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Potential model calculations and predictions for $c\bar{s}$ quarkonia

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We investigate the spectroscopy and decays of the charm-strange quarkonium system in a potential model consisting of a relativistic kinetic energy term, a linear confining term including its scalar and vector relativistic corrections, and the complete perturbative one-loop quantum chromodynamic short distance potential. The unperturbed wave functions of the various states are obtained using a variational technique. These are then used in a perturbative treatment of the potential to fit the mass spectrum of the $c\bar{s}$ system and calculate the radiative decay widths. Our results compare well with the available data for the spectrum of D_s states. We include a discussion of the effect of mixing and investigation of the Lorentz nature of the confining potential.

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I. INTRODUCTION

Recently we reported on a study of the charmonium and upsilon systems in a semirelativistic model which includes all v^2/c^2 and one-loop QCD corrections for the interaction of a quark and antiquark of equal mass [1]. This semirelativistic potential model successfully describes the spectra and leptonic and radiative decays of those systems. We have now extended this modeling approach to systems in which the quark and antiquark have different masses.

Interest in the modeling of light-heavy quarkonia is over 25 years old [2]. A variety of modeling approaches have been employed with varying success [3–8]. Renewed and continuing interest in the modeling of $c\bar{s}$ quarkonia is fueled by, in particular, the recent discovery of the 2^3S_1 state [9] as well as ongoing efforts to determine the masses and decays of the D_s mesons [10].

We have revised and extended the approach of our earlier papers in order to investigate the spectroscopy and decays of the D_s system, as well as to discuss other questions of modeling interest. In addition, we investigate the scalar/vector mixture of the phenomenological confining potential.

In the next section, we describe the potential model in some detail. This is followed, in Sec. III, by an outline of our calculational approach. In Sec. IV, we present our results for the D_s system, and then give some conclusions in Sec. V. The conventions we use for our treatment of the mixing of the $J = 1$ p states are given in the Appendix.

II. SEMIRELATIVISTIC MODEL

In our analysis, we use a semirelativistic Hamiltonian of the general form

$$H = \sqrt{\vec{p}^2 + m_1^2} + \sqrt{\vec{p}^2 + m_2^2} + Ar - \frac{4\alpha_S}{3r}\left[1 - \frac{3\alpha_S}{2\pi} + \frac{\alpha_S}{6\pi}(33 - 2n_f)(\ln(\mu r) + \gamma_E)\right] + V_L + V_S \quad (1)$$

$$= H_0 + V_L + V_S, \quad (2)$$

where m_1 and m_2 are the quark masses, μ is the renormalization scale, n_f is the effective number of light quark flavors, and γ_E is Euler's constant. V_L contains the v^2/c^2 corrections to the linear confining potential

$$V_L = -(1 - f_V)\frac{A}{4r}\left[\left(\frac{1}{m_1^2} + \frac{1}{m_2^2}\right)\vec{L}\cdot\vec{S} + \left(\frac{1}{m_1^2} - \frac{1}{m_2^2}\right)\times\vec{L}\cdot(\vec{S}_1 - \vec{S}_2)\right] + f_V\frac{A}{4r}\left[\frac{1}{m_1^2} + \frac{1}{m_2^2} + \frac{16}{3m_1m_2}\times\vec{S}_1\cdot\vec{S}_2 + \left(\frac{1}{m_1^2} + \frac{1}{m_2^2} + \frac{4}{m_1m_2}\right)\vec{L}\cdot\vec{S} + \left(\frac{1}{m_1^2} - \frac{1}{m_2^2}\right)\times\vec{L}\cdot(\vec{S}_1 - \vec{S}_2) + \frac{4}{3m_1m_2}(3\vec{S}_1\cdot\hat{r}\vec{S}_2\cdot\hat{r} - \vec{S}_1\cdot\vec{S}_2)\right], \quad (3)$$

where A is the linear coupling strength. The terms in Eq. (3) with coefficient $(1 - f_V)$ are the contributions from scalar exchange and those with a coefficient f_V are the contributions from vector exchange. Here, f_V represents the fraction of vector exchange in the interaction. The short distance potential is [2]

$$V_S = V_{HF} + V_{LS} + V_T + V_{SI} + V_{MIX}, \quad (4)$$

with

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$$V_{HF} = \frac{32\pi\alpha_S \vec{S}_1\cdot\vec{S}_2}{9m_1m_2}\left\{\left(1-\frac{19\alpha_S}{6\pi}\right)\delta(\vec{r}) - \frac{\alpha_S}{8\pi}\left(8\frac{m_1-m_2}{m_1+m_2}+\frac{m_1+m_2}{m_1-m_2}\ln\frac{m_2}{m_1}\right)\delta(\vec{r}) - \frac{\alpha_S}{24\pi^2}(33-2n_f)\nabla^2\left[\frac{\ln\mu r+\gamma_E}{r}\right] + \frac{21\alpha_S}{16\pi^2}\nabla^2\left[\frac{\ln(m_1m_2)^{1/2}r+\gamma_E}{r}\right]\right\}, \tag{5a}$$

