Geometric phase for a neutral particle in rotating frames in a cosmic string spacetime

Knut Bakke* and Claudio Furtado†

Departamento de Física, Universidade Federal da Paraíba, Caixa Postal 5008, 58051-970, João Pessoa, PB, Brazil

(Received 18 February 2009; published 24 July 2009)

We study of the appearance of geometric quantum phases in the dynamics of a neutral particle that possess a permanent magnetic dipole moment in rotating frames in a cosmic string spacetime. The relativistic dynamics of spin-1/2 particle in this frame is investigated and we obtain several contributions to relativistic geometric phase due rotation and topology of spacetime. We also study the geometric phase in the nonrelativistic limit. We obtain effects analogous to the Sagnac effect and Mashhoon effect in a rotating frame in the background of a cosmic string.

DOI: 10.1103/PhysRevD.80.024033 PACS numbers: 04.62.+v, 03.65.Vf, 04.25.Nx

I. INTRODUCTION

In the quantum theory, the phenomenon of the arising of topological phases in the wave function of the particles in interference experiments is one the most interesting quantum effects which have attracted a special attention in the last decades. The first quantum effect associated to the topological quantum phases, demonstrated by Aharonov and Bohm, is known as the Aharonov-Bohm (AB) effect [1]. Aharonov and Casher (AC) [2] have studied the quantum behavior of a neutral particle with a permanent magnetic dipole moment interacting with an external electric field and have obtained a topological quantum phase. The effect dual to the AC effect was proposed by He and MacKellar [3] and, independently, by Wilkens [4]: when they have investigated the quantum dynamics of an electric dipole in the presence of an external magnetic field, they have found that the wave function of the system acquires a quantum phase. Following this way, Dowling *et al.* [5] and Furtado and Duarte [6] have studied the effect dual to the AB effect using the Maxwell duality transformation and the quantum dynamics of a magnetic monopole in the presence of a electric solenoid, respectively. Recently, Horsley and Babiker have discussed the dual Aharonov-Bohm effect in the dynamics of a composite particle [7,8].

In the last decades, effects generated by the rotating frames have been studied within classical as well as within quantum mechanics. The well known effect arisen due to the rotation is the Sagnac effect [9]. The influence of the rotation was also discussed in some works [10,11] in the nonrelativistic quantum mechanics such as [12], where the influence of the rotation of the Earth gives contribution to the phase shift of the wave function. In Ref. [13], Anandan has investigated the interference of coherent beams of particles. The relativistic aspect of the rotation was studied in [14], where the Dirac equation in flat spacetime is written in the rotating frame. A review of the Sagnac effect and corresponding experiments is given in [15] and a detailed discussion of the Sagnac effect in the nonrelativistic and relativistic regimes is presented in [16]. Another effect arisen due to the rotating frames is the appearance of the Berry phase [17]. In [18], the Berry phase arises due to mechanical reasons, and possible experiments are suggested to detect this phase. In [19] a time-dependent Schröedinger equation is considered and a nonadiabatic Berry phase is shown to arise from the exact solutions of this equation of motion, thus it is proved that the Berry phase can be observed within rotating systems.

The Refs. [20,21] are devoted to discussion of the phase shift in the neutron wave function due to the rotation of the Earth, and a interesting relation between the angular momentum corresponding to the motion of the neutron around the center of the Earth and the angular velocity of the Earth is showed. In Ref. [22], Mashhoon has discussed the interference effects in rotating frames in flat spacetime and an important coupling of the spin of the particles with the angular velocity of the rotating frames, and this coupling is known as the Mashhoon effect. In Ref. [23], Hehl and Ni have studied the Dirac equation in a flat spacetime with accelerated and rotating frames showing Sagnac-type and a rotation-spin coupling effects. In Ref. [24] the origin of the Sagnac and Mashhoon effects is related to the application of Lorentz transformations. The nonrelativistic limit of the Dirac equation with accelerated and rotating frames was also studied in [23] showing redshift-like effect and the effects similar to those ones studied in Refs. [20–22]. In the presence of a gravitational field, through the weak field approximation, the Sagnac effect and the spin-rotation coupling are derived in [25] both in the relativistic and nonrelativistic dynamics of a spin one half particle.

The study of appearance of geometric phases for a neutral particle with permanent magnetic and electric dipole moments was done in some backgrounds. In [26,27], the electromagnetic effects affecting the electric and mag-

*kbakke@fisica.ufpb.br
†furtado@fisica.ufpb.br

netic dipoles moments were discussed in flat spacetime. In [28], the geometric phases for neutral particles were obtained in the context of the noncommutative quantum mechanics, and in [29], in the context of the Lorentz-symmetry violation. In the presence of a topological defect, the quantum dynamics of a charged particle with a magnetic dipole moment interacting with an electromagnetic field was investigated in [30]. The geometric phases for a neutral particle with permanent electric and magnetic dipole moments interacting with an external electric field in curved spacetime was studied in [31] and in curved spacetime and in the presence of torsion in [32].

