Semileptonic decays of double heavy baryons in a relativistic constituent three-quark model

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We study the semileptonic decays of double-heavy baryons using a manifestly Lorentz covariant constituent three-quark model. We present complete results on transition form factors between double-heavy baryons for finite values of the heavy quark/baryon masses and in the heavy quark symmetry limit, which is valid at and close to zero recoil. Decay rates are calculated and compared to each other in the full theory, keeping masses finite, and also in the heavy quark limit.

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I. INTRODUCTION

The semileptonic decays of double-heavy baryons provide yet another opportunity to measure the Cabibbo-Kobayashi-Maskawa matrix element V_{cb}. This is particularly so since the transition matrix elements between double-heavy baryons obey spin symmetry relations in the heavy quark limit in addition to a model independent zero recoil normalization of the relevant transition matrix elements. In this paper we study current-induced transitions between double-heavy baryons in a fully relativistic constituent three-quark model. In the heavy quark limit we recover the spin symmetry relations among the form factors valid at zero recoil and close to zero recoil including their zero recoil normalization. Since the model is formulated in terms of finite values of the quark and baryon masses we are able to calculate the corrections to the spin symmetry relations and the zero recoil normalization valid in the heavy quark limit.

Current-induced double-heavy baryon (DBH) transitions have been analyzed in a number of model approaches. These include effective field theories based on heavy quark spin symmetry [1–4], three-quark models [5–7], quark-diquark models [8,9], and nonrelativistic QCD sum rules [10,11]. The progress achieved up to now can be summarized as follows. In the heavy quark limit (HQL) or in the limit of infinitely heavy quarks ($m_{c,b} \to \infty$) double-heavy baryons can be viewed as heavy-light mesonlike states containing a heavy diquark in the $\bar{3}$ color state and a light quark in the 3 color state [1]. In the HQL the spins of the light quark and heavy diquark system decouple. This gives rise to relations between different transition form factors involving double-heavy baryons in the quark-diquark picture [1] as well as in the three-quark picture [3]. In particular, working in the near-zero recoil limit one can express all weak transition form factors between double-heavy baryons through a single universal function $\eta(\omega)$, which depends on the kinematical parameter $\omega = v \cdot v'$, where v and v' stand for the four-velocities of the initial and final double-heavy baryon, respectively.

In Ref. [5] we have analyzed double-heavy baryons for specific decay modes. We have restricted ourselves to spin $1/2$ to spin $1/2$ transitions using the same relativistic constituent three-quark model [5,12] as is being used in the present paper except that we now no longer have to rely on the impulse approximation. Differing from the approach of the present paper, in [5] we have treated the double-heavy baryons as bound states of a heavy b quark and a heavy-light (cq) diquark. In this paper we take double-heavy baryons to be bound states of a light quark and a double-heavy (bc) diquark. In particular, this means that the interpolating three-quark currents used in this paper have a different spin-flavor structure than the corresponding current in [5]. In the full theory this will lead to different predictions for the rates. We would like to emphasize, though, that, in the nonrelativistic limit, both currents are consistent with one another.

The relativistic constituent three-quark model can be viewed as an effective quantum field theory approach based on an interaction Lagrangian of hadrons interacting with their constituent quarks. From such an approach one can derive universal and reliable predictions for exclusive processes involving both mesons composed of a quark and antiquark and baryons composed of three quarks. The coupling strength of a hadron H to its constituent quarks is determined by the compositeness condition $Z_H = 0$ [13,14], where Z_H is the wave function renormalization constant of the hadron. The quantity $Z_H^{1/2}$ is the matrix element between a physical particle state and the corresponding bare state. The compositeness condition $Z_H = 0$ enables one to represent a bound state by introducing a hadronic field interacting with its constituents so that the renormalization factor is equal to zero. This does not mean that we can solve the QCD bound state equations but we are able to show that the condition $Z_H = 0$ provides an

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effective and self-consistent way to describe the coupling of a hadron to its constituents. One starts with an effective interaction Lagrangian written down in terms of quark and hadron variables. Then, by using Feynman rules, the S-matrix elements describing hadron-hadron interactions are given in terms of a set of quark level Feynman diagrams. In particular, the compositeness condition enables one to avoid the problem of double counting of quark and hadronic degrees of freedom. The approach is self-consistent and all calculations of physical observables are straightforward. There is a small set of model parameters: the values of the constituent quark masses and the scale parameters that define the size of the distribution of the constituent quarks inside a given hadron.

The main objective of the present paper is to present a comprehensive analysis of all possible current-induced spin transitions between double-heavy baryons containing both types of light quarks—nonstrange $q = u$, d and strange s. This involves the flavor transitions $bc \to cc$ and $bb \to bc$, where the transition $bc \to cc$ is treated as the generic process in the main text, while the results for the transition $bb \to bc$ are mainly relegated to tables. The paper is structured as follows: First, in Sec. II we present interpolating three-quark currents with the appropriate quantum numbers of the double-heavy baryons. We then write down the corresponding Lagrangians defining the couplings of these currents to double-heavy baryons. Second, we briefly discuss the calculational techniques of how to calculate transition matrix elements generated by the Lagrangian functions. In Sec. III, we consider the heavy quark limit of our transition matrix elements and recover the known heavy quark symmetry relations for the transition matrix elements between double-heavy baryons as well as the appropriate zero recoil normalization of the form factors. In particular, we compare the results of the full finite mass calculation with the results derived in the HQL. Fourth, in Sec. IV we present our numerical results, which are compared to predictions of other theoretical approaches. In particular, we compare the results of the full finite mass calculation with the results derived in the HQL. Finally, in Sec. IV we present a short summary of our results.

II. SEMILEPTONIC DECAYS OF DOUBLE-HEAVY BARYONS

A. Lagrangian

For the evaluation of the semileptonic decays we will consistently employ the relativistic constituent three-quark model. In the following we present details of the model, which is based on an interaction Lagrangian describing the coupling between baryons and their constituent quarks.

The coupling of a baryon $B(q_1q_2q_3)$ to its constituent quarks q_1, q_2, and q_3 is described by the Lagrangian

$$\mathcal{L}^{\rm str}_{\rm int}(x) = g_B\bar{B}(x)\int dx_1\int dx_2\int dx_3 F_B(x, x_1, x_2, x_3) \times J_B(x_1, x_2, x_3) + \text{H.c.}, \tag{1}$$

where $J_B(x_1, x_2, x_3)$ is the interpolating three-quark current with the quantum numbers of the relevant baryon B. One has

$$J_B(x_1, x_2, x_3) = \varepsilon^{a_1a_2a_3}\Gamma_1 q_1^{a_1}(x_1)q_2^{a_2}(x_2)C\Gamma_2 q_3^{a_3}(x_3), \tag{2}$$

where the $\Gamma_{1,2}$ are sets of Dirac matrices, C is the charge conjugation matrix $C = \gamma^0\gamma^2$, and the a_i ($i = 1, 2, 3$) are color indices. $F_B(x, x_1, x_2, x_3)$ is a nonlocal scalar vertex function, which characterizes the finite size of the baryon.

B. Vertex function

The vertex function F_B is related to the scalar part of the Bethe-Salpeter amplitude and characterizes the finite size of the baryon. To satisfy translational invariance the function F_B has to fulfill the identity

$$F_B(x + a, x_1 + a, x_2 + a, x_3 + a) = F_B(x, x_1, x_2, x_3) \tag{3}$$

for any given four-vector a. In the following we use a specific form for the vertex function

$$F_B(x, x_1, x_2, x_3) = N_B\delta^{(4)}\Big(x - \sum_{i=1}^{3} w_i x_i\Big)\Phi_B\Big(\sum_{i<j}(x_i - x_j)^2\Big), \tag{4}$$

where Φ_B is a nonlocal correlation function involving the three constituent quarks with masses m_1, m_2, m_3; $N_B = 9$ is a normalization factor. The variable w_i is defined by $w_i = m_i/(m_1 + m_2 + m_3)$. The vertex function (4) satisfies the translational identity (3).

The Fourier transform of the correlation function $\Phi_B(\sum_{i<j}(x_i - x_j)^2)$ can be calculated by using Jacobi coordinates. One has

$$\begin{aligned}\tilde{\Phi}_B(p_1, p_2, p_3) &= N_B\int dx e^{-ipx}\prod_{i=1}^{3}\int dx_i e^{ip_ix_i} \\ &\quad\times \delta^{(4)}\Big(x - \sum_{i=1}^{3} w_i x_i\Big)\Phi_B\Big(\sum_{i<j}(x_i - x_j)^2\Big) \\ &= (2\pi)^4\delta^{(4)}\Big(p - \sum_{i=1}^{3} p_i\Big)\bar{\Phi}_B(-l_1^2 - l_2^2),\end{aligned} \tag{5}$$

where the Jacobi coordinates are defined by

$$\begin{aligned} x_1 &= x + \frac{1}{\sqrt{2}}\xi_1 w_3 - \frac{1}{\sqrt{6}}\xi_2(2w_2 + w_3), \\ x_2 &= x + \frac{1}{\sqrt{2}}\xi_1 w_3 + \frac{1}{\sqrt{6}}\xi_2(2w_1 + w_3), \\ x_3 &= x - \frac{1}{\sqrt{2}}\xi_1(w_1 + w_2) + \frac{1}{\sqrt{6}}\xi_2(w_1 - w_2). \end{aligned} \tag{6}$$

The corresponding Jacobi momenta read

$$
\begin{aligned}
p &= p_1 + p_2 + p_3, \\
l_1 &= \frac{1}{\sqrt{2}} w_3 (p_1 + p_2) - \frac{1}{\sqrt{2}} (w_1 + w_2) p_3, \\
l_2 &= -\frac{1}{\sqrt{6}} (2w_2 + w_3) p_1 + \frac{1}{\sqrt{6}} (2w_1 + w_3) p_2 \\
&\quad + \frac{1}{\sqrt{6}} (w_1 - w_2) p_3,
\end{aligned} \tag{7}
$$

where, according to Eq. (4), $\sum_{i=1}^{3} w_i x_i = x$. Since the function $\Phi_B(\sum_{i<j}(x_i - x_j)^2)$ is invariant under translations, its Fourier transform only depends on two four-momenta. The function $\bar{\Phi}_B(-l_1^2 - l_2^2)$ in Eq. (5) will be modeled in our approach. The minus sign in the argument is chosen to emphasize that we are working in Minkowski space. A simple choice is the Gaussian form

$$\bar{\Phi}_B(-l_1^2 - l_2^2) = \exp(18(l_1^2 + l_2^2)/\Lambda_B^2), \tag{8}$$

where the parameter Λ_B characterizes the size of the double-heavy baryon. Since l_1^2 and l_2^2 turn into $-l_1^2$ and $-l_2^2$ in Euclidean space the form (8) has the appropriate falloff behavior in the Euclidean region.

