Resummation of infrared divergences in the free energy of spin-two fields

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We derive a closed form expression for the sum of all the infrared divergent contributions to the free energy of a gas of gravitons. An important ingredient of our calculation is the use of a gauge fixing procedure such that the graviton propagator becomes both traceless and transverse. This has been shown to be possible, in a previous work, using a general gauge fixing procedure, in the context of the lowest order expansion of the Einstein-Hilbert action, describing noninteracting spin-two fields. In order to encompass the problems involving thermal loops, such as the resummation of the free energy, in the present work, we have extended this procedure to the situations when the interactions are taken into account.

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I. INTRODUCTION

The usual perturbative expansion of the free energy of a gas of massless bosons at temperature T contains infrared divergences when three or more loops are taken into account. When all the infrared divergent diagrams are summed, the resulting expression is finite but nonanalytic in the coupling constant. This is well known in the context of scalar or spin-one gauge fields [1–3]. The same situation is expected for spin-two gauge fields when we apply the finite temperature field theory techniques to the graviton gauge field described by the weak field expansion of the Einstein-Hilbert action [4]. One of the main purposes of the present paper is to obtain the explicit full result for the summation of these so-called *ring diagrams* in the case of gravity.

In Fig. 1 we show a typical infrared divergent ring diagram containing two insertions of the graviton self-energy. It illustrates the two important ingredients in the analysis of any of the higher order ring contributions. First we need the full tensor structure of the dominant high-temperature contribution to the graviton self-energy (the blob in Fig. 1) in the static limit. This is a well known quantity which has been studied and shown to be gauge independent [5,6]. Second, we need the three-level graviton propagator (the curly line in Fig. 1) connecting the two self-energies in the ring diagram. Once we know these quantities any ring diagram can be obtained by multiple insertions of the self-energy in a closed loop of gravitons, in a rather straightforward manner. At this point the tensor properties of the free propagator are essential in order to obtain a closed form and simple result for the resummed free energy. As we will see, if the propagator satisfies the following traceless-transverse (TT) conditions:

$$\eta^{\mu\nu} D^{\rm TT}_{\mu\nu,\lambda\sigma}(k) = 0, \tag{1.1a}$$

$$k^{\mu} D^{\rm TT}_{\mu\nu,\lambda\sigma}(k) = 0, \tag{1.1b}$$

then the sum of the ring diagrams acquires a simple form in terms of the three TT projections of the static high-temperature limit of the graviton self-energy.

Taking into account the gauge independence of the leading contributions of the ring diagrams, one can choose the most convenient gauge for the graviton propagator. It has been shown in Ref. [7] that it is possible to choose a gauge such that the graviton propagator becomes TT. In principle we could just assume that the known gauge invariant result for the graviton self-energy could be used safely in the ring diagrams. However, since the derivation of the TT propagator is only possible if we consider some modifications in the usual Faddeev-Popov procedure, which necessarily leads to new ghosts and interactions, we have also performed the explicit calculation of the leading static graviton self-energy and verified that the result is indeed the correct gauge invariant one. Considering that this calculation involves some rather nontrivial cancellations of diagrams, it also constitutes an important test of the gauge fixing procedure introduced in [7].

This paper is organized as follows. In Sec. II we will review the generalization of the Faddeev-Popov formalism which leads to a traceless-transverse graviton propagator. We extend the procedure presented in Ref. [7] by taking into consideration interactions of gravitons and ghosts more explicitly. In Sec. III we present the calculation of the high-temperature static limit of the graviton self-

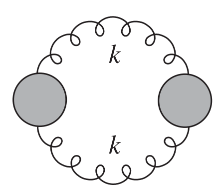

FIG. 1. The lowest order infrared divergent contribution to the free energy. The curly line represents the graviton propagator $D_{\mu\nu,\alpha\beta}$ and the blob represents the graviton self-energy.

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energy. Having verified that our gauge fixing procedure yields the correct gauge independent result for the self-energy, we proceed, in Sec. IV, to the calculation of the sum of all the infrared divergent contributions to the free energy. Finally, in Sec. V we discuss the main results.

II. GENERAL GAUGE FIXING AND FEYNMAN RULES

In this section we will follow the basic idea of Sec. II of Ref. [7] in order to derive the TT propagator. In addition to that derivation, we will also obtain the interaction vertices involving three types of ghosts and gravitons (see also Ref. [8] for an analogous derivation in the context of spin-one fields where it has been shown that a global gauge invariance, analogous to the Becchi-Rouet-Stora-Tyutin (BRST) invariance is present in the effective action). This involves a generalization of the well known Faddeev-Popov procedure [9,10] to the cases when the gauge fixing condition is nonquadratic.

In order to illustrate the main features of the generalization of the Faddeev-Popov procedure, it is convenient to consider the following integral over the components of a n-dimensional vector $\vec{h}$:

$$Z = \int d\vec{h} \exp S(\vec{h}), \tag{2.1}$$

where

$$S(\vec{h}) = -\vec{h}^T \underset{\sim}{M} \vec{h} + S_i(\vec{h}). \tag{2.2}$$

Later we will associate $\vec{h}$ with the graviton field $h_{\mu\nu}(x)$; the first and second terms in Eq. (2.2) will be identified, respectively, with the quadratic and interaction terms which arise from the weak field expansion of the Einstein-Hilbert action.

Let us now consider the interesting case when $S(\vec{h})$ is invariant under an infinitesimal transformation of the form

$$\vec{h} \rightarrow \vec{h} + \underset{\sim}{A}(\vec{h})\vec{\theta}, \tag{2.3}$$

where the operator $\underset{\sim}{A}(\vec{h})$ is of first order in the derivative operator ∂ as well as in $\vec{h}$. This symmetry makes the integration in Eq. (2.1) undefined so that we have to employ the Faddeev-Popov procedure which leads to the introduction of a "gauge fixing term," yielding a quadratic term of the form $\vec{h}\overline{\underset{\sim}{M}}\vec{h}$, such that $\underset{\sim}{D}^{(0)} = \overline{\underset{\sim}{M}}^{-1}$ is well defined. In the context of gauge field theories, $\underset{\sim}{D}^{(0)}$ is the free propagator which will be dependent on the specific choice of gauge fixing. In the case of gravity, some rather general gauge fixing conditions have been investigated previously [11]. However, in Ref. [7] it has been shown that it is not possible to obtain a graviton propagator satisfying the TT conditions given in (1.1), using the standard gauge fixing procedures. In what follows we will present the main steps which are involved in the generalized gauge fixing and them apply the results to the case of the Einstein-Hilbert action.

