

**Confined but chirally symmetric hadrons at large density and the Casher's argument**

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The Casher argument, which is believed to be quite general, states that in the confining regime chiral symmetry is necessarily broken. In the large- $N_c$  limit and at moderate and low temperatures, QCD is confining up to arbitrary large densities, and there should appear a quarkyonic matter. It has been demonstrated, within a manifestly confining and chirally symmetric model, which is a  $3 + 1$  dimensional generalization of the 't Hooft model, that, at zero temperature and at a density exceeding a critical one, the chiral symmetry is restored while quarks remain confined in color-singlet hadrons. This is in conflict with the Casher argument. Here, we explain the reason why the Casher argument fails and clarify the physical mechanism lying behind such confined but chirally symmetric hadrons.

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**I. INTRODUCTION**

A famous Casher argument [1] states that, in a confining domain, chiral symmetry should be necessarily broken in hadrons. The argument is simple, transparent, and relies on the constraints implied by requirements of confinement of quarks. It is believed to be rather general. By contrast, the 't Hooft anomaly matching conditions [2,3] state that, at zero temperatures and densities, confinement implies necessarily chiral symmetry breaking in the vacuum. These conditions look rather formal and do not suggest any physical picture that could lie behind such constraints. These two generic arguments, supplemented by various models, constituted the basis for the belief that the QCD phase diagram should contain two general phases: one with both confinement and broken chiral symmetry (hadronic phase) and the other one, at larger temperatures and/or densities, without confinement and with restored chiral symmetry (quark-gluon matter). Quite recently McLerran and Pisarski suggested the existence of another state of the matter—quarkyonic phase [4]. Their crucial observation is that, in the large- $N_c$  limit and at low and moderate temperatures, confinement in QCD survives up to arbitrarily high densities. Indeed, if the large- $N_c$  limit is taken first, then there are no dynamical quark loops and hence nothing screens the confining gluon propagator, whatever nature this propagator can be. Then the Wilson and Polyakov loop criteria of confinement for a pure gauge theory survive in this case. Therefore, in the large- $N_c$  dense matter confinement takes place exactly in the same way as in the vacuum simply because there is no screening of the linear confining potential between the static quark sources in the fundamental representation. They have also suggested that, since chiral symmetry is expected to be restored at some critical density, then there could appear a chirally symmetric but confined *subphase* within the quarkyonic matter. Existence of such a subphase would mean that, while deep in the quark Fermi sea, quark language is adequate, near the Fermi surface, confinement necessarily groups quarks

into color-singlet hadrons with the restored chiral symmetry. Then the only allowed excitation modes in this phase are confined but chirally symmetric hadrons. However, no microscopic mechanism of this phenomenon was suggested.

Shortly after this, it was shown [5,6], within a manifestly confining and chirally symmetric solvable model, that this was indeed possible. The following mechanism for the confining but chirally symmetric matter at large densities was observed. Indeed, if one assumes an instantaneous Coulomb-like confining interaction between quarks (which is seen in Coulomb-gauge studies of QCD [7] and in Coulomb-gauge lattice QCD simulations [8]) then a quark Green function, that is a solution of the gap equation, acquires not only a chiral symmetry breaking Lorentz-scalar part, but also a Lorentz-vector part, which preserves chiral symmetry. Both these parts are infrared-divergent, which guarantees that the quark is confined. In color-singlet hadrons, the infrared divergence cancels exactly, so the color-singlet hadrons are finite and well-defined quantities. At low temperatures and rather large densities, chiral symmetry is restored due to the Pauli blocking of the quark levels required for the existence of the quark condensate. This means that the Lorentz-scalar part of the quark Green function vanishes. Meanwhile, the Lorentz-vector part of the quark Green function is still there and is infrared divergent. Hence a single quark does not exist. At the same time, as was mentioned before, this infrared divergence cancels exactly in color-singlet hadrons, so that these manifestly chirally symmetric hadrons form exact chiral multiplets. The masses of such hadrons are generated only through chirally symmetric dynamics.

The chirally symmetric quarkyonic matter was also studied within the Polyakov-Nambu-Jona-Lasinio model [9–11]. This model is nonconfining, however, and the problem of the confined but chirally symmetric hadrons (excitations) cannot be formulated in its framework.