C. Three-quark currents

Double-heavy baryons are classified by the set of quantum numbers (J^P, S_d), where J^P is the spin-parity of the baryon state and S_d is the spin of the heavy diquark. There are two types of heavy diquarks—those with $S_d = 0$ (antisymmetric spin configuration $[Q_1 Q_2]$) and those with $S_d = 1$ (symmetric spin configuration $\{Q_1 Q_2\}$). Accordingly, there are two $J^P = 1/2^+$ double-heavy baryon states. We follow the standard convention and attach a prime to the $S_d = 0$ states, whereas the $S_d = 1$ states are unprimed. Note that the $J^P = 3/2^+$ states are in the symmetric heavy quark spin configuration. In Table I we list the quantum numbers of the double-heavy baryons including their mass spectrum as calculated in [9].

We pause for a moment to discuss some of the features of the mass spectrum obtained in [9], which are relevant for our calculation. One notes that there is a mass inversion in the $(1/2^+)$ mixed flavor states (Ξ_{bc}, Ξ'_{bc}) and $(\Omega_{bc}, \Omega'_{bc})$ in that $M(\Xi'_{bc}) > M(\Xi_{bc})$ and $M(\Omega'_{bc}) > M(\Omega_{bc})$ contrary to naive expectation even though the heavy triplet diquark state has a higher mass than the heavy singlet diquark state in the model of [9], i.e. $m(bc; S_d = 1) > m(bc; S_d = 0)$. The inverted mass hierarchy is at the origin of the prime notation mentioned above. We mention that the inverted mass hierarchy is a feature of all models that have attempted to calculate the mass spectrum of double-heavy baryons [6,9,11,16–18]. In particular, the inverted mass hierarchy implies that one can only expect substantial flavor-changing branching ratios for the two lowest-lying states Ξ_{bc} and Ω_{bc}, whereas the rates of the higher-lying states Ξ'_{bc} and Ξ^*_{bc}, and Ω'_{bc} and Ω^*_{bc} will be dominated by flavor-preserving one-photon transitions to the lowest-lying states Ξ_{bc} and Ω_{bc}. It will be interesting to analyze the strength of one-photon transitions between the $S_d = 0$ and $S_d = 1$ double-heavy baryon states, which are forbidden in the HQL since, in the HQL, the photon couples to the light quark only. For finite heavy quark masses one-photon transitions between the $S_d = 0$ and $S_d = 1$ double-heavy baryon states will occur at a somewhat reduced rate which, however, very likely will still exceed the flavor-changing weak decay rates of these states.

TABLE I. Classification and mass values of double-heavy baryons. Mass values are used from [9] except for the Ξ_{cc} mass which is taken from [15].

Notation	Content	J^P	S_d	Mass (GeV)
Ξ_{cc}	$q\{cc\}$	$1/2^+$	1	3.5189
Ξ_{bc}	$q\{bc\}$	$1/2^+$	1	6.933
Ξ'_{bc}	$q[bc]$	$1/2^+$	0	6.963
Ξ_{bb}	$q\{bb\}$	$1/2^+$	1	10.202
Ξ^*_{cc}	$q\{cc\}$	$3/2^+$	1	3.727
Ξ^*_{bc}	$q\{bc\}$	$3/2^+$	1	6.980
Ξ^*_{bb}	$q\{bb\}$	$3/2^+$	1	10.237
Ω_{cc}	$s\{cc\}$	$1/2^+$	1	3.778
Ω_{bc}	$s\{bc\}$	$1/2^+$	1	7.088
Ω'_{bc}	$s[bc]$	$1/2^+$	0	7.116
Ω_{bb}	$s\{bb\}$	$1/2^+$	1	10.359
Ω^*_{cc}	$s\{cc\}$	$3/2^+$	1	3.872
Ω^*_{bc}	$s\{bc\}$	$3/2^+$	1	7.130
Ω^*_{bb}	$s\{bb\}$	$3/2^+$	1	10.389

We construct the interpolating currents of the double-heavy baryon $B_{qQ_1Q_2}$ in the form of a light quark q^{a_1} coupled to a heavy diquark $d^{a_1}_{Q_1Q_2}$, *viz.*

$$
\begin{aligned}
J_{qQ_1Q_2} &= \Gamma_{Q_1Q_2} q^{a_1} d^{(Q_1Q_2)}_{a_1}, \\
d^{(Q_1Q_2)}_{a_1} &= \varepsilon^{a_1a_2a_3} (Q_1^{a_2} C \Gamma^{(d)}_{Q_1Q_2} Q_2^{a_3}).
\end{aligned} \tag{9}
$$

We shall only consider currents without derivatives. With this restriction one can construct three interpolating currents for the $(1/2^+, 0)$ states—the pseudoscalar J^P, scalar J^S and axial J^A currents

$$J^P_{qQ_1Q_2} = \varepsilon^{a_1a_2a_3} q^{a_1} (Q_1^{a_2} C\gamma_5 Q_2^{a_3}), \tag{10a}$$

$$J^S_{qQ_1Q_2} = \varepsilon^{a_1a_2a_3} \gamma^5 q^{a_1} (Q_1^{a_2} C Q_2^{a_3}), \tag{10b}$$

$$J^A_{qQ_1Q_2} = \varepsilon^{a_1a_2a_3} \gamma^\mu q^{a_1} (Q_1^{a_2} C\gamma_5\gamma_\mu Q_2^{a_3}). \tag{10c}$$

For the $(1/2^+, 1)$ states one has a vector J^V and a tensor J^T current

$$J^V_{qQ_1Q_2} = \varepsilon^{a_1a_2a_3} \gamma^\alpha \gamma^5 q^{a_1} (Q_1^{a_2} C\gamma_\alpha Q_2^{a_3}), \tag{11a}$$

$$J^T_{qQ_1Q_2} = \frac{1}{2} \varepsilon^{a_1a_2a_3} \sigma^{\mu\nu} \gamma^5 q^{a_1} (Q_1^{a_2} C\sigma_{\mu\nu} Q_2^{a_3}). \tag{11b}$$

Finally, for the $(3/2^+, 1)$ states one has the vector and tensor currents J^V_μ and J^T_μ

$$J^V_{qQ_1Q_2,\mu} = \varepsilon^{a_1a_2a_3} q^{a_1}(Q_1^{a_2} C\gamma_\mu Q_2^{a_3}), \tag{12a}$$

$$J^T_{qQ_1Q_2,\mu} = -i\varepsilon^{a_1a_2a_3} \gamma^\nu q^{a_1}(Q_1^{a_2} C\sigma_{\mu\nu} Q_2^{a_3}). \tag{12b}$$

Note that any double-heavy baryon current in the form of a heavy quark coupling to a heavy-light diquark can be transformed to a linear combination of the above form of currents using a Fierz transformation.

In the heavy quark limit the scalar current $J^S_{Q_1Q_2q}$ vanishes, while the other currents become degenerate in the following way:

$$J^P_{qQ_1Q_2} = J^A_{qQ_1Q_2} = \varepsilon^{a_1a_2a_3}\psi_q^{a_1}(\psi_{Q_1}^{a_2}\sigma_2\psi_{Q_2}^{a_3}), \tag{13a}$$

$$J^V_{qQ_1Q_2} = J^T_{qQ_1Q_2} = \varepsilon^{a_1a_2a_3}\vec\sigma\psi_q^{a_1}(\phi_{Q_1}^{a_2}\sigma_2\vec\sigma\psi_{Q_2}^{a_3}), \tag{13b}$$

$$\vec J^V_{qQ_1Q_2} = \vec J^T_{qQ_1Q_2} = \varepsilon^{a_1a_2a_3}\psi_q^{a_1}(\psi_{Q_1}^{a_2}\sigma_2\vec\sigma\psi_{Q_2}^{a_3}), \tag{13c}$$

where ψ_{q,Q_1,Q_2} are the upper components of the Dirac quark spinors and the σ_i are Pauli spin matrices. Excluding the scalar current $J^S_{Q_1Q_2q}$, which vanishes in the HQL, we remain with two currents for each of the double-heavy baryon states which, as shown above, become degenerate in the HQL. It is therefore reasonable to take only one of the interpolating currents each. Our choice is to take the simplest current from each pair—the pseudoscalar current for the $(1/2^+, 0)$ states and the vector currents for the $(1/2^+, 1)$ and $(3/2^+, 1)$ states. Note that the HQL coincides with the nonrelativistic limit. In the nonrelativistic limit our DHB currents have a one-to-one correspondence to the naive quark model baryon spin-flavor functions, which are displayed in Table II. Further details on the naive quark model and how to evaluate the semileptonic current-induced transition amplitudes in this framework can be found in Appendix A.

We now shall give explicit expressions for the three-quark currents that are needed for the calculation of the $bcq \to ccq$ semileptonic transition amplitudes:

$$J_{bcq} = \Gamma^{(q)}_{bc} q^{a_1} d^{a_1}_{(bc)}, \qquad d^{a_1}_{(bc)} = \varepsilon^{a_1a_2a_3}(b^{a_2}C\Gamma^{(d)}_{bc}c^{a_3}), \tag{14a}$$

$$\bar J_{bcq} = \bar d^{a_1}_{(bc)}\bar q^{a_1}\bar\Gamma^{(q)}_{bc}, \qquad \bar d^{a_1}_{(bc)} = \varepsilon^{a_1a_2a_3}(\bar c^{a_2}\bar\Gamma^{(d)}_{bc}C\bar b^{a_3}), \tag{14b}$$

TABLE II. Double-heavy baryon wave functions.

Baryon	Wave function	Baryon	Wave function
Ξ_{cc}	$qcc\chi_S$	Ω_{cc}	$scc\chi_S$
Ξ_{bb}	$qbb\chi_S$	Ω_{bb}	$sbb\chi_S$
Ξ_{bc}	$\frac{1}{\sqrt2}q(bc+cb)\chi_S$	Ω_{bc}	$\frac{1}{\sqrt2}s(bc+cb)\chi_S$
Ξ'_{bc}	$\frac{1}{\sqrt2}q(bc-cb)\chi_A$	Ω'_{bc}	$\frac{1}{\sqrt2}s(bc-cb)\chi_A$
Ξ^*_{cc}	$-qcc\chi^*_S$	Ω^*_{cc}	$-scc\chi^*_S$
Ξ^*_{bb}	$-qbb\chi^*_S$	Ω^*_{bb}	$-sbb\chi^*_S$
Ξ^*_{bc}	$-\frac{1}{\sqrt2}q(bc+cb)\chi^*_S$	Ω^*_{bc}	$-\frac{1}{\sqrt2}s(bc+cb)\chi^*_S$

where $\bar J = J^\dagger\gamma^0$ and $\bar\Gamma = \gamma^0\Gamma^\dagger\gamma^0$. The corresponding ccq currents are obtained by the obvious replacement $b \to c$. Note that $C\Gamma^{(d)}_{cc}C = -\Gamma^{(d)T}_{cc}$. Using a rather suggestive notation we specify the coupling content of the DHB currents for the $\Xi_{Q_1Q_2}$ baryons using the form $\Rightarrow \Gamma^{(q)} \otimes \Gamma^{(d)}$. We thus consider the currents

$$\Xi_{cc} \Rightarrow \gamma^\alpha\gamma^5 \otimes \gamma_\alpha, \qquad \Xi^*_{cc} \Rightarrow I \otimes \gamma^\nu, \tag{15a}$$

$$\Xi_{bc} \Rightarrow \gamma^\alpha\gamma^5 \otimes \gamma_\alpha, \qquad \Xi^*_{bc} \Rightarrow I \otimes \gamma^\nu, \qquad \Xi'_{bc} \Rightarrow I \otimes \gamma^5. \tag{15b}$$

We use the same set of currents for the double-heavy $\Omega_{Q_1Q_2}$-type baryons replacing the light u, d-quark by a s quark.

