\left(\frac{3-d}{2}, \frac{1}{2}; \frac{3}{2}; -\frac{1}{3}\right) \quad (72)$$

with the special cases

$$\begin{aligned} f(1) &= \frac{\pi}{2\sqrt{3}}, & f(2) &= \frac{1}{2}\sqrt{3}\ln 3, \\ f(3-2\epsilon) &= 1 + \epsilon\left(2 - \frac{\pi}{\sqrt{3}} - \ln\left(\frac{4}{3}\right)\right). \end{aligned} \quad (73)$$

These can be used to write out closed-form results for the local Casimir energy density in 1 + 1 dimensions (calculated previously in Ref. [21]) and for the

¹⁰The inclusion of this term corresponds to the finite renormalizations $m \rightarrow \bar{m}$ and $\lambda \rightarrow \bar{\lambda}$ in the final result (28) of Ref. [21].

2 + 1-dimensional domain wall. With $\int dx \phi_0^2 = \int dx \phi_B^2 = 1$ one can readily verify that the *total* energy obtained from $\int dx \epsilon_{\text{Cas}}^{(T=0)}(x)$ agrees with the result given in Eq. (19) of Ref. [30], which is finite for all $d \leq 4$.

But for the *local* energy density we have UV divergent contributions for $d = 3 - 2\epsilon$,

$$\epsilon_{\text{Cas}}^{\text{div}}(x)|_{d=3-2\epsilon} = \frac{1}{6\epsilon} \frac{m^3}{(4\pi)^2} [\phi_0^2(x) - \phi_B^2(x)], \quad (74)$$

which turns out to cancel incompletely with the UV divergence found above in Eq. (68),

$$\Delta \epsilon_{\text{Cas}}^{\text{div}}(x)|_{d=3-2\epsilon} = -\frac{1}{2\epsilon} \frac{m^3}{(4\pi)^2} [\phi_0^2(x) - \phi_B^2(x)]. \quad (75)$$

However, it is well known that the local energy density is ambiguous up to improvement terms to the energy-momentum tensor, and that an unimproved energy-momentum tensor in general needs composite operator renormalization [50–54]. This ambiguity signals that the local energy density is not a well-defined physical observable as such. If one defines the energy density as the 00-component of the gravitational energy-momentum tensor (variation of the Lagrangian with respect to the metric) then it will depend on the particular coupling to the gravitational field, which can also include a term in the Lagrangian of the form $-\frac{1}{2}\xi\sqrt{-g(x)}R(x)\varphi(x)^2$ where R is the Riemann scalar. In a flat spacetime, this produces an additional contribution to the local energy density given by

$$\epsilon_\xi(x) = -\xi \partial_x^2 \varphi^2(x), \quad (76)$$

which can contribute already at tree-level for a ‘‘nonminimal’’ coupling $\xi \neq 0$.¹¹ Even when $\xi = 0$ at tree level, as we have implicitly been assuming so far, quantum contri-

¹¹The associated integrated total energy vanishes for the kink, but in solitons involving long-range fields such as magnetic monopoles improvement terms to the energy-momentum tensor can also contribute to the total energy, see Ref. [55].

butions will in general produce such a term. Indeed, the combination $\phi_0^2 - \phi_B^2$ appearing in the divergent contributions Eqs. (74) and (75) is proportional to $\partial_x^2 \varphi_K^2(x)$,

$$\phi_0^2 - \phi_B^2 = \frac{6\lambda}{m^3} \partial_x^2 \varphi_K^2(x) \quad (77)$$

and thus can be removed by a counterterm $-\frac{1}{2} \delta\xi \sqrt{-g(x)} R(x) \varphi(x)^2$ with divergent $\delta\xi$.

Alternatively, having a nonzero ξ already at tree level produces further divergences of the form of (74) and (75), namely,

$$\Delta\epsilon_\xi = -\xi \partial_x^2 \langle \eta^2 \rangle(x) = -4\xi \Delta\epsilon_{\text{Cas}}. \quad (78)$$

This is UV divergent in 3 + 1 dimensions and also generally IR divergent because it involves the kink profile. By choosing $\xi = 1/2$ we can in fact make sure that the correspondingly modified local energy density is independent of $A_d(T)$ and thus always IR finite (but not UV finite in 3 + 1 dimensions). Alternatively, $\xi = 1/6$ leads to a UV finite (but generally IR divergent) one-loop local energy density for $d = 3$ without infinite renormalization of ξ . (In a massless theory, $\xi = \frac{d-1}{4d}$ preserves conformal invariance in a gravitational background and thus $\xi = +1/6$ is the value which corresponds to an improved stress tensor that is finite in 3 + 1 dimensions.)

