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Matter bounce in Hořava-Lifshitz cosmology

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Hořava-Lifshitz gravity, a recent proposal for a UV-complete renormalizable gravity theory, may lead to a bouncing cosmology. In this article we argue that Hořava-Lifshitz cosmology may yield a concrete realization of the matter bounce scenario and thus give rise to an alternative to inflation for producing a scale-invariant spectrum of cosmological perturbations. In this scenario, quantum vacuum fluctuations exit the Hubble radius in the prebounce phase, and the spectrum is transformed into a scale-invariant one on super-Hubble scales before the bounce because the long wavelength modes undergo squeezing of their wave functions for a longer period of time than shorter wavelength modes. The scale invariance of the spectrum of curvature fluctuations is preserved during and after the bounce. A distinctive prediction of this scenario is the amplitude and shape of the bispectrum.

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I. INTRODUCTION

Recently, Hořava (based on the pioneering work of [1]) proposed [2,3] a model for quantum gravity which is power-counting renormalizable and hence potentially UV complete. This model does not have the complete diffeomorphism invariance of general relativity, but the action has a fixed point in the IR which is that of general relativity with a negative cosmological constant. In the UV, however, the theory flows to a different fixed point, a fixed point at which space and time scale differently and which has much better UV behavior of perturbation theory. Since Hořava's theory is modeled after a scalar field model studied by Lifshitz [4] in which the full Lorentz symmetry also emerges only at an IR fixed point, the theory is now called Hořava-Lifshitz gravity.

Specific solutions of the simplest version of Hořava-Lifshitz gravity have recently been analyzed. In [5], homogeneous vacuum solutions with gravitational waves were studied. In [6,7], cosmological solutions with matter were explored, and in [8], black hole solutions were analyzed, As pointed out in [6,7], the analogs of the Friedmann equations in Hořava-Lifshitz gravity include a term which scales as dark radiation and contributes a negative term to the energy density. Thus, it is possible in principle to obtain a nonsingular cosmological evolution with the big bang of standard and inflationary cosmology replaced by a bounce.

In [2,7], it was argued that the different ultraviolet behavior of the theory might provide an alternative to cosmological inflation for solving the problems of standard cosmology such as the horizon and flatness problems. Specifically, the divergence of the speed of light in the far ultraviolet leads to the possibility of solving the horizon problem as proposed a while back in [9,10]. In [6] it was emphasized that if the wavelength of fluctuations penetrates the UV region, then the usual arguments for the origin of a scale-invariant spectrum of cosmological perturbations from an inflationary phase might break down, as suggested more generally in the context of the ''trans-Planckian problem'' for inflationary fluctuations [11,12].

However, if Hořava-Lifshitz cosmology leads to a cosmological bounce, then it is not necessary to invoke a period of inflationary expansion to produce the observed spectrum of cosmological perturbations. The purpose of this paper is to point out that Hořava-Lifshitz cosmology may provide a UV-complete realization of the ''matter bounce'' scenario (see [13] for an introduction to this scenario), an alternative to cosmological inflation for explaining the origin of the observed structure in the Universe.

As realized in [14–16], perturbations which start out as quantum vacuum fluctuations and exit the Hubble radius during a matter-dominated phase of contraction acquire a scale-invariant spectrum. Given a nonsingular bouncing background cosmology, the fluctuations can be followed unambiguously through the bounce. If the energy density at which the bounce occurs is smaller than the Planck scale, then the wavelength of the fluctuations which are being probed in today's observations are in the far IR (they are a fraction of a millimeter). Hence, the equations which describe these fluctuations are those of the IR fixed point of the theory, which is the Einstein action in the case of Hořava-Lifshitz cosmology. It has been shown [17–20] that, provided the duration of the bounce phase is short compared to the wavelength of the fluctuations being considered, then the spectrum of curvature fluctuations is not changed during the bounce. Thus, a scale-invariant spectrum of curvature perturbations will persist in the post-bounce expanding phase. Specific predictions of the matter bounce scenario include a specific form of the non-Gaussianities as measured by the amplitude and shape of the bispectrum [21].