D. Normalization

As described, e.g. in [5] we need the derivative of the mass operator for the double-heavy baryon $\Xi_{Q_1Q_2}$ in order to evaluate the coupling constants $g_B = g_{\Xi_{Q_1Q_2}}$. The mass operator is given by a two-loop Feynman diagram and reads

$$\begin{aligned}\tilde\Pi_{\Xi_{Q_1Q_2}}(p) = {} & \frac{N_{q_1q_2}}{(16\pi^2)^2}\int\frac{d^4k_1}{\pi^2 i}\int\frac{d^4k_2}{\pi^2 i}\bar\Phi^2_B(-l_1^2-l_2^2) \\ & \times \mathrm{tr}[\Gamma^{(d)}\tilde S_1(k_1+w_1p)\bar\Gamma^{(d)}\tilde S_2(k_2-w_2p)] \\ & \times \Gamma^{(q)}\tilde S_3(k_2-k_1+w_3p)\bar\Gamma^{(q)}, \\ l_1 = {} & \frac{1}{\sqrt2}(k_1-k_2), \qquad l_2 = -\frac{1}{\sqrt6}(k_1+k_2),\end{aligned} \tag{16}$$

where $q_1 = b$ or c, $q_2 = c$, $q_3 = q$ and $N_{bc} = 6$, $N_{cc} = 12$. The expression $\tilde S_i(k) = (m_{q_i} - \not k - i\epsilon)^{-1}$ denotes the free fermion propagator for the constituent quark with mass m_{q_i}. Integration momenta have been shifted in such a way so as to remove the external momentum from the vertex function. We have assigned outgoing momenta to the first and third outgoing quarks and an ingoing momentum for the second quark. Since the p dependence of the mass operator resides entirely in the propagators it is not difficult to calculate the derivative of the mass operator needed for the normalization condition

$$g^2_{\Xi_{Q_1Q_2}}\frac{d}{dp^\mu}\tilde\Pi_{\Xi_{q_1q_2}}(p) = \gamma^\mu. \tag{17}$$

The latter condition is known as a Ward identity, which is equivalent to the compositeness condition $Z_H = 0$ [13,14]. One obtains

$$\frac{d}{dp^\mu}\tilde{\Pi}_{\Xi_{Q_1Q_2}}(p) = \frac{N_{q_1q_2}}{(16\pi^2)^2}\int\frac{d^4k_1}{\pi^2 i}\int\frac{d^4k_2}{\pi^2 i}\bar{\Phi}_B^2(-l_1^2-l_2^2)\{w_1\,\mathrm{tr}[\Gamma^{(d)}\tilde{S}_1(k_1+w_1p)\gamma^\mu\tilde{S}_1(k_1+w_1p)\bar{\Gamma}^{(d)}\tilde{S}_2(k_2-w_2p)]$$
$$\times\Gamma^{(q)}\tilde{S}_3(k_2-k_1+w_3p)\bar{\Gamma}^{(q)} - w_2\,\mathrm{tr}[\Gamma^{(d)}\tilde{S}_1(k_1+w_1p)\bar{\Gamma}^{(d)}\tilde{S}_2(k_2-w_2p)\gamma^\mu\tilde{S}_2(k_2-w_2p)]$$
$$\times\Gamma^{(q)}\tilde{S}_3(k_2-k_1+w_3p))\bar{\Gamma}^{(q)} + w_3\,\mathrm{tr}[\Gamma^{(d)}\tilde{S}_1(k_1+w_1p)\bar{\Gamma}^{(d)}\tilde{S}_2(k_2-w_2p)]$$
$$\times\Gamma^{(q)}\tilde{S}_3(k_2-k_1+w_3p)\gamma^\mu\tilde{S}_3(k_2-k_1+w_3p)\bar{\Gamma}^{(q)}\}, \tag{18}$$

where the double-heavy $\Xi_{Q_1Q_2}$ baryon is taken to be on its mass shell.

E. Matrix elements of semileptonic decays

The generic matrix element describing the semileptonic transitions $\Xi_{bc}\to\Xi_{cc}$ reads

$$\langle\Xi_{cc}(p')|\int d^4x e^{iqx}\bar{c}(x)O^\mu b(x)|\Xi_{bc}(p)\rangle = (2\pi)^4\delta^{(4)}(p-p'-q)\bar{u}_{\Xi_{cc}}(p')\Lambda^\mu(p,p')u_{\Xi_{bc}}(p), \tag{19}$$

where

$$\Lambda^\mu(p,p') = 12\frac{g_{\Xi_{bc}}g_{\Xi_{cc}}}{(16\pi^2)^2}\int\frac{d^4k_1}{\pi^2 i}\int\frac{d^4k_2}{\pi^2 i}\bar{\Phi}_B(-l_1^2-l_2^2)$$
$$\times\bar{\Phi}_B(-l_1'^2-l_2'^2)\,\mathrm{tr}[\Gamma_{cc}^{(d)}\tilde{S}_4(k_1+\frac{1}{2}p')O^\mu$$
$$\times\tilde{S}_1(k_1+p-\frac{1}{2}p')\bar{\Gamma}_{bc}^{(d)}\tilde{S}_2(k_2-\frac{1}{2}p')]$$
$$\times\Gamma_{cc}^{(q)}\tilde{S}_3(k_2-k_1)\bar{\Gamma}_{bc}^{(q)}. \tag{20}$$

Here, $O^\mu=\gamma^\mu(1-\gamma^5)$ and $w_i'=m_i/(m_4+m_2+m_3)$ with $i=4,2,3$. For the masses one has $m_1=m_b$, $m_2=m_4=m_c$, $m_3=m_q$. The Jacobi momenta ω_i and ω_i' are chosen as

$$l_1=\frac{1}{\sqrt{2}}[k_1-k_2+w_3p],$$
$$l_2=-\frac{1}{\sqrt{6}}[k_1+k_2+(w_2-w_1)p+q],$$
$$l_1'=-\frac{1}{\sqrt{2}}[k_1-k_2+w_3'p'],$$
$$l_2'=\frac{1}{\sqrt{6}}[k_1+k_2+(w_2'-w_4')p']. \tag{21}$$

Note that the expressions for the normalization and the vertex given in Eq. (18) and (20), respectively, are exact in the sense that they are obtained directly from the Lagrangian Eq. (2) for an arbitrary translationally invariant vertex function F_B such as the one defined in Eq. (4). The two-loop integrals in Eqs. (18) and (20), are invariant under translations of the loop variables $k_i\to k_i+b_i$ $(i=1,2)$, where b_i are arbitrary momentum four-vectors. We have assigned the loop momenta such that the heavy quark limit can easily be taken. Calculational techniques for the two-loop quark integrals are given in some detail in our previous publication [5]. One uses Schwinger's parametrization to raise the denominator factors into exponential factors. The tensor integrals are dealt with by using differential representations of the numerator factors. After doing as many loop integrations analytically as possible one ends up with four-fold parameter integrations for the derivative of the normalization factor and the transition form factors, which are evaluated numerically.

III. HEAVY QUARK SPIN SYMMETRY

A. Structure of weak transitions in the HQL

In the HQL the spins of the double-heavy diquark and the light quark in a double-heavy baryon decouple. At zero recoil and close to zero recoil this leads to spin symmetry relations among transition form factors between double-heavy baryons and a zero recoil normalization for the form factors. The spin symmetry is exact at zero recoil and close to zero recoil where the near-zero recoil region is specified later on. Near-zero recoil one can choose the momenta as $p_1^\mu=m_{bc}v^\mu$ and $p_2^\mu=m_{cc}v'^\mu=m_{cc}v^\mu+r^\mu$, where r is a small residual momentum in the sense that $r^2\sim O(1)$ when $m_c\to\infty$. Since the final baryon is on mass shell one has $v\cdot r=-r^2/2m_{cc}\sim O(1/m_c)$. This imposes a restriction on the kinematical variable $w=v\cdot v'$ since $r^2=m_{cc}^2(v-v')^2=2m_{cc}^2(1-w)\sim O(1)$. From the last equation one obtains $w\sim 1+O(1/m_c^2)$. This situation is different compared with the case of baryons with a single heavy quark. A baryon with a single heavy quark possesses both a spin and a flavor symmetry. In what follows we will work near-zero recoil in the sense that we neglect terms of $O(v\cdot r)$. In order to keep things simple we assume that $m_{bc}=m_b+m_c$ and $m_{cc}=2m_c$.

Using the above assumptions the heavy mass propagators simplify in the HQL. One has

$$\tilde{S}_b(k_1+p-\frac{1}{2}p')\to\frac{1+\not{v}}{2}\frac{1}{-k_1v-i\epsilon}, \tag{22a}$$

$$\tilde{S}_c(k_1\pm\frac{1}{2}p')\to\frac{1\pm\not{v}'}{2}\frac{1}{\mp k_1v-i\epsilon}. \tag{22b}$$

Because of the simple form of the heavy quark propagators in the HQL the defining equation for the coupling constant Eq. (16) simplifies considerably. One obtains

$$1 = N_{Q_1Q_2}\frac{g^2_{\Xi_{Q_1Q_2}}}{(16\pi^2)^2}\,\mathrm{tr}\Big[\Gamma^{(d)}\frac{1+\not{v}}{2}\bar{\Gamma}^{(d)}\frac{1-\not{v}}{2}\Big]\int\frac{d^4k_1}{\pi^2 i}\times\int\frac{d^4k_2}{\pi^2 i}\bar{\Phi}^2_B(y_0)\frac{\Gamma^{(q)}\tilde{S}_3(k_2-k_1)\bar{\Gamma}^{(q)}}{(-k_1v-i\epsilon)^2(k_2v-i\epsilon)}. \quad (23)$$

A similar simplification occurs for the transition operator $\Lambda^\mu(p,p')$ in Eq. (20). One obtains

$$\Lambda^\mu(v,v') = 12\frac{g_{\Xi_{bc}}g_{\Xi_{cc}}}{(16\pi^2)^2}\,\mathrm{tr}\Big[\Gamma^{(d)}_{cc}\frac{1+\not{v}'}{2}O^\mu\frac{1+\not{v}}{2}\times\bar{\Gamma}^{(d)}_{bc}\frac{1-\not{v}'}{2}\Big]\int\frac{d^4k_1}{\pi^2 i}\int\frac{d^4k_2}{\pi^2 i}\bar{\Phi}_B(y_0)\bar{\Phi}_B(y_r)\times\frac{\Gamma^{(q)}_{cc}\tilde{S}_3(k_2-k_1)\bar{\Gamma}^{(q)}_{bc}}{(-k_1v-i\epsilon)(-k_1v'-i\epsilon)(k_2v'-i\epsilon)}, \quad (24)$$

where

$$y_r \equiv y(r) = -\frac{1}{2}(k_1-k_2)^2 - \frac{1}{6}(k_1+k_2-r)^2. \quad (25)$$

It is not difficult to see that once we put $v = v'(r=0)$ in Eq. (24) the two-loop integral and the trace factor in (24) reduce to the corresponding factors in Eq. (23). As a result all semileptonic transition matrix elements can be expressed in terms of a universal function $\eta(\omega)$ normalized to 1 at zero recoil with $\omega = 1$ in full consistency with the heavy quark spin symmetry results derived in [3].