In this paper, we discuss the relativistic and nonrelativistic behavior of a neutral particle with a permanent magnetic dipole moment interacting with an external electric field in a rotating frame in a cosmic string background. We show that, if we construct a local corotating frame where there are no torques on the dipole moment, the relativistic coupling between the spin of the particle and the rotation of the local reference frame arises naturally without need to use the weak field approximation. We also show that the wave function of the neutral particle acquires a relativistic phase shift due to the topology of the cosmic string spacetime, the interaction between the magnetic dipole moment with the external electric field, the spin-rotation coupling and due to the rotation of the local reference frame. In the nonrelativistic case we show that the wave function of the neutral particle acquires a nonrelativistic quantum phase due to five contributions: one due to the topology of the topological defect, one due to the interaction between the electric field and the magnetic dipole moment, one due to the coupling spin-rotation and two given by gauge fields arising due to the rotation of the local reference frame. We will also show that the spin-rotation coupling [22] arises in the nonrelativistic behavior of the neutral particle without using of the weak field approximation.

This work is structured as follows: In Sec. II, we present the cosmic string spacetime and the field configuration in a rotating frame. In Sec. III, we discuss the relativistic dynamics of the neutral particle with permanent magnetic dipole moment interacting with an external electric field in a rotating frame in a cosmic string spacetime and obtain the relativistic geometric phases acquired by the wave function of the neutral particle. In Sec. IV, we investigate the nonrelativistic behavior of this neutral particle in the presence of the topological defect in rotating frames and obtain the nonrelativistic geometric phases. In Sec. V, we present our conclusions.

II. COSMIC STRING SPACETIME AND FIELD CONFIGURATION IN THE ROTATING FRAME

In this section we describe the curved spacetime in the rotating frame. The chosen curved spacetime is the cosmic string spacetime, where the line element is given by

$$ds^2 = -dT^2 + dR^2 + \eta^2 R^2 d\Phi^2 + dZ^2, \quad (1)$$

where $\eta = 1 - 4\nu$ is a parameter associated with the deficit angle of cosmic string spacetime and is defined in the range $0 < \eta < 1$, with ν being the linear mass density, and we consider that $\hbar = c = G = 1$. The azimuthal angle varies in the interval: $0 \leq \varphi < 2\pi$. The parameter η can assume only values $\eta < 1$ (unlike of this, in [33,34], it can assume values greater than 1, which correspond to an anticonical spacetime with negative curvature). This geometry possesses a conical singularity represented by the following curvature tensor

$$R_{\rho,\varphi}^{\rho,\varphi} = \frac{1-\eta}{4\eta}\delta_2(\vec{r}), \quad (2)$$

where $\delta_2(\vec{r})$ is the two-dimensional delta function. This behavior of the curvature tensor is denominated as a conical singularity [35]. The conical singularity gives rise to the curvature concentrated on the cosmic string axis, in all other places the curvature is zero.

We are interesting in work out with a rotating frame, thus, we carry out the following coordinate transformation

$$T = t; \qquad R = \rho; \qquad \Phi = \varphi + \omega t; \qquad Z = z, \quad (3)$$

where ω is the constant angular velocity of the rotating frame which must satisfy $\omega\rho \ll 1$. With this transformation, the line element (1) becomes

$$ds^2 = -dt^2 + d\rho^2 + \eta^2\rho^2(d\varphi + \omega dt)^2 + dz^2 \quad (4)$$

$$= -(1 - \omega^2\eta^2\rho^2)dt^2 + 2\omega\eta^2\rho^2 d\varphi dt + d\rho^2 + \eta^2\rho^2 d\varphi^2 + dz^2. \quad (5)$$

With the line element given by the expression (5), we need to construct the local reference frame to which the observers will be associated. It is the local reference frame in which we can define the spinor in the curved spacetime. We can build the local reference frame through a non-coordinate basis $\hat{\theta}^a = e^a{}_\mu dx^\mu$, which its components $e^a{}_\mu(x)$ satisfy the following relation [36,37]

$$g_{\mu\nu}(x) = e^a{}_\mu(x) e^b{}_\nu(x)\eta_{ab}. \quad (6)$$

The components of the non-coordinate basis $e^a{}_\mu(x)$ are called *tetrads or vierbeins*, and they form our local reference frame. The tetrad has an inverse one defined as $dx^\mu = e^\mu{}_a\hat{\theta}^a$, where

$$e^a{}_\mu e^\mu{}_b = \delta^a{}_b e^\mu{}_a e^a{}_\nu = \delta^\mu{}_\nu. \quad (7)$$

We are interested to build a corotating frame where there is no torque on the magnetic dipole moment. Thus, we choose that the tetrad and its inverse are

$$e^a{}_\mu(x) = \begin{pmatrix} \sqrt{1-\beta^2} & 0 & -\frac{\omega\eta^2\rho^2}{\sqrt{1-\beta^2}} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{\eta\rho}{\sqrt{1-\beta^2}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}; \tag{8}$$
$$e^\mu{}_a(x) = \begin{pmatrix} \frac{1}{\sqrt{1-\beta^2}} & 0 & \frac{\omega\eta\rho}{\sqrt{1-\beta^2}} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{\sqrt{1-\beta^2}}{\eta\rho} & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

where $\beta = \omega\eta\rho$. Note that this choice makes the 1-axis of the local reference frame to be parallel to the ρ-axis of the spacetime and the 3-axis of the local reference frame—to be parallel to the z-axis of the spacetime. Let us write the tetrad and its inverse in the matrix form.