In this paper we give an overview of the Hořava-Lifshitz matter bounce cosmology, leaving details for future inves-

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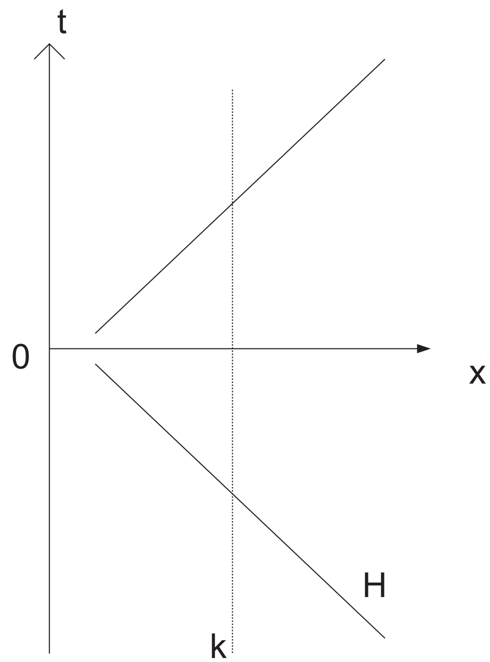

FIG. 1. A space-time sketch of the matter bounce scenario. The vertical axis is time, with $t = 0$ being the bounce time. The horizontal axis denotes comoving distance. The curve with the label H is the Hubble radius H^{-1}; in comoving coordinates, the vertical line labeled by k denotes the comoving wavelength of a fluctuation mode. This mode crosses the Hubble radius in the contracting phase before the time $-t_c$, when the period of matter domination ends.

tigations. We begin with a space-time sketch depicting the relevant phases (Fig. 1). The vertical axis is time, with $t = 0$ denoting the bounce time. In some early phase of contraction, the equation of state of matter is assumed to be dominated by nonrelativistic pressureless matter, in the same sense that our current expanding universe is. For times between $-t_m$ and the bounce, the equation of state can be different from that of pressureless matter. The horizontal axis denotes comoving spatial coordinates. Vertical lines correspond to fixed comoving wavelengths, and the dashed line is the comoving Hubble radius H^{-1}. Fluctuations which cross the Hubble radius during the matter phase of contraction acquire a scale-invariant spectrum, and those which cross later have a nontrivial spectral slope whose magnitude depends on the specific equation of state (see e.g. [22]).

The outline of this article is as follows. We review the action of Hořava-Lifshitz gravity in Sec. II. In Sec. III, we review the equations for cosmological backgrounds and study the possibility of obtaining a bouncing cosmology with a matter-dominated phase of contraction. In Sec. IV we review the evolution of fluctuations in the contracting phase of the matter bounce scenario, and in Sec. V we study how fluctuations pass through the bounce in Hořava-Lifshitz cosmology. Section VI concludes with a discussion of some of the many open issues.

II. REVIEW OF HORíAVA-LIFSHITZ GRAVITY

We begin with a brief review of Hořava-Lifshitz gravity.[1] The dynamical variables are the lapse and shift functions N and N_i, respectively, and the spatial metric g_{ij} (roman letters indicate spatial indices). In terms of these fields, the full metric is

$$ds^2 = -N^2 dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt), \qquad (1)$$

where the indices of N are raised and lowered using the spatial metric g_{ij}.

The scaling symmetry of the coordinates in the simplest version of Hořava-Liftshitz gravity is (we are following the notation of [7])

$$t \longrightarrow l^3 t \quad \text{and} \quad x^i \longrightarrow l x^i. \qquad (2)$$