Neglecting terms of $O(v\cdot r)$ one finds

$$\Lambda^\mu(\Xi_{bc}(v)\to\Xi_{cc}(v')) = \sqrt{2}\Big(\gamma^\mu - \frac{2}{3}\gamma^\mu\gamma^5\Big)\eta(w), \quad (26a)$$

$$\Lambda^\mu(\Xi'_{bc}(v)\to\Xi_{cc}(v')) = -\sqrt{\frac{2}{3}}\gamma^\mu\gamma^5\eta(w), \quad (26b)$$

$$\Lambda^\mu(\Xi_{bc}(v)\to\Xi^*_{cc}(v',\nu)) = -\sqrt{\frac{2}{3}}g^{\mu\nu}\eta(w), \quad (26c)$$

$$\Lambda^\mu(\Xi'_{bc}(v)\to\Xi^*_{cc}(v',\nu)) = \sqrt{2}g^{\mu\nu}\eta(w), \quad (26d)$$

$$\Lambda^\mu(\Xi^*_{bc}(v,\nu)\to\Xi_{cc}(v')) = -\sqrt{\frac{2}{3}}g^{\mu\nu}\eta(w), \quad (26e)$$

$$\Lambda^\mu(\Xi^*_{bc}(v,\nu)\to\Xi^*_{cc}(v',\nu')) = \sqrt{2}O^\mu g^{\nu\nu'}\eta(w), \quad (26f)$$

where

$$\eta(w) = \frac{J(w)}{J(1)}, \quad (27)$$

$$J(w) = \int\frac{d^4k_1}{\pi^2 i}\int\frac{d^4k_2}{\pi^2 i}\bar{\Phi}_B(y_0)\bar{\Phi}_B(y_r)\times\frac{m_q+(k_2-k_1)v}{(-k_1v-i\epsilon)^2(k_2v-i\epsilon)(m_q^2-(k_2-k_1)^2-i\epsilon)}. \quad (28)$$

By keeping the corrections of $O(v\cdot r)$ one can obtain explicit model dependent expressions for the corrections to the spin symmetry relations Eq. (26). These corrections will not be listed in the present paper.

As an extra bonus of our dynamical treatment the $bb\to bc$ transition matrix element can be related to the $bc\to cc$ transition in the HQL. One simply has to replace m_{cc} by m_{bb} in the functional form of the universal function $\eta(w)$ in (27), i.e. in the functions $\bar{\Phi}_B(y_0)$ and $\bar{\Phi}_B(y_r)$ appearing in (28).

B. Calculation of the universal function $\eta(w)$

It turns out that one can derive a closed-form expression for the universal Isgur-Wise (IW) function $\eta(w)$ if one uses a Gaussian ansatz for the three-quark correlation function $\bar{\Phi}_B$ as has been done in Eq. (8). We use the Laplace transformation

$$\bar{\Phi}_B(z) = \int_0^\infty ds\,\Phi_B^L(s)e^{-sz} \quad (29)$$

and the integral representation

$$\exp\Big(-\frac{s_1s_2}{s_1+s_2}x\Big) = \frac{s_1+s_2}{\pi}\int_{-\infty}^\infty dt_1\int_{-\infty}^\infty dt_2\times\exp(-s_1((t_1+\sqrt{x})^2+t_2^2) - s_2(t_1^2+t_2^2)). \quad (30)$$

In terms of the variable w one obtains

$$J(w) = -\frac{2}{\pi}\int_0^\infty\int_0^\infty\int_0^\infty\frac{d\alpha_1\alpha_1d\alpha_2d\alpha_3}{\Delta^2}\Big(m_q+\frac{\alpha_1+\alpha_2}{4\Delta}\Big)\times\int_{-\infty}^\infty dt_1\int_{-\infty}^\infty dt_2\bar{\Phi}'_B(z_w)\bar{\Phi}_B(z_1), \quad (31)$$

where

$$\Delta = \frac{3}{4}+\alpha_3, \qquad \bar{\Phi}'_B(z) = d\bar{\Phi}_B(z)/dz,$$
$$z_w \equiv z(w) = \Big(t_1+m_{cc}\sqrt{\frac{w-1}{3}}\Big)^2 + t_2^2 + \frac{2}{3}\Big(\alpha_3m_q^2 + \frac{(1+\alpha_3)(\alpha_1^2+\alpha_1\alpha_2+\alpha_2^2)+\alpha_1\alpha_2\alpha_3}{4\Delta}\Big), \quad (32)$$

and where m_q is the mass of the light quark in the DHB. The integral (31) can be evaluated in closed form using the Gaussian ansatz for the correlation function Eq. (8). One obtains a rather simple form for the universal $\eta(w)$ function

$$\eta(w) = \exp\Big(-3(w-1)\frac{m_{cc}^2}{\Lambda_B^2}\Big). \quad (33)$$

The dependence on the light quark masses has disappeared due to cancellation effects between the numerator $J(w)$ and the denominator $J(1)$.

For the slope ρ^2 of $\eta(w)$ defined by $\eta(w) = 1 - \rho^2(w-1) + \ldots$ one obtains

$$\rho^2 = -\frac{d\eta(w)}{dw}\bigg|_{w=1} = 3\frac{m_{cc}^2}{\Lambda_B^2}. \tag{34}$$

As mentioned before, the HQL results for the $bb \to bc$ transitions can be obtained by the replacement $m_{cc} \to m_{bb}$ in the IW function. Accordingly, the slope of the IW function for the $bb \to bc$ transitions is obtained from (34) by the replacement $m_{cc} \to m_{bb}$, i.e. the slope increases by the factor m_{bb}^2/m_{cc}^2 when going from the $bc \to cc$ case to the $bb \to bc$ transition if one uses the same size parameter Λ_B in both cases. One should stress that there exists a spin-flavor symmetry at zero recoil $w = 1$ giving $\eta(1) \equiv 1$, which means that the $bc \to cc$ transition is identical to the $bb \to bc$ one. Close to zero recoil there exists only spin symmetry, because the IW functions for $bc \to cc$ and $bb \to bc$ transitions explicitly contain the flavor factors m_{cc} and m_{bb}, respectively.

IV. RESULTS

We now proceed to present our numerical results. We first present results on the semileptonic rates using finite heavy quark masses, i.e. we do not take the HQL for the transition form factors. In the finite mass case we present results for both the electron/muon mode and the τ mode. We then present results using the zero recoil form factors in Eq. (26) and the universal function $\eta(\omega)$ of Eq. (33), i.e. we extent the validity of the near-zero recoil approach to the whole kinematic range $0 \leq q^2 \leq (M_1 - M_2)^2$. Also, we present estimates of the width using the nonrelativistic quark model, in which, as described before, the wave functions have the same spin-flavor structure as our relativistic current considered in the nonrelativistic limit. Note, in the naive quark model we drop the q^2 dependence of the corresponding form factors and use only their values at $q^2 = 0$. We choose the Gaussian form Eq. (8) for the correlation function of the double-heavy baryons. Our results depend on the following set of parameters: the constituent quark masses and the size parameter Λ_B. The parameters have been taken from a fit to the properties of light, single and double-heavy baryons in previous analyses [12]. One has

$$\begin{array}{cccccc} m_{u(d)} & m_s & m_c & m_b & \Lambda_B & \\ 0.42 & 0.57 & 1.7 & 5.2 & 2.5\text{–}3.5 & \text{GeV} \end{array}. \tag{35}$$

All our analytical calculations have been done using the computer program FORM [19]. For the numerical evaluation we have used FORTRAN.

In Table III we present results on the $q^2 = 0$ values of the transition form factors F_1^V and F_1^A in the naive quark model. In Table IV we compare our finite mass results with the theoretical approaches of Refs. [4,9] for the electron/muon mode. As indicated in Tables IV and V we allow the size parameter Λ_B to vary in the range $2.5 \leq \Lambda_B \leq 3.5$ GeV. The variation of our rates listed in Tables IV and V reflect via error bars the variation of the size parameter Λ_B. Note that a smaller value of Λ_B gives smaller

TABLE III. $F_1^V(0)$ and $G_1^V(0)$ in the nonrelativistic quark model.

Quantity	$\Xi_{bc} \to \Xi_{cc}$ $\Xi_{bb} \to \Xi_{bc}$	$\Xi'_{bc} \to \Xi_{cc}$ $\Xi_{bb} \to \Xi'_{bc}$	$\Xi_{bc} \to \Xi^*_{cc}$ $\Xi^*_{bb} \to \Xi_{bc}$	$\Xi'_{bc} \to \Xi^*_{cc}$ $\Xi^*_{bb} \to \Xi'_{bc}$	$\Xi^*_{bc} \to \Xi_{cc}$ $\Xi_{bb} \to \Xi^*_{bc}$	$\Xi^*_{bc} \to \Xi^*_{cc}$ $\Xi^*_{bb} \to \Xi^*_{bc}$
$F_1^V(0)$	$\sqrt{2}$	0	0	0	0	$\sqrt{2}$
$F_1^A(0)$	$\frac{1}{\sqrt{2}}$	$\sqrt{\frac{2}{3}}$	$\frac{2}{3}$	$-\frac{2}{\sqrt{3}}$	$\frac{\sqrt{2}}{3}$	$\frac{\sqrt{2}}{3}$

TABLE IV. Semileptonic decay widths of double-heavy baryons in units of 10^{-14} GeV. Comparison with other approaches in case of light leptons (e, μ) in the final state.

