PHYSICAL REVIEW D **80,** 034029 (2009)

Study of $\pi\pi$ phase shifts in $\psi(2S) \to \pi^+\pi^- J/\psi$

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(Received 20 April 2009; published 27 August 2009)

The $\pi\pi$ S-wave phase shifts below 0.6 GeV are extracted out from the published data of the decay $\psi(2S) \to \pi^+\pi^- J/\psi$. In fitting to the $m_{\pi\pi}$ distribution, several $\pi\pi$ production mechanisms are modeled in the amplitude. The fit results show that the amplitude including the $\pi\pi$ rescattering process with the minimal coupling $g_1\epsilon' \cdot \epsilon^*$ plus the direct nonresonant three-body decay process yields phase shifts consistent with those measured in scattering experiments and K_{e4} decays. This result agrees with the expectation of the Watson theorem.

DOI: 10.1103/PhysRevD.80.034029 PACS numbers: 13.20.Fc, 11.80.−m, 13.25.−k

I. INTRODUCTION

The $\pi\pi$ S-wave phase shifts below 1 GeV have well been established in experiment. A classical experiment to determine the $\pi\pi$ scattering amplitude was carried out by the CERN-Munich and CERN-Cracow-Munich Collaborations in the 1970s. From then on, the analysis of the CERN-Munich and CERN-Cracow-Munich data has been widely accepted as a standard of the phase shift of $\pi\pi$ scattering. A recent reanalysis of the CERN-Munich data yielded the $\pi\pi$ S-wave amplitude and phase in a $\pi\pi$ mass range from 0.6 to 1.6 GeV [1]. Many more experiments and analyses have been carried out to deduce the phase shift of the isoscalar scalar wave (e.g., see [1–3]). From the threshold of two pions to 0.6 GeV range, the most precise data came from $K^+ \to \pi\pi e^+\nu_e$ (K_{e4}) decays [4,5]. The general $\pi\pi$ phase shifts are observed as rising slowly with increasing mass $m_{\pi\pi}$ from the threshold to ~1 GeV, then increasing rapidly (from ~90° to ~240°) due to $f_0(980)$.

Theoretically, this behavior of $\pi\pi$ S-wave phase shifts below the first inelastic threshold is precisely known in the context of the chiral unitarity theory. Many analyses on the K_{e4} decays [5] and the π production experiments [6–8] are performed by implementing the chiral symmetry, analyticity and crossing symmetry (e.g., Roy's Equation), and the $\pi\pi$ S-wave phase shifts are well reproduced below 0.8 GeV. The general results from unitarity suggest that the phase of the $\pi\pi$ interaction below the first inelastic threshold in the weak or the electromagnetic production processes should be equivalent as in the elastic $\pi\pi$ scattering experiment. This agrees with the expectation of the famous Watson's theorem [9], which states that the $\pi\pi$ phase shifts in final-state interactions and the scattering are identical.

For the strong production modes, rescattering between final-state particles may lead to additional phase shift before $\pi\pi$ scattering happens, and thus make Watson's theorem inapplicable. But in most cases, it is believed that Watson's theorem works to a good extent, such as in the $J/\psi \to \omega\pi\pi$ decay [10]. Other examples are the recent analyses of the $\pi\pi$ S-wave phase shift extracted from D^+, $D_s^+ \to \pi^-\pi^+\pi^+$ decays by the E791 [11], FOCUS [12], and *BABAR* [13] Collaborations. These measurements turned out that the $\pi\pi$ S-wave phase shifts are basically consistent with each other within statistical errors. If one fixes the phase shift to 180° at the position of $f_0(980)$, the phase shifts below 1 GeV are compatible with the scattering data.

In analyses of the $m_{\pi\pi}$ spectrum in strong decays of $J/\psi \to \omega\pi^+\pi^-$ [14] and $\psi(2S) \to \pi^+\pi^- J/\psi$ by the BES Collaboration [15], it was found that it is necessary to include the σ particle to fit the $m_{\pi\pi}$ distribution. If the scalar meson σ is parametrized by the Breit-Wigner form in these two channels, the fits yield consistent results on the mass and decay width; the pole is quoted as $(541 \sim 552) - i(252 \sim 232)$ MeV. If there exists the σ particle having large coupling with the $\pi\pi$ channel, it must be seen in both of the scattering and production experiments. From the viewpoint of the $\pi\pi$ S-wave phase shift, they should be identical in both of the strong production and the scattering experiments if Watson's theorem is applicable. But until now the phase-shift analyses of these decays have not been available.