First, we introduce the following factors of "1" in the integrand of Eq. (2.1):

$$1 = \int d\vec{\theta}_1 \delta(\underset{\sim}{F}(\vec{h} + \alpha \underset{\sim}{A}\vec{\theta}_1)) \det(\alpha \underset{\sim}{F}\underset{\sim}{A}), \tag{2.4a}$$

$$1 = \int d\vec{\theta}_2 \delta(\underset{\sim}{G}(\vec{h} + \alpha \underset{\sim}{A}\vec{\theta}_2)) \det(\alpha \underset{\sim}{G}\underset{\sim}{A}), \tag{2.4b}$$

and

$$1 = (\alpha\pi)^{-n} \int d\vec{p}\, d\vec{q} \exp\left(-\frac{1}{\alpha}\vec{p}^T \underset{\sim}{N}\vec{q}\right) \det \underset{\sim}{N}, \tag{2.4c}$$

where the $\underset{\sim}{N}$ is a "Nielsen-Kallosh" factor [12–14], $\underset{\sim}{F}$ and $\underset{\sim}{G}$ are two independent operators, which we will assume that are of first order in ∂, and δ is the Dirac delta function. The use of two operators makes the gauge fixing prescription more general; the usual Faddeev-Popov procedure would introduce only two factors of 1, which corresponds to make the special identification $\underset{\sim}{F} = \underset{\sim}{G}$.

The next step consists in using the infinitesimal gauge transformation

$$\vec{h} \rightarrow \vec{h} - \alpha \underset{\sim}{A}\vec{\theta}_1. \tag{2.5}$$

After integrating out the $\vec{p}$ and $\vec{q}$ variables and setting, for simplicity, $\vec{N}$ equal to the identity operator, we obtain

$$Z = (\alpha)^{2n} \int d\vec{\theta}_1 \int d\vec{\theta} \int d\vec{h} \det(\underset{\sim}{F}\underset{\sim}{A}) \det(\underset{\sim}{G}\underset{\sim}{A}) \times \exp\left[-\vec{h}^T\left(\underset{\sim}{M} + \frac{1}{\alpha}\underset{\sim}{F}^T\underset{\sim}{G}\right)\vec{h} + S_i(\vec{h}) - \vec{h}^T\underset{\sim}{F}^T\underset{\sim}{G}\underset{\sim}{A}\vec{\theta}\right], \tag{2.6}$$

where $\vec{\theta} = \vec{\theta}_2 - \vec{\theta}_1$.

If we want to avoid the use of mixed propagators which would be generated by the last term in (2.6), then we also have to perform the shift

$$\vec{h} \rightarrow \vec{h} + \frac{1}{2}\left(\underset{\sim}{M} + \frac{1}{\alpha}\underset{\sim}{F}^T\underset{\sim}{G}\right)^{-1}(\underset{\sim}{F}^T\underset{\sim}{G}\underset{\sim}{A})\vec{\theta}, \tag{2.7}$$

so that (2.6) yields

$$Z = \int d\vec{\theta} \int d\vec{h} \det(\underset{\sim}{F}\underset{\sim}{A}) \det(\underset{\sim}{G}\underset{\sim}{A}) J \times \exp\left\{-\vec{h}^T\left(\underset{\sim}{M} + \frac{1}{\alpha}\underset{\sim}{F}^T\underset{\sim}{G}\right)\vec{h} + S_i\left(\vec{h} + \frac{1}{2}\left(\underset{\sim}{M} + \frac{1}{\alpha}\underset{\sim}{F}^T\underset{\sim}{G}\right)^{-1}(\underset{\sim}{F}^T\underset{\sim}{G}\underset{\sim}{A})\vec{\theta}\right) - \frac{1}{4}\vec{\theta}^T(\underset{\sim}{A}^T\underset{\sim}{G}^T\underset{\sim}{F})\left(\underset{\sim}{M} + \frac{1}{\alpha}\underset{\sim}{F}^T\underset{\sim}{G}\right)^{-1}(\underset{\sim}{F}^T\underset{\sim}{G}\underset{\sim}{A})\vec{\theta}\right\}, \tag{2.8}$$

where we have dropped the infinite normalization factors as well as the integration over the gauge orbit $\int d\vec{\theta}_1$. Since the transformation (2.7) is not a gauge transformation, its Jacobian may not be equal to one. For this reason, we have also introduced the Jacobian factor J in the integrand of (2.8).

The determinants in Eq. (2.8) can be exponentiated using the standard Berezin integral

$$\det \underset{\sim}{B} = \int d\vec{c} d\vec{\bar{c}} \exp(-\vec{\bar{c}}^T \underset{\sim}{B} \vec{c}), \tag{2.9}$$

where $\vec{c}$ and $\vec{\bar{c}}$ are Grassmann vectors; the first two determinants in Eq. (2.6) lead to "Faddeev-Popov"–like ghosts and the field $\vec{\theta}$ is a "bosonic" ghost. There would be also an extra ghost field associated with the determinant J. However, one can argue that in the applications to be considered in the present work, these extra ghost fields will not contribute. The final expression for Z can be written as

$$Z = \int d\vec{\theta} d\vec{c} d\vec{\bar{c}} d\vec{d} d\vec{\bar{d}} d\vec{h} J \exp[S_{\text{eff}}], \tag{2.10}$$

where

$$\begin{aligned} S_{\text{eff}} = & -\vec{h}^T \left(\underset{\sim}{M} + \frac{1}{\alpha} \underset{\sim}{F}^T \underset{\sim}{G} \right) \vec{h} \\ & + S_i \left(\vec{h} + \frac{1}{2} \left(\underset{\sim}{M} + \frac{1}{\alpha} \underset{\sim}{F}^T \underset{\sim}{G} \right)^{-1} (\underset{\sim}{F}^T \underset{\sim}{G} \underset{\sim}{A}) \vec{\theta} \right) \\ & - \frac{1}{4} \vec{\theta}^T (\underset{\sim}{A}^T \underset{\sim}{G}^T \underset{\sim}{F}) \left(\underset{\sim}{M} + \frac{1}{\alpha} \underset{\sim}{F}^T \underset{\sim}{G} \right)^{-1} (\underset{\sim}{F}^T \underset{\sim}{G} \underset{\sim}{A}) \vec{\theta} \\ & - \vec{\bar{c}}^T \underset{\sim}{F} \underset{\sim}{A} \vec{c} - \vec{\bar{d}}^T \underset{\sim}{G} \underset{\sim}{A} \vec{d}. \end{aligned} \tag{2.11}$$

Let us now consider the specific example when the general "action" in (2.11) describes the behavior of spin-two fields. In this case, the classical dynamics is obtained from the Einstein-Hilbert action

$$S_{\text{EH}} = \frac{2}{\kappa^2} \int d^d x \sqrt{-g} R, \tag{2.12}$$

where $\kappa^2 = 32\pi G$ (G has mass dimension $2 - d$), R is the Ricci scalar, $g = \det g_{\mu\nu}$ and

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu} \tag{2.13}$$

is the definition of the metric in terms of the graviton field $h_{\mu\nu}$, which is to be associated with $\vec{h}$. It is straightforward to obtain the corresponding expression for the quadratic operator $\underset{\sim}{M}$ as well as the first interaction terms (there will be an infinite number of graviton self-interactions) when (2.12) is expanded in powers of the κ. From the lowest order quadratic contribution one obtains the following tensor components of $\underset{\sim}{M}$:

$$\begin{aligned} (\underset{\sim}{M})^{\mu\nu,\alpha\beta} = & -\tfrac{1}{2}[\tfrac{1}{2}(\eta^{\mu\alpha}\eta^{\nu\beta} + \eta^{\mu\beta}\eta^{\nu\alpha}) - \eta^{\mu\nu}\eta^{\alpha\beta}]\partial^2 \\ & + \tfrac{1}{4}(\eta^{\mu\alpha}\partial^\nu\partial^\beta + \eta^{\mu\beta}\partial^\nu\partial^\alpha + \eta^{\nu\alpha}\partial^\mu\partial^\beta \\ & + \eta^{\nu\beta}\partial^\mu\partial^\alpha) - \tfrac{1}{2}(\eta^{\mu\nu}\partial^\alpha\partial^\beta + \eta^{\alpha\beta}\partial^\mu\partial^\nu). \end{aligned} \tag{2.14}$$