Decay mode	Ref. [9]	Ref. [4]	Our results	Decay mode	Ref. [9]	Ref. [4]	Our results
$\Xi_{bb} \to \Xi_{bc}$	1.63	$1.92^{+0.25}_{-0.05}$	0.80 ± 0.30	$\Omega_{bb} \to \Omega_{bc}$	1.70	$2.14^{+0.20}_{-0.02}$	0.86 ± 0.32
$\Xi_{bc} \to \Xi_{cc}$	2.30	$2.57^{+0.26}_{-0.03}$	2.10 ± 0.70	$\Omega_{bc} \to \Omega_{cc}$	2.48	$2.59^{+0.20}$	1.88 ± 0.62
$\Xi'_{bc} \to \Xi_{cc}$	0.88	$1.36^{+0.10}_{-0.03}$	1.10 ± 0.32	$\Omega'_{bc} \to \Omega_{cc}$	0.95	$1.36^{+0.09}$	0.98 ± 0.28
$\Xi_{bb} \to \Xi'_{bc}$	0.82	$1.06^{+0.13}_{-0.03}$	0.43 ± 0.12	$\Omega_{bb} \to \Omega'_{bc}$	0.83	$1.16^{+0.13}$	0.48 ± 0.14
$\Xi_{bb} \to \Xi^*_{bc}$	0.53	$0.61^{+0.04}$	0.25 ± 0.07	$\Omega_{bb} \to \Omega^*_{bc}$	0.55	$0.67^{+0.08}$	0.29 ± 0.10
$\Xi_{bc} \to \Xi^*_{cc}$	0.72	$0.75^{+0.06}$	0.64 ± 0.19	$\Omega_{bc} \to \Omega^*_{cc}$	0.74	$0.76^{+0.13}$	0.62 ± 0.19
$\Xi'_{bc} \to \Xi^*_{cc}$	1.70	$2.33^{+0.16}$	2.01 ± 0.62	$\Omega'_{bc} \to \Omega^*_{cc}$	1.83	$2.36^{+0.33}$	1.93 ± 0.60
$\Xi^*_{bb} \to \Xi_{bc}$	0.28	$0.35^{+0.03}$	0.14 ± 0.04	$\Omega^*_{bb} \to \Omega_{bc}$	0.29	$0.38^{+0.04}_{-0.02}$	0.15 ± 0.05
$\Xi^*_{bc} \to \Xi_{cc}$	0.38	$0.43^{+0.06}$	0.30 ± 0.08	$\Omega^*_{bc} \to \Omega_{cc}$	0.40	$0.44^{+0.06}$	0.27 ± 0.07
$\Xi^*_{bb} \to \Xi'_{bc}$	0.82	$1.04^{+0.06}$	0.36 ± 0.10	$\Omega^*_{bb} \to \Omega'_{bc}$	0.85	$1.13^{+0.11}_{-0.08}$	0.42 ± 0.14
$\Xi^*_{bb} \to \Xi^*_{bc}$	1.92	$2.09^{+0.16}$	1.05 ± 0.40	$\Omega^*_{bb} \to \Omega^*_{bc}$	2.00	$2.29^{+0.31}_{-0.04}$	1.11 ± 0.44
$\Xi^*_{bc} \to \Xi^*_{cc}$	2.69	$2.63^{+0.40}$	2.66 ± 0.86	$\Omega^*_{bc} \to \Omega^*_{cc}$	2.88	$2.79^{+0.60}$	2.51 ± 0.81

TABLE V. Detailed analysis of semileptonic decay widths of double-heavy baryons in units of 10^{-14} GeV.

Decay mode	Exact results e, μ modes	Exact results τ mode	HQS limit	NQM
$\Xi_{bb} \to \Xi_{bc}$	0.80 ± 0.30	0.46 ± 0.13	1.33 ± 0.61	9.58
$\Omega_{bb} \to \Omega_{bc}$	0.86 ± 0.32	0.49 ± 0.14	1.92 ± 1.15	9.69
$\Xi_{bc} \to \Xi_{cc}$	2.10 ± 0.70	0.97 ± 0.22	4.01 ± 1.21	8.57
$\Omega_{bc} \to \Omega_{cc}$	1.88 ± 0.62	0.80 ± 0.16	4.12 ± 1.10	7.73
$\Xi'_{bc} \to \Xi_{cc}$	1.10 ± 0.32	0.46 ± 0.10	1.94 ± 0.50	4.74
$\Omega'_{bc} \to \Omega_{cc}$	0.98 ± 0.28	0.38 ± 0.08	1.96 ± 0.46	4.60
$\Xi_{bb} \to \Xi'_{bc}$	0.43 ± 0.12	0.20 ± 0.04	0.76 ± 0.30	5.74
$\Omega_{bb} \to \Omega'_{bc}$	0.48 ± 0.14	0.22 ± 0.05	0.81 ± 0.32	4.87
$\Xi_{bb} \to \Xi^*_{bc}$	0.25 ± 0.07	0.12 ± 0.02	0.61 ± 0.15	1.39
$\Omega_{bb} \to \Omega^*_{bc}$	0.29 ± 0.10	0.13 ± 0.03	0.57 ± 0.23	1.32
$\Xi_{bc} \to \Xi^*_{cc}$	0.64 ± 0.19	0.20 ± 0.03	1.39 ± 0.34	2.13
$\Omega_{bc} \to \Omega^*_{cc}$	0.62 ± 0.19	0.20 ± 0.04	1.78 ± 0.64	2.19
$\Xi'_{bc} \to \Xi^*_{cc}$	2.01 ± 0.62	0.65 ± 0.12	4.40 ± 0.99	6.68
$\Omega'_{bc} \to \Omega^*_{cc}$	1.93 ± 0.60	0.65 ± 0.13	4.61 ± 1.10	7.34
$\Xi^*_{bb} \to \Xi_{bc}$	0.14 ± 0.04	0.06 ± 0.02	0.25 ± 0.10	1.12
$\Omega^*_{bb} \to \Omega_{bc}$	0.15 ± 0.05	0.07 ± 0.02	0.26 ± 0.10	1.12
$\Xi^*_{bc} \to \Xi_{cc}$	0.30 ± 0.08	0.13 ± 0.02	0.58 ± 0.14	0.45
$\Omega^*_{bc} \to \Omega_{cc}$	0.27 ± 0.07	0.11 ± 0.02	0.59 ± 0.13	0.42
$\Xi^*_{bb} \to \Xi'_{bc}$	0.36 ± 0.10	0.17 ± 0.03	0.76 ± 0.30	3.08
$\Omega^*_{bb} \to \Omega'_{bc}$	0.42 ± 0.14	0.20 ± 0.05	0.80 ± 0.31	2.93
$\Xi^*_{bb} \to \Xi^*_{bc}$	1.05 ± 0.40	0.55 ± 0.16	1.62 ± 0.73	1.12
$\Omega^*_{bb} \to \Omega^*_{bc}$	1.11 ± 0.44	0.59 ± 0.18	1.72 ± 0.77	6.70
$\Xi^*_{bc} \to \Xi^*_{cc}$	2.66 ± 0.86	1.00 ± 0.21	4.63 ± 1.23	5.32
$\Omega^*_{bc} \to \Omega^*_{cc}$	2.51 ± 0.81	0.98 ± 0.20	4.95 ± 1.26	5.43

rates and *vice versa*. In Table V we present detailed results on semileptonic rates of double-heavy baryons in the exact finite mass approach for the electron/muon and tau-lepton modes. These are compared to the corresponding results in the heavy quark spin symmetry limit and in the naive quark model. One can see that the naive quark model gives larger rates due to the omission of form factor effects that would result from the nonlocal structure of semileptonic transitions. The results in the heavy quark symmetry (HQS) limit are calculated using the IW function (33) with $m_{cc} = 2m_c = 3.4$ GeV for the $bc \to cc$ transitions and $m_{bb} = 2m_b = 10.4$ GeV for the $bb \to bc$ transitions.

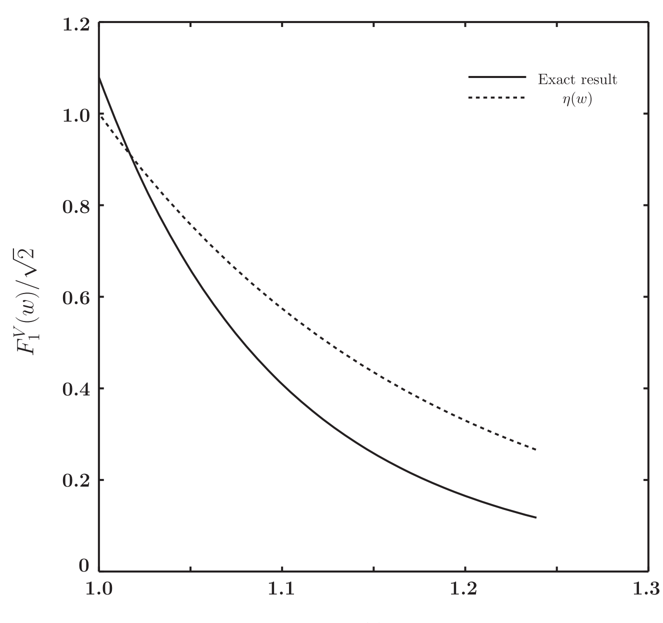

FIG. 1. Form factor $F_1^V/\sqrt{2}$ describing the $\Xi_{bc} \to \Xi_{cc}$ transitions: exact result and IW function $\eta(w)$.

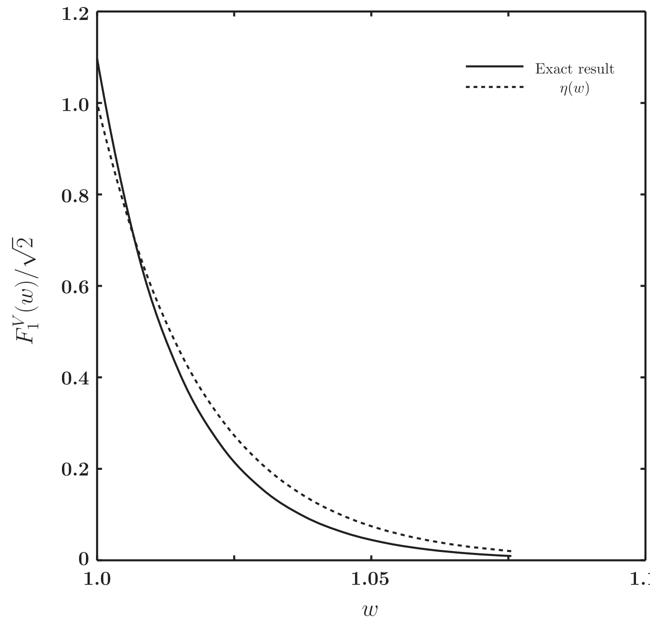

FIG. 2. Form factor $F_1^V/\sqrt{2}$ describing the $\Xi_{bb} \to \Xi_{bc}$ transitions: exact result and IW function $\eta(w)$.

In order to check on the quality of the HQS limit in Figs. 1 and 2 we show plots of the ω dependence of the form factor $F_1^V(\omega)/\sqrt{2}$ for the exact finite mass result and the HQS result using the IW function $\eta(\omega)$. As examples we take the two transitions $\Xi_{bc} \to \Xi_{cc}$ and $\Xi_{bb} \to \Xi_{bc}$. From Table V and Figs. 1 and 2 one can see that HQS limit is better justified for $bb \to bc$ transitions, while there is a larger difference between the finite mass results and the HQS results for the $bc \to cc$ mode as is evident in Fig. 1 and Table V.