Study of the $\pi\pi$ S-wave phase shift in $\psi(2S) \to \pi^+\pi^- J/\psi$ is helpful to disentangle strong interactions from $\pi\pi$ final-state interactions (FSIs). In this decay, the FSIs get more simple, where the $\pi\pi$ rescattering is expected to dominate FSIs. Since the pions and the J/ψ have the different quark components, the rescattering effects induced by exchanging light hadrons between them are negligible, and the J/ψ can be regarded as a spectator in the FSIs. Hence the phase shift of the amplitude is due to the $\pi\pi$ FSI. In this case, the Watson theorem is applicable, and the $\pi\pi$ phase shifts extracted are expected to be comparable with those obtained in the scattering experiments, and thus will shed light on the strong production mechanism of scalar meson σ in $\psi(2S) \to \pi^+\pi^- J/\psi$ decays.

In Sec. II, the amplitudes by modeling the $\psi(2S)$ strong decay mechanism plus the $\pi\pi$ S-wave rescattering are

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formulated. The fit results of the extracted phase shift of the $\pi\pi$ S-wave are given in Sec. III. The comparison of these results with other measurements and Roy's Equation result are presented in the last section, together with some remarks about this result.

II. FORMALISM

A. $\pi\pi$ phase shift

Before we go ahead to construct the amplitude for the production process, we recapitulate some points associated with the phase shift.

The amplitude T describing the elastic process $\pi\pi \to \pi\pi$ must respect unitarity. In the spirit of the Bethe-Salpeter equation, the dynamical content of a unitary elastic scattering amplitude can be realized as a series involving $\pi\pi$ scattering kernels and propagators. This idea is represented in Fig. 1, where the iteration of the kernel K by means of a two-body mesonic Green's function yields the full amplitude T. The amplitude T can be parametrized in terms of the phase shift $\delta(s)$ as

$$T(s) = \frac{\sin\delta(s)}{\rho(s)} \exp[i\delta(s)], \qquad (1)$$

where s is the energy squared of two pions in their center-mass-system (CMS), and the phase space factor $\rho(s) = \frac{1}{16\pi}\sqrt{1 - \frac{4m_\pi^2}{s}}$. Unitary is implemented automatically in this parametrization, by means of the real function $\delta(s)$, which encompasses the full dynamical content of the iteration.

B. Model of production amplitude

The $\pi\pi$ phase shift induced by the FSIs implies that the measurement of phase shifts needs systematically to investigate the $\pi\pi$ production mechanism. We consider the $\pi\pi$ production in three cases. The first is the direct nonresonant three-body decay process as shown in Fig. 2(a), where

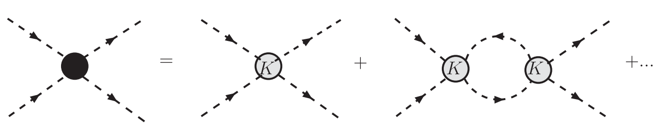

FIG. 1. Illustration of constructing the full amplitude with $\pi\pi$ scattering kernel and propagators.

the two pions are directly produced from the $\psi(2S)$ transition to J/ψ without the FSIs, which is called the "direct process" in the following part. The experimental analysis indicates that the direct process is necessary to destroy nonresonance backgrounds required by fitting to the data to extract σ contributions [15]. The second is the process including the FSI which leads to the $\pi\pi$ phase shift (see Fig. 2(b)), and the last is the process that the $\pi\pi$ pair is produced via the intermediate state σ as shown in Fig. 2(c); it has been argued that this process is necessary in fitting to data, e.g., in extracting the $\pi\pi$ S-wave phase shift from the analysis of $D^+ \to \pi^-\pi^+\pi^+$ data [16].