The Einstein-Hilbert action (2.12) is invariant under the space-time-dependent coordinate transformation $x^\mu \to x^\mu - \theta^\mu(x)$, where $\theta^\mu(x)$ are the generators of the transformation. This induces the following gauge transformation of the graviton field:

$$\kappa h_{\mu\nu} \to \kappa h_{\mu\nu} + (\underset{\sim}{A})_{\mu\nu,\lambda}(h)\theta^\lambda, \tag{2.15}$$

where

$$\begin{aligned} (\underset{\sim}{A})_{\mu\nu,\lambda} = & \eta_{\mu\lambda}\partial_\nu + \eta_{\nu\lambda}\partial_\mu + \kappa(h_{\mu\lambda}\partial_\nu + h_{\nu\lambda}\partial_\mu \\ & + (\partial_\lambda h_{\mu\nu})) \end{aligned} \tag{2.16}$$

can now be identified with the tensor components of $\underset{\sim}{A}$ in (2.11).

Let us now introduce the tensor components of the gauge fixing conditions $\underset{\sim}{F}$ and $\underset{\sim}{G}$. Following the analysis performed in Ref. [7] we choose

$$(\underset{\sim}{F})^{\lambda,\mu\nu} = g_1 \eta^{\mu\nu}\partial^\lambda + \eta^{\mu\lambda}\partial^\nu, \tag{2.17a}$$

$$(\underset{\sim}{G})^{\lambda,\mu\nu} = g_2 \eta^{\mu\nu}\partial^\lambda + \eta^{\mu\lambda}\partial^\nu, \tag{2.17b}$$

where g_1 and g_2 are two independent gauge parameters. It has been shown that this choice is sufficiently general in order to make the propagator TT when the limit $\alpha \to 0$ is taken. In addition, the conditions defined by (2.17) also interpolate continuously between other usual gauges such as the de Donder gauge in which case $g_1 = g_2 = -1/2$ and $\alpha = 1$.

From the explicit expressions for $\underset{\sim}{M}$, $\underset{\sim}{A}$, $\underset{\sim}{F}$ and $\underset{\sim}{G}$ given, respectively, by Eqs. (2.14), (2.16), and (2.17), together with Eq. (2.11) and the relevant expressions for S_i, the momentum space Feynman rules can now be obtained. Notice also that the multiplication rules for the operators $\underset{\sim}{F}$, $\underset{\sim}{G}$ and $\underset{\sim}{A}$ are such that

$$(\underset{\sim}{F}^T \underset{\sim}{G})^{\mu\nu,\alpha\beta} = F_{\lambda,}{}^{\mu\nu} G^{\lambda,\alpha\beta}, \tag{2.18}$$

$$(\underset{\sim}{G}\underset{\sim}{A})^\delta{}_\lambda = G^{\delta,\mu\nu} A_{\mu\nu,\lambda}, \tag{2.19}$$

$$(\underset{\sim}{F}^T \underset{\sim}{G} \underset{\sim}{A})^{\mu\nu}{}_\lambda = F_{\delta,}{}^{\mu\nu} G^{\delta,\alpha\beta} A_{\alpha\beta,\lambda} \tag{2.20}$$

with analogous relations for the other terms in Eq. (2.11). Also, the individual terms in the action are such that, for instance,

$$\vec{h}^T \underset{\sim}{M} \vec{h} = \int d^d x h_{\mu\nu} (\underset{\sim}{M})^{\mu\nu\alpha\beta} h_{\alpha\beta} \tag{2.21}$$

and

$$\vec{\bar{c}} (\underset{\sim}{F}\underset{\sim}{A}) \vec{c} = \int d^d x \bar{c}^\lambda (\underset{\sim}{F}\underset{\sim}{A})^\delta{}_\lambda c_\delta. \tag{2.22}$$

The momentum space Feynman rules can now be derived in the usual fashion using the above identifications for each term in the action (2.11). (This tedious but straightforward task has been done with the help of the computer algebra program HIP [15].) The expressions for the graviton and the θ ghosts propagators are explicitly given in Ref. [7]. For completeness (and also to define the present normalizations and notations) let us briefly rederive those expressions. Here we will also give the explicit expressions for the c and d ghost propagators.

The free graviton propagator in momentum space, $D^h_{\mu\nu,\alpha\beta}(k)$, can be readily obtained from the first term in Eq. (2.11) as the solution of

$$D^h_{\mu\nu,\alpha\beta}\left(\underset{\sim}{M} + \frac{1}{\alpha}\underset{\sim}{F}^T\underset{\sim}{G}\right)^{\alpha\beta;\lambda\gamma}_{\text{symm}} = I^{\lambda\gamma}_{\mu\nu},, \tag{2.23}$$

where the subscript "symm" indicates that we have taken into account the bosonic symmetry of the graviton field $h_{\mu\nu}$. Also, it is implicit that we have already made the Fourier transformations to the momentum space. From the symmetry properties under interchange of tensor indices, it follows that we can parametrize the solution of Eq. (2.23) in terms of five quantities $\mathbf{C}_i$, $i = 1, \ldots, 5$, as

$$D^h_{\mu\nu,\alpha\beta}(k) = \frac{1}{2k^2}\sum_{i=1}^{5}\mathbf{C}^i T^i_{\mu\nu,\alpha\beta}(k), \tag{2.24}$$

where

$$\begin{aligned}
T^1_{\mu\nu,\alpha\beta} &= \eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\alpha}\eta_{\nu\beta},\\
T^2_{\mu\nu,\alpha\beta} &= \eta_{\mu\nu}\eta_{\alpha\beta},\\
T^3_{\mu\nu,\alpha\beta} &= \frac{1}{k^2}(\eta_{\mu\alpha}k_\nu k_\beta + \eta_{\mu\beta}k_\nu k_\alpha) + \mu \leftrightarrow \nu,\\
T^4_{\mu\nu,\alpha\beta} &= \frac{1}{k^2}(\eta_{\mu\nu}k_\alpha k_\beta + \eta_{\alpha\beta}k_\mu k_\nu),\\
T^5_{\mu\nu,\alpha\beta} &= \frac{1}{k^4}k_\mu k_\nu k_\alpha k_\beta.
\end{aligned} \tag{2.25}$$

In terms of this tensor basis, the solutions for $\mathbf{C}^i$ are given by Eqs. (48) of Ref. [7]. They depend on g_1, g_2, α and the space-time dimension d, in such a way that the TT property (1.1) is fulfilled when the limit $\alpha \longrightarrow 0$ is taken. In this case, the propagator can be expressed as

$$D^{\text{TT}}_{\mu\nu,\alpha\beta} = \frac{1}{k^2}\left(\frac{1}{2}P_{\mu\alpha}P_{\nu\beta} + \frac{1}{2}P_{\nu\alpha}P_{\mu\beta} - \frac{1}{d-1}P_{\mu\nu}P_{\alpha\beta}\right), \tag{2.26}$$

where

$$P_{\mu\nu} = \eta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \tag{2.27}$$

[notice that the transversality $k^\mu P_{\mu\nu} = 0$, idempotency $P_{\mu\alpha}P^\alpha_\nu = P_{\mu\nu}$ and the trace $\eta_{\mu\nu}P^{\mu\nu} = d - 1$ guarantee the TT condition (1.1)].

