From Cavendish to PLANCK: Constraining Newton's gravitational constant with CMB temperature and polarization anisotropy

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We present new constraints on cosmic variations of Newton's gravitational constant by making use of the latest CMB data from WMAP, BOOMERANG, CBI and ACBAR experiments and independent constraints coming from big bang nucleosynthesis. We found that current CMB data provide constraints at the ~10% level, that can be improved to ~3% by including big bang nucleosynthesis data. We show that future data expected from the Planck satellite could constrain G at the ~1.5% level while an ultimate, cosmic variance limited, CMB experiment could reach a precision of about 0.4%, competitive with current laboratory measurements.

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I. INTRODUCTION

Since Cavendish's first measurement in 1798 ([1]), Newton's gravitational constant remains one of the most elusive constants in physics. The past two decades did not succeed in substantially improving our knowledge of its value from the precision of 0.05% reached in 1942 (see [2]). To the contrary, the variation between different measurements forced the CODATA committee,[1] which determines the internationally accepted standard values, to increase the uncertainty from 0.013% for the value quoted in 1987 to the 1 order of magnitude larger uncertainty of 0.15% for the 1998 "official" value ([3]). Recent laboratory measurements (see e.g. [4]) point towards an uncertainty at the level of ~0.4%, while other works claim an improved precisions below 0.01% ([5]). Analysis of the secular variation of the period of nonradial pulsations of the white dwarf G117-B15A ([6]) has produced complementary constraints at ~0.1% level.

Measurements of the cosmic microwave background (CMB, hereafter) temperature and polarization anisotropy have been suggested as a possible tool for determining the value of G (see [7]). In recent years, CMB temperature and polarization anisotropy have been measured with great precision from experiments as WMAP ([8,9]), BOOMERANG ([10]), CBI [11], and ACBAR ([12]). The impressive agreement between those measurements and the expectations of the standard model of structure formation have paved the way to the use of cosmology as a new laboratory where to test physical hypothesis at energies and scales not reachable on earth. Since a variation in G affects CMB temperature and polarization anisotropy, changing the position and the amplitude of the acoustic peaks present in the corresponding angular power spectra, it is indeed possible to infer new and independent constraints on G from CMB data.

In this paper we follow this timely line of investigation. Respect to previous works (most notably [7]) we update the CMB constraints on G by using the most recent CMB data (most notably, WMAP) and by also including complementary information from big bang nucleosynthesis (hereafter, BBN, see [13] for a complete review). As already shown in several papers (see e.g. [14,15]), any variation in G changes the Hubble parameter at BBN given by $H \sim \sqrt{G g_*} T^2$ where g_* counts the number of relativistic particles species and T is the temperature of the Universe. Since the predicted amount of light elements depends crucially on the comparison between the expansion rate H and, for example, the neutron-proton conversion rate $\Gamma_{\rm np} \sim G_F^2 T^5$, where G_F is the Fermi constant, any change in G can be strongly constrained by combining BBN predictions with observations of primordial elements. Moreover, we also discuss the ability of next CMB experiments as Planck ([16]) to constrain G, including the possibility of a "cosmic variance limited" survey.

Any cosmological constraint is, however, indirect and, in the case of the CMB data, depends on the assumed

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theory of structure formation. The major caveat in our case is the assumption of a cosmological constant, or dark energy component, the nature of which is puzzling and unknown (for a recent review, see e.g. [17–19]). While the derived constraints will therefore be model dependent, it is interesting that a major alternative to a dark energy component, i.e. modified gravity theories, could be parametrized by introducing an effective value of Newton's constant $G_{\rm eff}$, that could not only be different from the local value of G but also spatial and time dependent (see e.g. [20–22]). Moreover, if dark energy interacts with dark matter, there is a change in the background evolution of the universe leading to an effective $G_{\rm eff}$ for the matter component (see e.g. [23]) and to a possible change in the cosmic bound on G.

In this respect, the search for variations in Newton's constant using cosmological data could also play a role in the understanding of the dark sector. If the Newton's constant inferred from cosmology will turn out to be different from the local value, then this may suggest a modification of gravity at large scale or a more complex interacting dark energy scenario. Since an interacting dark energy or a modified gravity theory could be responsible for a variation of G in the late universe, we also consider the possibility of a redshift dependence of G.