With the information about the choice of the local reference frame, we can obtain the one-form connection $\omega^a{}_b = \omega_\mu{}^a{}_b dx^\mu$ through the Maurer-Cartan's structure equation [37]. In the absence of the torsion field, the Maurer-Cartan's structure equation may be written as

$$d\hat{\theta}^a + \omega^a{}_b \wedge \hat{\theta}^b = 0, \tag{9}$$

where the operator d is the exterior derivative and the symbol $\wedge$ means the external product. So, the nonzero components of the one-form connection are

$$\omega_t{}^0{}_1 = \omega_t{}^1{}_0 = -\frac{\omega^2\eta^2\rho}{\sqrt{1-\beta^2}}; \qquad \omega_t{}^1{}_2 = -\omega_t{}^2{}_1 = -\frac{\omega\eta}{\sqrt{1-\beta^2}}; \tag{10}$$

$$\omega_\rho{}^0{}_2 = \omega_\rho{}^2{}_0 = \frac{\omega\eta}{(1-\beta^2)}; \tag{11}$$

$$\omega_\varphi{}^0{}_1 = \omega_\varphi{}^1{}_0 = -\frac{\omega\eta^2\rho}{\sqrt{1-\beta^2}}; \qquad \omega_\varphi{}^1{}_2 = -\omega_\varphi{}^2{}_1 = -\frac{\eta}{\sqrt{1-\beta^2}}. \tag{12}$$

Now we suggest that there is an electric charge density λ_e concentrated in the z-axis of the cosmic string. This distribution of charges creates a cylindrically symmetric electric field in the rest frame of the observer ($a = 0$), given by [31,32]

$$E^\rho_{(rf)} = \frac{\lambda_e}{\sqrt{-g}}, \tag{13}$$

where $g = \det(g_{\mu\nu})$. So, with the 1-axis of the local reference frame is parallel to the ρ-axis of the spacetime, we have $E^1 = e^1{}\rho E^\rho_{(rf)} = \lambda_e/\sqrt{-g}$. However, in the accelerated reference frame associated with the observer, the fields are given by [38–40]

$$F^{\mu\nu} = e^\mu{}_a e^\nu{}_b F^{ab}. \tag{14}$$

Thus, the components of the electric and magnetic fields in the corotating frame of the observer are

$$E^\rho = \frac{1}{\sqrt{1-\beta^2}}\frac{\lambda_e}{\eta\rho}; \qquad E^\varphi = 0; \qquad E^z = 0; \qquad B^\rho = 0; \qquad B^\varphi = 0; \qquad B^z = 0. \tag{15}$$

Here we had just used the local reference frame given by the expression (8). In that way, we get all information about the topology of the cosmic string and the rotating frame. Our next step is to study the dynamics of a neutral particle in this background.

III. RELATIVISTIC DYNAMICS

Now, let us study the dynamics of a spin one half neutral particle in the cosmic string background where the observers are associated to the rotating frame. The Dirac equation of a neutral particle with a permanent magnetic dipole moment interacting with an external electric field arises due to the introduction of a nonminimal coupling [41]. In that way, the Dirac equation in curved spacetime with the interaction of the magnetic dipole moment of the neutral particle with an external electric field is given by the following expression [31]:

$$i\gamma^a e^\mu{}_a \partial_\mu \psi + i\gamma^\mu \Gamma_\mu \psi + \frac{\mu}{2} F_{\mu\nu}\Sigma^{\mu\nu}\psi = m\psi, \tag{16}$$

with Γ_μ being the spinor connection of the form [36,37]

$$\Gamma_\mu = \frac{i}{4}\omega_{\mu ab}\Sigma^{ab}, \tag{17}$$

and $\Sigma^{ab} = \frac{i}{2}[\gamma^a, \gamma^b]$, and the indices $(a, b, c = 0, 1, 2, 3)$ indicate the local reference frame. The γ^a matrices are defined in the local reference frame and are identical to the Dirac matrices in the flat spacetime, *i.e.*,

$$\gamma^0 = \hat{\beta} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}; \qquad \gamma^i = \hat{\beta}\hat{\alpha}^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}; \qquad \Sigma^i = \begin{pmatrix} \sigma^i & 0 \\ 0 & \sigma^i \end{pmatrix}, \tag{18}$$

with $\vec{\Sigma}$ being the spin vector and σ^i are the Pauli matrices satisfying the relation $(\sigma^i\sigma^j + \sigma^j\sigma^i) = 2\eta^{ij}$, where $\eta^{ab} = \text{diag}(- + ++)$ is the Minkowski metric tensor and the index $i, j, k = (1, 2, 3)$ denotes the spacial index of the local reference frame. With the one-form connection given in (12) we obtain the following spinor connections

$$\Gamma_t = -\frac{1}{2}\frac{\omega^2\eta^2\rho}{\sqrt{1-\beta^2}}\hat{\alpha}^1 - \frac{i}{2}\frac{\omega\eta}{\sqrt{1-\beta^2}}\Sigma^3; \tag{19}$$

$$\Gamma_\rho = \frac{1}{2}\frac{\omega\eta}{(1-\beta^2)}\hat{\alpha}^2; \tag{20}$$

$$\Gamma_\varphi = -\frac{1}{2}\frac{\omega\eta^2\rho}{\sqrt{1-\beta^2}}\hat{\alpha}^1 - \frac{i}{2}\frac{\eta}{\sqrt{1-\beta^2}}\Sigma^3. \tag{21}$$