IV. CONCLUSION

We have calculated one-loop corrections to the profile of sine-Gordon and CP^1 kinks and φ^4 domain walls at zero and finite temperature. Using dimensional regularization, we have reproduced results for the one-loop field profile of 1 + 1-dimensional (bosonic) kinks of Refs. [12,21], and have extended them to include thermal contributions, and also to cover higher-dimensional φ^4 kink domain walls. We have shown that a renormalization condition which defines thermal screening masses self-consistently at negative wave vector squared simplifies the result and removes certain artifacts in the resulting one-loop corrected kink profile.

However, in the case of domain walls, we have encountered divergences in local quantities that are not taken care of by standard renormalization of the parameters in the Lagrangian as summarized in Table I. On the one hand, we

have found infrared singularities caused by the massless modes which are the higher-dimensional analogs of the translational zero mode of the 1 + 1-dimensional kink. In the 2 + 1-dimensional domain wall, these infrared singularities lead to a logarithmic sensitivity of the correction to the classical kink profile on the system size. This corresponds to the phenomenon of interfacial roughening in statistical physics, where the same model has been studied in a 3-dimensional Euclidean setting [27]. In 3 + 1 dimensions, the one-loop kink profile at zero temperature turns out to be infrared finite in dimensional regularization, whereas thermal contributions lead to logarithmic infrared singularities, thus exhibiting the phenomenon of interface roughening in the context of a relativistic field theory at finite temperature.

In the case of the φ^4 kink and the corresponding domain walls, we have also calculated the local energy profile and discussed its ambiguities. Depending on the underlying energy-momentum tensor, in particular, the parameter ξ in a possible $-\frac{1}{2}\xi\sqrt{-g}R\varphi^2$ term in the Lagrangian, we generally found both infrared and ultraviolet divergences. In 2 + 1 and 3 + 1 dimensions, the infrared divergences are the same as those found in the field profile, although for the choice $\xi = \frac{1}{2}$ they can be eliminated from the energy profile. Ultraviolet divergences arise in 3 + 1 dimensions whenever $\xi \neq \frac{1}{6}$, corresponding to an unimproved energy-momentum tensor, and these divergences can then be canceled by improvement terms. However, both infrared and ultraviolet divergences drop out in the integrated energy density. Considering the manifestly finite thermal corrections to the energy density we have found that the contributions from the massless modes are crucial in ensuring that the energy density vanishes in the limit $m \rightarrow 0$, i.e. when the domain wall has melted in a second-order phase transition. In the case of the field profile, excluding these massless modes (as done e.g. in Ref. [46]) would have resulted in nonrenormalizable ultraviolet-divergences in place of the (physical) infrared divergences.

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TABLE I. Divergences left after standard renormalization in the field profile $\phi_1(x)$, the local energy density $\epsilon(x)$, and total energy (mass or surface tension) for the various dimensions. ‘‘IR’’ denotes IR divergences which introduce a dependence on system size; ‘‘UV’’ denotes divergences requiring composite operator renormalization through improvement terms for the energy-momentum tensor.

Dimension	Profile $\phi_1(x)$	Energy density $\epsilon(x)$	Total energy M
1 + 1	Finite	Finite	Finite
2 + 1	IR (all T)	IR (all T , $\xi \neq \frac{1}{2}$)	Finite
3 + 1	IR ($T > 0$)	IR ($T > 0$, $\xi \neq \frac{1}{2}$), UV ($\xi \neq \frac{1}{6}$)	Finite

