

**$J/\psi$  polarization in photoproduction up to the next-to-leading order of QCD**Chao-Hsi Chang,<sup>1,2,3</sup> Rong Li,<sup>1,3</sup> and Jian-Xiong Wang<sup>1,3</sup><sup>1</sup>*Institute of High Energy Physics, Chinese Academy of Sciences, P.O. Box 918(4), Beijing, 100049, China*<sup>2</sup>*Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing, 100080, China*<sup>3</sup>*Theoretical Physics Center for Science Facilities, Chinese Academy of Sciences, Beijing, 100049, China*

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We investigate the  $J/\psi$  polarization in photoproduction at the Hadron Electron Ring Accelerator (HERA) up to the next-to-leading order of QCD. The results show that the transverse momentum  $p_t$  and energy fraction  $z$  distributions of  $J/\psi$  production do not agree with the observed ones very well. The theoretical uncertainties for the  $z$  distributions of the  $J/\psi$  polarization parameters with respect to various choices of the renormalization and factorization scales are too large to give a definite predication. The uncertainties for the  $p_t$  distributions of these parameters are small when  $p_t > 3$  GeV and the obtained  $p_t$  distributions cannot describe the experimental data even in this region.

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The heavy-quarkonium systems ( $c\bar{c}$ ) and ( $b\bar{b}$ ), especially  $J/\psi$  and  $\Upsilon$ , being flavor hidden, have received much attention from both experimental and theoretical sides, since they were discovered. Their heavy masses set a “large” scale that makes the effective field theory, non-relativistic quantum chromodynamics (NRQCD) [1] applicable for their production and decay processes. In terms of factorization, NRQCD manages the expansion on the strong coupling constant  $\alpha_s$  in the hard part and the  $v$  (velocity between the heavy quark and antiquark in quarkonium) in the soft part properly. Indeed, NRQCD achieves much success by taking high Fock state contributions into account. It reconciles several discrepancies between the theoretical predictions and experimental data, but there are still some problems. Of the problems, the discrepancies between the theoretical predictions at leading order (LO) and experimental data on the polarization of  $J/\psi$  and  $\Upsilon$  hadroproduction are outstanding. Recent reviews on the situation can be found in Ref. [2].

Many works [3–5] indicate that the next-to-leading-order (NLO) QCD corrections under the NRQCD framework drastically change the features of LO theoretical predictions in the heavy-quarkonium production in various cases. Especially, it is reported in Ref. [3] that with the color-singlet model (CSM) alone the NLO prediction on  $J/\psi$  photoproduction at HERA can explain the experimental data well. Also with the CSM alone, the NLO results on the charmonium production moderate, even resolve, the discrepancies between theoretical predictions and experimental measurements at  $B$  factories [4]. Recently, the NLO QCD corrections on the hadronic production of  $J/\psi$  and  $\Upsilon$  in the CSM have also been studied by several groups [6–8] and the results show that the  $p_t$  distribution of  $J/\psi$  production is largely enhanced. Furthermore, the NLO QCD corrections on the hadronic production of  $J/\psi$  in the color-octet mechanism (COM) have been completed in Ref. [9] and the results show that the  $p_t$  distribution of  $J/\psi$  production is changed slightly, whereas the polarization of the

produced quarkonium still is an open problem. Namely, the results at NLO in the CSM give a longitudinal polarization contrary to the transverse one as LO does [7], and the NLO results in the COM give almost unchanged polarization as the LO ones [9]. To add the NLO CSM and NLO COM results together, the data for polarization of  $J/\psi$  hadroproduction at Tevatron cannot be described properly, although the data for  $p_t$  distribution of hadroproduction can be fit quite well [7,9].