Let us now consider the quadratic ghost sectors of the action. From the last three terms in Eq. (2.11) one can obtain the quadratic and interaction terms for the three ghost fields. The quadratic terms can be readily obtained considering the contribution of the first two terms in (2.16). From these quantities the ghost propagators associated to the fields c^μ and d^μ are given, respectively, by

$$D^c_{\mu\nu}(k) = \frac{(2g_1 + 1)(\frac{k_\mu k_\nu}{k^2} - \eta_{\mu\nu}) - \eta_{\mu\nu}}{2(g_1 + 1)k^2} \tag{2.28a}$$

and

$$D^d_{\mu\nu}(k) = \frac{(2g_2 + 1)(\frac{k_\mu k_\nu}{k^2} - \eta_{\mu\nu}) - \eta_{\mu\nu}}{2(g_2 + 1)k^2}. \tag{2.28b}$$

Similarly the θ sector of (2.11) yields the following expression for the propagator of the θ ghost:

$$D^{\theta}{}_{\mu\nu} = \frac{2}{\alpha k^4}\left\{\eta_{\mu\nu} - \left[1 - \frac{1}{4(g_1+1)(g_2+1)} - \frac{1(d-1)}{8\alpha(d-2)}\left(\frac{1}{g_1+1} - \frac{1}{g_2+1}\right)^2\right]\frac{k_\mu k_\nu}{k^2}\right\}. \tag{2.29}$$

The interaction vertices can also be derived directly from Eq. (2.11). Let us recall that the quantity S_i represents all the interaction terms, starting with the three-graviton vertex, which arise from the expansion of the Einstein-Hilbert action in powers of κ. Some of the expressions for the graviton self-interaction vertices have been derived before up to the five-graviton vertex [16]. Since the argument of S_i in Eq. (2.11) has been shifted by a θ-dependent quantity, there will also be additional interaction terms between the θ field and the graviton. In Fig. 2 we show some of the new vertices involving this type of θ-graviton interactions. The numbers inside the blobs are meant to indicate when the corresponding vertex comes from the cubic or quartic terms of S_i. Here we are only considering the vertices which will contribute to the one-loop graviton self-energy. The third term in Eq. (2.11) also yields new θ-graviton interactions. In this case, there are only two diagrams which are shown in Fig. 3. Finally, in Fig. 4 we show the cubic and quartic graviton self-interactions, as well as the two diagrams involving the interaction of the

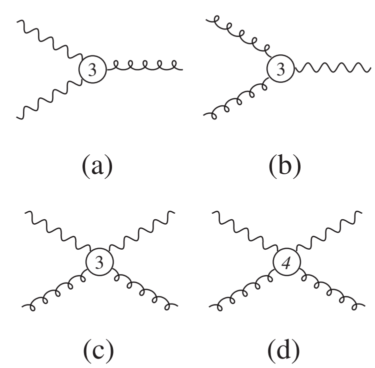


FIG. 2. Interactions between the graviton and the θ ghost arising from the second term in Eq. (2.11). The graviton and the ghost fields are, respectively, depicted by curly and wavy lines. Graphs (a), (b) and (c) arise from the shifted cubic term and graph (d) from the shifted quartic term.

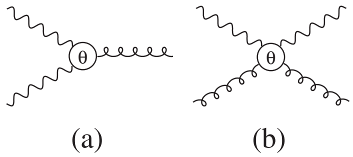


FIG. 3. Interactions between the graviton and the θ ghost arising from the third term in Eq. (2.11).

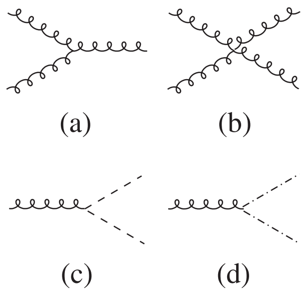


FIG. 4. (a) and (b) represent the graviton self-interactions from the Einstein-Hilbert action and (c) and (d) represent the ghost-graviton interactions from the last two terms in Eq. (2.11).

graviton with the two types of fermionic ghosts. In the next section all the vertices shown in the above figures will be employed in order to obtain the known gauge invariant result for the leading high-temperature limit of the static graviton self-energy.

III. THE STATIC SELF-ENERGY AT FINITE TEMPERATURE

The static graviton self-energy at finite temperature is a well known quantity and one of the simplest examples exhibiting gauge independence [5,6]. Therefore, it can be used as a rather nontrivial test of the gauge fixing procedure presented in the previous section. In fact, there is an even simpler example, namely, the one-graviton function. This has been considered in Ref. [7], and used as a test of the vertices involving only the interactions of three particles. In the case of the self-energy, the contribution of all the vertices shown in the previous section will be taken into account as it can be seen in Figs. 5 and 6. Therefore, one of the main results of this section will be the verification of the gauge independence of the static graviton self-energy, which will be employed in the next section, together with the TT graviton propagator, in order to derive the resummation of the infrared divergences of the free energy.

Because of the algebraic complexity involved in the calculation of some of these diagrams, we have considered the special case when the external momentum vanishes. Nonetheless, for the dominant high-temperature contribution, this special choice has the interesting property of being identical to the static limit, as we have found explicitly in the early stages of the present investigation. More recently, this equivalence of the static and the zero four-

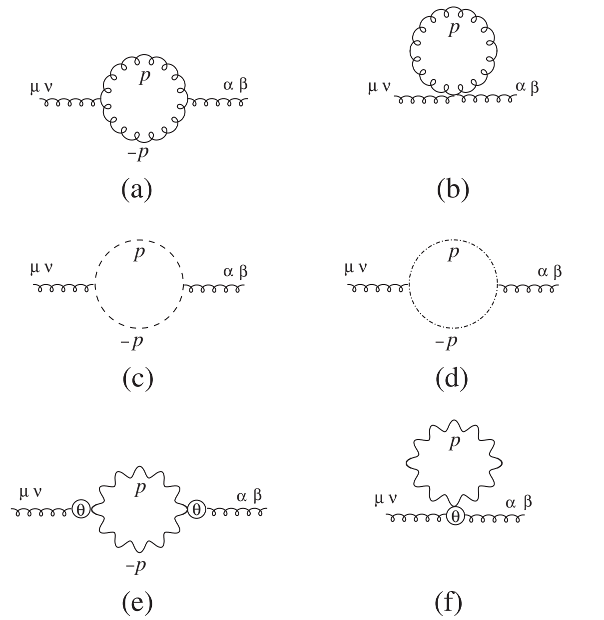


FIG. 5. Diagrams which contribute the static limit of the graviton self-energy. The curly and wave lines represent, respectively, gravitons and the θ ghost. The dashed and dotted-dashed lines represent the two types of fermionic ghosts. Graphs (a), (b), (e) and (f) have a symmetry factor $1/2$. A factor of (-1) is associated with the fermionic ghost loops in (c) and (d).

momenta limits has been proved to be true for all the thermal Green's functions [17] (this is not so, however, for the long wavelength limit [18]).