The full action of this version of Hořava-Lifshitz gravity is

$$S = \int dt d^3x \sqrt{g} N \bigg[\frac{2}{\kappa^2}(K_{ij}K^{ij} - \lambda K^2) - \frac{\kappa^2}{2w^4} C_{ij}C^{ij} + \frac{\kappa^2 \mu}{2w^2} \frac{\epsilon^{ijk}}{\sqrt{g}} R_{il} \nabla_j R^l_k - \frac{\kappa^2 \mu^2}{8} R_{ij}R^{ij} + \frac{\kappa^2 \mu^2}{8(1-3\lambda)} \bigg(\frac{1-4\lambda}{4} R^2 + \Lambda R - 3\Lambda^2 \bigg) \bigg], \qquad (3)$$

where

$$K_{ij} = \frac{1}{2N}(\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i), \qquad (4)$$

and C_{ij} is the Cotton tensor

$$C^{ij} = \frac{\epsilon^{ijk}}{\sqrt{g}} \nabla_k \bigg(R^j_i - \frac{1}{4} R \delta^j_i \bigg). \qquad (5)$$

The tensor ϵ^{ijk} is the totally antisymmetric unit tensor, λ is a dimensionless constant, and Λ is related to the cosmological constant in the IR limit. The variables κ, w, and μ are constants with mass dimensions -1, 0, and 1, respectively.

In the IR limit, the action reduces to

$$S_E = \int dt d^3x \sqrt{g} N[\alpha(K_{ij}K^{ij} - \lambda K^2) + \xi R + \sigma], \qquad (6)$$

with

$$\alpha = \frac{2}{\kappa^2}, \qquad (7)$$

$$\xi = \frac{\kappa^2 \mu^2}{8(1-3\lambda)} \Lambda, \qquad (8)$$

and

[1]This section is based completely on the analysis of [7].

$$\sigma = -3\frac{\kappa^2\mu^2}{8(1-3\lambda)}\Lambda^2. \tag{9}$$

In order to obtain the Einstein action, we require $\lambda = 1$. In this case, the variables of the Hořava-Lifshitz action go over into the following expressions for the speed of light c, Newton's gravitational constant G, and the effective cosmological constant Λ_E:

$$c = \sqrt{\frac{\xi}{\alpha}}, \tag{10}$$

$$16\pi G = \sqrt{\frac{\xi}{\alpha^3}}, \tag{11}$$

$$\Lambda_E = -\frac{\sigma}{2\alpha}. \tag{12}$$

In the following we will consider scalar field matter in the contracting phase. The action for matter is

$$S_M = \int dt d^3x\sqrt{g}N\mathcal{L}_m, \tag{13}$$

where the matter Lagrangian $\mathcal{L}_m$ depends on the scalar matter field φ and the metric. In the IR limit, this action reduces to the usual scalar field matter action in curved space-time. The form of the scalar field Lagrangian, valid also in the UV, is given in [7] but will not be used in this paper.

III. MATTER BOUNCE IN HORíAVA-LIFSHITZ COSMOLOGY

To obtain the equations for Hořava-Lifshitz cosmology we assume that the metric is homogeneous and isotropic, i.e.

$$N = N(t), \qquad N_i = 0, \quad \text{and} \quad g_{ij} = a^2(t)\gamma_{ij}, \tag{14}$$

where γ_{ij} is a maximally symmetric constant curvature metric. We will denote the spatial curvature parameter by $\bar{k}$.

The equations of motion are obtained by varying the action with respect to N, a, and φ, and setting $N = 1$ at the end of the calculation. The resulting equations are

$$H^2 = -\frac{\bar{k}}{a^2} - \frac{\Lambda_E}{3} - \frac{2\bar{k}^2(\zeta + 3\eta)}{\alpha a^4} + \frac{\rho}{6\alpha}, \tag{15}$$

$$\left[\dot{H} + \frac{3}{2}H^2\right] = -\frac{\bar{k}}{2a^2} - \frac{\Lambda_E}{2} + \frac{\bar{k}^2(\zeta + 3\eta)}{\alpha a^4} - \frac{p}{4\alpha}, \tag{16}$$

and

$$\ddot{\varphi} + 3H\dot{\varphi} + V' = 0, \tag{17}$$

where $H = \dot{a}/a$, p, and ρ are the pressure and energy density of the scalar matter field, respectively; a prime denotes the derivative with respect to φ, the dimensionless constant η