It may provide  $ep$  and  $\gamma p$  processes to investigate the production of  $J/\psi$  at HERA. The transverse momentum  $p_t$  distributions of  $J/\psi$  production and polarization at LO were studied in the CSM a long time ago [10–13]. As mentioned above, it was reported in Ref. [3] that with the CSM alone the NLO results can give a proper description for the photoproduction of  $J/\psi$  at HERA, including the energy fraction distribution  $d\sigma/dz$  in the intermediate  $z$  region and transverse momentum  $p_t$  distribution, where the energy fraction  $z$  is defined by  $z \equiv \frac{(p_{J/\psi} \cdot p_p)}{(p_\gamma \cdot p_p)}$  and  $p_{J/\psi}$ ,  $p_\gamma$ , and  $p_p$  are the momenta of  $J/\psi$ ,  $\gamma$ , and protons, respectively. Then, to investigate the  $J/\psi$  inelastic photoproduction, many works based on NRQCD at LO and NLO in the CSM and COM followed [14]. In Ref. [15] the photoproduction of polarized  $J/\psi$  was studied at LO in the CSM and COM. In Ref. [16] the theoretical predictions based on the  $k_T$  factorization formula were also given at LO. However, the latest experimental results on the photoproduction of polarized  $J/\psi$  [17] do not favor the LO predictions in the CSM and COM, even those obtained with the  $k_t$  factorization formula. So, up to now there is no satisfactory explanation on the measurement results of photoproduction of polarized  $J/\psi$  at HERA. In view of the proper description of the HERA data on  $p_t$  and  $z$  distributions by the CSM NLO prediction [3], we focus the theoretical predictions on  $J/\psi$  polarization with NLO QCD corrections in the CSM in this paper.

To calculate the NLO QCD correction for the photoproduction of polarized  $J/\psi$  in the CSM, there are direct

and resolved processes to be considered. The existing LO calculation [15] shows that the contribution from resolved processes is about 2 orders of magnitude smaller than that of the direct one for the  $p_i$  distribution of  $J/\psi$ , and only in the lower  $z$  ( $z < 0.2$ ) region is the contribution from resolved processes comparable with that from the direct one. Therefore, with the cut condition  $z > 0.2$  for the experimental measurements [17,18], only the direct processes are investigated in this paper. To perform the lengthy analytic evaluation for all the processes, the computer program package, Feynman Diagram Calculation (FDC), is used. FDC was developed and well tested for many years [19], and recently the functions for manipulating one-loop quantum corrections in analytic reduction and numerical calculations were completed and tested in many aspects [20].

To calculate the  $J/\psi$  photoproduction at NLO, the following relevant processes,

$$\gamma + g \rightarrow J/\psi + g, \quad (1)$$

$$\gamma + g \rightarrow J/\psi + g + g, \quad (2)$$

$$\gamma + g \rightarrow J/\psi + q + \bar{q}, \quad (3)$$

$$\gamma + q(\bar{q}) \rightarrow J/\psi + g + q(\bar{q}), \quad (4)$$

need to be considered. For process (1), there are 6 Feynman diagrams at LO and 111 at NLO, and there are the ultraviolet (UV), infrared (IR), and Coulomb singularities. By using dimensional regularization and introducing a small relative velocity between the quark and antiquark, we can separate the singularities out. The UV and Coulomb singularities are absorbed into the redefinition of mass, coupling constant, fields of quark and gluons, and wave function of  $J/\psi$  by renormalization, for which the same renormalization scheme as in Ref. [7] is applied. By using the phase space slicing method [21], we separate out the IR singularities in the real processes (2)–(4). Finally to add all the contributions from the real and virtual parts together, a finite result, which is free from all the singularities, is obtained.

As the main purpose of this paper, we extract the information of the  $J/\psi$  polarization through the angular distribution of the lepton  $l^+$  in  $J/\psi \rightarrow l^+l^-$  decay. As in Ref. [15], the decay angular distribution of the outgoing  $l^+$  can be parametrized in the  $J/\psi$  rest frame as

$$\begin{aligned} \frac{d\sigma}{d\Omega dy} &\propto 1 + \lambda(y)\cos^2\theta + \mu(y)\sin 2\theta \cos\phi \\ &+ \frac{\nu(y)}{2}\sin^2\theta \cos 2\phi, \end{aligned} \quad (5)$$

where  $y$  stands for a suitable variable (such as transverse momentum  $p_t$ , energy fraction  $z$ , etc.).  $\theta$  and  $\phi$  are the polar and azimuthal angles of the outgoing  $l^+$ , respectively. The polarization parameters,  $\lambda$ ,  $\mu$ , and  $\nu$ , are related to the density matrix of  $J/\psi$  production as

$$\begin{aligned} \lambda(y) &= \frac{d\sigma_{11}/dy - d\sigma_{00}/dy}{d\sigma_{11}/dy + d\sigma_{00}/dy}, \\ \mu(y) &= \frac{\sqrt{2}\text{Re}d\sigma_{10}/dy}{d\sigma_{11}/dy + d\sigma_{00}/dy}, \\ \nu(y) &= \frac{2d\sigma_{1-1}/dy}{d\sigma_{11}/dy + d\sigma_{00}/dy}. \end{aligned} \quad (6)$$