For the direct production process of $\psi(2S) \times (\epsilon', p') \to \pi^+(p_+)\pi^-(p_-)J/\psi(\epsilon, p)$, the leading term of the effective Lagrangian can be written as $g_1\psi(2S)_\mu\psi^\mu\partial_\nu\phi\partial^\nu\phi'$, where $\phi(\phi')$ denotes the $\pi^+(\pi^-)$ field, and ϵ' and ϵ are the polarization vectors for $\psi(2S)$ and J/ψ, respectively; and p', p, p_-, and p_+ are the four-vector momentum for $\psi(2S)$, J/ψ, π^-, and π^+, respectively. The amplitude with a minimal coupling can be expressed by

$$\mathcal{M}_a(s) = g_1\epsilon^* \cdot \epsilon' p_+ \cdot p_- = \frac{1}{2} g_1 \epsilon^* \cdot \epsilon'(s - 2m_\pi^2), \qquad (2)$$

where g_1 is the coupling constant.

In Fig. 2(b), the FSIs arise from the pion pair with the isospin $I = 0$ due to the isospin selection rule. In the center mass system, the angular-momentum between the two pions is limited to be 0 or 2, i.e., the S wave or the D wave. The experimental analysis on the BESII data indicates that the D wave component is so small that it could be negligible ($< 1\%$) [15], so we ignore the D wave component and only consider the S wave $\pi\pi$ FSI. In the analysis, the phase space limits the kinematic region of the $\pi\pi$ invariant mass in a region below 0.59 GeV, where the chiral unitarity approach is regarded as the effective tool to model the $\pi\pi$ FSI. The amplitude can be written in terms of the $\pi\pi$ amplitude obtained by using the factorization method as

$$\mathcal{M}_b^{\mathrm{ch}}(s) = \mathcal{M}_a(s)[1 + 2G(s)T^{I=0}_{\pi\pi,\pi\pi}(s)], \qquad (3)$$

where $T^{I=0}_{\pi\pi,\pi\pi}$ denotes the $\pi\pi$ rescattering amplitude with the constraint $I = 0$, and G represents the $\pi\pi$ propagator loop, which can be reduced to a explicit expression after performing the loop integration [17], i.e.,

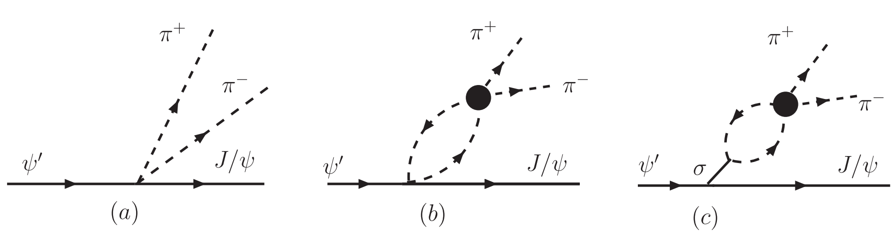

FIG. 2. Modeling $\pi\pi$ production in the transition of $\psi(2S) \to \pi^+\pi^- J/\psi$, (a) direct production, (b) rescattering process, and (c) σ resonance.

$$G(s) = \frac{1}{8\pi^2}\left\{\sigma(s)\arctan\frac{1}{\lambda\sigma(s)} - \ln\left[\frac{q_{\max}}{m_\pi}(1+\lambda)\right]\right\}, \quad (4)$$

where $q_{\max}$ is the momentum cutoff, conventionally taken as $q_{\max} \approx 1$ GeV. And $\sigma(s) = i16\pi\rho(s)$, $\lambda = \sqrt{\frac{m_\pi^2}{q_{\max}^2} + 1}$.

Using Eq. (1), the amplitude is rewritten in terms of the $\pi\pi$ phase shift $\delta(s)$

$$\mathcal{M}_b^{\rm ch}(s) = \mathcal{M}_a(s)\left(1 + 2G\frac{\sin\delta(s)}{\rho(s)}\exp[i\delta(s)]\right), \quad (5)$$

where $\rho(s)$ is the $\pi\pi$ phase space factor.

In the phenomenological method, the $\pi\pi$ production amplitude can be parametrized as an overall complex constant multiplication by the scattering amplitude as given in Eq. (1). In the measurement of $\pi\pi$ phase shifts from decays of the D meson into three pions [11,12,18], the amplitude is taken as

$$\mathcal{M}_b^{\rm ph}(s) = g_2\epsilon^*\cdot\epsilon'\frac{\sin\delta(s)}{\rho(s)}\exp[i\delta(s)], \quad (6)$$

where g_2 is the coupling constant, which can be taken as a complex value.