Our paper is therefore organized as follows: in the next section we briefly describe the effects of a variation in G on CMB temperature and polarization anisotropy. In Sec. III we describe our method of analysis and the data sets considered. In Sec. IV we present our results and, finally, in Sec. V we derive our conclusions.

II. THE IMPACT OF G ON RECOMBINATION AND THE CMB

Following [7] we parametrize the deviations from Newton's gravitational constant by introducing a dimensionless parameter λ_G such that

$$G \longrightarrow \lambda_G^2 G \tag{1}$$

As showed in [7], expressing the perturbed quantities in Fourier space, a variation in Newton's gravitational constant is equivalent in a simple rescaling of the wave numbers. No preferred cosmological scale is introduced by varying G and the density fluctuations produced by a mode of wave vector $\mathbf{k}$ in a universe with $\lambda_G \neq 1$ have equivalent dynamics of a mode with $k' = k/\lambda_G$ in a universe with $\lambda_G = 1$.

However the physics of recombination does introduce a preferred time scale and it will actually change when varying λ_G. This is clearly shown in Fig. 1 where the ionization fraction x_e at different redshift z, computed with a modified version of RECFAST [24], is plotted for different values of λ_G. The ionization fraction x_e is just the free electron number density n_e divided by the total number density of hydrogen nuclei (free and bound) n_H. As we

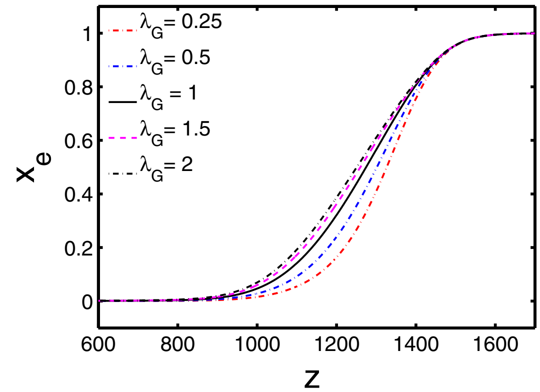

FIG. 1 (color online). Ionization fraction in function of redshift for different values of λ_G.

can see, higher (lower) values of the gravitational constant yields a delayed (accelerated) period of recombination. A change in the number density of free electrons n_e in function of the conformal time τ, changes the visibility function $g(\tau)$, written in terms of the opacity for Thomson scattering κ as

$$g(\tau) = \dot{\kappa}\exp(-\kappa) = -d/d\tau \exp(-\kappa) \tag{2}$$

with

$$\kappa = \sigma_T \int_\tau^{\tau_0} a n_e(\tau) d\tau, \tag{3}$$

where σ_T is the Thomson scattering cross section, a is the scale factor and $\dot{\kappa} = \sigma_T a n_e$.

This clearly affects the CMB temperature anisotropy that can be written as an integral along the line of sight over sources,

$$\Delta T(\hat{\mathbf{n}}, \mathbf{k}) = \int_0^{\tau_0} d\tau S(k, \tau) e^{i\mathbf{k}\cdot\hat{\mathbf{n}}D(\tau)} g(\tau), \tag{4}$$

where $S(k, \tau)$ is the anisotropy source term (see [25]) and $D(\tau)$ is the distance from the observer to a point along the line of sight at conformal time τ.

In Fig. 2 we plot the CMB temperature and polarization spectra computed from a modified version of the CAMB [26] code. The effect of modified recombination is clear. Namely, varying λ_G changes the recombination process, shifting $g(\tau)$ along the conformal time τ. The net effect is a damping or enhancement of the acoustic oscillations and a shift of the Doppler peaks in the angular scales. This mechanism could mimic an extra injection or absorption of Lyman-α photons at last scattering, as already analyzed in several recent papers (see e.g. [27]), and it would be difficult to disentangle the two scenarios.