All the zero momentum diagrams (a)–(j), as well as the sum of the diagrams (k) and (l), in Figs. 5 and 6 are such

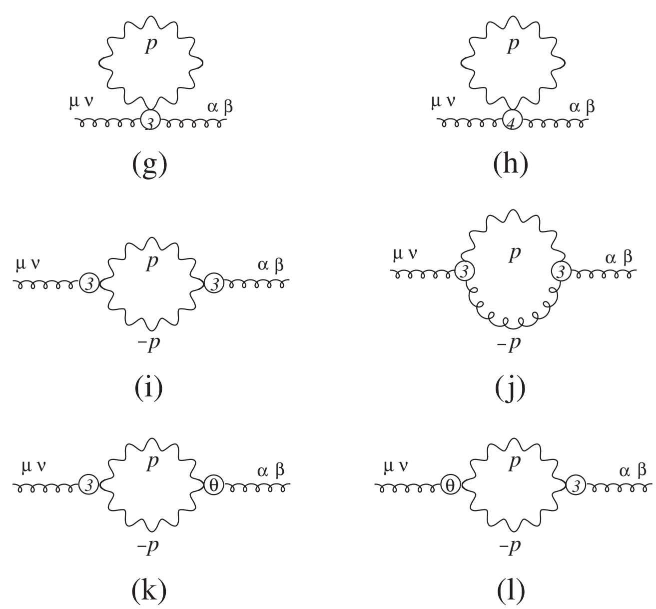


FIG. 6. Contributions involving of the θ ghosts (wavy lines) to the static limit of the graviton self-energy. Except for graph (j), all graphs have a symmetry factor $1/2$.

that their integrands have a tensor symmetry which allows one to parametrize then in terms of the basis given in Eqs. (2.25). Therefore, we can express the static thermal self-energy as

$$\Pi_{\mu\nu,\alpha\beta} = \sum_{I=1}^{11} \Pi^{I}_{\mu\nu,\alpha\beta} = \sum_{I=1}^{11} \int d^{d-1}pT \sum_{n=-\infty}^{\infty} \left(\sum_{i=1}^{5} C_I^i T^i_{\mu\nu,\alpha\beta}(p) \right), \quad (3.1)$$

where $I = 1, \ldots, 10$ and $I = 11$ labels the contributions of the diagrams from (a) to (j) and the one from the sum of diagrams (k) and (l), respectively. We are using the imaginary time formalism [1–3], so that the integrand depends on n through the Matsubara frequencies $p_0 = 2\pi nT$.

Once we compute the contributions to integrands of $\Pi^{I}_{\mu\nu,\alpha\beta}$, the expressions for C_I^i can be obtained in a straightforward manner contracting the integrand of $\Pi^{I}_{\mu\nu,\alpha\beta}$ with the five tensors $T^i_{\mu\nu,\alpha\beta}$ and solving the system of five equations. Notice that the momentum independence of the tensors $T^1_{\mu\nu,\alpha\beta}$ and $T^2_{\mu\nu,\alpha\beta}$ may require a prescription such as

$$\int d^{n-1}p \sum_{n=-\infty}^{\infty} 1 = \int d^{n-1}p \lim_{\epsilon\to 0}\left(1 + 2\sum_{n=0}^{\infty} \frac{1}{n^{\epsilon}}\right) = \int d^{n-1}p(1 + 2\zeta(0)) = 0. \quad (3.2)$$

However, for our present purpose, it would be interesting if the quantities

$$C^i = \sum_{I=1}^{11} C_I^i \quad (3.3)$$

happen to be gauge independent even before the sum and integration is performed. In order to investigate this possibility, let us first write down the results which one would obtain when the static self-energy is directly computed in the de Donder gauge. The calculation is much simpler in this case; only the diagrams (a) and (b) as well as one of the fermionic ghost loops contribute to C^i. A straightforward calculation yields

$$C^1 = \frac{d(d-5)}{16} - \frac{1}{2}, \qquad C^2 = C^4 = 0, \qquad C^3 = -\frac{d-3}{8}, \qquad C^5 = -2C^3, \quad (3.4)$$

where the second term in C^1 is the only nonvanishing contribution which comes from the ghost loop.

Let us now consider the individual contributions of the diagrams in Figs. 5 and 6 which arise in the case of a general gauge fixing. The simplest diagrams are the ones shown in Figs. 5(c) and 5(d). Using the results of the previous section, we find that each of these ghost loops yield an identical contribution such that

$$C_3^1 = C_4^1 = -\tfrac{1}{2}, \qquad C_3^i = C_4^i = 0, \qquad i = 2, \ldots, 5. \quad (3.5)$$

The θ ghost diagrams (e) and (f) in Fig. 5 have propagators and vertices which depend on the three gauge parameters as described in the previous section. Despite this, the calculation yields a result such that all gauge parameter dependence cancels out and we are left with

$$C_5^1 + C_6^1 = \tfrac{1}{2}, \qquad C_5^i + C_6^i = 0, \qquad i = 2, \ldots, 5. \quad (3.6)$$

Notice that the integrand of one of the ghost loops in Eq. (3.5) cancels with the corresponding θ loop in Eq. (3.6) in such a way that $C_3^1 + C_4^1 + C_5^1 + C_6^1 = -1/2$, which is the same as the result that one would obtain in the de Donder gauge for the single fermionic ghost loop.

Let us now introduce the quantities

$$\Delta^i = \sum_{I=1}^{6} C_I^i - C^i, \qquad i = 1, \ldots, 5, \quad (3.7)$$

where C^i are given by (3.4). If the gauge independence manifests at the integrand level, then

$$\Delta^i + \sum_{I=7}^{11} C_I^i = 0, \qquad i = 1, \ldots, 5, \quad (3.8)$$

should be verified. In the appendix we display the results for Δ^i and C_I^i ($I = 7, \ldots, 11$) and from these results it can be verified that (3.8) is indeed satisfied. The expressions in the appendix show how individual diagrams can have a very involved dependence on the three gauge parameters α, g_1 and g_2. This constitutes a rather nontrivial example of the consistence of the general gauge fixing procedure presented in the previous section.