$$\eta = \frac{\kappa^2\mu^2(1-4\lambda)}{32(1-3\lambda)} \tag{18}$$

is the coefficient of the R^2 term in the gravitational action, and the dimensionless constant ζ is given by

$$\zeta = -\frac{\kappa^2\mu^2}{8}. \tag{19}$$

The key new term in the cosmological equations of motion is the second to last term on the right-hand sides of (15) and (16). This term corresponds to "dark radiation" with a negative energy density. This term is present only if the spatial curvature of the metric is nonvanishing. If the energy density of regular matter increases less fast than a^{-4} as the scale factor decreases, the dynamics will lead to a cosmological bounce provided that[2]

$$\left(\frac{\rho}{4} - p\right) > 0. \tag{20}$$

To obtain a bounce in Hořava-Lifshitz cosmology we will assume that matter in the prebounce epoch is described by a scalar field φ with a potential

$$V(\varphi) = \tfrac{1}{2}m^2\varphi^2. \tag{21}$$

As studied in detail in quintom bounce [19,20,23] scenarios, we take the scalar field to be oscillating during the contracting phase, with an amplitude $\mathcal{A}(t)$.

As the Universe contracts, the amplitude $\mathcal{A}(t) \sim a(t)^{-3/2}$ will increase. Once the amplitude reaches the value

$$\mathcal{A}_{\text{crit}} = (12\pi)^{-1/2}m_{\text{pl}}, \tag{22}$$

where m_{pl} is the Planck mass, then the field oscillations will stop and φ will enter a "slow-climb" phase, the time reversal of the inflationary slow-roll phase. During this phase, the matter energy density is approximately constant but the scale factor is rapidly decreasing. Hence, the dark radiation term in the Hubble equation rapidly catches up with the matter energy term. Since in the slow-climb phase the pressure of matter is negative, the condition (20) is satisfied. Thus, we obtain a cosmological bounce.

Note that the slow-climb phase is unstable with respect to the presence of the second mode in the scalar field equation of motion, a mode which is exponentially decaying in an inflationary slow-roll phase and thus ensures that the slow-roll trajectory in large-field inflation is a local attractor [24]. In the slow-climb phase, the second mode is increasing. Thus, the slow-climb trajectory is a repeller.[3] However, coming out of the oscillatory phase, the initial

[2] The contributions scaling like curvature or a cosmological constant can be neglected.

[3] We thank Misao Sasaki and Takahiro Tanaka for discussions on this point.

amplitude of the unstable mode is sufficiently small such that the instability does not have time to develop before the bounce takes place.

IV. FLUCTUATIONS IN THE MATTER BOUNCE SCENARIO

In the following we assume that the contracting phase before the bounce was first dominated by cold matter, matter with an equation of state $w = 0$, where w is the ratio of pressure p divided by energy density ρ. We will now review how an initial vacuum spectrum of cosmological perturbations on sub-Hubble scales in the contracting phase develops into a scale-invariant spectrum for wavelengths which exit the Hubble radius in the matter-dominated phase.

As has proven to be convenient in inflationary cosmology, we track the cosmological fluctuations in terms of the variable $\mathcal{R}$, the curvature fluctuations in comoving coordinates [25–28]. This variable is conserved at phase transitions and is constant on super-Hubble scales in an expanding universe.

If we work in longitudinal gauge in which the metric in the absence of anisotropic stress takes the form

$$ds^2 = a^2(\eta)[(1 + 2\Phi)d\eta^2 - (1 - 2\Phi)d\mathbf{x}^2], \tag{23}$$

where η is conformal time, $\mathbf{x}$ are comoving spatial coordinates, and $\Phi(\mathbf{x}, \eta)$ describes the metric fluctuations, then $\mathcal{R}$ is given by (modulo terms which are suppressed on super-Hubble scales)

$$\mathcal{R} = \frac{2}{3}(\mathcal{H}\Phi' + \Phi)\frac{1}{1 + w} + \Phi, \tag{24}$$

$\mathcal{H}$ denoting the Hubble expansion rate in conformal time and a prime indicating the derivative with respect to η.