Another important aspect to consider is a possible redshift dependence of G. If interacting dark energy or a modification to general relativity are responsible for the current accelerated expansion of the universe, it is indeed possible that this could result in an observed cosmic value

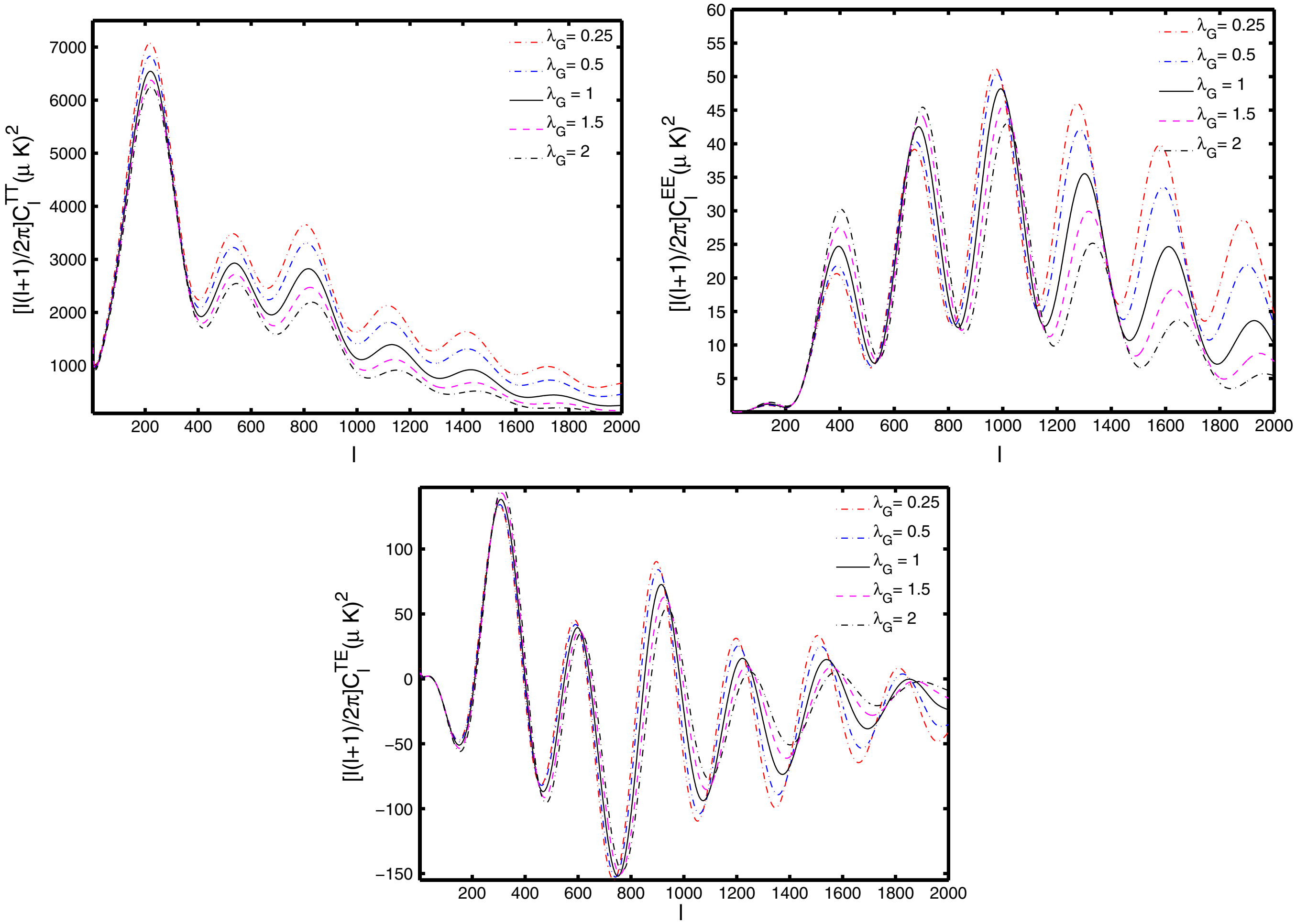

FIG. 2 (color online). From Top to Bottom: Temperature, Polarization and cross Temperature-Polarization power spectra in function of variations in λ_G.

of G different from the one obtained from local measurements. Moreover, it is plausible to think that this kind of deviation of G will be triggered by acceleration, i.e., to be conservative, will appear at redshift $0.1 < z < 2$.

