An estimate of the branching fraction of $\tau \to \pi\eta'\nu_\tau$

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We calculate the expected branching fraction of the second-class-current decay $\tau \to \pi\eta'\nu_\tau$, motivated by a recent experimental upper-limit determination of this quantity. The largest contribution to the branching fraction is due to the intermediate $a_0(980)$ scalar meson, assuming it is a $\bar{u}d$ state. Smaller contributions arise from $a_0(1450)$, $\rho(770)$, and $\rho(1450)$. Our calculated values are substantially below the experimental upper limit, and are smaller still if the $a_0(980)$ is a four-quark state, as often suggested. Thus, a precise measurement or tight upper limit has the potential to determine the nature of the $a_0(980)$, as well as provide information about new scalar interactions.

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I. INTRODUCTION

In a recent paper [1], we considered the branching fraction of the isospin-violating decay $\tau \to \pi\eta\nu_\tau$. We found an expected branching fraction of

$$\mathcal{B} \equiv \mathcal{B}(\tau \to \pi\eta\nu_\tau) = (0.3\text{–}1.0) \times 10^{-5}, \tag{1}$$

in rough agreement with a detailed chiral perturbation theory calculation [2] and other evaluations [3], which yielded central values in the range

$$\mathcal{B} = (1.2\text{–}1.6) \times 10^{-5}. \tag{2}$$

The experimental bound on this branching fraction, $\mathcal{B} < 1.4 \times 10^{-4}$ [4], was obtained by CLEO with an e^+e^--collision data sample of 3.5 fb^{-1}, a fraction of a percent of currently available integrated luminosity. The only related high-luminosity measurement is a stringent *BABAR* upper limit on the branching fraction of $\tau \to \pi\eta'\nu_\tau$ [5],

$$\mathcal{B}' \equiv \mathcal{B}(\tau \to \pi\eta'\nu_\tau) < 7.2 \times 10^{-6} @ 90\%\text{CL}, \tag{3}$$

obtained with an integrated luminosity of 384 fb^{-1}.

The fact that the experimental limit is lower than the results summarized in Eq. (2) raises the question of a possible discrepancy between theory and experiment. Therefore, our goal in this article is to calculate the expected value of $\mathcal{B}'$ and compare it to the experimental limit. We adapt the methods used in Ref. [1] to the present case, noting that a chiral perturbation theory calculation of this process, as performed for $\tau \to \pi\eta\nu_\tau$ by Neufeld and Rupertsberger [2], would be very useful.

First, we note several similarities and differences between the calculations of $\mathcal{B}'$ and $\mathcal{B}$:

(i) The $\bar{u}u + \bar{d}d$ fraction of the wave function which, unlike the $\bar{s}s$ and gg parts, contributes to the decay amplitude, may be smaller for the η'. While it appears that the magnitude of the $\bar{s}s$ part in relation to that of the light quarks is very similar for both states, the current estimate of the gg fraction of the η' wave function, Z_{gg}, is $|Z_{gg}|^2 = 0.3 \pm 0.2$ [6]. In our calculations we take $Z_{gg} = 0$, as this yields the most conservative limits on $\mathcal{B}'$, and since the modification for finite values of Z_{gg} is straightforward.

(ii) Calculations of $\mathcal{B}$ in Refs. [1–3] rely on extrapolations utilizing intermediate, low-mass $J^{PC} = 1^{--}$ and 0^{++} hadrons. Obvious intermediate states for the decay $\tau \to \pi\eta\nu_\tau$ are the ground-state mesons $\rho(770)$ and $a_0(980)$. In the case of $\tau \to \pi\eta'\nu_\tau$, these are off-shell processes, and the contributions of these resonances are suppressed. On the other hand, we do have now on-shell decays involving the next 1^{--} and 0^{++} states. These are the $\rho' \equiv \rho(1450)$ and $a_0(1450)$, which contribute to the P- and S-wave components of the decay, respectively.