A natural question arises. Existence of such hadrons is in conflict with the Casher argument. What is wrong? Here

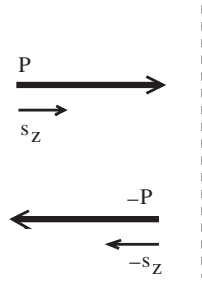


FIG. 1. Right-handed quark before and after turning point.

we demonstrate that the Casher argument is not general enough and in reality it does not preclude existence of confined but chirally symmetric hadrons at large density.

## II. THE CASHER ARGUMENT

Suppose we have a quark with a 3-momentum  $\vec{p}$  moving along the  $z$ -axis. Its helicity (chirality) is fixed. Let us choose, for simplicity, its spin to be parallel to the quark momentum  $\vec{p}$ —see Fig. 1. Confinement means that, at some point, this quark must turn back and start moving right in opposite direction. If chiral symmetry is unbroken, then the quark helicity (chirality) is conserved. Hence, at the turning point, the quark spin has to be flipped,  $\Delta S_z = -1$ . Since the angular momentum is conserved, then this spin flip must be compensated somehow. The only object which could be responsible for this spin compensation is the QCD string. This string does not have  $L_z$  and thus cannot support conservation of the total angular momentum. This implies that, if chiral symmetry is unbroken, the quark never turns, i.e., there is no confinement. The only possibility to turn the quark back and, at the same time, not to violate the angular momentum conservation is to keep the spin direction fixed. This requires the quark helicity (chirality) to be changed from  $+1$  before the turning point to  $-1$  after the turning point. Therefore, at the turning point, there must appear a chiral symmetry breaking term in the quark Green function. In other words, confinement of quarks requires dynamical breaking of chiral symmetry. Essentially the same picture takes place in the bag model [12].

## III. CONFINED BUT CHIRALLY SYMMETRIC HADRONS AT HIGH DENSITY

First, let us overview briefly the essentials of the model [5,6]. A global chiral symmetry of this large- $N_c$  model is  $U(2)_L \times U(2)_R$ . We assume that there is a linear Coulomb-like instantaneous Lorentz-vector potential between quarks. Hence this model can be considered as a straightforward generalization of the  $1 + 1$  dimensional 't Hooft model [13], that is QCD in the large- $N_c$  limit in two dimensions. The 't Hooft model is exactly solvable. Under an appropriate choice of the gauge, the only interaction between quarks is an instantaneous Coulomb poten-

tial, that is a linear Lorentz-vector confining potential in two dimensions. Instantaneous Lorentz-vector Coulomb or Coulomb-like interaction between fermions is one of the most important elements of both QED and QCD in the Coulomb gauge. Of course, in four dimensions, gluodynamics is much richer, so that solving QCD with full gluodynamics, even in the large- $N_c$ , looks hopeless. It is postulated within the model that there exists an instantaneous Coulomb-like confining potential, like that in the 't Hooft model in  $1 + 1$  dimensions, which is seen in lattice simulations in four dimensions, indeed. Clearly, such a model represents a certain simplification of real QCD because gluonic interactions beyond the Coulomb-like part are neglected. Nevertheless, such a model contains all principal elements of QCD, such as confinement of quarks, dynamical breaking of chiral symmetry, Goldstone bosons, etc. Hence, it can be used as a toy model related to some aspects of confinement and chiral symmetry breaking.

The problem of chiral symmetry breaking within this model was addressed long ago [14–18]. It actually reduces to solving the gap (Schwinger-Dyson) equation in the rainbow approximation, which is exact in the large- $N_c$  limit.

The Fourier transform of the linear potential and loop integrals are infrared-divergent. Hence an infrared regularization is required. Any physical observables, such as hadron masses, etc., must be independent of the infrared regulator  $\mu_{\text{IR}}$  in the infrared limit (i.e., when  $\mu_{\text{IR}} \rightarrow 0$ ).