The variable $\mathcal{R}$ is closely related to the variable v (see [29] for an in-depth review of the theory of cosmological fluctuations and [30] for an introductory overview) in terms of which the action for cosmological fluctuations has the canonical kinetic term:

$$\mathcal{R} = \frac{v}{z}, \tag{25}$$

where z is a function of the background which is proportional to the scale factor a as long as the equation of state of matter is constant.

The equation of motion for the Fourier mode v_k of v is

$$v_k'' + \left(k^2 - \frac{z''}{z}\right)v_k = 0. \tag{26}$$

This shows that on length scales larger than the Hubble radius, where the k^2 term is negligible, v does not oscillate, its time evolution being determined by the gravitational background, whereas on sub-Hubble scales v_k is oscillating with approximately constant amplitude.

On super-Hubble scales, the equation of motion for v_k in a universe which is undergoing matter-dominated contraction is

$$v_k'' = 2\eta^{-2}v_k, \tag{27}$$

which has the general solution

$$v_k(\eta) = c_1\eta^{-1} + c_2\eta^2, \tag{28}$$

where c_1 and c_2 are constants. Since for a matter-dominated phase

$$a(\eta) \sim \eta^2, \tag{29}$$

it follows that the c_2 mode is the mode for which $\mathcal{R}$ is constant on super-Hubble scales, whereas for the c_1 mode $\mathcal{R}$ scales as η^{-3}, which is decaying in an expanding universe but growing in a contracting phase. It is this growth which is responsible for turning an initial vacuum spectrum of fluctuations into a scale-invariant one.

To see this, let us compute the power spectrum of $\mathcal{R}$ on super-Hubble scales late in the contracting phase:

$$\begin{aligned} P_{\mathcal{R}}(k, \eta) &\sim k^3|v_k(\eta)|^2a^{-2}(\eta) \\ &\sim k^3|v_k(\eta_H(k))|^2\left(\frac{\eta_H(k)}{\eta}\right)^2 \sim k^{3-1-2} \\ &\sim \text{const.} \end{aligned} \tag{30}$$

In this first step, we have used the definition of the power spectrum, replaced $\mathcal{R}$ by v via (25) and used the scaling $z(\eta) \sim a(\eta)$. In the second step, we made use of the growth of the c_1 mode, the dominant mode, to relate the value of v at late times to its value at the time $\eta_H(k)$ when the mode k crosses the Hubble radius. Finally, in the last step we insert the vacuum spectrum for v on sub-Hubble scales and the Hubble radius crossing condition $\eta_H(k) \sim k^{-1}$.

V. EVOLVING FLUCTUATIONS THROUGH THE BOUNCE

If the bounce phase is short compared to the wavelength of the fluctuations which are being followed, then the spectrum of $\mathcal{R}$ is unchanged through the bounce. This result can be obtained by explicitly evolving fluctuations through a nonsingular bounce using the equations of motion for fluctuations which follow from Einstein's theory (see e.g. [17–20]). This result also agrees with what is obtained by replacing the bounce phase by a matching surface and making use of the Hwang-Vishniac [31] (Deruelle-Mukhanov [32]) matching conditions.

If the energy density at the bounce is of the order of $(10^{16}\ \text{GeV})^4$, the wavelength of a mode which corresponds to the current Hubble radius is about 1 mm, i.e. in the far IR. Since in the Hořava-Lifshitz bounce, the bounce time is set by the UV scale, it is well justified to assume that in the context of the use of the Einstein equations for the gravitational fluctuations the spectrum of $\mathcal{R}$ does not change across the bounce.

In addition, again since the scales we are interested in are in the far IR, it should be justified to use the IR limit of Hořava-Lifshitz cosmology to propagate the fluctuations.[4]

Thus, we argue that the scale invariance of the spectrum of cosmological perturbations will be preserved after the bounce. Since $\mathcal{R}$ is constant on super-Hubble scales in the postbounce expanding phase, it then immediately follows that the spectrum of fluctuations at late times will be scale invariant.