(iii) The ρ and ρ' vectors are the quark-model $\bar{u}d$, S-wave 1^{--} ground state and first radial excitation, respectively. However, the theoretical assignment of the $a_0(980)$ (and, consequently, that of the $a_0(1450)$ as well) is ambiguous, generating the largest uncertainty in both $\mathcal{B}$ and $\mathcal{B}'$. Conversely, information on these branching fractions can help resolve the long-standing dilemma of the "$\bar{K}K$-threshold" state $a_0(980)$. The significant branching fractions of $a_0(980)$ and $f_0(980)$ decays to $\bar{K}K$, despite the very small phase space, seem inconsistent with these mesons being the ground states of the quark-model scalar nonet, motivating a four-quark ($\bar{u}d\bar{s}s$) interpretation [7]. In this case, the $\bar{u}d$ scalar ground state should most likely be identified with $a_0(1450)$. However, this would make the scalar 190 MeV heavier than the axial vector state $a_1(1260)$, implying a pattern of $L \cdot S$ splitting that is different from what is observed in any other $L = 1$, $\bar{Q}q$ or $\bar{Q}Q$ sysetm. The more appealing possibility, namely, that the two 980-MeV states are indeed just $\bar{u}d$ states, may have been partially resurrected in recent work [8], in which 'tHooft's $\bar{u}u\bar{d}d\bar{s}s$ six-quark vertex was utilized to admix the 2- and 4- quark states.

The plan of this note is as follows. As we did in Ref. [1], we discuss separately our estimates of the P- and S-wave contributions to $\mathcal{B}'$. In Sec. II we present the more robust results for the P-wave part, calculating upper bounds on the contributions of the ρ and ρ' using recently published experimental data involving η' and τ decays. In Sec. III we present the less clear-cut estimate of the S-wave component. This contribution depends most strongly on whether the $a_0(980)$ is a 4-quark state or the $\bar{u}d$ ground state. In any event, our predictions for $\mathcal{B}(\tau \to \pi\eta'\nu_\tau)$ lie significantly below the *BABAR* limit [5]. A brief summary and future outlook are given in Sec. IV.

II. THE $L = 1$ CONTRIBUTION

In Ref. [1], we obtained the $L = 1$ contribution to $\mathcal{B}$ assuming that it was dominated by the ρ, an assumption justified by the large branching fraction $\mathcal{B}(\tau \to \rho\nu_\tau)$. We compared this branching fraction to $\mathcal{B}$ using the ratio of coupling constants $g_{\eta\rho\pi}/g_{\rho\pi\pi}$, where $g_{\rho\pi\pi}$ was related to the width of the ρ, and $g_{\eta\rho\pi}$ was obtained by analyzing the Dalitz-plot distribution of the decay $\eta \to \pi^+\pi^-\pi^0$, taking the scalar contribution to $\eta \to \pi^+\pi^-\pi^0$ from $\mathcal{B}(\eta \to \pi^0\pi^0\pi^0)$.

This procedure is not directly applicable to $\mathcal{B}'$, since there is no experimental information on the Dalitz-plot distribution of the decay $\eta' \to \pi^+\pi^-\pi^0$, nor a measurement of $\mathcal{B}(\eta' \to \pi^0\pi^0\pi^0)$. Therefore, we make use of the fact that the branching fraction $\mathcal{B}(\eta' \to \pi^+\pi^-\pi^0)$ depends on the coupling constant $g_{\eta'\rho\pi}$, under the conservative assumption that the $\rho^\pm$ states dominate the decay $\eta' \to \pi^+\pi^-\pi^0$. This yields a conservative upper bound on $g_{\eta'\rho\pi}$, from which we obtain an upper bound on the ρ contribution to $\tau \to \pi\eta'\nu_\tau$. We discuss the likelihood of this assumption and its implications below.