In Fig. 2(c), the amplitude associated with the intermediate resonance σ and $\pi\pi$ FSIs can be explicitly written in terms of the phase shift as given in Ref. [19], i.e.,

$$\mathcal{M}_c(s) = g_3\epsilon^*\cdot\epsilon'(s-2m_\pi^2)\frac{\cos\delta(s)}{m_\sigma^2 - s}\left[1 + \frac{\tan\delta(s)}{\tan\omega(s)}\right] \times \exp[i\delta(s)], \quad (7)$$

where $\epsilon^*\cdot\epsilon'$ arises from the vertex $\psi(2S) - \psi - \sigma$, and the factor $(s-2m_\pi^2)$ arises from the vertex $\sigma - \pi - \pi$, and g_3 is an overall coupling constant taken as a real number; m_σ is the σ mass; in fitting to the data, we take $m_\sigma = 0.552$ [15]. And $\omega(s)$ is given by $\tan\omega(s) = \frac{\pi\rho(s)}{\mathcal{R}[L(s)-L(m_\sigma^2)]}$, where $\rho(s)$ is the phase space factor, and $\mathcal{R}L(s) = \rho(s)\log[\frac{\sqrt{s}-\sqrt{s-4m_\pi^2}}{\sqrt{s}+\sqrt{s-4m_\pi^2}}]$.

In fitting to the data, we will consider a different $\pi\pi$ production mechanism in the amplitude to extract the $\pi\pi$ phase shift, and this will illustrate the role played by the $\pi\pi$ production mechanism.

1. Fit I

As studied in the three-body decay of heavy meson $D^+ \to \pi^-\pi^+\pi^+$ [20], the $\pi\pi$ phase shifts due to FSIs can be parametrized as an S-wave amplitude with an overall coupling constant to describe the $\pi\pi$ production. The amplitude including the direct production (Fig. 2(a)) and the FSI (Fig. 2(b)) subprocesses can be written as:

$$\text{Fit I: } \mathcal{M}(s) = \mathcal{M}_a(s) + \mathcal{M}_b^{\rm ph}(s) = \frac{1}{2}g_1\epsilon^*\cdot\epsilon'(s-2m_\pi^2) + g_2\epsilon^*\cdot\epsilon'\frac{\sin\delta(s)}{\rho(s)}\exp[i(\delta(s)+\gamma)], \quad (8)$$

where g_1 and g_2 are real values, and the constant γ denotes the phase difference relative to the direct production subprocess.

2. Fit II

In Fit I, the coupling g_2 is taken as a constant; in general it may be dependent on the energy of the pion pair. A possible coupling is taken in the form of the direct production process. So the amplitude is obtained naively by replacing g_2 with $g_2 p_+ \cdot p_-$ in Eq. (8), then the amplitude is expressed as

$$\text{Fit II: } \mathcal{M}(s) = \frac{1}{2}g_1\epsilon^*\cdot\epsilon'(s-2m_\pi^2)\left\{1 + g_2\frac{\sin\delta(s)}{\rho(s)} \times \exp[i(\delta(s)+\gamma)]\right\}, \quad (9)$$

where the coupling constants g_1 and g_2 are real numbers.

3. Fit III

This fit is motivated by the chiral unitarity calculation. The amplitude is chosen as

$$\text{Fit III: } \mathcal{M}(s) = \mathcal{M}_b^{\rm ch}(s) = -g\epsilon'\cdot\epsilon^*(s-2m_\pi^2) \times \left\{1 + 2G(s)\frac{\sin\delta(s)}{\rho(s)}\exp[i\delta(s)]\right\}, \quad (10)$$

where g is the coupling constant, and the coupling constant g is taken as a real number.

4. Fit IV

As studied by the BES Collaboration, the chiral unitarity amplitude fitting to the data also needs a small component of the $\pi\pi$ direct production [15]. The amplitude is chosen as the sum of the direct process plus the rescattering process as shown in Figs. 2(a) and 2(b)

$$\text{Fit IV: } \mathcal{M}(s) = \mathcal{M}_a(s) + \mathcal{M}_b^{\rm ch}(s) = -g\epsilon'\cdot\epsilon^*(s-2m_\pi^2)\left\{c_0 + 2G(s)\frac{\sin\delta(s)}{\rho(s)} \times \exp[i\delta(s)]\right\}, \quad (11)$$

where $c_0 = |c_0|e^{i\gamma}$, and the coupling constant g is taken as a real number.