A bouncing cosmology in the context of Hořava-Lifshitz gravity can thus provide an alternative to inflation for providing a scale-invariant spectrum of cosmological perturbations, provided that we begin in the contracting phase with quantum vacuum fluctuations and provided that the relevant scales exit the Hubble radius in a period of cold matter domination (the case of initial thermal fluctuations is analyzed in [35]).

Since the curvature perturbation $\mathcal{R}$ grows on super-Hubble scales, a matter bounce leads to a larger amplitude of non-Gaussianities than slow-roll single-field inflation. Since it is a different mode of $\mathcal{R}$ which dominates, the shape of the non-Gaussianities is also different from what is obtained in slow-roll single-field inflation models. The specific predictions for the amplitude and shape of the three-point function (the "bispectrum") were worked out in [21]. In particular, the predicted amplitude of the bispectrum is very close to the level which could be detected using the Planck satellite experiment.

VI. CONCLUSIONS AND DISCUSSION

In this paper we have shown how to obtain a matter bounce in Hořava-Lifshitz gravity. Such a bounce is obtained because of a dark radiation term which appears in the equations of motion for cosmological solutions, a term which stems from the terms in the quantum gravity theory which appear in the UV and help render the theory renormalizable. Note, however, that the presence of the dark radiation term requires nonvanishing spatial curvature.

To obtain a cosmological bounce it is important that no source of matter is present which redshifts equally fast or faster than that of the dark radiation term. Only in this case, can the energy density of dark radiation grow with respect to the regular matter energy, a condition which is required to obtain a bounce. This condition appears to be rather restrictive since it even rules out regular radiation before the bounce.

We have presented a model in which a bounce can be obtained. In this model, matter is modeled by a scalar field with a standard mass term. The scalar field oscillates at early times in the contracting phase, leading to a matter equation of state which is that of cold matter. Once the amplitude of scalar field oscillations reaches a critical value, the field enters a deflationary slow-climb phase during which its energy density is approximately constant and its pressure is negative. Hence, a bounce occurs.

In the matter bounce model thus constructed, initial quantum vacuum fluctuations which exit the Hubble radius in the contracting matter-dominated phase acquire a scale-invariant spectrum of curvature fluctuations, as already envisioned in [14–16] and recently studied in detail in [20]. Thus, one of the main messages of this article is that it is not necessary to force a period of inflationary expansion into Hořava-Lifshitz cosmology. The alternative matter bounce scenario predicts an amplitude of the normalized bispectrum which is the order of 1, and a specific shape of this three-point function, as studied in detail in [21]. These specific predictions are potentially within the reach of upcoming cosmic microwave background missions such as PLANCK.

To obtain a successful late-time cosmology, the model presented here must be supplemented with a mechanism to transfer the energy at late times to standard model matter and radiation. If we include such matter in the basic Lagrangian, then an initial condition problem arises: in order for a bounce to occur, the initial energy density of radiation must be so low that the radiation never comes to dominate during the contracting phase. All of these issues deserve further study.

While this article was being prepared for submission, a very interesting paper by Mukohyama [36] appeared showing that in the UV region, fluctuations of a scalar field in Hořava-Lifshitz gravity acquire a scale-invariant spectrum, a spectrum which can be later transformed to curvature fluctuations via a transfer from entropy to adiabatic modes such as the curvaton mechanism. Thus, one can obtain a scale-invariant spectrum of cosmological fluctuations. An advantage of this mechanism is that it also operates in a background cosmology without a bounce (e.g. in the case of zero spatial curvature). However, in the case of a matter bounce background, it is unclear whether the scaling of the correlation functions used in [36] would extend to the large IR scales required to match with observations, the scales on which our mechanism works nicely.

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[4]This claim should be justified with an explicit calculation in the same way that the corresponding claim was justified [33] in the bouncing scenario obtained by using the special higher derivative gravitational action of [34] which is ghost free about Minkowski space-time.

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