In this way, when the charges are concentrated in the symmetry axis of the cosmic string, the Dirac equation in a cosmic string background with rotating frames has the form

$$\begin{aligned} m\psi = {}& \frac{i}{\sqrt{1-\beta^2}}\gamma^0\frac{\partial\psi}{\partial t} + i\gamma^2\frac{\omega\eta\rho}{\sqrt{1-\beta^2}}\frac{\partial\psi}{\partial t} \\ & + i\gamma^1\left(\partial_\rho - \mu\hat{\beta}E_\rho + \frac{1}{2\rho}\right)\psi + \frac{i}{\sqrt{1-\beta^2}}\frac{\gamma^2}{\eta\rho}\frac{\partial\psi}{\partial\varphi} \\ & - i\gamma^2\frac{\omega^2\eta\rho}{\sqrt{1-\beta^2}}\frac{\partial\psi}{\partial\varphi} + i\gamma^3\frac{\partial\psi}{\partial z} \\ & - \frac{1}{2}\frac{\eta}{(1-\beta^2)}\gamma^0\vec{\omega}\cdot\vec{\Sigma}\psi. \end{aligned} \tag{22}$$

For the Dirac equation in cosmic string spacetime given by the equation above, we obtain the term $\vec{\omega}\cdot\vec{\Sigma}$ which is related to the spin-rotation coupling as pointed out in [23]. The relativistic quantum phase acquired by the wave function of the particle is given by the Dirac phase factor [42,43]

$$\psi = e^{i\phi}\psi_0 \tag{23}$$

Substituting the ansatz above into the Dirac Eq. (22), we find that the wave function of the neutral particle acquires four independent contributions for the relativistic geometric quantum phase. The first contribution is generated by the topology of the cosmic string space-time, and is given by

$$\Phi_{R1} = \oint \frac{1}{2}\frac{\eta}{\sqrt{1-\beta^2}}\Sigma^3 d\varphi, \tag{24}$$

this term is related to the parameter η that characterize the cosmic string spacetime. Notice that in the limit $\eta \longrightarrow 1$, absence of topological defect, the contribution (24) is due rotating frame in a Minkwoski spacetime. The second contribution is generated by the interaction between the permanent magnetic moment and the external electric field and is given by

$$\phi_{R2} = -\mu\hat{\beta}\oint\frac{(\vec{\Sigma}\times\vec{E})_\varphi}{\sqrt{1-\beta^2}}d\varphi. \tag{25}$$

This contribution is due the relativistic Aharonov-Casher [2] coupling in rotating frame in the presence of a topological defect. In the limit $\eta \longrightarrow 1$ we obtain the relativistic Aharonov-casher effect in a rotating frame in a flat spacetime. The contributions given by (24) and (25) are identical to the relativistic phase obtained in [31] up to the correction term $(1-\beta^2)^{1/2}$ corresponding to rotation of the local reference frame of the observers. If we take $\omega = 0$, we recuperate the same relativistic geometric phase obtained in the rest frame of the observers in cosmic string spacetime as in [31]. The third contribution for the relativistic quantum phase is given by the spin-rotation coupling

$$\phi_{R3} = -\frac{\eta}{2}\oint\frac{\vec{\omega}\cdot\vec{\Sigma}}{\sqrt{1-\beta^2}}dt, \tag{26}$$

where the correction term $(1-\beta^2)^{1/2}$ again arises due to the rotation of the local reference frame, and the dependence on the parameter η characterize the influence of topological defect also in this phase shift. The last contribution for the relativistic quantum phase will be

$$\phi_{R4} = \frac{1}{2}\oint\frac{(\vec{\Sigma}\times\vec{\mathcal{E}})_\varphi}{(1-\beta^2)^{3/2}}d\varphi, \tag{27}$$

where $\vec{\mathcal{E}} = (\omega^2\eta^2\rho)\hat{\rho}$, with $\hat{\rho}$ being the unit vector in the ρ-direction. If we integrate the last relativistic quantum phase (27), we have

$$\Phi_{R4} = \frac{\omega^2\eta^2}{(1-\beta^2)^{3/2}}\vec{A}\cdot\vec{\Sigma}, \tag{28}$$

where $\vec{A} = A\hat{n}$, with A being the area perpendicular to the symmetry axis of the cosmic string and $\hat{n}$ the unitary vector perpendicular to the area A. In this way, we have obtained the relativistic phase shift associated a rotating frame in a cosmic string background without make the weak field approximation. We can see, in the limit of $\eta \longrightarrow 1$, that we obtain a phase shift due to the rotation frame analogous to that one obtained by Anandan and Suzuki in [25] without make the weak field approximation, up to the correction term $(1-\beta^2)^{3/2}$ arising due to the rotation of the local reference frame of the observers.