Here  $d\sigma_{\lambda\lambda'}/dy$  are the differential density matrix elements and defined as

$$\begin{aligned} \frac{d\sigma_{\lambda\lambda'}}{dy} &= \frac{1}{F} \int \prod_{i=1}^n \frac{d^3 p_i}{2E_i} \delta^4\left(p_a + p_b - \sum_{i=1}^n p_i\right) \delta(y - y(p_i)) \\ &\times M(\lambda)M^*(\lambda'), \end{aligned} \quad (7)$$

where  $M(\lambda)$  is the matrix element of polarized  $J/\psi$  production,  $\lambda$  and  $\lambda'$  stand for the polarization,  $F$  includes the flux factor and spin average factor, and  $p_i$  is the momentum of corresponding particles. In the calculation, the polarization of  $J/\psi$  must be explicitly retained and the treatment for it is the same as in Ref. [7]. Obviously these polarization parameters depend on the coordinate system choice. In the following, we calculate them in the target frame with the  $Z$  axis defined as the inverse direction of the initial proton and the polarization vectors of  $J/\psi$  defined in the appendix of Ref. [15]. We also compute the polarization parameters of  $J/\psi$  in the helicity-base frame where the  $Z$  axis is defined as the  $J/\psi$  flight direction in the laboratory frame, and the angular distribution of  $l^+$  is parametrized as

$$\frac{d\sigma}{d\cos\theta dy} \propto 1 + \alpha(y)\cos^2\theta. \quad (8)$$

Here the parameter  $\alpha$  is related to the polarized cross sections of  $J/\psi$  production by

$$\alpha(y) = \frac{d\sigma_T/dy - 2d\sigma_L/dy}{d\sigma_T/dy + 2d\sigma_L/dy}, \quad (9)$$

where  $\sigma_T$  and  $\sigma_L$  are the cross sections of transverse and longitudinal polarized  $J/\psi$ , respectively.  $\alpha = -1$  corresponds to fully longitudinal polarization and  $\alpha = 1$  to fully transverse polarization.

To replace the polarization vectors for photons or gluons by their corresponding momentum in the numerical calculation, the gauge invariance is obviously observed. Since the two phase space cutoffs are chosen to handle the IR singularities of the real processes, we numerically check the independence of the results on the cutoffs.

In the numerical calculation, we use  $\alpha = 1/137$ ,  $m_c = 1.5$  GeV, and  $M_{J/\psi} = 2m_c$ . The CTEQ6L1 and CTEQ6M [22] are used in the calculations at LO and NLO, respectively, with the corresponding  $\alpha_s$  running formula in the CTEQ6 being used. In Ref. [3] a fixed value of  $\alpha_s$  is taken for numerical calculations. Whereas the running coupling constant is chosen in most literatures, here we use the running  $\alpha_s$  in all our calculations. The renormalization scale  $\mu_r$