$$V_{LS} = \frac{\alpha_S\vec{L}\cdot\vec{S}}{3m_1^2m_2^2r^3}\left\{[(m_1+m_2)^2+2m_1m_2]\left[1-\frac{3\alpha_S}{2\pi}+\frac{\alpha_S}{6\pi}(33-2n_f)(\ln\mu r+\gamma_E-1)\right] + \frac{\alpha_s}{2\pi}(m_1+m_2)^2\left[\frac{8}{3}-6(\ln(m_1m_2)^{1/2}r+\gamma_E-1)\right]-\frac{3\alpha_s}{2\pi}(m_1^2-m_2^2)\ln\frac{m_2}{m_1}\right\}, \tag{5b}$$

$$V_T = \frac{4\alpha_S(3\vec{S}_1\cdot\hat{r}\vec{S}_2\cdot\hat{r}-\vec{S}_1\cdot\vec{S}_2)}{3m_1m_2r^3}\left\{1+\frac{4\alpha_S}{3\pi}+\frac{\alpha_S}{6\pi}\left[(33-2n_f)\left(\ln\mu r+\gamma_E-\frac{4}{3}\right)-18\left(\ln(m_1m_2)^{1/2}r+\gamma_E-\frac{4}{3}\right)\right]\right\}, \tag{5c}$$

$$V_{SI} = \frac{2\pi\alpha_S}{3}\left(\frac{1}{m_1^2}+\frac{1}{m_2^2}\right)\left\{\left(1-\frac{3\alpha_S}{2\pi}\right)\delta(\vec{r})-\frac{\alpha_S}{24\pi^2}(33-2n_f)\nabla^2\left[\frac{\ln\mu r+\gamma_E}{r}\right]-\frac{\alpha_S}{6\pi r^2}\left[\frac{9(m_1+m_2)^2-8m_1m_2}{m_1m_2(m_1+m_2)}\right]\right\}, \tag{5d}$$

$$V_{MIX} = -\frac{\alpha_S\vec{L}\cdot(\vec{S}_1-\vec{S}_2)}{3m_1^2m_2^2r^3}\left\{(m_1^2-m_2^2)\left[1-\frac{\alpha_S}{6\pi}+\frac{\alpha_S}{6\pi}(33-2n_f)(\ln\mu r+\gamma_E-1)-\frac{3\alpha_S}{\pi}(\ln(m_1m_2)^{1/2}r+\gamma_E-1)\right] - \frac{3\alpha_s}{2\pi}(m_1+m_2)^2\ln\frac{m_2}{m_1}\right\}. \tag{5e}$$

We have chosen H_0 such that it contains the relativistic kinetic energy and the leading order spin-independent portions of the long-range confining potential and the one-loop QCD short-range potential. It is important to recall that the potential given by Eq. (4) does not reduce to the potential in Ref. [1], due to the presence of annihilation terms in the equal-mass quark-antiquark potential. It should also be noted that in calculating the matrix elements of the $\delta(\vec{r})$ terms in Eqs. (5a) and (5d), we "soften" their singularity by adopting the quasistatic approximation of Ref. [4], which leads to the replacement

$$\delta(\vec{r}) \to \frac{\omega^2}{\pi r}e^{-2\omega r}, \tag{6}$$

where $\omega^2 = 2m_1^2m_2^2/(m_1^2+m_2^2)$. This softening helps the stability of the eigenvalue calculation.

III. CALCULATIONAL APPROACH

The $c\bar{s}$ mass spectrum and corresponding wave functions are obtained using the variational approach described in Ref. [1]. In this approach, we expand the wave functions as

$$\psi^m_{j\ell s}(\vec{r}) = \sum_{k=0}^{n} C_k\left(\frac{r}{R}\right)^{k+\ell}e^{-r/R}\mathcal{Y}^m_{j\ell s}(\Omega), \tag{7}$$

where $\mathcal{Y}^m_{j\ell s}(\Omega)$ denotes the orbital-spin wave function for a specific total angular momentum j, orbital angular momentum ℓ, and total spin s. The C_k's are determined by minimizing

$$E = \frac{\langle\psi|H|\psi\rangle}{\langle\psi|\psi\rangle} \tag{8}$$

with respect to variations in these coefficients. This procedure results in a linear eigenvalue equation for the C_k's and the energies, and is equivalent to solving the Schrödinger equation. The wave functions corresponding to different eigenvalues are orthogonal and the kth eigenvalue λ_k is an approximation to the true eigenvalue E_k. For $n = 14$, the lowest four eigenvalues for any ℓ are stable to one part in 10^6. We performed a perturbative calculation, using H_0 as the unperturbed Hamiltonian and all other terms treated as first-order perturbations.