5. Fit V

This fit encompasses all subprocesses as shown in Fig. 2. The amplitude is written as

$$\text{Fit V: } \mathcal{M}(s) = \mathcal{M}_a(s) + \mathcal{M}_b^{\mathrm{ch}}(s) + \mathcal{M}_c(s) = g\epsilon' \cdot \epsilon^*(s - 2m_\pi^2)\Big\{c_0 + 2G(s)t^{I=0}_{\pi\pi,\pi\pi} + c_1 \frac{\cos\delta(s)}{m_\sigma^2 - s}\Big[1 + \frac{\tan\delta(s)}{\tan\omega(s)}\Big]\exp[i\delta(s)]\Big\}, \quad (12)$$

where $c_k (k = 0, 1)$ is taken as the complex number, i.e., $c_k = |c_k|e^{i\gamma_k}$, and the coupling constant g is taken as a real number.

III. RESULTS OF FIT

For the three-body decay, the differential decay width is given by [21]

$$d\Gamma(s) = \frac{1}{(2\pi)^5}\frac{1}{16M_{\psi(2S)}}\bar{\sum_{\lambda,\lambda'}}|\mathcal{M}(s)|^2 dm_{\pi\pi}|\mathbf{p}| \times |\mathbf{p}^*|d\Omega_1^* d\Omega_3, \quad (13)$$

where $\bar{\Sigma}_{\lambda,\lambda'}$ stands for taking an average over the $\psi(2S)$ polarization $\lambda'(\pm 1)$, and taking the sum over the J/ψ polarization $\lambda(0, \pm 1)$. In this equation, $m_{\pi\pi}$ is the invariant mass of π^+ and π^- with $s = m_{\pi\pi}^2$, and $\mathbf{p}$ is the three-vector momentum of J/ψ in the $\psi(2S)$ rest system with a solid angle Ω, and $\mathbf{p}^*$ is the three-vector momentum of π^+ or π^- in the two-pion center mass system with a solid angle Ω^*.

Using the relation

$$\bar{\sum}_{\lambda,\lambda_i}\epsilon' \cdot \epsilon^* = 1 + \frac{|\mathbf{p}|^2}{2m_\psi}(1 - \cos^2\theta), \quad (14)$$

one gets the $m_{\pi\pi}$ distribution

$$\frac{d\Gamma(s)}{dm_{\pi\pi}} = \frac{1}{32\pi^2 M^2_{\psi(2S)}}|\mathcal{M}_R(s)|^2|\mathbf{p}||\mathbf{p}^*|\Big(1 + \frac{|\mathbf{p}|^2}{2m_\psi}\Big), \quad (15)$$

where the reduced amplitude $\mathcal{M}_R$ is defined as $\mathcal{M}(s) = \epsilon' \cdot \epsilon^* \mathcal{M}_R(s)$.

We make use of the likelihood function ($\mathcal{L}$) to fit the experimental distribution of $m_{\pi\pi}$, in which the detection efficiency is considered. The involved parameters encompass the coupling constants and the phase shift $\delta_k(s_k)$, where the s_k is a series of the interpolated points. A second-order spline interpolation is used to define the phase shift δ_k at the interpolation point $s_k = m_{\pi\pi}^2(k)$. Within the allowed phase space region $m_{\pi\pi} \in [0.26, 0.6]$ GeV, eight points are interpolated. To get parameter values, we make use of the standard procedure MINUIT in the CERN library to minimize the function $-\ln\mathcal{L}$.

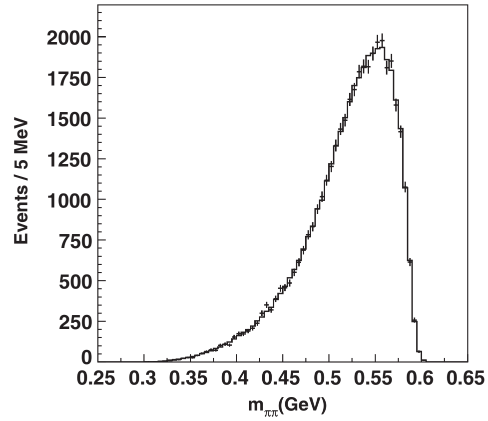

FIG. 3. Fit results of the $m_{\pi\pi}$ distribution. The points with error bars are experimental measurements [15], and the histogram is the fit results.