V. SUMMARY

In this paper we have analyzed the semileptonic decay of double-heavy baryons using a manifestly Lorentz covariant constituent quark model approach. Our main results are summarized as follows:

(i) We have derived results for the matrix elements of the semileptonic decays of double-heavy baryons for finite values of the heavy quark/baryon masses and for the HQS limit, which is valid at and close to zero recoil;

(ii) We have presented a detailed numerical analysis of the decay rates for the two (e/μ)- and τ-lepton modes in the exact finite mass approach and in the HQS limit;

(iii) We have compared our results with the predictions of other theoretical approaches.

We hope that the results of this paper can be used to extract the value of the Cabibbo-Kobayashi-Maskawa matrix element V_{cb} from future experiments on the semileptonic decays of double-heavy baryons.

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APPENDIX A: NONRELATIVISTIC QUARK MODEL: SPIN-FLAVOR WAVE FUNCTIONS AND SEMILEPTONIC DECAY CONSTANTS OF DOUBLE-HEAVY BARYONS

In this Appendix we present results on the $q^2 = 0$ values of the double-heavy transition form factors $F_1^V(0)$ and $F_1^A(0)$ in the nonrelativistic quark model. As emphasized before the nonrelativistic quark model is based on the spin-flavor wave functions, which arise in the nonrelativistic limit of the relativistically covariant double-heavy three-quark currents with quantum numbers $J^P = 1/2^+$ and $3/2^+$. The corresponding quark model spin-flavor wave functions are given in Table II, where we use the following notation for the antisymmetric χ_A and symmetric (χ_S, χ_S^*) spin wave functions with $S_z = +1/2$ (spin projection on z axis):

$$\chi_A = \sqrt{\frac{1}{2}}\{\uparrow(\uparrow\downarrow - \downarrow\uparrow)\}, \qquad \chi_S = \sqrt{\frac{1}{6}}\{\uparrow(\uparrow\downarrow + \downarrow\uparrow) - 2\downarrow\uparrow\uparrow\},$$
$$\chi_S^* = \sqrt{\frac{1}{3}}\{\uparrow\uparrow\downarrow + \uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow\}. \tag{A1}$$

We derive the expressions for the semileptonic decay constants $F_1^V(0)$ and $F_1^A(0)$ of double-heavy baryons using the master formulas:

$$F_1^V(0) = \langle B'| \sum_{i=1}^{3} [I_{bc}]^{(i)} |B\rangle$$
$$\text{and } F_1^A(0) = \langle B'| \sum_{i=1}^{3} [\sigma_3 I_{bc}]^{(i)} |B\rangle,$$

where σ_3 is the z component of the Pauli spin matrix and I_{bc} is the flavor matrix responsible for the $b \to c$ semileptonic transitions. In Table III we list the results for the $\Xi_{Q_1Q_2}$-type baryon decay modes. Corresponding results for the $\Omega_{Q_1Q_2}$-type baryon decay modes can be obtained by replacing the light nonstrange quark q by the strange quark s.

APPENDIX B: SPIN KINEMATICS OF SEMILEPTONIC DECAYS

In this Appendix we first write down covariant expressions for the current-induced transitions involving the $(1/2^+)$ and $(3/2^+)$ baryons, which allows us to define sets of vector and axial vector invariant transition form factors. We then calculate all helicity amplitudes expressed in terms of linear combinations of the invariant form factors. The advantage of using helicity amplitudes is that one obtains very compact expressions for the decay rates including lepton mass effects. We mention that the use of helicity amplitudes allows one also to derive very compact expressions for single angular decay distributions and for joint angular decay distributions of the decay products (see e.g. [20,21]).

The momenta and masses in the semileptonic decays of double-heavy baryons are denoted by

$$B_1(p_1, M_1) \to B_2(p_1, M_2) + l(p_l, m_l) + \nu_l(p_\nu, 0), \tag{B1}$$

where $p_1 = p_2 + q$ and $q = p_l + p_\nu$. The matrix elements of the vector and axial vector currents $J_\mu^{V(A)}$ between the baryon states with spin 1/2 or 3/2 are written as Transition $1/2^+ \to 1/2^+$:

$$M_\mu^V = \langle B_2|J_\mu^V|B_1\rangle = \bar{u}(p_2, s_2)\left[\gamma_\mu F_1^V(q^2) - i\sigma_{\mu\nu}\frac{q_\nu}{M_1}F_2^V(q^2) + \frac{q_\mu}{M_1}F_3^V(q^2)\right]u(p_1, s_1), \tag{B2a}$$

$$M_\mu^A = \langle B_2|J_\mu^A|B_1\rangle = \bar{u}(p_2, s_2)\left[\gamma_\mu F_1^A(q^2) - i\sigma_{\mu\nu}\frac{q_\nu}{M_1}F_2^A(q^2) + \frac{q_\mu}{M_1}F_3^A(q^2)\right]\gamma_5 u(p_1, s_1). \tag{B2b}$$

Transition $1/2^+ \to 3/2^+$:

$$M^V_\mu = \langle B_2^*|J^V_\mu|B_1\rangle = \bar u^\alpha(p_2,s_2)\bigg[g_{\alpha\mu}F^V_1(q^2) + \gamma_\mu\frac{p_{1\alpha}}{M_1}F^V_2(q^2) + \frac{p_{1\alpha}p_{2\mu}}{M_1^2}F^V_3(q^2) + \frac{p_{1\alpha}q_\mu}{M_1^2}F^V_4(q^2)\bigg]\gamma_5 u(p_1,s_1), \tag{B3a}$$

$$M^A_\mu = \langle B_2^*|J^A_\mu|B_1\rangle = \bar u^\alpha(p_2,s_2)\bigg[g_{\alpha\mu}F^A_1(q^2) + \gamma_\mu\frac{p_{1\alpha}}{M_1}F^A_2(q^2) + \frac{p_{1\alpha}p_{2\mu}}{M_1^2}F^A_3(q^2) + \frac{p_{1\alpha}q_\mu}{M_1^2}F^A_4(q^2)\bigg]u(p_1,s_1). \tag{B3b}$$

Transition $3/2^+ \to 1/2^+$:

$$M^V_\mu = \langle B_2|J^V_\mu|B_1^*\rangle = \bar u(p_2,s_2)\bigg[g_{\alpha\mu}F^V_1(q^2) + \gamma_\mu\frac{p_{2\alpha}}{M_2}F^V_2(q^2) + \frac{p_{2\alpha}p_{1\mu}}{M_2^2}F^V_3(q^2) + \frac{p_{2\alpha}q_\mu}{M_2^2}F^V_4(q^2)\bigg]\gamma_5 u^\alpha(p_1,s_1), \tag{B4a}$$

$$M^A_\mu = \langle B_2|J^A_\mu|B_1^*\rangle = \bar u(p_2,s_2)\bigg[g_{\alpha\mu}F^A_1(q^2) + \gamma_\mu\frac{p_{2\alpha}}{M_2}F^A_2(q^2) + \frac{p_{2\alpha}p_{1\mu}}{M_2^2}F^A_3(q^2) + \frac{p_{2\alpha}q_\mu}{M_2^2}F^A_4(q^2)\bigg]u^\alpha(p_1,s_1). \tag{B4b}$$

Transition $3/2^+ \to 3/2^+$:

$$\begin{aligned} M^V_\mu = \langle B_2^*|J^V_\mu|B_1^*\rangle = \bar u^\alpha(p_2,s_2)\bigg[& g_{\alpha\beta}\bigg(\gamma_\mu F^V_1(q^2) - i\sigma_{\mu\nu}\frac{q_\nu}{M_1}F^V_2(q^2) + \frac{q_\mu}{M_1}F^V_3(q^2)\bigg) \\ & + \frac{q_\alpha q_\beta}{M_1^2}\bigg(\gamma_\mu F^V_4(q^2) - i\sigma_{\mu\nu}\frac{q_\nu}{M_1}F^V_5(q^2) + \frac{q_\mu}{M_1}F^V_6(q^2)\bigg) + \frac{g_{\alpha\mu}q_\beta - g_{\beta\mu}q_\alpha}{M_1}F^V_7(q^2)\bigg]u^\beta(p_1,s_1), \end{aligned} \tag{B5a}$$

$$\begin{aligned} M^A_\mu = \langle B_2^*|J^A_\mu|B_1^*\rangle = \bar u^\alpha(p_2,s_2)\bigg[& g_{\alpha\beta}\bigg(\gamma_\mu F^A_1(q^2) - i\sigma_{\mu\nu}\frac{q_\nu}{M_1}F^A_2(q^2) + \frac{q_\mu}{M_1}F^A_3(q^2)\bigg) \\ & + \frac{q_\alpha q_\beta}{M_1^2}\bigg(\gamma_\mu F^A_4(q^2) - i\sigma_{\mu\nu}\frac{q_\nu}{M_1}F^A_5(q^2) + \frac{q_\mu}{M_1}F^A_6(q^2)\bigg) + \frac{g_{\alpha\mu}q_\beta - g_{\beta\mu}q_\alpha}{M_1}F^A_7(q^2)\bigg]\gamma_5 u^\beta(p_1,s_1), \end{aligned} \tag{B5b}$$

where $\sigma_{\mu\nu} = (i/2)(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)$ and all γ matrices are defined as in Bjorken-Drell.

Next we express the vector and axial helicity amplitudes $H^{V,A}_{\lambda_2\lambda_W}$ in terms of the invariant form factors $F^{V,A}_i$, where $\lambda_W = t$, ±1, 0 and $\lambda_2 = \pm 1/2$, $\pm 3/2$ are the helicity components of the $W_{\text{off-shell}}$ and the daughter baryon, respectively. Since lepton mass effects are taken into account in this paper we need to retain the temporal component "t" of the four-currents $J^{V,A}_\mu$. We need to calculate the expressions

$$H^{V,A}_{\lambda_2\lambda_W} = M^{V,A}_\mu(\lambda_2)\bar\epsilon^{*\mu}(\lambda_W). \tag{B6}$$

Note that the helicity of the parent baryon (λ_1) is fixed by the relation $\lambda_1 = \lambda_2 - \lambda_W$. We shall work in the rest frame of the parent baryon B_1 with the daughter baryon B_2 moving in the positive z direction: $p_1 = (M_1, \vec{\mathbf{0}})$, $p_2 = (E_2, 0, 0, |\mathbf{p}_2|)$, and $q = (q_0, 0, 0, -|\mathbf{p}_2|)$, where $E_2 = Q_+/(2M_1)$, $|\mathbf{p}_2| = \sqrt{Q_+Q_-}/(2M_1)$, $q_0 = M_1 - E_2$, and $Q_\pm = (M_1 \pm M_2)^2 - q^2$.