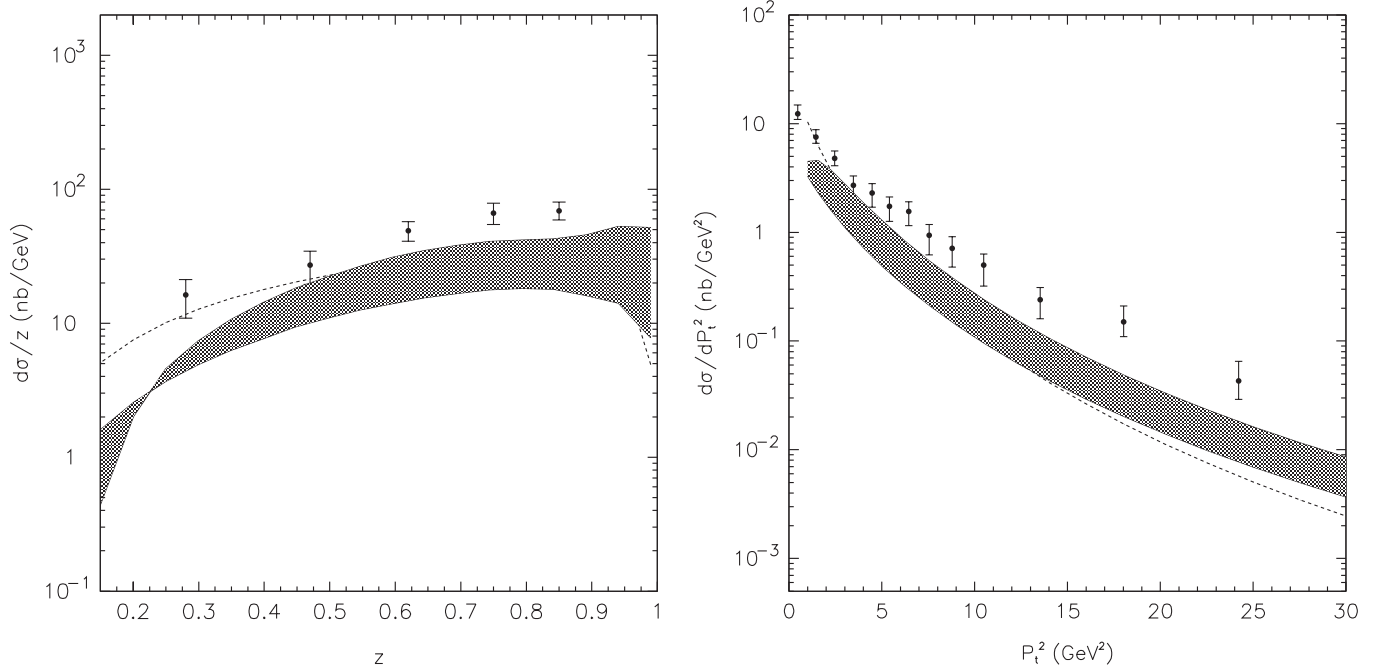


FIG. 1. The energy fraction  $z$  distribution of  $J/\psi$  photoproduction with  $P_t > 1$  GeV (left panel). The transverse momentum  $p_t$  distribution of  $J/\psi$  with  $0.4 < z < 0.9$  (right panel). The dotted lines in each one are the LO results. The upper bound of the shaded band is obtained with  $\mu_r = \mu_f = \frac{1}{2}\sqrt{(2m_c)^2 + p_t^2}$  and  $m_c = 1.4$  GeV, and the lower one with  $\mu_r = \mu_f = 2\sqrt{(2m_c)^2 + p_t^2}$  and  $m_c = 1.6$  GeV. The experimental data (filled circles with bars) are taken from Ref. [18].

and the factorization scale  $\mu_f$  are set as  $\mu_r = \mu_f = \sqrt{(2m_c)^2 + p_t^2}$ . The value for  $J/\psi$  wave function at the origin is extracted from the leptonic decay width with the NLO formula, the value  $\Gamma_{J/\psi \rightarrow ee} = 5.55$  keV, and  $\alpha_s(M_{J/\psi}) = 0.26$ . The typical HERA photon-proton center-of-mass energy  $\sqrt{s_{\gamma p}} = 100$  GeV is chosen. In addition, the experimental cut conditions  $0.4 < z < 0.9$  and  $p_t > 1$  for the  $p_t$  and  $z$  distributions on  $J/\psi$  production are applied, respectively, and  $0.4 < z < 1$  and  $p_t > 1$  are ap-

plied for the  $J/\psi$  polarization. Furthermore, all the differential cross sections are calculated directly in numerical calculation by using the corresponding analytic phase space treatment just like that used in Ref. [7].

The final results are presented in the figures. From Fig. 1, we can see that the results for  $p_t$  and  $z$  distributions of  $J/\psi$  production at NLO moderate the discrepancies between theoretical predictions and experimental measurements substantially. The  $p_t$  and  $z$  distributions of the polarization parameters in Eq. (5) is presented in Fig. 2.