We tried to fit the $m_{\pi\pi}$ distribution with the amplitudes given in Fits I to V; it turned out that all fits yield a good reproduction of the spectrum. The extracted phase shifts are summarized in Table I. The fit results of the $m_{\pi\pi}$ distribution are shown in Fig. 3, where points with an error bar are the experimental data, and the $m_{\pi\pi}$ distributions of each fit are almost the same. The log-likelihood values ($-\ln\mathcal{L}$) for each fit are nearly identical, varying from -16114.8 (Fit II) to -16116.6 (Fit V), and together with the coupling constants are given in Table II.

The extracted phase shifts are also shown in Fig. 4, where the point sets from bottom to top correspond to Fit sets I to V, respectively, and the pentacle points with blue bars are taken from other experiments [3,4,22] for comparison, and the curve is the result of Roy's Equation taken from Ref. [8]. The results exhibit some common features. The first is that all values of the phase shifts increase from

TABLE I. The extracted results of phase shifts (in unit of degree) for each fit, and $m_{\pi\pi}$ is the invariant mass of two pions in unit of MeV.

$m_{\pi\pi}$	281.3	323.8	366.3	408.8	451.3	493.8	536.3	578.8
FIT I	0.4 ± 0.0	2.0 ± 0.3	7.5 ± 0.8	15.0 ± 1.3	24.2 ± 1.8	35.2 ± 2.2	48.1 ± 2.4	64.1 ± 2.6
FIT II	0.0 ± 0.0	10.3 ± 0.8	25.9 ± 0.9	39.1 ± 1.0	50.6 ± 1.4	60.3 ± 1.8	67.7 ± 2.6	70.9 ± 3.1
FIT III	1.1 ± 0.0	15.4 ± 0.9	31.0 ± 0.8	44.1 ± 0.7	55.5 ± 0.8	65.6 ± 0.7	73.7 ± 1.3	76.2 ± 0.5
FIT IV	0.0 ± 0.0	17.3 ± 0.7	32.9 ± 0.7	46.0 ± 0.7	56.8 ± 0.8	65.8 ± 0.9	72.4 ± 1.2	77.6 ± 0.9
FIT V	0.9 ± 0.0	35.1 ± 3.4	51.8 ± 2.8	65.3 ± 2.4	76.0 ± 1.9	84.6 ± 1.4	89.8 ± 0.1	90.3 ± 0.1

TABLE II. The fitting results of the coupling constants and the log-likelihood $-\ln\mathcal{L}$.

Fits	Parameters	$-\ln\mathcal{L}$
FIT I	$g_2/g_1 = 4.6 \pm 2.9$, $\gamma = 2.8 \pm 1.2$ (radian)	-16115.1
FIT II	$g_2/g_1 = 17.0 \pm 4.0$, $\gamma = -3.0 \pm 1.8$(radian)	-16114.8
FIT III	-	-16115.7
FIT IV	$c_0/\lvert g\rvert = 1.0 \pm 0.2$, $\gamma = 0.5 \pm 0.2$ (radian)	-16115.4
FIT V	$\lvert c_0\rvert/g = 3.3 \pm 0.4$, $\gamma_1 = 0.46 \pm 0.07\Lambda$ (radian)	
	$\lvert c_1\rvert/g = 0.01 \pm 0.01$, $\gamma_2 = 1.4 \pm 0.7$ (radian)	-16116.6

0 degrees to a maximum of 90 degrees (fit V) with increasing $m_{\pi\pi}$ from the threshold to ~0.6 GeV; the second is that the extracted phase shifts are sensitive to the choice of $\pi\pi$ production mechanisms assumed; the last is that the Fit I yields the phase shifts consistent with the scattering experiment and K_{e4} decays.