- [1] R. Rajaraman, *Solitons and Instantons* (North-Holland, Amsterdam, 1982); *Solitons and Particles*, edited by C. Rebbi and G. Soliani (World Scientific, Singapore, 1984).
- [2] R. Dashen, B. Hasslacher, and A. Neveu, Phys. Rev. D **10**, 4130 (1974).
- [3] R. F. Dashen, B. Hasslacher, and A. Neveu, Phys. Rev. D **11**, 3424 (1975).
- [4] J. Zinn-Justin, *Quantum Field Theory and Critical Phenomena* (Clarendon, Oxford, 2002).
- [5] T. W. B. Kibble, J. Phys. A **9**, 1387 (1976).
- [6] R. K. Kaul and R. Rajaraman, Phys. Lett. **131B**, 357 (1983).
- [7] C. Imbimbo and S. Mukhi, Nucl. Phys. **B247**, 471 (1984).
- [8] H. Yamagishi, Phys. Lett. **147B**, 425 (1984).
- [9] A. Rebhan and P. van Nieuwenhuizen, Nucl. Phys. **B508**, 449 (1997).
- [10] H. Nastase, M. Stephanov, P. van Nieuwenhuizen, and A. Rebhan, Nucl. Phys. **B542**, 471 (1999).
- [11] N. Graham and R. L. Jaffe, Nucl. Phys. **B544**, 432 (1999).
- [12] M. A. Shifman, A. I. Vainshtein, and M. B. Voloshin, Phys. Rev. D **59**, 045016 (1999).
- [13] A. Rebhan, P. van Nieuwenhuizen, and R. Wimmer, Nucl. Phys. **B648**, 174 (2003).
- [14] A. Rebhan, P. van Nieuwenhuizen, and R. Wimmer, Phys. Lett. B **594**, 234 (2004).
- [15] A. Rebhan, P. van Nieuwenhuizen, and R. Wimmer, J. High Energy Phys. 06 (2006) 056.
- [16] R. K. Kaul, Phys. Lett. **143B**, 427 (1984).
- [17] C. Imbimbo and S. Mukhi, Nucl. Phys. **B249**, 143 (1985).
- [18] N. Seiberg and E. Witten, Nucl. Phys. **B426**, 19 (1994).
- [19] A. S. Goldhaber, A. Rebhan, P. van Nieuwenhuizen, and R. Wimmer, Phys. Rep. **398**, 179 (2004); A. Rebhan, P. van Nieuwenhuizen, and R. Wimmer, arXiv:0902.1904.
- [20] M. Shifman and A. Yung, Rev. Mod. Phys. **79**, 1139 (2007).
- [21] A. S. Goldhaber, A. Litvintsev, and P. van Nieuwenhuizen, Phys. Rev. D **67**, 105021 (2003).
- [22] D. Boyanovsky, F. Cooper, H. J. de Vega, and P. Sodano, Phys. Rev. D **58**, 025007 (1998).
- [23] M. Salle, Phys. Rev. D **69**, 025005 (2004).
- [24] Y. Bergner and L. M. A. Bettencourt, Phys. Rev. D **69**, 045002 (2004).
- [25] C. A. A. de Carvalho, Phys. Rev. D **65**, 065021 (2002); **66**, 049901(E) (2002).
- [26] J. Rudnick and D. Jasnow, Phys. Rev. B **17**, 1351 (1978).
- [27] M. H. Köpf and G. Münster, J. Stat. Phys. **132**, 417 (2008).
- [28] C. Aragao de Carvalho, G. C. Marques, A. J. da Silva, and I. Ventura, Nucl. Phys. **B265**, 45 (1986).
- [29] A. Bessa, C. A. A. de Carvalho, and E. S. Fraga, Phys. Rev. D **69**, 125013 (2004).
- [30] A. Rebhan, P. van Nieuwenhuizen, and R. Wimmer, New J. Phys. **4**, 31 (2002).
- [31] G. Münster, Nucl. Phys. **B324**, 630 (1989).
- [32] P. Arnold, Phys. Rev. D **46**, 2628 (1992).
- [33] P. Arnold and O. Espinosa, Phys. Rev. D **47**, 3546 (1993).
- [34] M. B. Einhorn and D. R. T. Jones, Nucl. Phys. **B398**, 611 (1993).
- [35] G. Bimonte, D. Iniguez, A. Tarancon, and C. L. Ullod, Nucl. Phys. **B490**, 701 (1997).
- [36] M. Dine and W. Fischler, Phys. Lett. B **105**, 207 (1981).
- [37] J. O. Andersen, D. Boer, and H. J. Warringa, Phys. Rev. D **69**, 076006 (2004).
- [38] F. P. Buff, R. A. Lovett, and F. H. Stillinger, Phys. Rev. Lett. **15**, 621 (1965).
- [39] M. Gelfand and M. Fisher, Physica A (Amsterdam) **166**, 1 (1990).
- [40] M. Müller and G. Münster, J. Stat. Phys. **118**, 669 (2005).
- [41] S. Coleman, Phys. Rev. D **11**, 2088 (1975).
- [42] J. L. Gervais and B. Sakita, Phys. Rev. D **11**, 2943 (1975).
- [43] S. R. Coleman, Commun. Math. Phys. **31**, 259 (1973).
- [44] C. G. Bollini and J. J. Giambiagi, Report No. CBPF-NF-085/83.
- [45] A. Parnachev and L. G. Yaffe, Phys. Rev. D **62**, 105034 (2000).
- [46] N. Graham, Phys. Lett. B **529**, 178 (2002).
- [47] A. K. Rebhan, Phys. Rev. D **48**, R3967 (1993).
- [48] H. A. Weldon, Phys. Rev. D **47**, 594 (1993).
- [49] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products* (Academic Press, Orlando, 1980).
- [50] C. G. Callan, Jr., S. Coleman, and R. Jackiw, Ann. Phys. (Leipzig) **59**, 42 (1970).
- [51] D. Z. Freedman, I. J. Muzinich, and E. J. Weinberg, Ann. Phys. (Leipzig) **87**, 95 (1974).
- [52] D. Z. Freedman and E. J. Weinberg, Ann. Phys. (Leipzig) **87**, 354 (1974).
- [53] J. C. Collins, Phys. Rev. D **14**, 1965 (1976).
- [54] L. S. Brown, Ann. Phys. (Leipzig) **126**, 135 (1980).
- [55] A. Rebhan, R. Schöfbeck, P. van Nieuwenhuizen, and R. Wimmer, Phys. Lett. B **632**, 145 (2006).