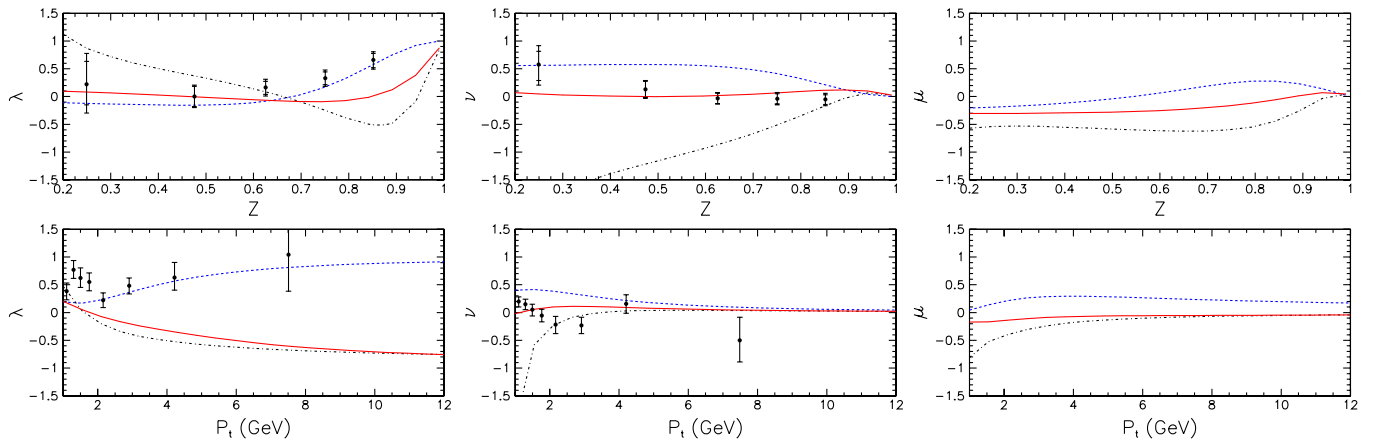


FIG. 2 (color online). The energy fraction  $z$  distributions of polarization parameters  $\lambda$ ,  $\nu$ , and  $\mu$  with  $p_t > 1$  GeV, and transverse momentum  $p_t$  distributions with  $0.4 < z < 1$ . Dashed lines are the LO results and the dot-dashed lines are the NLO results. Solid lines present the results with  $\mu_r = \mu_f = 8m_c$ . The experimental data (filled circles with bars) are taken from Ref. [17].

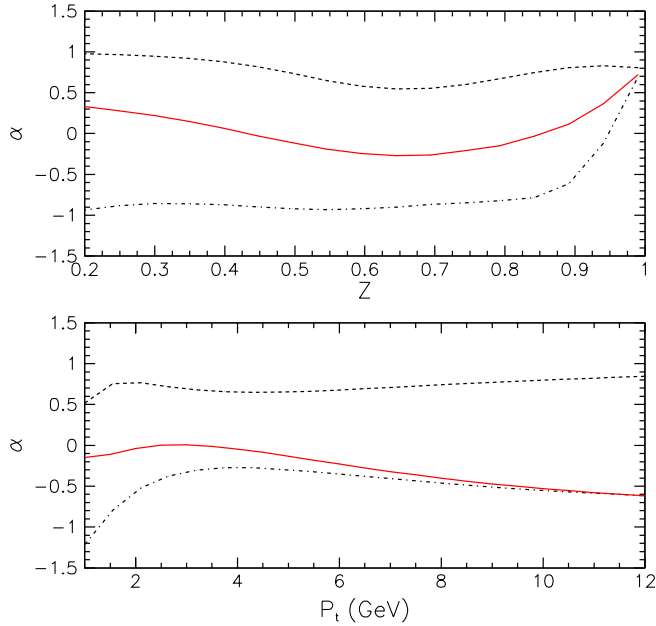


FIG. 3 (color online). The polarization parameter  $\alpha$  distributions as functions of  $P_t$  and  $z$  in the helicity basis. Dashed lines are the results at LO and the dot-dashed lines are the ones at NLO. The results with  $\mu_r = \mu_f = 8m_c$  are presented by the solid lines.