The differential branching fraction of $\eta' \to \pi^+\pi^-\pi^0$ as a function of the Dalitz-plot position is given by

$$\frac{d\Gamma_{\eta'\to\pi^+\pi^-\pi^0}}{\Gamma_{\eta'}} = \frac{(g_{\eta'\rho\pi}g_{\rho\pi\pi})^2}{384\sqrt{3}\pi^3}\frac{Q^2}{m_{\eta'}\Gamma_{\eta'}}|\overline{\mathcal{M}}|^2 dXdY, \quad (4)$$

where

$$Q \equiv m_{\eta'} - 3m_\pi \quad (5)$$

is the kinetic energy in the decay, and

$$X \equiv \frac{\sqrt{3}}{Q}(T_+ - T_-), \qquad Y \equiv \frac{3}{Q}T_0 - 1 \quad (6)$$

are the Dalitz-plot variables, with T_c being the kinetic energy of the pion with charge c. Assuming ρ dominance, we obtain from Eq. (15) of Ref. [1] the reduced matrix element

$$\overline{\mathcal{M}} = -2\frac{rY - \frac{1}{3}r^2(Y^2 + X^2)}{1 - \frac{2}{3}rY + \frac{1}{3}r^2(\frac{1}{3}Y^2 - X^2)}, \quad (7)$$

where

$$r = \frac{m_{\eta'}Q}{m_\rho^2 - \frac{1}{3}m_{\eta'}^2 - m_\pi^2 - i\Gamma_\rho m_\rho} = 1.6 + 0.7i. \quad (8)$$

The product $(g_{\eta'\rho\pi}g_{\rho\pi\pi})^2$ is then found by integrating Eq. (4) over the Dalitz plot. In the $\eta \to \pi^+\pi^-\pi^0$ case, we exploited the small value of r to simplify the expression by expanding in r. Because of the $O(1)$ value of r for $\eta' \to \pi^+\pi^-\pi^0$, we resort to numerical integration, which yields

$$\int |\overline{\mathcal{M}}|^2 dXdY = 2.4. \quad (9)$$

From this we obtain, using $\mathcal{B}(\eta' \to \pi^+\pi^-\pi^0) = 3.7 \times 10^{-3}$ [9] and $g_{\rho\pi\pi} = 6.0$ [1],

$$g_{\eta'\rho\pi} < 0.025. \quad (10)$$

As a cross check, we apply the procedure to the decay $\eta \to \pi^+\pi^-\pi^0$, obtaining $g_{\eta\rho\pi} < 0.52$. This value is to be compared to the one obtained from the more precise Dalitz-plot analysis in Ref. [1], $g_{\eta\rho\pi} \approx 0.085$. The factor of 6 ratio between the results reflects the fact that the procedure used here yields but a conservative upper bound, obtained by assuming that the decay $\eta' \to \pi^+\pi^-\pi^0$ is dominated by the $\rho^\pm$ resonances. This assumption is manifestly false, as the $\eta' \to \pi^+\pi^-\pi^0$ Dalitz-plot distribution is in much better agreement with a flat distribution than with that expected from $\rho^\pm$ dominance [9]. By contrast, in Ref. [1], the value of $g_{\eta\rho\pi}$ obtained from the Dalitz-plot distribution yielded good agreement between the expected and measured values of $\mathcal{B}(\eta \to \pi^+\pi^-\pi^0)$.

With this point in mind, we proceed to use the upper bound on $g_{\eta'\rho\pi}$ to calculate the upper bound on the ρ contribution to $\mathcal{B}(\tau \to \pi\eta'\nu_\tau)$. We do this by relating $\mathcal{B}(\tau \to \rho_{(\pi\eta')}\nu_\tau)$ to $\mathcal{B}(\tau \to \rho_{(\pi\eta)}\nu_\tau)$ via the ratio of coupling constants and phase-space factors

$$\frac{\mathcal{B}(\tau \to \rho_{(\pi\eta')}\nu_\tau)}{\mathcal{B}(\tau \to \rho_{(\pi\eta)}\nu_\tau)} \approx \left(\frac{g_{\eta'\rho\pi}}{g_{\eta\rho\pi}}\right)^2 \frac{V(\tau \to \rho_{(\pi\eta')}\nu_\tau)}{V(\tau \to \rho_{(\pi\eta)}\nu_\tau)}, \quad (11)$$