In this way, in this section we have obtained a similar coupling term in the Dirac equation discussed in Ref. [23], for a flat space case, in the quantum dynamics of a neutral particle in a rotating frame in a cosmic string background. We can see that all contribution depends on the parameter η and the ω fact that indicate the influence of a topology of the spacetime and the rotating frame. We can see that each independent contribution for the relativistic quantum phase is independent of the velocity of the neutral particle, which characterizes a nondispersive quantum phases in the terms established in [44–46].

IV. THE NON-RELATIVISTIC LIMIT

In this section we study the nonrelativistic behavior of the neutral particle with a permanent magnetic dipole moment interacting with an external electric field in rotating frame. We start this section writing the Dirac equation in cosmic string background (22) in the form

$$i\frac{\partial\psi}{\partial t} = H\psi. \tag{29}$$

Thus, the Dirac Eq. (22) becomes

$$i\frac{\partial\psi}{\partial t} + i\beta\hat{\alpha}^2\frac{\partial\psi}{\partial t} = \sqrt{1-\beta^2}m\hat{\beta}\psi + \sqrt{1-\beta^2}\vec{\alpha}\cdot\vec{\pi}\psi + \frac{1}{2}\frac{\eta}{\sqrt{1-\beta^2}}\vec{\omega}\cdot\vec{\Sigma}\psi, \tag{30}$$

where we defined $\vec{\pi} = \vec{p} + i\mu\hat{\beta}\vec{E} - i\vec{\xi}$ as in [31], with $p_i = -ie^{\lambda}{}_i\partial_\lambda$, and the components of the vector $\vec{\xi}$ are

$$\xi_k = \frac{i}{2}e^{\varphi}{}_k\omega_{\varphi ij}\Sigma^{ij} = -\frac{i}{2\rho}\Sigma^3\delta_{k2}. \tag{31}$$

The nonrelativistic dynamics of the neutral particle can be obtained when we extract the temporal dependence of the wave function through the ansatz

$$\psi = e^{-imt}\begin{pmatrix}\Phi\\ X\end{pmatrix}, \tag{32}$$

so, from the Dirac Eq. (30) we have

$$\begin{aligned} i\frac{\partial\Phi}{\partial t} &+ m\Phi + i\beta\sigma^2\frac{\partial X}{\partial t} + m\beta\sigma^2 X \\ &= \sqrt{1-\beta^2}[m + \vec{\sigma}\cdot(\vec{p} + i\mu\vec{E} - i\vec{\xi})]X \\ &\quad + \frac{1}{2}\frac{\eta}{\sqrt{1-\beta^2}}\vec{\omega}\cdot\vec{\sigma}\Phi \\ i\frac{\partial X}{\partial t} &+ mX + i\beta\sigma^2\frac{\partial\Phi}{\partial t} + m\beta\sigma^2\Phi \\ &= \sqrt{1-\beta^2}[m + \vec{\sigma}\cdot(\vec{p} - i\mu\vec{E} - i\vec{\xi})]\Phi \\ &\quad + \frac{1}{2}\frac{\eta}{\sqrt{1-\beta^2}}\vec{\omega}\cdot\vec{\sigma}X. \end{aligned} \tag{33}$$

We find that $\omega\eta\rho \ll 1$, thus, we make the approximation $\sqrt{1-\beta^2} \approx 1 - \frac{1}{2}\beta^2 + \ldots$ and rewrite (33) as

$$\begin{aligned} i\frac{\partial\Phi}{\partial t} &+ \frac{1}{2}m\beta^2\Phi - \frac{\eta}{2}\vec{\omega}\cdot\vec{\sigma}\Phi + \frac{1}{4}\beta^2\omega\eta\sigma^2\Phi \\ &= \vec{\sigma}\cdot(\vec{p} + i\mu\vec{E} - i\vec{\xi})X - \frac{1}{2}\beta^2\vec{\sigma}\cdot(\vec{p} + i\mu\vec{E} - i\vec{\xi})X \\ &\quad - m\beta\sigma^2 X - i\beta\sigma^2\frac{\partial X}{\partial t}, \end{aligned} \tag{34}$$

where we considering X being the "small" components of the wave function, so, we have

$$X = \frac{1}{2m}\left[\vec{\sigma}\cdot(\vec{p} - i\mu\vec{E} - i\vec{\xi}) - \frac{1}{2}\beta^2\vec{\sigma}\cdot(\vec{p} - i\mu\vec{E} - i\vec{\xi}) - m\beta\sigma^2\right]\Phi - \frac{i}{2m}\beta\sigma^2\frac{\partial\Phi}{\partial t}. \tag{35}$$

In this way, substituting the expression (35) into (34), one can obtain the nonrelativistic Hamiltonian which describes the interaction between a neutral particle with a permanent magnetic dipole moment and an external electric field in the rotating frame in the cosmic string spacetime:

$$H_{NR} = \frac{1}{2m}(\vec{p} + \vec{\Xi})^2 + \frac{\mu}{2m}\vec{\nabla}\cdot\vec{E} - \frac{\mu^2E^2}{2m} - m\mathcal{A}_0 - \frac{\eta}{2}\vec{\omega}\cdot\vec{\sigma} + \frac{\mu\lambda_e\eta\omega^2}{4m} + \mathcal{O}\left(\frac{\beta^2}{2m}\right), \tag{36}$$

where the vector $\vec{\Xi}$ has the following components

$$\Xi_k = \mu(\vec{\sigma}\times\vec{E})_k - \frac{1}{2}\frac{\eta}{\eta\rho}\sigma^3\delta_{k2} - m\mathcal{A}_k - \frac{1}{2}(\vec{\sigma}\times\vec{\mathcal{E}})_k. \tag{37}$$