An optimal set of the parameters $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_n)$ was found by minimizing the χ^2 function

$$\chi^2 = \sum_{i=1}^{N}\frac{(\mathcal{O}_{\mathrm{exp}i}-\mathcal{O}_{\mathrm{th}}(\alpha)_i)^2}{\sigma_i^2}, \tag{9}$$

where the $\mathcal{O}_i$ denote the experimental and theoretical values of some quarkonium observable and the σ_i are the associated errors. In this work, the $\mathcal{O}_{\mathrm{exp}i}$ consist of a subset of the measured D_s masses. For the masses, the σ_i are taken to be the actual experimental error and a common intrinsic theoretical error added in quadrature. The latter error reflects the theory uncertainty associated with omitting corrections beyond one loop and is estimated by requiring the χ^2/degree of freedom to be approximately unity. Typically, this error is a few MeV. The minimization of χ^2 with respect to variations of the parameters α is accomplished using the search program STEPIT [11].

In QCD the values of α_S and quark masses at different energy scales are related through the renormalization group (RG) equations, and depend not only on the scale but also on the renormalization scheme employed. We note that we use the Gupta-Radford scheme [12], which is particularly well suited for potential calculations. In Ref. [1] we obtained α_S and m_C at $\mu_{c\bar{c}} = 2.60$ GeV as

0.334 and 1.51 GeV, respectively. The values of α_S and m_C in this paper were kept consistent with the results of RG running these parameters by using a two-step procedure. First, we allowed STEPIT to find best fit values for A, m_S, μ, and f_V while requiring $\alpha_S(\mu)$ and $m_C(\mu)$, to run according to the RG relations. We then relaxed these requirements by refitting the spectrum with α_S and m_C as parameters constrained by introducing a Gaussian prior in the χ^2 function for each one, centered on the RG-preferred values. The final value for m_C is about 5% high compared to the RG-preferred value. However, the fitted value of α_S is about 17% low, although consistent with the direction of the RG running.

For the case of the long-range coupling coefficient, A, the fit was constrained with a Gaussian prior centered at 0.155 GeV^2. This value was used assuming a slight scale dependence following the trend found in Ref. [1]: $A(b\bar{b}) = 0.177$ GeV^2; $A(c\bar{c}) = 0.166$ GeV^2. The lower value of A found here was strongly preferred by the fitting routine.

In this perturbative treatment, as in the equal-mass case [1], the value of f_V, the long-range scalar/vector mixing parameter, is $f_V = 0$. We could set this mixing equal to zero at the outset and the quality of the fix to the spectrum would not be affected in any substantial way.

IV. RESULTS

We summarize our results in the following tables. The parameters resulting from our fit are given in Table I. The results for our determination of the D_s levels are shown in Table II [13]. Overall our fit to the spectrum is quite good.

As is usual in potential model treatments [5,7,14–16], the radiative widths were calculated in the dipole approximation. We obtained the E_1 matrix elements by using the variational radial wave functions to construct initial and final state wave functions with the appropriate angular dependence and explicitly performing the angular integration. Our results are

$$\Gamma_{fi} = \frac{4\alpha}{9}\left(\frac{q_1 m_2 - q_2 m_1}{m_1 + m_2}\right)^2 \omega^3 |\langle f|r|i\rangle|^2 \frac{E_f}{M_i} \times \begin{cases} 1 & \text{for } {}^3P_J \to {}^3S_1, \\ 1 & \text{for } {}^1P_1 \to {}^1S_0, \\ (2J+1)/3 & \text{for } {}^3S_1 \to {}^3P_J, \\ 3 & \text{for } {}^1S_0 \to {}^1P_1, \end{cases} \tag{10}$$

TABLE I. Fitted parameters for the $c\bar{s}$ system.

A (GeV^2)	0.123
α_S	0.384
m_c (GeV)	1.72
m_S (GeV)	0.406
μ (GeV)	1.43
f_V	0.00

TABLE II. Results for the D_s spectrum are shown. The fit uses all the indicated states of the D_s system except for the $n = 2$ $D_s(2503)$.