where $\rho_{(\pi\eta')}$ indicates that the ρ is observed in the $\pi\eta'$ final state, and $V(X)$ is the integral over the Dalitz plot of the three-body decay X. The ratio of phase-space integrals is 0.06, with up to 15% variation depending on whether one uses Blatt-Weisskopf and s-dependent widths for the ρ and on the choice of angular distribution. Using $\mathcal{B}(\tau \to \rho_{(\pi\eta)}\nu_\tau) = 3.6 \times 10^{-6}$ [1], we obtain

$$\mathcal{B}(\tau \to \rho_{(\pi\eta')}\nu_\tau) < 2 \times 10^{-8}, \quad (12)$$

more than 2 orders of magnitude below the *BABAR* upper limit, Eq. (3).

Next, we evaluate the contribution of the on-shell ρ'. One expects that this state, being a radial excitation and hence having a node in its wave-function, couples to the ground-state particles η and π more weakly than the ρ. We hypothesize that this ρ' suppression mechanism works equally strongly for the final states $\pi\eta'$ and $\pi\pi$, leading to an equality of the ratios of the squared matrix elements

$$\frac{\mathcal{B}(\tau \to \rho'_{(\pi\eta')}\nu_\tau)}{\mathcal{B}(\tau \to \rho_{(\pi\eta')}\nu_\tau)} \frac{V(\tau \to \rho_{(\pi\eta')}\nu_\tau)}{V(\tau \to \rho'_{(\pi\eta')}\nu_\tau)} \approx \frac{\mathcal{B}(\tau \to \rho'_{(\pi\pi)}\nu_\tau)}{\mathcal{B}(\tau \to \rho_{(\pi\pi)}\nu_\tau)} \frac{V(\tau \to \rho_{(\pi\pi)}\nu_\tau)}{V(\tau \to \rho'_{(\pi\pi)}\nu_\tau)}. \tag{13}$$

The relevant phase-space integral ratios are

$$\frac{V(\tau \to \rho_{(\pi\eta')}\nu_\tau)}{V(\tau \to \rho'_{(\pi\eta')}\nu_\tau)} \approx 0.06, \qquad \frac{V(\tau \to \rho_{(\pi\pi)}\nu_\tau)}{V(\tau \to \rho'_{(\pi\pi)}\nu_\tau)} \approx 2.5. \tag{14}$$

We use the upper bound of Eq. (12) and the central value plus 1 standard deviation of the recent Belle result [10]

$$\sqrt{\frac{\mathcal{B}(\tau \to \rho'_{(\pi\pi)}\nu_\tau)}{\mathcal{B}(\tau \to \rho_{(\pi\pi)}\nu_\tau)}} = 0.15 \pm 0.05^{+0.15}_{-0.04} \tag{15}$$

to obtain the conservative upper limit

$$\mathcal{B}(\tau \to \rho'_{(\pi\eta')}\nu_\tau) < 8 \times 10^{-8}. \tag{16}$$

We note that this is an upper bound both due to the way we use Eq. (15) and since Eq. (12) is an upper bound.

III. THE $L = 0$ CONTRIBUTION

Calculating the $L = 0$ contributions to $\mathcal{B}'$ is not as straightforward as the $L = 1$ case, where one can make use of the dominant ρ coupling to the leptonic vector current. Therefore, it is important to evaluate the scalar component using different methods, as has been done for the $\tau \to \pi\eta\nu_\tau$ decay [1–3]. It should be noted that these calculation are performed under the assumption that the relevant scalar resonances are $\bar{u}d$ states. The coupling of a 4-quark state to the $\bar{u}d$ scalar current is "Zweig-Rule" suppressed, making it significantly smaller than the predictions.