Finally, let us substitute the gauge invariant results given in Eq. (3.4) into Eq. (3.1) and perform the sum and integration. This straightforward and standard calculation yields the following temperature-dependent expression for the static graviton self-energy in d space-time dimensions:

$$\Pi^{\text{term}}_{\mu\nu,\alpha\beta} = \sum_{i=1}^{5} \mathcal{C}^i T^i_{\mu\nu,\alpha\beta}(u), \quad (3.9)$$

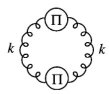
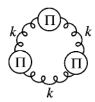
where u is the heat bath four velocity and

$$\begin{aligned} \mathcal{C}^1 &= \frac{d(d-3)2^{(4-d)}\pi^{7-d/2}}{(d-1)\Gamma(\frac{d-1}{2})} G\rho_d, \\ \mathcal{C}^2 &= -\mathcal{C}^1, \\ \mathcal{C}^3 &= 0, \\ \mathcal{C}^4 &= d\mathcal{C}^1, \\ \mathcal{C}^5 &= -d(d+2)\mathcal{C}^1; \\ \rho_d &\equiv \frac{\Gamma(d)\zeta(d)}{\pi^2} T^d, \end{aligned} \quad (3.10)$$

with Γ and ζ, respectively, Euler's and Riemann's functions. From these expressions we obtain, for $d = 4$, the results given in Eqs. (3.8) of Ref. [5] (notice that in the present work we have labeled the tensors T^i in a different order, so that the constants c_i of Ref. [5] are such that $c_1 = \mathcal{C}^1/\kappa^2 = \rho_4/12$, $c_2 = \mathcal{C}^3 = 0$, $c_3 = \mathcal{C}^5/\kappa^2 = -2\rho_4$, $c_4 = \mathcal{C}^2/\kappa^2 = -\rho_4/12$ and $c_5 = \mathcal{C}^4/\kappa^2 = \rho_4/3$ when $d = 4$). It is also possible to include contributions of other thermal particles, such as scalars or fermions. Such contributions will only modify all the $\mathcal{C}^i$ by a common integer factor which counts the number of degrees freedom associated with each field.

IV. RESUMMATION OF INFRARED DIVERGENCES IN THE FREE ENERGY

The sum of all the infrared divergent contributions to the free energy can be represented graphically as

$$\Omega(T) = -\frac{1}{2}\left[\frac{1}{2}\right.$$

$$+\frac{1}{3}$$

$$\left. + \cdots\right], \tag{4.1}$$

where the quantity Π stands for the static graviton self-energy computed in the previous section. It is implicit in (4.1) that we are considering only the *zero mode* contribution of the graviton propagator (denoted by curly lines), so that $k^2 = -|\vec{k}|^2$. The reason for this is because each graviton propagator in (4.1) introduces a factor

$$\frac{1}{k^2} = -\frac{1}{(2\pi n T)^2 + |\vec{k}|^2}, \tag{4.2}$$

so that the zero mode ($n = 0$) of each individual ring diagram in (4.1) yields an infrared divergent contribution, when $d = 4$, from the momentum integration $\int d^3k \ldots$. In scalar theories, as well as in gauge theories of spin-one fields, it is well known that the sum of all such infrared divergent contributions yields a finite result which is nonanalytic in the coupling constant [2]. In what follows we will employ the results of the last two sections in order to investigate how a similar result may be achieved in the case of spin-two fields. The details of this analysis show explicitly that the use of the TT graviton propagator allows one to obtain an explicit and compact result.

Since we are going to employ the TT graviton propagator, the computation of the right-hand side of Eq. (4.1) becomes much simpler if we express the static self-energy in terms of its traceless and transverse components. However, at finite temperature there are three distinct TT tensors which may depend on the momentum k and the heat bath four velocity u. In the static case, when $k \cdot u = k_0 = 0$ (we are adopting the rest frame of the heat bath), these tensors can be written in d dimensions as

$$T^A_{\mu\nu,\lambda\sigma}(u,k) = \frac{1}{2}\left(\bar{\eta}_{\mu\lambda}\bar{\eta}_{\nu\sigma} + \bar{\eta}_{\nu\lambda}\bar{\eta}_{\mu\sigma} - \frac{2}{d-2}\bar{\eta}_{\mu\nu}\bar{\eta}_{\lambda\sigma}\right), \tag{4.3a}$$

$$T^B_{\mu\nu,\lambda\sigma}(u,k) = \frac{1}{2}P_{\mu\lambda}P_{\nu\sigma} + \frac{1}{2}P_{\nu\lambda}P_{\mu\sigma} - \frac{1}{d-1}P_{\mu\nu}P_{\lambda\sigma} - T^A_{\mu\nu,\lambda\sigma}(u,k) - T^C_{\mu\nu,\lambda\sigma}(u,k), \tag{4.3b}$$

$$T^C_{\mu\nu,\lambda\sigma}(u,k) = \frac{1}{(d-1)(d-2)} \times\left[(d-1)u_\mu u_\nu + \frac{k_\mu k_\nu}{k^2} - \eta_{\mu\nu}\right] \times\left[(d-1)u_\lambda u_\sigma + \frac{k_\lambda k_\sigma}{k^2} - \eta_{\lambda\sigma}\right], \tag{4.3c}$$

where

$$\bar{\eta}_{\mu\nu} = \eta_{\mu\nu} - u_\mu u_\nu - \frac{k_\mu k_\nu}{k^2}. \tag{4.4}$$

For $d = 4$ the above expressions reduces to the static limit of the corresponding ones given in Ref. [5]. Notice that the sum of the three traceless-transverse tensors coincides with the numerator of the propagator in Eq. (2.26), which represents the only traceless-transverse tensor available at zero temperature, so that we can write

$$D^{\mathrm{TT}}_{\mu\nu,\alpha\beta} = \frac{1}{k^2}(T^A_{\mu\nu,\lambda\sigma} + T^B_{\mu\nu,\lambda\sigma} + T^C_{\mu\nu,\lambda\sigma}). \tag{4.5}$$

In terms of its TT components the dominant contribution to the static self-energy can then be expressed as

$$\Pi^{\mathrm{term}}_{\mu\nu,\alpha\beta} = \sum_{I=A,B,C}\bar{\mathcal{C}}^I T^I_{\mu\nu,\alpha\beta} + \bar{\mathcal{C}}^4 T^4_{\mu\nu,\alpha\beta} + \cdots, \tag{4.6}$$

where the ellipsis represents terms which are orthogonal to the TT tensors and the constants $\bar{\mathcal{C}}^I$ and $\bar{\mathcal{C}}^4$ can be solved in terms the ones given in Eq. (3.10) as follows:

$$\bar{\mathcal{C}}^A = \bar{\mathcal{C}}^B = 2\mathcal{C}^1, \tag{4.7a}$$

$$\bar{\mathcal{C}}^C = 2\mathcal{C}^1 + \frac{d-2}{d-1}\mathcal{C}^5, \tag{4.7b}$$

$$\bar{\mathcal{C}}^4 = \mathcal{C}^4 + \frac{\mathcal{C}^5}{d-1}. \tag{4.7c}$$

Notice that the dependence of each individual TT tensor T^I on the momentum variable $k = (0, \vec{k})$ is canceled when all the terms in (4.6) are taken into account so that the result becomes identical to the one in Eq. (3.9). This apparently unnecessary complication is very convenient in order to perform the calculation of the ring diagrams in (4.1). This happens because the TT tensors in (4.3) are not only traceless and transverse, but also enjoy of some very important properties. First, they are idempotent so that

$$T^I_{\mu\nu,\lambda\delta}T^{I\lambda\delta}{}_{\alpha\beta} = T^I_{\mu\nu,\alpha\beta} \qquad (I = A, B, C). \tag{4.8}$$

