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Primordial magnetic fields and gravitational baryogenesis in nonlinear electrodynamics

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The amplification of the primordial magnetic fields and the gravitational baryogenesis, a mechanism that allows one to generate the baryon asymmetry in the Universe by means of the coupling between the Ricci scalar curvature and the baryon current, are reviewed in the framework of the nonlinear electrodynamics. To study the amplification of the primordial magnetic field strength, we write down the gauge invariant wave equations and then solve them (in the long wavelength approximation) for three different eras of the Universe: de Sitter, the reheating, and the radiation-dominated era. Constraints on parameters entering the nonlinear electrodynamics are obtained by using the amplitude of the observed galactic magnetic fields and the baryon asymmetry, which are characterized by the dimensionless parameters $r \sim 10^{-37}$ and $\eta_B \lesssim 9 \times 10^{-11}$, respectively.

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I. INTRODUCTION

With the aim to build up a classically singularity-free theory of the electron, that is a theory in which infinite physical quantities are avoided, Born and Infeld [1] proposed a model in which additional terms or modifications of the standard electrodynamics were included. To prevent the infinite self energy of point particles (as follows from standard electrodynamics), they introduced an upper limit on the electric field strength and considered the electron as an electric particle with finite radius. In successive investigations, other examples of nonlinear electrodynamic Lagrangians were proposed by Plebanski, who also showed that the Born-Infeld model satisfies physically acceptable requirements [2]. Consequences of nonlinear electrodynamics have been studied in many contexts, for example, cosmological models [3], black holes and wormhole physics [4,5], and astrophysics [6].

Recently, the nonlinear electrodynamics has been also invoked as an available framework for generating the primordial magnetic fields in the Universe [7,8]. The latter, indeed, is a still open problem of the modern cosmology, and although many mechanisms have been proposed, this issue is far from being solved. Seeds of magnetic fields may arise in different contexts, e.g. cosmological phase transitions of the early Universe [9], string cosmology [10], inflationary models of the Universe [11,12], nonminimal electromagnetic-gravitational coupling [13–15], gauge invariance breakdown [12,13,16], density perturbations [17], gravitational waves in the early Universe [18], lorentz violation [19], cosmological defects [20], electroweak anomaly [21], temporary electric charge nonconservation [22], trace anomaly [23], parity violation of the weak interactions [24], and Biermann type battery seed effect [25]. Once these seeds are generated, they must be amplified by means of some mechanism. Promising candidates are the dynamo mechanism [26,27] and the protogalactic collapse and differential rotation [28]. The first mechanism allows an enhancement of the (preexisting) magnetic strength from $\sim 10^{-20}$ G to $\sim 10^{-6}$–10^{-5} G, the present (observed in galaxies and galaxy clusters) strength, the second one instead allows an amplification from $\sim 10^{-10}$ G to $\sim 10^{-6}$–10^{-5} G. For a review, see [29]. Moreover, the presence of magnetic fields in the Universe has important cosmological consequences as, for example, the generation of anisotropies in CMB [30], and the primordial abundances of the light elements [big bang nucleosynthesis (BBN)] [31].

In this paper, besides studying the amplification of primordial magnetic fields in the context of nonlinear electrodynamics, we also discuss the possibility that nonlinear electrodynamics might provide a framework for the so-called *gravitational* baryogenesis. The latter is related to the origin of the baryon number asymmetry, which is, as is well known, a still open problem of the particle physics and cosmology [32]. BBN [33] and measurements of CMB combined with the large structure of the Universe [34] indicate that the order of magnitude of such an asymmetry is

$$\eta_B \equiv \frac{n_B - n_{\bar{B}}}{s} \lesssim 9\, 10^{-11},$$

where n_B ($n_{\bar{B}}$) is the baryon (antibaryon) number density, and s the entropy of the Universe. Conventionally, the necessary requirements for a (CPT invariant) theory able to generate the baryon asymmetry are dictated by Sakharov's conditions [35]: (1) there must exist processes that violate the baryon number; (2) the discrete symmetries C and CP must be violated; (3) departure from thermal equilibrium. However, none of the Sakharov's conditions are obligatory [36]. In fact, as shown in [37], a dynamical violation of *CPT* (which implies a different spectrum of particles and antiparticles) may give rise to the baryon number asymmetry also in a regime of thermal equilibrium.

The paper is organized as follows. In next section we study the amplification of the primordial magnetic fields in the framework of the nonlinear electrodynamics. We shall investigate the case in which the Lagrangian is of the form $L \sim X + \gamma X^{\delta}$, where $X = F_{\mu\nu}F^{\mu\nu}/4$, and γ and δ are free parameters. In Sec. III, after a short review of the gravitational baryogenesis mechanism [38], we investigate the possibility to generate, during the radiation-dominated era, the observed baryon asymmetry if effects of nonlinear electrodynamics are taken into account. Section IV is devoted to the analysis of the amplification of primordial magnetic fields and of the gravitational baryogenesis for the nonlinear electrodynamics whose Lagrangian is of the form $\mathcal{L} \sim X + \gamma/X$. Conclusions are shortly discussed in Sec. V.