We have therefore considered two possible parametrizations for a redshift-dependent gravitational constant. A first parametrization, that somewhat ties the change in G with the appearance of dark energy is to consider:

$$G(z) = G + \Delta G(1 - a), \tag{5}$$

where the variation ΔG is equal to $G(\lambda_G^2 - 1)$. This parametrization, similar to the one proposed in [28] for the dark energy equation of state, has the advantage of a smooth transition between the value of G today to $\lambda_G^2 G$ in the past, when $z \gg 1$. However the redshift of transition between these two values is not an independent variable.

We have therefore considered a second possible parametrization as:

$$G(z) = G[1 - (1 - \lambda_G^2)H(z - z_t)] \tag{6}$$

where $H(x)$ is the Heaviside function ($H(x)$ for $x < 0$ and $H(x) = 1$ for $x > 0$) and z_T is the redshift of transition between the two values (local and past) of G.

In Fig. 3 we plot different power spectra computed considering the two parametrizations described using a fixed value of $\lambda_G = 0.9$. As we can see, introducing a redshift-dependent variation in G increases the CMB anisotropy at large angular scales. On sub-Hubble scales, the Einstein equations in an expanding space-time reduce to the Poisson equation

$$\Delta\Phi = 4\pi G\rho a^2\delta \tag{7}$$

that relates the gravitational potential Φ to the density contrast δ. If a redshift variation in G occurs, this will clearly change the gravitational potential, the density growth function and large scale CMB anisotropy through the Integrated Sachs Wolfe effect (ISW hereafter, see e.g. [29]). Since a large ISW signal is at odds with current WMAP data, a varying with redshift G is strongly constrained, as we will see in the next section.

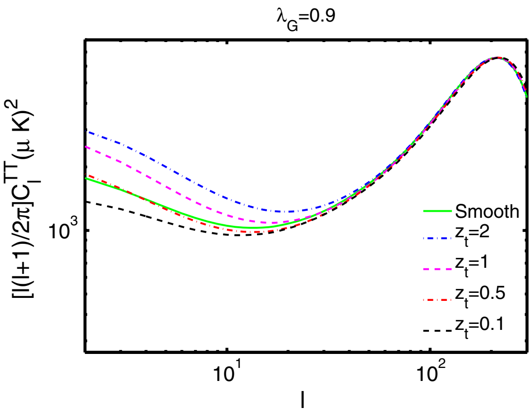

FIG. 3 (color online). Temperature power spectrum when the gravitational constant varies such that $\lambda_G = 0.9$. The graph shows the effects of the smooth transition parametrization described in Eq. (5) (green line) and of the Heaviside parametrization of Eq. (6) for different redshifts of transition z_T between 0.1 and 2.

III. ANALYSIS METHOD

We constrain variations in the Newton's constant with current CMB data by making use of the publicly available Markov Chain Monte Carlo package cosmomc [30]. Other than λ_G we sample the following set of cosmological parameters, adopting flat priors on them: the physical baryon and CDM densities, $\omega_b = \Omega_b h^2$ and $\omega_c = \Omega_c h^2$, the Hubble parameter, H_0, the scalar spectral index, n_s, the normalization, $\ln 10^{10} A_s(k = 0.05/\mathrm{Mpc})$ and the reionization optical depth τ.

As discussed in the previous section, we will also consider the possibility of a variation with redshift in G and we will consider as extra parameter the redshift of transition z_T.

The MCMC convergence diagnostic tests are performed on 4 chains using the Gelman and Rubin "variance of chain mean"/"mean of chain variances" $R - 1$ statistic for each parameter. Our $1 - D$ and $2 - D$ constraints are obtained after marginalization over the remaining "nuisance" parameters, again using the programs included in the cosmomc package. We use a cosmic age top-hat prior as $10\ \mathrm{Gyr} \le t_0 \le 20\ \mathrm{Gyr}$. We include the five-year WMAP data [9] (temperature and polarization) with the routine for computing the likelihood supplied by the WMAP team (we will refer to this analysis as WMAP5).

Moreover, in order to test the effect of current polarization measurements on constraining λ_G we also considered the combination of the WMAP data with the polarization results coming from the BOOMERANG ([10]) and CBI ([11]) experiments. We will refer to this analysis as WMAP5 + POL.

Together with the WMAP data we also consider the small-scale CMB measurements of ACBAR [12] (we will refer to this analysis as WMAP5 + ACBAR).