They are also orthogonal:

$$T^I_{\mu\nu,\lambda\delta} T^{I'\lambda\delta}{}_{\alpha\beta} = 0; \qquad I \neq I'. \tag{4.9}$$

Their "norm" is given by

$$T^A_{\mu\nu,\lambda\delta} T^{A\mu\nu\lambda\delta} = \frac{d(d-3)}{2}, \tag{4.10a}$$

$$T^B_{\mu\nu,\lambda\delta} T^{B\mu\nu\lambda\delta} = d-2, \tag{4.10b}$$

$$T^C_{\mu\nu,\lambda\delta} T^{C\mu\nu\lambda\delta} = 1. \tag{4.10c}$$

Finally, the tensors T^A and T^B also satisfy

$$T^A_{\mu\nu,\lambda\delta} u^\lambda = T^B_{\mu\nu,\lambda\delta} u^\lambda u^\delta = 0. \tag{4.11}$$

Using these properties, as well as Eqs. (4.5) and (4.6) it is straightforward to show that the integrand of the first ring diagram in Eq. (4.1) is given by

$$D^{\mathrm{TT}\mu\nu,\mu_1\nu_1} \Pi^{\mathrm{term}}_{\mu_1\nu_1,\lambda_1\sigma_1} D^{\mathrm{TT}\lambda_1\sigma_1,\mu_2\nu_2} \Pi^{\mathrm{term}}_{\mu_2\nu_2,\mu\nu} = \frac{1}{k^4}\left(\frac{d(d-3)}{2}(\bar{\mathcal{C}}^A)^2 + (d-2)(\bar{\mathcal{C}}^B)^2 + (\bar{\mathcal{C}}^C)^2\right). \tag{4.12}$$

Similarly, in the case of the higher order graphs we obtain

$$D^{\mathrm{TT}\mu\nu,\mu_1\nu_1} \Pi^{\mathrm{term}}_{\mu_1\nu_1,\lambda_1\sigma_1} \ldots D^{\mathrm{TT}\lambda_{n-1}\sigma_{n-1},\mu_n\nu_n} \Pi^{\mathrm{term}}_{\mu_n\nu_n,\mu\nu} = \frac{1}{(-|\vec{k}|^2)^n}\left(\frac{d(d-3)}{2}(\bar{\mathcal{C}}^A)^n + (d-2)(\bar{\mathcal{C}}^B)^n + (\bar{\mathcal{C}}^C)^n\right), \tag{4.13}$$

where we have used the zero mode condition $k^2 = -|\vec{k}|^2$. Notice that the components of the self-energy which are not traceless and transverse drop out in the final result. This is consistent with the fact that the TT components are physical.

We have now all the ingredients to compute the sum of the infrared divergent contributions to the free energy in Eq. (4.1). From Eq. (4.13) we can see that the sum of the ring diagrams is composed of three similar structures, each one having the same form as $-\sum_{n=2}^{\infty}(-x)^n/n = \log(1+x) - x$, so that the free energy can be written as

$$\Omega(T) = \frac{T}{2}\frac{1}{(2\pi)^{d-1}}\left[\frac{d(d-3)}{2} I(\bar{\mathcal{C}}^A) + (d-2) I(\bar{\mathcal{C}}^B) + I(\bar{\mathcal{C}}^C)\right], \tag{4.14}$$

where

$$I(c) \equiv \int d^{d-1}k\left[\log\left(1 + \frac{c}{|\vec{k}|^2}\right) - \frac{c}{|\vec{k}|^2}\right] \tag{4.15}$$

is a familiar integral which arises also in the context of scalar or vector fields and it can be done in a closed form. Performing the $d-2$ angular integral and using the change of variable $z = |\vec{k}|/\sqrt{c}$, we obtain

$$I(c) = \frac{2(\pi c)^{(d-1)/2}}{\Gamma(\frac{d-1}{2})}\int_0^\infty dz z^{d-2}\left[\log\left(1 + \frac{1}{z^2}\right) - \frac{1}{z^2}\right]. \tag{4.16}$$

Using integration by parts, and performing the resulting integral yields

$$I(c) = -\frac{4(\pi c)^{(d-1)/2}}{(d-1)\Gamma(\frac{d-1}{2})}\int_0^\infty dz \frac{z^{d-4}}{z^2+1} = -\frac{4\Gamma(\frac{5-d}{2})}{(d-1)(d-3)}(\pi c)^{(d-1)/2}. \tag{4.17}$$

Substituting (4.17) into (4.14) we obtain the following result:

$$\Omega(T) = -\frac{2\Gamma(\frac{5-d}{2})T}{(d-1)(d-3)(2\pi)^{d-1}}\left[\frac{d(d-3)}{2}(\pi\bar{\mathcal{C}}^A)^{(d-1)/2} + (d-2)(\pi\bar{\mathcal{C}}^B)^{(d-1)/2} + (\pi\bar{\mathcal{C}}^C)^{(d-1)/2}\right]. \tag{4.18}$$

Finally, using Eqs. (3.10) and (4.7) this expression can be written as

$$\Omega(T) = -\frac{2\Gamma(\frac{5-d}{2})}{(d-1)(d-3)(2\pi)^{d-1}}\left[\frac{2^{(5-d)}\pi^{5-d/2}\Gamma(d+1)\zeta(d)}{\Gamma(\frac{d-1}{2})}\right]^{(d-1)/2} \times\left\{\frac{d(d-3)}{2}\left(\frac{d-3}{d-1}\right)^{(d-1)/2} + (d-2)\left(\frac{d-3}{d-1}\right)^{(d-1)/2} + \left[\left(1 - \frac{(d-2)d(d+2)}{2(d-1)}\right)\left(\frac{d-3}{d-1}\right)\right]^{(d-1)/2}\right\} \times (GT^{d-2})^{(d-1)/2} T^d. \tag{4.19}$$

V. DISCUSSION

In this work we have investigated the possibility of extending the general gauge fixing procedure of Ref. [7] in order to take into account interacting spin-two fields. As an explicit example, we have computed the dominant one-loop contributions to the thermal self-energy of the graviton, and verified that it agrees with the known gauge invariant result. Then, using a decomposition of the self-energy in terms of the three traceless and transverse tensors which arise at finite temperature, as well as the TT graviton propagator, we were able to obtain a closed form expression for the sum of the infrared divergent contributions to the free energy.

We note that the above gauge invariant result for the graviton thermal self-energy satisfies a simple Ward identity [19] (not just a BRST identity) which is a consequence of the fact that the ghost self-energies are subleading at high temperature. Consequently, on dimensional grounds, the thermal self-energies associated with the c_μ and d_μ ghost fields must be of order $GT^{d-2}k^2$, whereas the θ_μ ghost self-energy would be proportional to $GT^{d-2}k^4$, where the factor GT^{d-2} is dimensionless. The presence of these powers of k in the thermal ghost self-energies ensures that infrared divergences will be absent in ring diagrams involving ghost propagators. This justifies the neglect of such diagrams in the evaluation of the leading infrared divergent contributions to the free energy of spin-two fields. (A similar behavior occurs also in the case of the free energy in QCD [1]).