$m_{c\bar{s}}$ (MeV)	Model	Experiment
D_s	1968.8	1968.49 ± 0.34
D_s^*	2111.8	2112.3 ± 0.5
$D_{s0}(2317)$	2317.1	2317.8 ± 0.6
$D_{s1}(2460)$	2460.8	2459.6 ± 0.6
$D'_{s1}(2536)$	2534.4	2535.35 ± 0.34
$D_{s2}(2573)$	2574.8	2572.6 ± 0.9
$D_s(2503)$	2503.7	
$D_s^*(2690)$	2654.8	2690. ± 7

for E_1 transitions. Here, ω is the photon energy, q_1 and q_2 are the quark charges in units of the proton charge, E_f is the energy of the final quarkonium state, M_i is the mass of the initial quarkonium state, and m_1 and m_2 are the quark masses.

We also take into account the mixing between the ${}^1P_1$ and ${}^3P_1$ eigenstates of the $c\bar{s}$ Hamiltonian due to the $\vec{L}\cdot(\vec{S}_1 - \vec{S}_2)$ terms in Eqs. (3) and (5e) of the perturbative potential. This mixing yields the two $J = 1$ states D_{s1} and D'_{s1}. They are, explicitly,

$$|D_{s1}(2460)\rangle = \sin(\theta)|{}^3P_1\rangle + \cos(\theta)|{}^1P_1\rangle, \tag{11a}$$
$$|D'_{s1}(2536)\rangle = \cos(\theta)|{}^3P_1\rangle - \sin(\theta)|{}^1P_1\rangle, \tag{11b}$$

where

$$\tan(\theta) = -\frac{V_{31}}{E_+ - E({}^1P_1)}, \tag{12}$$

with V_{31} denoting the expectation value of the mixing terms and E_+ denoting the larger of the two eigenvalues of the mixing matrix. Note that, because of the $1/m_2^2$ behavior of these terms, the mixing is quite sensitive to the strange quark mass. The conventions used in parametrizing the mixing are given in the Appendix.

For M_1 transitions, a parallel calculation, using the fact that the singlet and triplet s states have the same wave functions in the perturbative treatment, yields

$$\Gamma_{fi} = \frac{4\alpha}{3}\omega^3\left(\frac{q_1}{2m_1} - \frac{q_2}{2m_2}\right)^2 \frac{E_f}{M_i} \tag{13}$$

for the $D_s^* \to D_s + \gamma$. In the case of the p-state magnetic transitions $D_{s1} \to D_{s0} + \gamma$ and $D'_{s1} \to D_{s0} + \gamma$, both the singlet and triplet states of the mixtures in Eq. (11) contribute to the widths. If we use the perturbative wave functions, then the relative phase of the triplet contribution with respect to the singlet contribution is $\pi/2$. The widths in this case are

TABLE III. The radiative decays of the D_s mesons are shown. These widths are computed using the mass values obtained directly from our calculation. This includes the $n = 2$ pseudoscalar and vector states, the latter of which has recently been observed with a higher mass [13]. The widths are from [13]. Included in the table are the results from Refs. [5,7] for comparison.

Γ_γ (keV)	Model	Ref. [5]	Ref. [7]	Experiment
$D_s^* \to D_s$	1.12		0.43	$<1.9 \times 10^3$
$D_{s0}(2317) \to D_s^*$	3.37	1.9	1.74	
$D_{s1}(2460) \to D_s$	8.3	6.2	5.08	BR $= 0.18 \pm 0.04$
$D_{s1}(2460) \to D_s^*$	11.0	5.5	4.66	BR < 0.08
$D_{s1}(2460) \to D_{s0}(2317)$	4.70		2.74	
$D'_{s1}(2536) \to D_s$	37.7	15.0		
$D'_{s1}(2536) \to D_s^*$	5.74	5.6		Possibly seen
$D'_{s1}(2536) \to D_{s0}(2317)$	5.62			
$D_{s2}(2575) \to D_s^*$	30.5	19.0		
$D_s^*(2655) \to D_{s0}(2317)$	5.64	3.4		
$D_s^*(2655) \to D_{s1}(2460)$	2.66	2.3		
$D_s^*(2655) \to D'_{s1}(2536)$	0.26	0.5		
$D_s^*(2655) \to D_{s2}(2573)$	0.48	1.5		
$D_s(2503) \to D_{s1}(2460)$	0.04			

$$\Gamma_{fi} = \frac{4\alpha}{9}\omega^3 \frac{E_f}{M_i} \begin{cases} 2\sin^2(\theta)\left(\frac{q_1}{2m_1} + \frac{q_2}{2m_2}\right)^2 + \cos^2(\theta)\left(\frac{q_1}{2m_1} - \frac{q_2}{2m_2}\right)^2 & \text{for } D_{s1} \to D_{s0} + \gamma, \\ 2\cos^2(\theta)\left(\frac{q_1}{2m_1} + \frac{q_2}{2m_2}\right)^2 + \sin^2(\theta)\left(\frac{q_1}{2m_1} - \frac{q_2}{2m_2}\right)^2 & \text{for } D'_{s1} \to D_{s0} + \gamma. \end{cases} \tag{14}$$

The resulting radiative widths are shown in Table III.