Finally, we forecast future constraints on λ_G simulating a set of mock data with a fiducial model given by the best fit WMAP5 model with $\lambda_G = 1$ and experimental noise described by:

$$N_\ell = \left(\frac{w^{-1/2}}{\mu\mathrm{K} - \mathrm{rad}}\right)^2 \exp\left[\frac{\ell(\ell+1)(\theta_{\mathrm{FWHM}}/\mathrm{rad})^2}{8\ln 2}\right], \quad (8)$$

where $w^{-1/2}$ is the temperature noise level (we consider a factor $\sqrt{2}$ larger for polarization noise) and θ is the beam size. We considered two future data sets. The first, based on the experimental specifications of the PLANCK SURVEYOR mission, with $w^{1/2} = 58\ \mu\mathrm{K}$ and $\theta_{\mathrm{FWHM}} = 7.1'$ equivalent to the 143 GHz channel (see [16]). The second data set is a cosmic variance limited experiment (CVL hereafter) with no experimental noise for both temperature and polarization anisotropy and $\ell_{\max} = 2500$.

Constraints on λ_G are also computed using standard BBN theoretical predictions as provided by the new numerical code described in [31,32], which includes a full updating of all rates entering the nuclear chain based on the most recent experimental results on nuclear cross sections. The BBN predictions are compared with the D/H abundance ratio of [33] obtained including a new measurement in a metal poor damped Lyman-α system along the line of sight of QSO SDSS1558-0031

$$\mathrm{D/H} = (2.82^{+0.27}_{-0.25}) \times 10^{-5}. \quad (9)$$

We use the uncertainty as quoted in [33], computed by a jackknife analysis.

IV. RESULTS

A. Constant *G* with redshift

We report in Table I the constraints obtained on λ_G analyzing the data sets mentioned in the previous section. As we can see, current CMB data only provide a constraints at about $\sim 15\%$ level. The WMAP constraint is improved by $\sim 10\%$ when temperature and polarization anisotropy data from BOOMERANG and CBI is included and by $\sim 30\%$ when the small-scale temperature angular spectrum data from ACBAR is added. However, as we can see from the Table, the major improvement comes from

TABLE I. Constraints on λ_G from current WMAP and BBN observations and future constraints achievable from the Planck satellite mission and from a cosmic variance limited experiment.

Experiment	Constraints on λ_G at 68% c.l.
WMAP	1.01 ± 0.16
WMAP + POL	0.97 ± 0.13
WMAP + ACBAR	1.03 ± 0.11
WMAP + BBN	0.98 ± 0.03
PLANCK	1.01 ± 0.015
CVL	1.002 ± 0.004

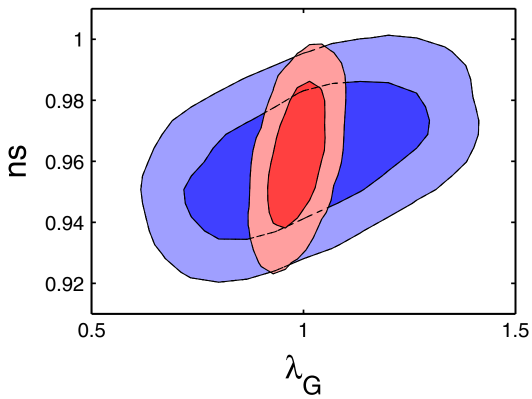

FIG. 4 (color online). 68% and 95% likelihood contour plots on the λ_G-n_S plane using present CMB data with and without BBN constraints.

BBN: in this case the constraint WMAP + BBN reaches the ~3% level.

It is interesting to consider possible correlations between λ_G and more usual cosmological parameters. In Figs. 4 and 5 we plot the 1 and 2 σ's confidence level on the n_S-λ_G and ω_b-λ_G planes, respectively. As we can see there is a strong degeneracy between these parameters. Increasing (decreasing) G would yield higher (lower) values of n_S and lower (higher) values for ω_b more consistent with CMB data.

The degeneracy with the scalar spectral index is clear since increasing λ_G delays recombination, damping the small angular scale oscillations. This effect could be counterbalanced by increasing n_S and the small-scale power of primordial perturbations. This will also change the relative amplitude between odd and even peaks, affecting the constraints on the baryon density.