Here we have used the same notations of the Ref. [25] to define the 4-vector $\mathcal{A}^\mu$ and the vector $\vec{\mathcal{E}}$, whose nonzero components are

$$\begin{aligned} \mathcal{A}_0 &= \frac{1}{2}\eta^2(\vec{\omega}\times\vec{r})^2 = \frac{1}{2}\omega^2\eta^2\rho^2; \\ \mathcal{A}_\varphi &= \eta(\vec{\omega}\times\vec{r})_\varphi, \qquad \mathcal{E}^\rho = \omega^2\eta^2\rho. \end{aligned} \tag{38}$$

The Hamiltonian (36) describes the nonrelativistic behavior of a neutral particle with a permanent magnetic dipole moment interacting with an external electric field in a rotating frame with the presence of a topological defect. The influence of the rotating frame in this nonrelativistic dynamics can be viewed easily in the expression (36) through the spin-rotation coupling given by the term $\vec{\omega}\cdot\vec{\sigma}$, which is known as Mashhoon effect [22], and by the terms $m\mathcal{A}^0$ and $m\vec{\mathcal{A}}$ in (37), which arise directly from the construction of the local reference frame of the observers and is pointed out as a component of a gauge field for rotating frame in [25]. The influence of the topology of the defect can be observed in the spin-orbit coupling and in the terms $m\mathcal{A}^0$ and $m\vec{\mathcal{A}}$. The other contribution due to the topology of the defect is given by the second term of the expression (37). If we take the limit $\eta \to 1$, we can recuperate the spin-rotation coupling in flat spacetime given in [22] and the gauge field built in the weak field approximation given in [25]. The last term of the nonrelativistic Hamiltonian which we observe, is $(\vec{\sigma}\times\vec{E})$, which corresponds to the interaction between the magnetic dipole moments of the neutral particle with the external electric field.

The nonrelativistic geometric phases can be obtained via the ansatz given by

$$\psi = e^{i\phi}\psi_0 \tag{39}$$

which corresponds to the application of the Dirac phase factor [42,43]. Since the terms $\vec{\nabla}\cdot\vec{E}$, E^2 and $m\mathcal{A}^0$ are local terms, they do not contribute to the geometric phase

[26,27]. Hence, the terms which contribute to the nonrelativistic geometric phase are $\vec{\Xi}$ and the spin-rotation coupling $\vec{\omega}\cdot\vec{\sigma}$, with ψ_0 being the solution of the equation

$$-\frac{1}{2m}\nabla^2\psi_0-\frac{\mu^2E^2}{2m}+m\mathcal{A}^0\psi_0=0. \tag{40}$$

In this way, the wave function of the neutral particle acquires five independent contributions to the nonrelativistic geometric phase. The first contribution is given by the topology of the spacetime, that is, this contribution is given by the angle deficit of the topological defect,

$$\phi_{NR1}=\oint(-i\xi_k)e^k{}_\varphi d\varphi=-\frac{1}{2}\oint\frac{1}{\rho}\sigma^3\delta_{2k}e^k{}_\varphi d\varphi$$
$$=-\pi\eta\sigma^3, \tag{41}$$

where we have neglected the terms of order β^2, The expression (41) gives us a quantum phase identical to that one obtained in [31] in a nonrotating reference frame. If we add 2π in (41) to remove the effects arisen due to an arbitrary rotation of $\hat{\theta}^2=e^2{}_\varphi d\varphi$, when we transport the local reference frame around a closed path, this contribution to the phase shift becomes

$$\phi'_{NR1}=\frac{1}{2}\sigma^3\left(-\oint\frac{\eta}{\eta\rho}\delta_{2k}e^k{}_\varphi d\varphi+2\pi\right)=(1-\eta)\pi\sigma^3$$
$$=\frac{1}{2}(8\pi G\nu\sigma^3), \tag{42}$$

which gives us the half-value of the flux generated by the gravitational Aharonov-Casher effect obtained in Refs. [47,48] for cosmic string case without making the weak field approximation. The second contribution is given by the gauge field $m\vec{\mathcal{A}}$ from (38)

$$\phi_{NR2}=-m\oint\mathcal{A}_ke^k{}_\varphi d\varphi=-2m\eta\vec{\omega}\cdot\vec{A} \tag{43}$$

where $A=\pi\eta\rho^2$ is the area enclosed by the path of the neutral particle, perpendicular to the angular velocity ω. This phase shift is originated from the rotation frame and give us the same impact as the Sagnac effect [15,16,25] in the presence of the topological defect.