Here we perform a more detailed version of the calculation used in Ref. [1]. We begin with the ratio of branching fractions

$$R_{a_1}^{a_0} \equiv \frac{\mathcal{B}(\tau \to a_0\nu_\tau)}{\mathcal{B}(\tau \to a_1\nu_\tau)} = \frac{p_{a_0}}{p_{a_1}} \times \frac{|\langle a_0|V_{h\mu}|0\rangle\langle\nu_\tau|J_l^\mu|\tau\rangle|^2}{|\langle a_1|A_{h\mu}|0\rangle\langle\nu_\tau|J_l^\mu|\tau\rangle|^2}, \tag{17}$$

where a_0 stands for either $a_0(980)$ or $a_0(1450)$, a_1 is the $a_1(1260)$, p_X is the τ-rest-frame momentum of the products of the decay $\tau \to X\nu_\tau$, $V_{h\mu} \equiv \bar{\psi}_u(x)\gamma_\mu\psi_d(x)$ is the hadronic vector current, $A_{h\mu} \equiv \bar{\psi}_u(x)\gamma_\mu\gamma^5\psi_d(x)$ is the hadronic axial vector current, and $J_l^\mu \equiv \bar{\psi}_{\nu_\tau}(x)\gamma^\mu(1-\gamma^5)\psi_\tau(x)$ is the leptonic current. The calculation of the leptonic parts of this ratio is well defined, while all the uncertainty in the hadronic parts comes down to a single parameter ξ, which shall be defined shortly. With this in mind, we can take the a_0 matrix element to be

$$\langle a_0|V_{h\mu}|0\rangle = f_0 \frac{q_\mu}{m_{a_0}}\langle a_0|S_h|0\rangle, \tag{18}$$

where f_0 is an isospin-violation suppression factor, and $S_h \equiv \bar{\psi}_u(x)\psi_d(x)$ is the scalar current operator. The weak vector current is conserved up to the difference between the u- and d-quark masses, plus a smaller electromagnetic part that we neglect. Therefore,

$$\partial^\mu V_{h\mu} \approx (m_d - m_u)S_h. \tag{19}$$

Using this relation in Eq. (18) yields

$$f_0 = \frac{m_d - m_u}{m_{a_0}}. \tag{20}$$

We use the fact that both the a_0 and the a_1 are P-wave states to relate the axial and scalar decay constants

$$\langle a_1|A_\mu|0\rangle = \xi\epsilon_\mu^*\langle a_0|S|0\rangle. \tag{21}$$

We note that this is reminiscent of applying $SU(6)$ [11] or, in this case, just $SU(4)$ [12] flavor-spin symmetry to the ($L = 0$) 15-plet plus singlet containing the π, ρ, η, and ω, or the ($L = 1$) states a_0, a_1, f_0, and h_1.

Naively, one expects ξ in Eq. (21) to be of order unity. However, this parameter incorporates all the hadronic uncertainty in our procedure. With Eqs. (18)–(21), Eq. (17) becomes, after spin averaging and index contraction,

$$R_{a_1}^{a_0} = |\xi|^2 \frac{p_{a_0}}{p_{a_1}}\left(\frac{m_d - m_u}{m_{a_0}}\right)^2 \frac{m_\tau^2 - m_{a_0}^2}{m_\tau^2 - m_{a_1}^2}\left(\frac{m_{a_1}}{m_{a_0}}\right)^2 \times \frac{1}{1 + 2(m_\tau/m_{a_1})^2}. \tag{22}$$

This yields the branching fractions

$$\begin{aligned} \mathcal{B}(\tau \to a_0(980)\nu_\tau) &= 1.6 \times 10^{-6}|\xi|^2, \\ \mathcal{B}(\tau \to a_0(1450)\nu_\tau) &= 6.4 \times 10^{-8}|\xi|^2, \end{aligned} \tag{23}$$

where, as in Ref. [1], we chose the mass difference of the two light quarks to be 4 MeV [13] and, assuming that the $\tau \to 3\pi\nu_\tau$ decay is dominated by the a_1, we took $\mathcal{B}(\tau \to a_1\nu_\tau) = 0.18$. We compare $\mathcal{B}(\tau \to a_0(980)\nu_\tau)$ of Eq. (23) with the value $\mathcal{B} = 1.2 \times 10^{-5}$, obtained from the more elaborate calculation of Ref. [2], minus the ρ contribution to $\mathcal{B}$, which is 3.6×10^{-6} [1]. This yields $|\xi|^2 \approx 5$, from which we conclude