As already described in [7], another possible degeneracy is present with the running of the spectral index α_s. We have therefore considered an extra analysis including possible variations in α_s. Considering the WMAP data only we found $\lambda_G = 0.96 \pm 0.19$.

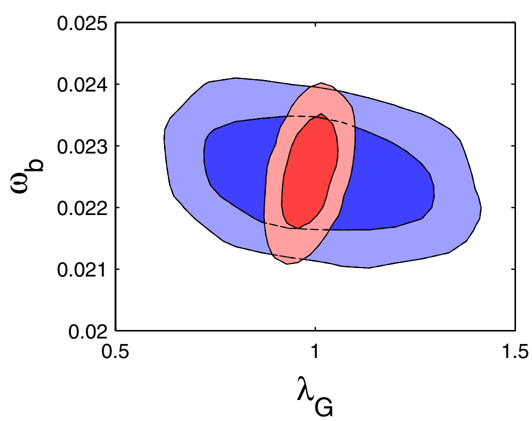

FIG. 5 (color online). 68% and 95% likelihood contour plots on the λ_G-ω_b plane using present CMB data with and without BBN constraints.

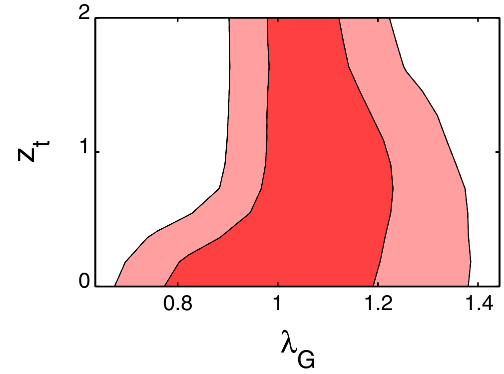

FIG. 6 (color online). 68% and 95% likelihood contour plots on the λ_G-z_T plane using present CMB data.

As we can see from Table I, future experiments can substantially improve the current constraints on λ_G. The PLANCK Surveyor mission is expected to provide constraints at the ~1.5% level. As already discussed in [7], the inclusion of polarization data is crucial in breaking the degeneracy between λ_G and inflationary parameters as n_s and α_s; we found that neglecting polarization data from Planck yields weaker constrains by a factor of ~4. Polarization data are therefore extremely useful in constraining λ_G.

The ultimate constraint achievable by a cosmic variance limited experiment is 0.4%, competitive with current laboratory bounds.

B. Varying G with redshift

Here we consider possible constraints on G allowing for variations in redshift. Using the simple parametrization in Eq. (5) we found that the WMAP data alone yields the constraint $\lambda_G = 1.01 \pm 0.1$ at 68% c.l.. This constraint is better by ~40% respect the corresponding bound obtained with constant G. The reason is due to the extra ISW effect that increases the large angular scale CMB spectra, in disagreement with the WMAP observations.

We have then considered a redshift dependence as in Eq. (6) with a flat prior $0 < z_T < 2$. In Fig. 6 we plot the 68% and 95% confidence levels on the λ_G-z_T plane using only the WMAP data. As we can see, for larger values of z_T the constraints on λ_G are stronger. Again, the presence of the ISW effect, irrelevant for $z_T \sim 0$ but sizable for larger values, helps in constraining λ_G.

V. CONCLUSIONS

In this paper we have updated the constraints from current CMB data on Newton's gravitational constant G. We have found no evidence for variation in this constant with a constraint of $\lambda_G = 1.03 \pm 0.11$ at 68% c.l. from WMAP + ACBAR ($\lambda_G = 0.98 \pm 0.03$ when BBN data is considered). BBN plays therefore a crucial role in constraining G. However, even without considering the possibility of systematics in current observations of primordial elements, the BBN constraints relies on the perfect knowledge of the amount of relativistic degrees of freedom g_*. Since $g_* = 5.5 + \frac{7}{4}N_\nu^{\rm eff}$ any possible extra background of relativistic particles, parametrized by the effective number of neutrino species $N_\nu^{\rm eff}$ would drastically change the BBN bound. Moreover, CMB and BBN probe completely different physics and epochs. While the agreement between the two results is reassuring, it is clear that it would be preferable to have an improved and independent CMB constraint.