II. FIELD EQUATIONS IN NONLINEAR ELECTRODYNAMICS AND THE PRIMORDIAL MAGNETIC FIELD

In this section we shall study the amplification of the primordial magnetic field for the case in which the electromagnetic field is described by a nonlinear electrodynamics. The Lagrangian density we consider is [39]

$$L(X) = -CX - \gamma X^{\delta}. \tag{2.1}$$

where γ and δ are free parameters that with the appropriate choice reproduce the well know Lagrangian already studied in the literature. γ has dimensions $[\text{energy}]^{4(1-\delta)}$. The case $C = 1$ and $\gamma = 0$ corresponds to the standard linear electrodynamics. The primordial magnetic field in nonlinear electrodynamics has been studied recently by Kunze [7] and Campanelli *et al.* [8]. Their study refers mainly to the inflationary era of the Universe's evolution. Our approach follows the paper [12], in which the electromagnetic field evolution is analyzed during the de Sitter, reheating, and radiation-dominated eras. Moreover, we derive a wave equation for the electromagnetic field strength tensor $F_{\mu\nu}$.

In the seminal paper by Turner and Widrow [12], it was suggested that a magnetic field might be generated by quantum fluctuations during an inflationary epoch, and it could be sustained after the wave length of interest crossed beyond the horizon giving the observed field today [12]. This model invokes a coupling among the electromagnetic field and the scalar (R) and (Ricci and Riemann) tensor curvatures, which break the conformal invariance. According to Turner and Widrow's paper, since the Universe is a good conductor (during its evolution), one expects that the magnetic flux is preserved even if the primordial magnetic field evolves. This physical behavior suggests the definition of the parameter $r = \rho_B/\rho_\gamma$, which remains (with good approximation) constant and provides an invariant measure of the magnetic field strength. Here ρ_B is the energy density of the magnetic field, and $\rho_\gamma = \pi^2 T^4/25$ is the energy density of the cosmic microwave background radiation. In order to explain the present value of $r \approx 1$ for galaxies, one needs a pregalactic magnetic field to which corresponds to $r \simeq 10^{-37}$ if dynamo amplifications are invoked, and $r \simeq 10^{-8}$ if the galactic magnetic fields are generated, in the collapse of the protogalactic cloud, by means of the compression of the primordial magnetic field. In the last case, the dynamo processes are not necessary (see, for example, Refs. [7,12]).

The action we consider is the electromagnetic field minimally coupled to gravity

$$S = \int d^4x \sqrt{-g}\left(\frac{R}{16\pi G} + \frac{1}{4\pi} L(X)\right). \tag{2.2}$$

The nonlinearity breaks the conformal invariance, which is the necessary condition for amplifying the primordial magnetic fields (in fact, the minimal coupling of electromagnetic fields to a four-dimensional background is invariant under conformal transformations of the metric; therefore, the time evolution of the conformally flat metric, as the Friedamn-Robertson-Walker metric, does not affect the electromagnetic fluctuations, and no amplifications occur).

The field equations for the electromagnetic fields are

$$\nabla_\rho F^{\rho\sigma} = -\frac{\nabla_\mu L_X}{L_X} F^{\mu\sigma}, \tag{2.3}$$

$$\nabla_\mu F_{\nu\lambda} + \nabla_\nu F_{\lambda\mu} + \nabla_\lambda F_{\mu\nu} = 0. \tag{2.4}$$

Equation (2.4) is the Bianchi identities and $L_X = dL/dX$. The wave equation for $F_{\mu\nu}$ follows by applying ∇_λ to Eq. (2.4) and then using Eq. (2.3). One gets

$$\begin{aligned}\Box F_{\nu\lambda} + [\nabla^\mu, \nabla_\nu]F_{\lambda\mu} - [\nabla^\mu, \nabla_\lambda]F_{\nu\mu} \\ = -\nabla_\nu\left(\frac{\nabla_\alpha L_X}{L_X} F^\alpha{}_\lambda\right) + (\nu \leftrightarrow \lambda),\end{aligned} \tag{2.5}$$

where $\Box = \nabla^\mu \nabla_\mu$ and $[.,.]$ is the commutator.

Using (1) the cyclic identities of the Riemann tensor $R_{\rho\alpha\beta\gamma} + R_{\rho\gamma\alpha\beta} + R_{\rho\beta\gamma\alpha} = 0$, (2) the Ricci identity $[\nabla^\mu, \nabla_\nu]F_{\alpha\mu} = R_{\rho\alpha\nu\mu}F^{\rho\mu} + R^\rho{}_\nu F_{\alpha\rho}$, (3) the fact that the Riemann tensor can be written in terms of the Ricci tensor and the scalar curvature R as (this is true because in a system of coordinates in which the metric is conformal to the Minkowski one, the Weyl tensor $C_{\lambda\mu\nu\rho}$ vanishes [40])