$$\mathcal{B}(\tau \to a_0(1450)\nu_\tau) \approx 3 \times 10^{-7}. \tag{24}$$

The $a_0(1450)$ contribution to $\tau \to \pi\eta'\nu_\tau$ depends also on the branching fraction $\mathcal{B}(a_0(1450) \to \pi\eta')$, regarding which there is only partial information. However, from the branching-fraction measurements that have been made [13], it is clear that $\mathcal{B}(a_0(1450) \to \pi\eta') < 0.3$. Hence

$$\mathcal{B}(\tau \to a_0(1450)_{(\pi\eta')}\nu_\tau) < 1 \times 10^{-7}. \tag{25}$$

If the $a_0(1450)$ is a radial excitation, which is the case if the $a_0(980)$ is the $\bar{u}d$ ground state, then $\mathcal{B}(\tau \to a_0(1450)_{(\pi\eta')}\nu_\tau)$ should be suppressed by an additional wave-function overlap factor.

Next, we look at the contribution of the $a_0(980)$ to $\tau \to \pi\eta'\nu_\tau$, which can be extracted from the relation

$$\frac{\mathcal{B}(\tau \to \nu a_0(980)_{(\pi\eta')})}{\mathcal{B}(\tau \to \nu a_0(980)_{(\pi\eta)})} = \frac{V(\tau \to \nu a_0(980)_{(\pi\eta')})}{V(\tau \to \nu a_0(980)_{(\pi\eta)})} R_\eta^{\eta'}, \tag{26}$$

where

$$R_\eta^{\eta'} \equiv \left| \frac{\mathcal{M}(a_0(980) \to \pi\eta')}{\mathcal{M}(a_0(980) \to \pi\eta)} \right|^2 \tag{27}$$

is the square of the ratio between the relevant hadronic-decay matrix elements. We assume that $R_\eta^{\eta'}$ equals the corresponding ratio of $a_0(1450)$-decay matrix elements, and is hence obtained from

$$R_\eta^{\eta'} \approx \frac{\mathcal{B}(a_0(1450) \to \pi\eta')}{\mathcal{B}(a_0(1450) \to \pi\eta)} \times \frac{p_\eta}{p_{\eta'}}, \tag{28}$$

where p_X is the $a_0(1450)$-rest-frame momentum of the products of the decay $a_0(1450) \to \pi X$. Given the $\sim 50\%$ error [13] on the ratio of branching fractions appearing in Eq. (28) and the uncertainty on the $a_0(1450)$ width, $R_\eta^{\eta'}$ comes out in the range [0.25, 1.25]. The ratio of the phase-space integrals in Eq. (26) is 0.06, with some dependence on what one takes for the $a_0(980)$ width. Using the range for $\mathcal{B}$ from Eq. (2), we obtain

$$\mathcal{B}(\tau \to a_0(980)_{(\pi\eta')}\nu_\tau) \approx [0.2 \text{ to } 1.2] \times 10^{-6}. \tag{29}$$

IV. CONCLUSIONS

Combining Eqs. (12), (16), (25), and (29), we obtain the branching fraction limit

$$\mathcal{B}(\tau \to \pi\eta'\nu_\tau) < 1.4 \times 10^{-6}, \tag{30}$$

in no conflict with the experimental upper limit, Eq. (3), which is about 5 times greater. Our result is dominated by the $a_0(980)$ contribution, assuming it is a $\bar{u}d$ state.

The experimental limit was obtained with only a third of the currently available *BABAR* and Belle data sets, and with the η reconstructed only in the $\gamma\gamma$ final state. Therefore, an improvement in the limit can be expected from the current generation of *B* factories, but probably not to the level of Eq. (30). By contrast, a Super *B* factory [14], with 2 orders of magnitude more luminosity, will be able to use $\mathcal{B}$ and $\mathcal{B}'$ to investigate the nature of the $a_0(980)$ and to search for new interactions mediated by heavy scalars [1].

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