The $J = 1/2$ baryon spinors are given by

$$\begin{aligned} \bar u_2\bigg(p_2, \pm\frac{1}{2}\bigg) &= \sqrt{E_2+M_2}\bigg(\chi^\dagger_\pm, \frac{\mp|\mathbf{p}_2|}{E_2+M_2}\chi^\dagger_\pm\bigg), \\ u_1\bigg(p_1, \pm\frac{1}{2}\bigg) &= \sqrt{2M_1}\begin{pmatrix}\chi_\pm \\ 0\end{pmatrix}, \end{aligned} \tag{B7}$$

where

$$\chi_+ = \begin{pmatrix}1\\0\end{pmatrix}$$

and

$$\chi_- = \begin{pmatrix}0\\1\end{pmatrix}$$

are two-component Pauli spinors.

The $J = \frac{3}{2}$ baryon spinors are defined as

$$u_\mu(p, s^*) = \sum_{\lambda,s}\bigg\langle 1\lambda\frac{1}{2}s\,\bigg|\,\frac{3}{2}s^*\bigg\rangle\epsilon_\mu(p,\lambda)u(p,s), \tag{B8}$$

where $\langle 1\lambda 1/2 s|3/2 s^*\rangle$ is the projection matrix element of the spin 3/2 onto spin 1/2; $\epsilon_\mu(p,\lambda)$ is the polarization vector and $u(p,s)$ are the usual $J = 1/2$ spinors defined above. In particular, the $J = 3/2$ spinors with helicities $\lambda = \pm 3/2$, $\pm 1/2$ read

$$\begin{aligned} u_\mu\bigg(p, \pm\frac{3}{2}\bigg) &= \epsilon_\mu(p,\pm1)u\bigg(p,\pm\frac{1}{2}\bigg), \\ u_\mu\bigg(p, \pm\frac{1}{2}\bigg) &= \sqrt{\frac{2}{3}}\epsilon_\mu(p,0)u\bigg(p,\pm\frac{1}{2}\bigg) \\ &\quad + \sqrt{\frac{1}{3}}\epsilon_\mu(p,\pm1)u\bigg(p,\mp\frac{1}{2}\bigg). \end{aligned} \tag{B9}$$

The polarization vectors corresponding to the parent and daughter $J = 3/2$ baryons are given by:

$$\begin{aligned} &\epsilon(p_1,0) = (0,0,0,1), \quad \epsilon(p_1,\pm1) = \frac{1}{\sqrt{2}}(0,\mp1,-i,0), \\ &\epsilon^*(p_2,0) = \frac{1}{M_2}(|\mathbf{p}_2|,0,0,E_2), \quad \epsilon^*(p_2,\pm1) = \frac{1}{\sqrt{2}}(0,\mp1,i,0). \end{aligned} \tag{B10}$$

The polarization vectors of the $W_{\text{off-shell}}$ are written as

$$\bar{\epsilon}^{*\mu}(t) = \frac{1}{\sqrt{q^2}}(q_0, 0, 0, -|\mathbf{p}_2|),$$
$$\bar{\epsilon}^{*\mu}(\pm 1) = \frac{1}{\sqrt{2}}(0, \pm 1, i, 0), \tag{B11}$$
$$\bar{\epsilon}^{*\mu}(0) = \frac{1}{\sqrt{q^2}}(|\mathbf{p}_2|, 0, 0, q_0).$$

They satisfy the conditions

$$q_\mu \bar{\epsilon}^{*\mu}(\pm 1, 0) = 0, \qquad q_\mu \bar{\epsilon}^{*\mu}(t) = \sqrt{q^2}. \tag{B12}$$

Using the above formulas for the spin wave functions with definite helicities one can then calculate the helicity amplitudes $H_{\lambda_2\lambda_W} = H^V_{\lambda_2\lambda_W} - H^A_{\lambda_2\lambda_W}$, where the vector and axial components for the different spin transitions are defined by: Transition $1/2^+ \to 1/2^+$:

$$\begin{aligned} H^V_{\frac{1}{2}t} &= \alpha^V_{\frac{1}{2}t}\left(F^V_1 M_- + F^V_3 \frac{q^2}{M_1}\right), \qquad H^V_{\frac{1}{2}0} = \alpha^V_{\frac{1}{2}0}\left(F^V_1 M_+ + F^V_2 \frac{q^2}{M_1}\right), \qquad H^V_{\frac{1}{2}1} = \alpha^V_{\frac{1}{2}1}\left(-F^V_1 - F^V_2 \frac{M_+}{M_1}\right), \\ H^A_{\frac{1}{2}t} &= \alpha^A_{\frac{1}{2}t}\left(F^A_1 M_+ - F^A_3 \frac{q^2}{M_1}\right), \qquad H^A_{\frac{1}{2}0} = \alpha^A_{\frac{1}{2}0} c\left(F^A_1 M_- - F^A_2 \frac{q^2}{M_1}\right), \qquad H^A_{\frac{1}{2}0} = \alpha^A_{\frac{1}{2}1}\left(-F^A_1 + F^A_2 \frac{M_-}{M_1}\right), \end{aligned} \tag{B13}$$

where

$$\begin{aligned} &M_\pm = M_1 \pm M_2, \qquad q^2 = 2M_1M_2(w_{\max} - w), \qquad w = \frac{p_1 p_2}{M_1 M_2} = \frac{M_1^2 + M_2^2 - q^2}{2M_1M_2}, \qquad w_{\max} = \frac{M_1^2 + M_2^2}{2M_1M_2}, \\ &\alpha^V_{\frac{1}{2}t} = \alpha^A_{\frac{1}{2}0} = \sqrt{\frac{2M_1M_2(w+1)}{q^2}}, \qquad \alpha^V_{\frac{1}{2}0} = \alpha^A_{\frac{1}{2}t} = \sqrt{\frac{2M_1M_2(w-1)}{q^2}}, \\ &\alpha^V_{\frac{1}{2}1} = 2\sqrt{M_1M_2(w-1)}, \qquad \alpha^A_{\frac{1}{2}1} = 2\sqrt{M_1M_2(w+1)}. \end{aligned} \tag{B14}$$

From parity or from explicit calculations one has for the $1/2^+ \to 1/2^+$ helicity amplitudes

$$H^V_{-\lambda_2,-\lambda_W} = H^V_{\lambda_2,\lambda_W}, \qquad H^A_{-\lambda_2,-\lambda_W} = -H^A_{\lambda_2,\lambda_W}. \tag{B15}$$

Transition $1/2^+ \to 3/2^+$:

$$\begin{aligned} H^V_{\frac{1}{2}t} &= -\sqrt{\frac{2}{3}}\alpha^V_{\frac{1}{2}t}(w-1)\left(F^V_1 M_1 - F^V_2 M_+ + F^V_3 \frac{M_2}{M_1}(M_1 w - M_2) + F^V_4 \frac{q^2}{M_1}\right), \\ H^V_{\frac{1}{2}0} &= -\sqrt{\frac{2}{3}}\alpha^V_{\frac{1}{2}0}(F^V_1(M_1 w - M_2) - F^V_2(w+1)M_- + F^V_3(w^2-1)M_2), \\ H^V_{\frac{1}{2}1} &= \frac{1}{\sqrt{6}}\alpha^V_{\frac{1}{2}1}(F^V_1 - 2F^V_2(w+1)), \qquad H^V_{\frac{3}{2}1} = -\frac{1}{\sqrt{2}}\alpha^V_{\frac{1}{2}1}F^V_1, \\ H^A_{\frac{1}{2}t} &= \sqrt{\frac{2}{3}}\alpha^A_{\frac{1}{2}t}(w+1)\left(F^A_1 M_1 + F^A_2 M_- + F^A_3 \frac{M_2}{M_1}(M_1 w - M_2) + F^A_4 \frac{q^2}{M_1}\right), \\ H^A_{\frac{1}{2}0} &= \sqrt{\frac{2}{3}}\alpha^A_{\frac{1}{2}0}(F^A_1(M_1 w - M_2) + F^A_2(w-1)M_+ + F^A_3(w^2-1)M_2), \\ H^A_{\frac{1}{2}1} &= \frac{1}{\sqrt{6}}\alpha^A_{\frac{1}{2}1}(F^A_1 - 2F^A_2(w-1)), \qquad H^A_{\frac{3}{2}1} = \frac{1}{\sqrt{2}}\alpha^A_{\frac{1}{2}1}F^A_1. \end{aligned} \tag{B16}$$

From parity or from explicit calculations one has for the $1/2^+ \to 1/3^+$ helicity amplitudes

$$H^V_{-\lambda_2,-\lambda_W} = -H^V_{\lambda_2,\lambda_W}, \qquad H^A_{-\lambda_2,-\lambda_W} = H^A_{\lambda_2,\lambda_W}. \tag{B17}$$

Transition $3/2^+ \to 1/2^+$:

$$
\begin{aligned}
&H^V_{\frac12 t} = -\sqrt{\frac{2}{3}}\alpha^V_{\frac12 t}(w-1)\bigg(F_1^V M_2 + F_2^V M_+ - F_3^V \frac{M_1}{M_2}(M_1 - M_2 w) - F_4^V \frac{q^2}{M_2}\bigg),\\
&H^V_{\frac12 0} = -\sqrt{\frac{2}{3}}\alpha^V_{\frac12 0}(F_1^V(M_1 - M_2 w) + F_2^V(w+1)M_- - F_3^V(w^2-1)M_1),\\
&H^V_{\frac12 1} = \frac{1}{\sqrt{6}}\alpha^V_{\frac12 1}(F_1^V + 2F_2^V(w+1)), \qquad H^V_{\frac12 -1} = \frac{1}{\sqrt{2}}\alpha^V_{\frac12 1}F_1^V,\\
&H^A_{\frac12 t} = \sqrt{\frac{2}{3}}\alpha^A_{\frac12 t}(w+1)\bigg(F_1^A M_2 - F_2^A M_- - F_3^A \frac{M_1}{M_2}(M_1 - M_2 w) - F_4^A \frac{q^2}{M_2}\bigg),\\
&H^A_{\frac12 0} = -\sqrt{\frac{2}{3}}\alpha^A_{\frac12 0}(-F_1^A(M_1 - M_2 w) + F_2^A(w-1)M_+ + F_3^A(w^2-1)M_1),\\
&H^A_{\frac12 1} = \frac{1}{\sqrt{6}}\alpha^A_{\frac12 1}(-F_1^A + 2F_2^A(w-1)), \qquad H^A_{\frac32 1} = -\frac{1}{\sqrt{2}}\alpha^A_{\frac12 1}F_1^A,
\end{aligned}
\tag{B18}
$$

From parity or from explicit calculations one has for the $\frac{3}{2}^+ \to \frac{1}{2}^+$ helicity amplitudes

$$
H^V_{-\lambda_2,-\lambda_W} = -H^V_{\lambda_2,\lambda_W}, \qquad H^A_{-\lambda_2,-\lambda_W} = H^A_{\lambda_2,\lambda_W}. \tag{B19}
$$