The result presented in Eq. (4.19) has some interesting features which we would like to stress. First, for odd space-time dimensions it is a real and singular function. On the other hand, for even space-time dimensions, it is a finite and nonanalytic function of GT^{d-2} as one would expect for a nonperturbative quantity. However, in this case it acquires an imaginary part. For instance, for $d = 4$ the third term inside the curly brackets, which can be traced back to the T^C component of the self-energy, is equal to $(-7/3)^{3/2}$. As a result, one would conclude that the gravitational C mode is unstable, since the imaginary part of the free energy is connected with the decay rate of the quantum vacuum [20]. However, a detailed investigation shows that the graviton self-energy, which is proportional to GT^4, is of the same order as the solution of the Einstein equation for the curvature tensor, when the thermal energy momentum tensor is taken into account. Therefore, by consistency, one should also take into account the curvature corrections in the analysis of instabilities of gravity at finite temperature. These corrections [5,6] have the effect of adding some extra contributions to the self-energy in such a way that the C-mode contribution to $\Omega(T)$ would change the third term of the curly bracket of Eq. (4.19) to $(-7/3 + 5/27)^{3/2}$, which is still imaginary. This term may be related to an imaginary value of a thermal Jeans mass [5,6], which reflects the instability of the system due to the universal attractive nature of gravity.

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APPENDIX

Here we display the expressions for the quantities introduced in Eq. (3.8).

$$\Delta^1 = \frac{(3d-7)(g_1-g_2)^2}{8[(d-1)(g_1-g_2)^2 + 2\alpha(d-2)(g_1+1)(g_2+1)]}, \tag{A1a}$$

$$C_7^1 = 0, \tag{A1b}$$

$$G_8^1 = \frac{(d^2-11d+22)(g_1-g_2)^2}{8(d-2)[(d-1)(g_1-g_2)^2 + 2\alpha(d-2)(g_1+1)(g_2+1)]}, \tag{A1c}$$

$$G_9^1 = 0, \tag{A1d}$$

$$G_{10}^1 = -\frac{(d-3)^2(g_1-g_2)^2}{2(d-2)[(d-1)(g_1-g_2)^2 + 2\alpha(d-2)(g_1+1)(g_2+1)]}, \tag{A1e}$$

$$G_{11}^1 = 0, \tag{A1f}$$

$$\Delta^2 = -\frac{(g_1-g_2)^2[4(d-3)(g_1-g_2)^2 + \alpha(d-1)(d-2)(g_1+1)(g_2+1)]}{4[(d-1)(g_1-g_2)^2 + 2\alpha(d-2)(g_1+1)(g_2+1)]^2}, \tag{A2a}$$

$$C_7^2 = 0, \tag{A2b}$$

$$C_8^2 = -\frac{(d^2-11d+26)(g_1-g_2)^2}{8(d-2)[(d-1)(g_1-g_2)^2 + 2\alpha(d-2)(g_1+1)(g_2+1)]}, \tag{A2c}$$

$$C_9^2 = \frac{(d-5)^2(g_1-g_2)^4}{8[(d-1)(g_1-g_2)^2 + 2\alpha(d-2)(g_1+1)(g_2+1)]^2}, \tag{A2d}$$

$$C_{10}^2 = \frac{(g_1-g_2)^2[2(d-3)^2(g_1-g_2)^2 + \alpha(d-2)(d^2-7d+14)(g_1+1)(g_2+1)]}{2(d-2)[(d-1)(g_1-g_2)^2 + 2\alpha(d-2)(g_1+1)(g_2+1)]^2}, \tag{A2e}$$

$$C_{11}^2 = 0, \tag{A2f}$$

$$\Delta^3 = -\frac{(d-3)(g_1-g_2)^2}{4[(d-1)(g_1-g_2)^2 + 2\alpha(d-2)(g_1+1)(g_2+1)]}, \tag{A3a}$$

$$C_7^3 = 0, \tag{A3b}$$

$$C_8^3 = -\frac{(d-3)(d-4)(g_1-g_2)^2}{4(d-2)[(d-1)(g_1-g_2)^2 + 2\alpha(d-2)(g_1+1)(g_2+1)]}, \tag{A3c}$$

$$C_9^3 = 0, \tag{A3d}$$

$$C_{10}^3 = \frac{(d-3)^2(g_1-g_2)^2}{2(d-2)[(d-1)(g_1-g_2)^2 + 2\alpha(d-2)(g_1+1)(g_2+1)]}, \tag{A3e}$$

$$C_{11}^3 = 0, \tag{A3f}$$

$$\Delta^4 = \frac{(d-3)(g_1-g_2)^2[2(g_1-g_2)^2 + \alpha(d-2)(g_1+1)(g_2+1)]}{2[(d-1)(g_1-g_2)^2 + 2\alpha(d-2)(g_1+1)(g_2+1)]^2}, \tag{A4a}$$

$$C_7^4 = -\frac{(d-5)(g_1-g_2)^2}{2[(d-1)(g_1-g_2)^2 + 2\alpha(d-2)(g_1+1)(g_2+1)]}, \tag{A4b}$$

$$C_8^4 = \frac{(d-6)(d-3)(g_1-g_2)^2}{4(d-2)[(d-1)(g_1-g_2)^2 + 2\alpha(d-2)(g_1+1)(g_2+1)]}, \tag{A4c}$$

$$C_9^4 = -\frac{(d-3)(d-5)(g_1-g_2)^4}{4[(d-1)(g_1-g_2)^2 + 2\alpha(d-2)(g_1+1)(g_2+1)]^2}, \tag{A4d}$$

$$C_{10}^4 = -\frac{(d-3)(g_1-g_2)^2[(d-3)(g_1-g_2)^2 + \alpha(d-2)(d-4)(g_1+1)(g_2+1)]}{(d-2)[(d-1)(g_1-g_2)^2 + 2\alpha(d-2)(g_1+1)(g_2+1)]^2}, \tag{A4e}$$

$$C_{11}^4 = \frac{(d-5)(g_1-g_2)^2}{2[(d-1)(g_1-g_2)^2 + 2\alpha(d-2)(g_1+1)(g_2+1)]}, \tag{A4f}$$

$$\Delta^5 = \frac{(d-3)^2(g_1-g_2)^4}{2[(d-1)(g_1-g_2)^2 + 2\alpha(d-2)(g_1+1)(g_2+1)]^2}, \tag{A5a}$$

$$C_7^5 = \frac{(2d-6)(g_1-g_2)^2}{[(d-1)(g_1-g_2)^2 + 2\alpha(d-2)(g_1+1)(g_2+1)]}, \tag{A5b}$$

$$C_8^5 = 0, \tag{A5c}$$

$$C_9^5 = \frac{(d-3)^2(g_1-g_2)^4}{2[(d-1)(g_1-g_2)^2 + 2\alpha(d-2)(g_1+1)(g_2+1)]^2}, \tag{A5d}$$

$$C_{10}^5 = -\frac{(d-3)^2(g_1-g_2)^4}{[(d-1)(g_1-g_2)^2 + 2\alpha(d-2)(g_1+1)(g_2+1)]^2}, \tag{A5e}$$

$$C_{11}^5 = -\frac{(2d-6)(g_1-g_2)^2}{[(d-1)(g_1-g_2)^2 + 2\alpha(d-2)(g_1+1)(g_2+1)]}. \tag{A5f}$$

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