The third contribution is obtained by the vector $\vec{\mathcal{E}}$ defined in (38)

$$\phi_{NR3}=-\frac{1}{2}\int(\vec{\sigma}\times\vec{\mathcal{E}})_\varphi d\varphi=-\omega^2\eta^2\vec{A}\cdot\vec{\sigma}, \tag{44}$$

this phase shift is due the to the rotating frame of the observers and to the presence of the topological defect. Notice that in limit $\eta\to 1$ we obtain similar result pointed out in [25] through the weak field approximation being an analog to the phase shift in neutron interferometry due to the interaction between the effective electric $\mathcal{E}$ field and the magnetic dipole moment. Here, we obtained this phase shift without making the weak field approximation.

The fourth contribution for the nonrelativistic geometric phase is obtained through the spin-rotation coupling known as Mashhoon effect [22]

$$\phi_{NR4}=\frac{\eta}{2}\int\vec{\omega}\cdot\vec{\sigma}dt=\frac{\eta T}{2}\vec{\omega}\cdot\vec{\sigma}, \tag{45}$$

with T being the time which the particle spent to travel along a closed path around the symmetry axis of the topological defect. Notice that, this phase depend on parameter η that demonstrate the influence of cosmic string in this contribution. The last contribution is given by the interaction between the permanent magnetic dipole moment of the neutral particle and the external electric field (15)

$$\phi_{NR5}=\mu\oint(\vec{\sigma}\times\vec{E})_\varphi d\varphi=2\pi\mu\lambda_e\sigma^3. \tag{46}$$

This expression is identical to that one obtained in [26,27] in flat spacetime and in [31,32]—in cosmic string spacetime. When we consider a linear distribution of electric charges on the symmetry axis of the topological defect in this rotating frame, the phase shift on the wave function becomes

$$\phi_{\mathrm{AC}}=2\pi\mu\lambda_e\sigma^3+(1-\eta)\pi\sigma^3-2m\eta\vec{\omega}\cdot\vec{A}$$
$$-\omega^2\eta^2\vec{A}\cdot\vec{\sigma}+\frac{\eta}{2}\vec{\omega}\cdot\vec{\sigma}T, \tag{47}$$

which is the result analogous to the Aharonov-Casher (AC) effect with a rotating frame in the cosmic string background. We can see that if $\omega\to 0$, we recuperate the analog of the AC effect in a topological defect spacetime as in [31], and with $\eta\to 1$ and $\omega=0$, we recuperate the AC effect [2] in flat spacetime. However, if we consider $\eta\to 1$ and $\omega\neq 0$, we have the analog of the AC effect in the flat spacetime with a rotating frame. We can also observe that each independent contribution for the nonrelativistic geometric quantum phase is independent of the velocity of the neutral particle, which is a characteristic of a nondispersive quantum phase [44–46].

V. CONCLUSIONS

We studied the interference effects for the wave function of a neutral particle with a permanent magnetic dipole moment interacting with an external electric field in the presence of a topological defect with rotating frames both in relativistic regime and in nonrelativistic one. We built a corotating frame for the observers where no torques act on the magnetic dipole moment and consider a field configuration in this corotating frame generated by a linear distribution of electric charges on the symmetry axis of the topological defect.

In the relativistic dynamics of the neutral particle in the cosmic string spacetime we obtained four independent contributions for the geometric quantum phase. The first contribution was given by the topology of the defect. The second contribution arose due to the interaction between the external electric field and the magnetic dipole moment. The third and fourth contributions were generated by the rotation of the local reference frame of the observers. In the third contribution, we saw the effect due to the spin-rotation coupling and the topology of the defect. In the fourth contribution, we obtained a effect analogous to the interaction between the effective field and the dipole moment. This effect is analogous Aharonov-Casher effect due rotating frame. We saw that each of these contributions is nondispersive because they are independent of the velocity of the neutral particle. In the limit of $\eta \rightarrow 1$ we obtain the results for the relativistic geometric phase for neutral in a rotating frame in flat spacetime.

In the nonrelativistic dynamics of the neutral particle in cosmic string background with rotating frame, we obtained five independent contributions for the geometric quantum phase and an effect analogous to the AC effect in the rotating frame. The first contribution for the nonrelativistic phase shift was generated by the topology of the defect and gives us a flux similarly to the gravitational Aharonov-Casher effect [47,48]. The second, third, and fourth contributions were generated by the rotating frame and the topology of the defect. In the second contribution we obtained an analog of Sagnac effect, and in third contribution we obtained a phase shift analogous to the interaction between the dipole moment and the electric field, without making the weak field approximation. The fourth contribution arises from the spin-rotation coupling which is known as Mashhoon effect [22] and from the deficit angle. The last contribution for the nonrelativistic geometric phase was given by the interaction between the magnetic dipole moment of the neutral particle and the external electric field [2]. We also saw that each contribution of the nonrelativistic geometric phases is nondispersive.

Finally, when we consider a linear charge distribution on the symmetry axis of the topological defect in the nonrelativistic dynamics, we discussed the influence of the rotating frame which generated an effect analogous to the AC effect. We see that when $\omega = 0$, we recuperate the same effect analogous to the AC effect in cosmic string background as it was discussed in [31,32]. When we consider $\eta \rightarrow 1$, we see the influence of the rotating frame in the AC effect in flat spacetime.