V. CONCLUSIONS

We have shown that a potential model consisting of the relativistic kinetic energy, a linear long-range confining potential together with its v^2/c^2 relativistic corrections, and the full v^2/c^2 plus one-loop QCD corrected short distance potential is capable of providing extremely good fits to the spectra of the D_s states by treating them as states of the $c\bar{s}$ system. We find that in this perturbative treatment the long-range potential must be entirely due to scalar exchange.

The single photon widths can be obtained from the variational wave functions, but, apart from some branching ratio measurements, there are relatively little data available. Our theoretical results are compared with those given in Refs. [5,7] in Table III. With respect to Ref. [7], the difference in the calculated widths might be attributable to the larger value of m_s (480 MeV) used by these authors. In Ref. [5], the strange quark mass is taken to be $m_s =$ 419 Mev, which is comparable to our value given in Table I. The differences in the $E1$ widths must, therefore, be attributable to differences in the dipole matrix elements and/or the choice of the mixing angle. For the most part, these differences can be reconciled by using our values of the dipole matrix elements, $\langle 1S|r|1P\rangle = 2.72\ \text{GeV}^{-1}$ and $\langle 1P|r|2S\rangle = 3.04\ \text{GeV}^{-1}$, together with those given in Ref. [5] to rescale our radiative widths. Keeping in mind that our mixing angles differ, the resulting rescaled widths are quite comparable to those in Ref. [5]. In every case, efforts to model these states will be greatly improved by the availability of additional data.

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APPENDIX: DETAILS OF MIXING

The mixing of the $^3P_1$ and $^1P_1$ states is obtained by diagonalizing the 2×2 matrix

$$\begin{pmatrix} E_3 & V_{31} \\ V_{31} & E_1 \end{pmatrix}, \tag{A1}$$

where E_3 is the $^3P_1$ energy, E_1 is the $^1P_1$ energy, and V_{31} is the mixing matrix element. In perturbation theory, this is relatively simple since all of these matrix elements can be calculated using the unperturbed wave functions that are all the same. The energy eigenvalues are

$$E_\pm = \tfrac{1}{2}(E_3 + E_1) \pm \tfrac{1}{2}\sqrt{(E_3 - E_1)^2 + 4V_{31}^2}, \tag{A2}$$

and we fit the $D'_{s1}(2536)$ and $D_{s1}(2460)$ to E_+ and E_-. Note that as $V_{31} \to 0$, $E_+ \to E_3$, and $E_- \to E_1$. To define the mixing angles in terms of known parameters, we assume that the eigenvector ψ_+ corresponding to E_+ behaves as

$$\psi_+ \overset{V_{31}\to 0}{\to} \begin{pmatrix} 1 \\ 0 \end{pmatrix}. \tag{A3}$$

With this assumption, ψ_+ is

$$\psi_+ = \frac{1}{\sqrt{(E_+ - E_1)^2 + V_{31}^2}} \begin{pmatrix} E_+ - E_1 \\ V_{31} \end{pmatrix}. \tag{A4}$$

By writing ψ_+ as

$$\psi_+ = \begin{pmatrix} \cos(\theta) \\ -\sin(\theta) \end{pmatrix}, \tag{A5}$$

with

$$\tan(\theta) = -\frac{V_{31}}{E_+ - E_1}, \tag{A6}$$

we arrive at the decomposition Eq. (11). It is possible to obtain an estimate of the mixing angle by using the branching ratios of the $D_{s1}(2460) \to D_s\gamma$ and $D_{s1}(2460) \to D_s^*\gamma$, whose ratio gives

$$\frac{\Gamma(D_{s1}(2460) \to D_s^*\gamma)\omega^3}{\Gamma(D_{s1}(2460) \to D_s\gamma)\omega^{*3}} = \tan^2(\theta), \tag{A7}$$

where ω is the momentum of the photon in the D_s transition and ω^* is the corresponding photon momentum in the D_s^* transition. Using the published branching ratio information, with favorable assumptions, the data are consistent with $\theta = \pm 50°$. Our calculation gives $\theta = 60.7°$.

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