We have then considered the constraints achievable from ongoing and future satellite experiments. For the Planck Surveyor satellite mission we have found a future constraints of the order of 1.5% using only CMB data. Next, cosmic variance limited experiments as, for example, the future EPIC satellite proposal (see [34]), could probe Newton's constant with a ~0.4% precision, i.e. with grossly the same accuracy currently reached from local experiments.

It is important to stress that the accuracy on λ_G achievable by the CMB is limited by how precisely we treat the recombination process. Current recombination codes should be accurate enough for the Planck mission (see e.g. [35]) but this may provide an intrinsic limit for the next, beyond Planck, CMB surveys. Moreover, recombination could be modified by nonstandard mechanisms as dark matter decay or variations in the fine structure constant α. High frequency measurements of the blackbody CMB spectrum, where recombination absorption lines are expected, could be helpful in disentangling the two effects. However, galactic foregrounds at those frequencies largely dominate over the CMB signal.

In this paper we followed a conservative approach by considering only future CMB data. It is clear that the inclusion of complementary cosmological data, as expected from future galaxy, weak lensing and 21 cm surveys, will further break the degeneracies between the parameters and substantially improve the constraints. We plan to discuss this in more detail in a future paper ([36]).

Moreover, we have considered a variation of G with redshift, parametrizing its variation either with a smooth transition between G and $\lambda_G^2 G$, or with a simple step function at a transition redshift z_T. The ISW effect arising from redshift variations in G is at odds with the low CMB quadrupole measured by WMAP and therefore yields stronger constraints on λ_G. Current data, also in this case, do not exhibit a deviation from the standard value. In this respect, the constraints obtained under the assumption of constant with redshift G could be considered as more conservative.

Finally, since G is a dimensional constant and since the definition of a system of units and the value of the fundamental constants (and thus the status of their constancy) are entangled, it could be preferable to consider the variation of dimensionless ratios. For example, in a Cavendish-type experiment, one can not distinguish a variation in G from a variation in m. That is the unambiguous quantity is Gm_1m_2. Thus a variation of G in this case must require the assumption of fixed masses. It is therefore important to clarify what quantities are assumed fixed in our analysis, i.e. what is the true dimensionless quantity (involving G) which is being constrained. The answer to this question is not straightforward since G enters in the background and in the perturbation equations. Considering the Friedmann equation, the dimensionless quantity tested is $G\rho/H^2$. As already discussed in [7], the energy density is constrained by the CMB anisotropies since they are sensitive to variation in the redshift of equality $z_{\rm eq}$, fully defined by the matter density ω_m once the energy density in the relativistic particles is fixed. Rescaling the Hubble parameter affects recombination and, again, the shape of the CMB anisotropies. Our constraints on G therefore relies both in the assumption of a standard background of relativistic particles and of standard recombination. Since variations in other fundamental constants, as the fine structure constant α. affect recombination, it is useful to investigate possible degeneracies between G and α. We therefore changed the CAMB code in order to allow for variations in α. As we can see from Fig. 7 a variation in α does notreally affect our constraints on G, since the two have different effects on the temperature spectra. In particular, for example, an higher value of G and a smaller value of α would both decrease the amplitude of the temperature power spectrum, shift the peaks position in opposite ways.

Future cosmological data will therefore substantially improve the bounds on G and on its possible variations with time, space and redshift. By comparing local and cosmic measurements, the Newton's constant will be less elusive and may shed light on the late accelerated evolution of the universe.

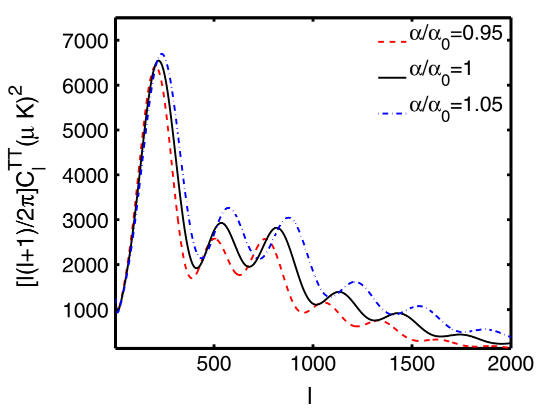

FIG. 7 (color online). Dependence on variation from the fine structure constant α of the temperature anisotropy angular spectra.

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