ACKNOWLEDGMENTS

Authors are grateful to A. Yu. Petrov for some criticism on the manuscript. This work was partially supported by PRONEX/FAPESQ-PB, FINEP, CNPq and CAPES/PROCAD

[1] Y. Aharonov and D. Bohm, Phys. Rev. **115**, 485 (1959).
[2] Y. Aharonov and A. Casher, Phys. Rev. Lett. **53**, 319 (1984).
[3] X.-G. He and B. H. J. McKellar, Phys. Rev. A **47**, 3424 (1993).
[4] M. Wilkens, Phys. Rev. Lett. **72**, 5 (1994).
[5] J. P. Dowling, C. Williams, and J. D. Franson, Phys. Rev. Lett. **83**, 2486 (1999).
[6] C. Furtado and G. Duarte, Phys. Scr. **71**, 7 (2005).
[7] S. A. R. Horsley and M. Babiker, Phys. Rev. Lett. **99**, 090401 (2007).
[8] S. A. R. Horsley and M. Babiker, Phys. Rev. A **78**, 012107 (2008).
[9] M. G. Sagnac, C. R. Acad. Sci. (Paris)**157**, 708 (1913); **157**, 1410 (1913).
[10] K. Varjú, and L. H. Ryder, Phys. Lett. A **250**, 263 (1998).
[11] K. Varjú and L. H. Ryder, Phys. Rev. D **62**, 024016 (2000).
[12] J. J. Sakurai, Phys. Rev. D **21**, 2993 (1980).
[13] J. Anandan, Phys. Rev. D **15**, 1448 (1977).
[14] B. R. Iyer, Phys. Rev. D **26**, 1900 (1982).
[15] E. J. Post, Rev. Mod. Phys. **39**, 475 (1967).
[16] J. Anandan, Phys. Rev. D **24**, 338 (1981).
[17] M. V. Berry, Proc. R. Soc. A **392**, 45 (1984).
[18] S.-M. Cui and H.-H Xu, Phys. Rev. A **44**, 3343 (1991).
[19] S.-M. Cui, Phys. Rev. A **45**, 5255 (1992).
[20] L. A. Page, Phys. Rev. Lett. **35**, 543 (1975).
[21] S. A. Werner, J.-L. Staudenmann, and R. Colella, Phys. Rev. Lett. **42**, 1103 (1979).
[22] B. Mashhoon, Phys. Rev. Lett. **61**, 2639 (1988).
[23] F. W. Hehl and W.-T. Ni, Phys. Rev. D **42**, 2045 (1990).
[24] I. D. Soares and J. Tiomno, Phys. Rev. D **54**, 2808 (1996).
[25] J. Anandan and J. Suzuki, arXiv:quant-ph/0305081.
[26] J. Anandan, Phys. Lett. A **138**, 347 (1989).
[27] J. Anandan, Phys. Rev. Lett. **85**, 1354 (2000).
[28] E. Passos, L. R. Ribeiro, C. Furtado, and J. R. Nascimento, Phys. Rev. A **76**, 012113 (2007).
[29] L. R. Ribeiro, E. Passos, C. Furtado, and J. R. Nascimento, arXiv:hep-th/0710.5858.
[30] E. R. Bezerra de Mello, J. High Energy Phys. 06 (2004) 016.
[31] K. Bakke and C. Furtado, Phys. Rev. D **78**, 064012 (2008);
[32] K. Bakke and C. Furtado, Eur. Phys. J. C **60**, 501 (2009).
[33] M. O. Katanaev and I. V. Volovich, Ann. Phys. (N.Y.) **216**, 1 (1992).
[34] C. Furtado and F. Moraes, Phys. Lett. A **188**, 394 (1994).
[35] D. D. Sokolov and A. A. Starobinskii, Dokl. Akad. Nauk SSSR **234**, 1043 (1977)[Sov. Phys. Dokl. **22**, 312 (1977)].
[36] J. Birrell and P. Davies, *Quantum Fields in Curved Space* (Cambridge University Press, Cambridge, England, 1982).
[37] M. Nakahara, *Geometry, Topology and Physics* (Institute of Physics Publishing, Bristol, 1998).
[38] T. C. Mo, J. Math. Phys. (N.Y.) **11**, 2589 (1970).

[39] J. L. Anderson and J. W. Ryon, Phys. Rev. **181**, 1765 (1969).
[40] J. van Bladel, Proceedings of the IEEE **64**, 301 (1976).
[41] J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill Book Company, New York, 1964).
[42] P. A. M. Dirac, Proc. R. Soc. A **133**, 60 (1931).
[43] M. V. Berry, Eur. J. Phys. **1**, 240 (1980).
[44] G. Badurek, H. Weinfruter, R. Gähler, A. Kollmar, S. Wehinger, and A. Zeilinger, Phys. Rev. Lett. **71**, 307 (1993).
[45] M. Peshkin and H. J. Lipkin, Phys. Rev. Lett. **74**, 2847 (1995).
[46] M. Peshkin, Found. Phys. **29**, 481 (1999).
[47] B. Resnik, Phys. Rev. D **51**, 3108 (1995).
[48] L. Anandan, arXiv:gr-qc/9504002.