If the quark self-energy operator is parametrized in the form

$$\Sigma(\vec{p}) = A_p + (\vec{\gamma} \cdot \hat{p})[B_p - p], \quad (1)$$

where the functions  $A_p$  and  $B_p$  are to be found, then, for an instantaneous interaction, the Schwinger-Dyson equation for the self-energy operator (see Fig. 2) reduces to a nonlinear gap equation for the chiral (Bogoliubov) angle  $\varphi_p$ ,

$$A_p \cos \varphi_p - B_p \sin \varphi_p = 0, \quad (2)$$

where

$$A_p = \frac{1}{2} \int \frac{d^3 k}{(2\pi)^3} V(\vec{p} - \vec{k}) \sin \varphi_k, \quad (3)$$

$$B_p = p + \frac{1}{2} \int \frac{d^3 k}{(2\pi)^3} (\hat{p} \cdot \hat{k}) V(\vec{p} - \vec{k}) \cos \varphi_k. \quad (4)$$

The functions  $A_p$  and  $B_p$ , i.e., the quark self-energy, are singular. However, in the gap equation (2), these singularities cancel against each other exactly. In Refs. [5,6,19–22], the infrared regularization of the linear potential is chosen in such a way that both functions  $A_p$  and  $B_p$  as well as the linear potential contain divergent contributions  $1/\mu_{\text{IR}}$ . This guarantees that a single quark cannot be observed and is therefore confined. There exist other regularization prescriptions that lead to the same result for the

$$S = S_0 + S_0 \Sigma S_0 + S_0 \Sigma S_0 \Sigma S_0 + \dots = S_0 + S_0 \Sigma S$$

$$\Sigma = S_0 + S_0 + \dots = S$$

FIG. 2. Dressed quark Green function and Schwinger-Dyson equation.

color-singlet observables, and physics, of course, does not depend on a particular regularization scheme.

The chiral symmetry breaking is signaled by a nontrivial solution for the chiral angle, nonzero quark condensate, and by the dynamical momentum-dependent “mass” of quarks

$$\langle \bar{q}q \rangle = -\frac{N_C}{\pi^2} \int_0^\infty dp p^2 \sin \varphi_p, \quad M(p) = p \tan \varphi_p. \quad (5)$$

The dynamical mass is finite at small momenta and vanishes at large momenta.

Given a dressed quark Green function, the homogeneous Bethe-Salpeter equation for a quark-antiquark bound state in the rest frame with the instantaneous interaction can be written in the ladder approximation, which is exact in the large- $N_c$  limit (see Fig. 3):

$$\chi(m, \vec{p}) = -i \int \frac{d^4 q}{(2\pi)^4} V(|\vec{p} - \vec{q}|) \gamma_0 S(q_0 + m/2, \vec{p} - \vec{q}) \times \chi(m, \vec{q}) S(q_0 - m/2, \vec{p} - \vec{q}) \gamma_0. \quad (6)$$

Here  $m$  is the meson mass and  $\vec{p}$  is the relative momentum. The infrared divergence cancels exactly in this equation and it can be solved either in the infrared limit or for very small values of the infrared regulator [6,21,22]. Consequently meson masses are well-defined, finite quantities. The spectrum exhibits a fast effective chiral restoration in excited mesons at  $J \rightarrow \infty$ —for a review see Ref. [23].

In a dense matter and at low temperatures we assume a quark Fermi surface with a Fermi momentum  $p_f$ . Hence, one is to remove from the integration both in the Schwinger-Dyson (gap) and Bethe-Salpeter equations all intermediate quark momenta below  $p_f$  since they are

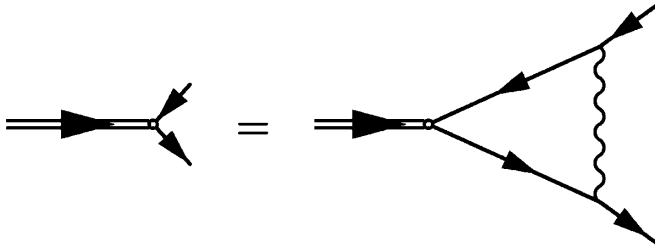


FIG. 3. Homogeneous Bethe-Salpeter equation for the quark-antiquark bound states.

Pauli-blocked. The modified gap equation is then the same as in (2)–(4), but the integration starts not from  $k = 0$ , but from  $k = p_f$ . Similarly, the integration in  $q$  in the Bethe-Salpeter equation also starts from  $q = p_f$ .