In the presentation, we make a cutoff for the region  $z \leq 0.2$ , because in this region the denominator of Eq. (6) crosses zero at a certain point so that the values of the parameters change dramatically in the neighborhood of the point. It means that the perturbation expansion is very bad and the obtained result cannot be trusted in this region. Comparing with the results at LO, the NLO QCD corrections greatly change the distributions. The parameters  $\lambda$  obtained at LO and NLO are very different at the large and small  $z$  regions (there is a cross at about  $z \approx 0.67$ ). The  $z$  distribution of parameter  $\nu$  changes drastically in the small and intermediate regions, and  $\mu$  also changes in the intermediate region of  $z$ . As for the  $p_t$  distribution of the parameters, from the figure one can see that the NLO QCD correction changes that of the  $\lambda$  parameter from positive values to negative ones and makes it tend to  $-0.8$  as  $p_t$  increases. For the  $\nu$  parameter, there is little difference between the results of LO and NLO in the large  $p_t$  region. The influence of the NLO correction on the  $p_t$  distribution of  $\mu$  is also quite large. For comparison, the available experimental data at HERA [17] are plotted in the figures. It is clearly shown that both LO and NLO results do not fit the polarization measurement, and the NLO results are even worse. In these figures, the  $z$  and  $p_t$  distributions of  $\lambda$ ,  $\mu$ , and  $\nu$  with  $\mu_r = \mu_f = 8m_c$  at NLO are also presented.

The  $p_t$  and  $z$  distributions of  $\alpha$  in the helicity basis are presented in Fig. 3. We also make a cutoff for the region  $z \leq 0.2$ , due to the same reason as that in the case of the target frame. It is similar to the situation in the  $J/\psi$  hadroproduction: NLO QCD corrections change the  $p_t$  distribution of  $\alpha$  from positive values to negative ones. The  $z$  distribution of  $\alpha$  changes similar to that on the  $p_t$  distribution except the end point region near  $z = 1$ . We also present the results with  $\mu_r = \mu_f = 8m_c$ .

In summary, we have investigated the photoproduction of  $J/\psi$  at QCD NLO at HERA. The results show that the NLO QCD corrections cannot give a very good description on the  $p_t$  and  $z$  distributions. Furthermore, for the  $J/\psi$  polarization, the NLO QCD corrections make a drastic change on certain polarization parameters, but they cannot give a satisfied prescription for the available experimental data. In contrast to the LO calculation for polarization, the NLO results are even worse. There are two conclusions on the results at QCD NLO. One is that the theoretical uncertainties for the  $z$  distributions of the  $J/\psi$  polarization parameters  $\lambda$ ,  $\mu$ ,  $\nu$ , and  $\alpha$  on different choices of the renormalization and factorization scales are too large to give reasonable predictions to compare with the experimental measurement. Another is that the theoretical uncertainties for the  $p_t$  distributions of  $\lambda$ ,  $\mu$ ,  $\nu$ , and  $\alpha$  on different choices of the scales are small when  $p_t > 3$  GeV and the obtained  $p_t$  distributions cannot describe the experimental data even just in this region. Therefore, there is still no satisfactory theoretical description even at the NLO level on the  $p_t$  or  $z$  distributions of  $J/\psi$  polarization as well as the  $p_t$  or  $z$  distributions of  $J/\psi$  production at HERA. It will be interesting to know the situation by considering NLO color-octet contributions, or higher order QCD corrections in a future study.

While this paper was being prepared, we were informed of the same process also being considered by Artoisenet *et al.* [23]. Comparing our results in Figs. 1 and 2 with theirs, there are quantitative discrepancies between them. But our results on the  $z$  distributions of  $\lambda$  and  $\nu$  with  $\mu_r = \mu_f = 8m_c$  at NLO are consistent with theirs with  $\sqrt{s_{\gamma p}} = 100$  GeV and  $\mu_r = \mu_f = 8m_c$ .<sup>1</sup> It can be inferred that the discrepancies mainly come from the different choices of renormalization and factorization scales.

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