Transition $3/2^+ \to 3/2^+$:

$$
\begin{aligned}
&H^V_{\frac12 t} = -\frac{1+2w}{3}H^V_{\frac12 t}(F_1^V, F_2^V, F_3^V) + \frac{2}{3}(w^2-1)\frac{M_2}{M_1}H^V_{\frac12 t}(F_4^V, F_5^V, F_6^V),\\
&H^V_{\frac12 0} = -\frac{1+2w}{3}H^V_{\frac12 0}(F_1^V, F_2^V, F_3^V) + \frac{2}{3}(w^2-1)\frac{M_2}{M_1}H^V_{\frac12 0}(F_4^V, F_5^V, F_6^V) - \frac{1}{3}\alpha^A_{\frac12 1}(w+1)\frac{\sqrt{2q^2}}{M_1}F_7^A,\\
&H^V_{\frac12 1} = -\frac{2w}{3}H^V_{\frac12 1}(F_1^V, F_2^V, F_3^V) + \frac{2}{3}(w^2-1)\frac{M_2}{M_1}H^V_{\frac12 1}(F_4^V, F_5^V, F_6^V) + \frac{1}{3}\alpha^V_{\frac12 1}(w+1)\frac{M_+}{M_1}F_7^V,\\
&H^V_{\frac32 1} = -\frac{1}{\sqrt{3}}H^V_{\frac12 1}(F_1^V, F_2^V, F_3^V) + \frac{1}{\sqrt{3}}\alpha^V_{\frac12 1}(w+1)\frac{M_2}{M_1}F_7^V, \qquad H^V_{\frac12 -1} = -\frac{1}{\sqrt{3}}H^V_{\frac12 1}(F_1^V, F_2^V, F_3^V) + \frac{1}{\sqrt{3}}\alpha^V_{\frac12 1}(w+1)F_7^V,\\
&H^V_{\frac32 t} = -H^V_{\frac12 t}(F_1^V, F_2^V, F_3^V), \qquad H^V_{\frac32 0} = -H^V_{\frac12 0}(F_1^V, F_2^V, F_3^V),\\
&H^A_{\frac12 t} = \frac{1-2w}{3}H^A_{\frac12 t}(F_1^V, F_2^V, F_3^V) + \frac{2}{3}(w^2-1)\frac{M_2}{M_1}H^A_{\frac12 t}(F_4^A, F_5^A, F_6^A),\\
&H^A_{\frac12 0} = \frac{1-2w}{3}H^A_{(1/2)0}(F_1^A, F_2^A, F_3^A) + \frac{2}{3}(w^2-1)\frac{M_2}{M_1}H^A_{\frac12 0}(F_4^A, F_5^A, F_6^A) + \frac{1}{3}\alpha^A_{\frac12 1}(w-1)\frac{\sqrt{2q^2}}{M_1}F_7^A,\\
&H^A_{\frac12 1} = -\frac{2w}{3}H^A_{\frac12 1}(F_1^A, F_2^A, F_3^A) + \frac{2}{3}(w^2-1)\frac{M_2}{M_1}H^A_{\frac12 1}(F_4^A, F_5^A, F_6^A) - \frac{1}{3}\alpha^A_{\frac12 1}(w-1)\frac{M_-}{M_1}F_7^A,\\
&H^A_{\frac32 1} = -\frac{1}{\sqrt{3}}H^A_{\frac12 1}(F_1^A, F_2^A, F_3^A) - \frac{1}{\sqrt{3}}\alpha^A_{\frac12 1}(w-1)\frac{M_2}{M_1}F_7^A, \qquad H^A_{\frac12 -1} = \frac{1}{\sqrt{3}}H^A_{\frac12 1}(F_1^A, F_2^A, F_3^A) - \frac{1}{\sqrt{3}}\alpha^A_{\frac12 1}(w-1)F_7^A,\\
&H^A_{\frac32 t} = -H^A_{\frac12 t}(F_1^A, F_2^A, F_3^A), \qquad H^A_{\frac32 0} = -H^A_{\frac12 0}(F_1^A, F_2^A, F_3^A),
\end{aligned}
\tag{B20}
$$

where

$$
\begin{aligned}
&H^V_{\frac12 t}(x,y,z) = \alpha^V_{\frac12 t}\bigg(xM_- + z\frac{q^2}{M_1}\bigg), \qquad H^V_{\frac12 0}(x,y,z) = \alpha^V_{\frac12 0}\bigg(xM_+ + y\frac{q^2}{M_1}\bigg), \qquad H^V_{\frac12 1}(x,y,z) = -\alpha^V_{\frac12 1}\bigg(x + y\frac{M_+}{M_1}\bigg),\\
&H^A_{\frac12 t}(x,y,z) = \alpha^A_{\frac12 t}\bigg(xM_+ - z\frac{q^2}{M_1}\bigg), \qquad H^A_{\frac12 0}(x,y,z) = \alpha^A_{\frac12 0}\bigg(xM_- - y\frac{q^2}{M_1}\bigg), \qquad H^A_{\frac12 1}(x,y,z) = \alpha^A_{\frac12 1}\bigg(-x + y\frac{M_-}{M_1}\bigg).
\end{aligned}
\tag{B21}
$$

From parity or from explicit calculations one has for the $3/2^+ \to 3/2^+$ helicity amplitudes

$$
H^V_{-\lambda_2,-\lambda_W} = H^V_{\lambda_2,\lambda_W}, \qquad H^A_{-\lambda_2,-\lambda_W} = -H^A_{\lambda_2,\lambda_W}. \tag{B22}
$$

The decay width is given by the expression

$$\Gamma_{s_1\to s_2} = N_{s_1 s_2}\frac{G_F^2|V_{\rm CKM}|^2}{192\pi^3}\frac{M_2}{M_1^2}\int_{m_l^2}^{M_-^2}\frac{dq^2}{q^2}(q^2-m_l^2)^2\sqrt{w^2-1}\mathcal{H}_{s_1\to s_2} \tag{B23}$$

where $N_{s_1s_2} = 1$ for $s_1 = 1/2$ and $1/2$ for $s_1 = 3/2$; $\mathcal{H}_{s_1\to s_2}$ are the bilinear combinations of the helicity amplitudes

$$\mathcal{H}_{\frac12\to\frac12} = |H_{(1/2)1}|^2 + |H_{-\frac12-1}|^2 + |H_{\frac12 0}|^2 + |H_{-\frac12 0}|^2 + \frac{m_l^2}{2q^2}(3|H_{\frac12 t}|^2 + 3|H_{-\frac12 t}|^2 + |H_{\frac12 1}|^2 + |H_{-\frac12-1}|^2 + |H_{\frac12 0}|^2 + |H_{-\frac12 0}|^2), \tag{B24a}$$

$$\mathcal{H}_{\frac12\to\frac32} = |H_{\frac12 1}|^2 + |H_{-\frac12-1}|^2 + |H_{\frac32 1}|^2 + |H_{-\frac32-1}|^2 + |H_{\frac12 0}|^2 + |H_{-\frac12 0}|^2 + \frac{m_l^2}{2q^2}(3|H_{\frac12 t}|^2 + 3|H_{-\frac12 t}|^2 + |H_{\frac12 1}|^2 + |H_{-\frac12-1}|^2 + |H_{\frac32 1}|^2 + |H_{-\frac32-1}|^2 + |H_{\frac12 0}|^2 + |H_{-\frac12 0}|^2), \tag{B24b}$$

$$\mathcal{H}_{\frac32\to\frac12} = |H_{\frac12 1}|^2 + |H_{-\frac12-1}|^2 + |H_{\frac12-1}|^2 + |H_{-\frac12 1}|^2 + |H_{\frac12 0}|^2 + |H_{-\frac12 0}|^2 + \frac{m_l^2}{2q^2}(3|H_{\frac12 t}|^2 + 3|H_{-\frac12 t}|^2 + |H_{\frac12 1}|^2 + |H_{-\frac12-1}|^2 + |H_{\frac12-1}|^2 + |H_{-\frac12 1}|^2 + |H_{\frac12 0}|^2 + |H_{-\frac12 0}|^2), \tag{B24c}$$

$$\mathcal{H}_{\frac32\to\frac32} = |H_{\frac12 1}|^2 + |H_{-\frac12-1}|^2 + |H_{\frac32 1}|^2 + |H_{-\frac32-1}|^2 + |H_{\frac12-1}|^2 + |H_{-\frac12 1}|^2 + |H_{\frac12 0}|^2 + |H_{-\frac12 0}|^2 + |H_{\frac32 0}|^2 + |H_{-\frac32 0}|^2 + \frac{m_l^2}{2q^2}(3|H_{\frac12 t}|^2 + 3|H_{-\frac12 t}|^2 + 3|H_{\frac32 t}|^2 + 3|H_{-\frac32 t}|^2 + |H_{\frac12 1}|^2 + |H_{-\frac12-1}|^2 + |H_{\frac32 1}|^2 + |H_{-\frac32-1}|^2 + |H_{\frac32 0}|^2 + |H_{-\frac32 0}|^2 + |H_{\frac12-1}|^2 + |H_{-\frac12 1}|^2 + |H_{\frac12 0}|^2 + |H_{-\frac12 0}|^2). \tag{B24d}$$

In the zero recoil limit the expressions for the rates simplify considerably. One obtains

$$\Gamma_{\frac12\to\frac12} = \beta_W\int_1^{w_{\rm max}} dw\sqrt{w^2-1}(l_1^+(w)F^2(w) + l_1^-(w)G^2(w)), \tag{B25a}$$

$$\Gamma_{\frac12\to\frac32} = \beta_W\int_1^{w_{\rm max}} dw\sqrt{w^2-1}\,l_2(w)G^2(w), \tag{B25b}$$

$$\Gamma_{\frac32\to\frac12} = \frac12\beta_W\int_1^{w_{\rm max}} dw\sqrt{w^2-1}\,l_3(w)G^2(w), \tag{B25c}$$

$$\Gamma_{\frac32\to\frac12} = \frac12\beta_W\int_1^{w_{\rm max}} dw\sqrt{w^2-1}(l_4^+(w)F^2(w) + l_4^-(w)G^2(w)), \tag{B25d}$$

where $F = F_1^V$, $G = F_1^A$, $r = M_2/M_1$, $w = (1+r^2)/(2r)$ and

$$\beta_W = \frac{G_F^2|V_{\rm CKM}|^2}{12\pi^3}M_1^5 r^4, \qquad l_1^\pm(w) = (w\mp 1)(3w_{\rm max} \pm 1 - 2w), \qquad l_2(w) = 2(w+1)\left(w_{\rm max} - w + \frac{w^2-1}{6r}\right),$$
$$l_3(w) = 2(w+1)\left(w_{\rm max} - w + \frac{(w^2-1)r}{6}\right), \qquad l_4^\pm(w) = \frac49\left(l_1^\pm(w)\frac{3+2w^2}{2} \pm (w_{\rm max}\pm 1)(w^2-1)\right). \tag{B26}$$

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