At a critical value  $p_f^{\text{cf}}$ , the gap equation exhibits a chiral restoration phase transition [5,24]. Hence chiral symmetry gets restored, so that  $\varphi_p = 0$ . The quark condensate and the dynamical quark mass vanish as well,  $\langle \bar{q}q \rangle = 0$ ,  $M(q) = 0$ , as it follows from (5). At  $\varphi_k = 0$ , the Lorentz-scalar self-energy of quarks vanishes identically,  $A_p = 0$ . The Lorentz spatial-vector self-energy integral  $B_p$  does not vanish at  $\varphi_k = 0$ , however, and remains in fact infrared-divergent. Hence, a single quark is confined at any chemical potential. As a matter of fact, all color-nonsinglet objects are infrared divergent and, hence, are confined. Within the color-singlet hadrons or, in general, in a matter, the infrared divergence is canceled exactly [6]. The only allowed (infrared-finite) excitations are color-singlet hadrons. The spectrum represents a complete set of exact chiral multiplets [5]. Masses of these excitations are manifestly chirally symmetric and come from the manifestly chirally-symmetric dynamics.

#### IV. WHY THE CASHER ARGUMENT DOES NOT EXCLUDE EXISTENCE OF CHIRALLY SYMMETRIC HADRONS AT LARGE DENSITY

The spectrum of the color-singlet hadrons (excitations) at densities above the chiral restoration phase transition, obtained in Ref. [5], is manifestly chirally symmetric. This is certainly in conflict with the Casher qualitative argument. Then it is important to clarify where the Casher argument fails in the present situation.

In this model, as well as in the 't Hooft model, a motion of a quark and an antiquark within a meson is highly synchronous. This is because the interaction is instantaneous (see Fig. 4). When the quark scatters off the confin-

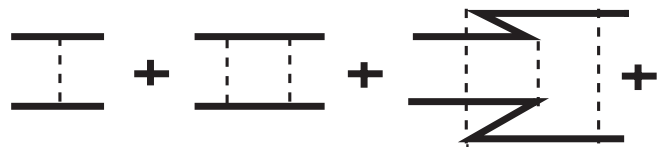


FIG. 4. Synchronous motion of a quark and an antiquark in a meson.

ing potential, the same happens simultaneously with the antiquark.

Consider, as an example, the motion of a quark and an antiquark in a spin-zero meson in a chirally restored regime. At the quark turning point, chiral symmetry requires a quark spin flip,  $\Delta S_z = -1$ . The same turning undergoes the antiquark and it happens simultaneously. The quark and the antiquark interact to each other at the turning point via the chirally symmetric Coulomb-like instantaneous interaction. In the meson rest frame, the momenta of the quark and the antiquark are just opposite, so that the flip of the antiquark spin is necessarily  $\Delta S_z = +1$ . Consequently, the total angular momentum in the quark-antiquark system is conserved, because spin flips of the quark and the antiquark mutually cancel. This analysis for the  $J = 0$  meson can be extended straightforwardly to mesons with arbitrary  $J$ 's. Then it makes it clear why the Bethe-Salpeter equation admits solutions in any nonexotic channel with fixed quantum numbers  $J^{PC}$  even when the quark Green function does not contain the chiral symmetry breaking self-energy part  $A_p$ , as it happens in a dense matter above the chiral restoration transition. At the same time a single quark is removed from the spectrum, because the chirally symmetric part of its self-energy,  $B_p$ , is always infrared divergent.

This simple picture demonstrates explicitly that the Casher argument is not general enough to forbid the existence of confined but chirally symmetric hadrons at large densities. A physical picture outlined above has obvious limitations. It relies on the Coulomb gauge where the presence of an instantaneous interaction is guaranteed. It remains a puzzle how this physical mechanism looks in other gauges. In addition, the Coulomb gauge is not covariant, so physics in a moving frame should look differently. However, we do know from the 't Hooft model that, while all "intermediate" results are manifestly gauge-dependent and look differently in different gauges, the final results for color-singlet quantities are gauge and Lorentz invariant. What mechanism will take place for the chirally symmetric quarkyonic matter in QCD within an alternative gauge remains to be seen. (Lorentz invariance is manifestly broken in a medium, however.)

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