IV. REMARKS AND SUMMARY

It is instructive to make a comparison between our results and those obtained in the scattering experiments. In Fig. 4, the pentacle points are taken from πp scattering experiments [3,4,22]. The results of Fit I appear to be close to the $\pi\pi$ scattering data. This indicates that if the amplitude of $\pi\pi$ production in $\psi(2S)$ decays is parametrized as the minimal coupling $g_1\epsilon' \cdot \epsilon^*$ plus the $\pi\pi$ rescattering processes, the extracted $\pi\pi$ phase shift confirms the expectation of the Watson theorem. Other sets of fits yield the $\pi\pi$ phase shift moving higher up to 90° at the mass near $m_{\pi\pi} = 0.6$ GeV. The fits using chiral amplitudes including or excluding the direct process yield almost the same phase shifts. This result confirms the BES analysis result that the direct process makes a little contribution to the chiral amplitude [15].

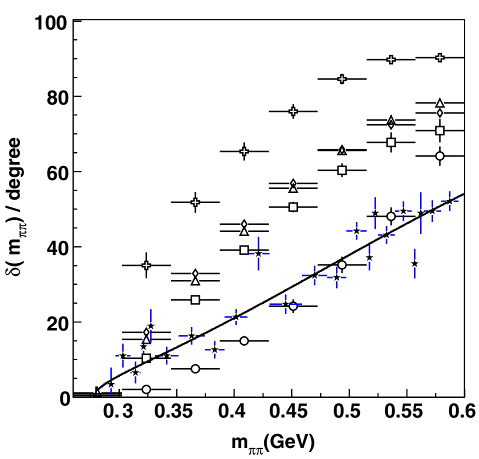

FIG. 4 (color online). The extracted phase shifts for each fit. Fit I: ○; Fit II: □; Fit III: △; Fit IV: ◇; Fit V: +. The pentacle points (★) are taken from other experiments [3,4,22], and the curve is the result of Roy's Equation taken from Ref. [8].

Our results show that the $\pi\pi$ phase shifts extracted from the $m_{\pi\pi}$ invariant mass spectrum are sensitive to the choice of the model to parametrize the $\pi\pi$ production mechanism. Since the $\pi\pi$ S-wave phase shifts are precisely known theoretically and experimentally below 0.8 GeV [6], the comparison of our results with the $\pi\pi$ scattering and K_{e4} experiments is helpful to pin down the strong production mechanism of the soft pion-pair in the decay of $\psi(2S) \to \pi^+\pi^- J/\psi$ by applying Watson's theorem. In Fit I, the production of the pion-pair is described by a minimal coupling, and then the $\pi\pi$ S-wave phase shift arises only due to the FSIs between the pion-pair. The fitting to the $m_{\pi\pi}$ mass distribution yields the phase shifts consistent with other experiments and the results of Roy's Equation between $0.4 < m_{\pi\pi} < 0.75$ GeV. The phase shifts deviate from Roy's Equation at other points due to the low statistics of the data (see Fig. 3). The total phase shifts receive contributions from the direct process (see Fig. 2(a)) and the $\pi\pi$ FSIs (see Fig. 2(b)). The direct process provides a constant phase shift γ, the fit yields $\gamma = 2.8° \pm 1.2°$. Since the $\pi\pi$ S-wave dynamical information has been parametrized by the phase shift $\delta(s)$, the amplitude of rescattering process cannot have the redundant $\pi\pi$ S-wave factor as $p_+ \cdot p_- = \frac{1}{2}(s - 2m_\pi^2)$. That's the reason why Fit II yields an inconsistent phase shift with other experiments. Similarly, the $\pi\pi$ production via the two-pion loop (Fit III and IV) and the σ resonance (Fit V) cannot reproduce the $\pi\pi$ S-wave shift as measured in the scattering experiments and K_{e4} decays.

In summary, the $\pi\pi$ S-wave phase shifts below 0.6 GeV are extracted out using data of the decay $\psi(2S) \to \pi^+\pi^- J/\psi$; the $\pi\pi$ production mechanisms are modeled by three subprocesses: direct process, the $\pi\pi$ rescattering, and the σ intermediate resonance. Several attempts to fit the $m_{\pi\pi}$ distribution by modeling the $\pi\pi$ production mechanism in the amplitude are tried. The results show that the amplitude including the $\pi\pi$ rescattering process with the minimal coupling $g_1\epsilon' \cdot \epsilon^*$ plus the direct process yield phase shifts consistent with whose measured in scattering experiments and K_{e4} decays.

ACKNOWLEDGMENTS

This work is partly supported by the National Natural Science Foundation of China under Grant No. 10575083.

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