$$\begin{aligned}R_{\lambda\mu\nu\rho} = {}& \frac{1}{2}(g_{\lambda\nu}R_{\mu\rho} - g_{\lambda\rho}R_{\mu\nu} - g_{\mu\nu}R_{\lambda\rho} + g_{\mu\rho}R_{\lambda\nu}) \\ & - \frac{R}{6}(g_{\lambda\nu}g_{\mu\rho} - g_{\lambda\rho}g_{\mu\nu}),\end{aligned}$$

one can rewrite Eq. (2.5) as

$$\Box F_{\alpha\lambda} - \frac{R}{3}F_{\alpha\lambda} = \frac{1}{a^2}\Box_\eta F_{\alpha\lambda} = -\nabla_\alpha\left(\frac{\nabla_\mu L_X}{L_X}F^\mu{}_\lambda\right) + (\alpha \leftrightarrow \lambda), \quad (2.6)$$

where $\Box_\eta = \eta^{\mu\nu}\partial_\mu\partial_\nu$ is the D'Alambertian in the Minkowski spacetime. Equation (2.6) is gauge invariant.

We work in the conformal Friedman-Robertson-Walker metric

$$g_{\mu\nu} = a^2(\eta)\mathrm{diag}(1, -1, -1, -1), \quad (2.7)$$

where $a(\eta)$ is the scale factor. The field strength tensor $F_{\mu\nu}$ in a curved spacetime has components

$$F_{\mu\nu} = a^2(\eta)\begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & B_z & -B_y \\ E_y & -B_z & 0 & B_x \\ E_z & B_y & -B_x & 0 \end{pmatrix}. \quad (2.8)$$

We shall set $E_i = 0$.

Equations (2.6) are very involved. To evaluate the magnetic field strength during the three eras we are interested in, i.e. de Sitter (dS), reheating (RH), and radiation-dominated (RD) phases of the Universe, we are concerned with the evolution of the magnetic field fluctuations whose wavelengths are well outside the horizon, $L_{\mathrm{phys}} = aL \gg H^{-1}$ or $k\eta \ll 1$ [12]. In this approximation, all spatial derivatives will be neglected (long wavelength approximation). Moreover, we shall assume that the direction of the magnetic field is fixed. Therefore, using the relations $F_{ij} = \varepsilon_{ijk}(a^2 B_k)$, and the notation $|\mathbf{F}(\eta)| \equiv F = a^2(\eta)|\mathbf{B}(\eta)|$, the field equation (2.6) reduces to the form

$$\left[C + \gamma\delta\left(\frac{F^2}{2a^4}\right)^{\delta-1}\right]F'' + \gamma\delta(\delta-1)\left(\frac{F^2}{2a^4}\right)^{\delta-2}\left(\frac{F^2}{2a^4}\right)'\mathcal{H}F = 0. \quad (2.9)$$

The prime means derivative with respect to the conformal time η and $\mathcal{H} = a'/a$ is the Hubble parameter.

It turns out to be convenient to express Eq. (2.9) in terms of the scale factor a. Since the scale factor varies as $a(\eta) = a_{(\alpha)}\eta^\alpha$, where $\alpha = -1, +2, +1$ during the dS, RH, and RD eras, respectively, while the constants $a_{(\alpha)}$, that are different for three eras, are explicitly specified in (2.12), (2.15), and (2.18), we get

$$\frac{d^2F}{da^2} + \left[1 - \frac{1}{\alpha} + 4(\delta-1)\mathcal{F}\right]\frac{1}{a}\frac{dF}{da} - 8(\delta-1)\frac{\mathcal{F}}{a^2}F = 0, \quad (2.10)$$

where

$$\mathcal{F} \equiv \frac{\gamma\delta(\frac{F^2}{2a^4})^{\delta-1}}{C + \gamma\delta(\frac{F^2}{2a^4})^{\delta-1}}.$$

The complex structure of the differential equation (2.10) does not allow one to determine exact solutions. We therefore assume that during the dS, RH, and RD eras the (F^2/a^4) term is dominant, which means $\mathcal{F} \sim 1$. In this regime, a solution of (2.10) is of the form

$$F(\eta) = F_{(\beta)}a^\beta, \quad (2.11)$$

where $F_{(\beta)}$, which is a constant, and β assume different values for each different phase of the Universe evolution.

A. Inflationary de Sitter (dS) phase ($\alpha = -1$)

The scale factor for this epoch of the Universe is

$$a(\eta) = -a_{\mathrm{dS}}\eta^{-1} \sim -\frac{1}{H_{\mathrm{dS}}\eta}, \quad (2.12)$$

where $H_{\mathrm{dS}} \sim 3 \times 10^{24}$ eV. Equation (2.11) reads

$$F \sim a^{\beta_{\mathrm{dS}}}. \quad (2.13)$$

The exponent β_{dS} is given by

$$\beta_{\mathrm{dS}} \equiv p_\pm = \frac{3}{2} - 2\delta \pm \sqrt{4\delta^2 + 2\delta - \frac{23}{4}}. \quad (2.14)$$

B. Reheating (RH) phase ($\alpha = 2$)

The scale factor for this stage of the Universe is given by [19]

$$a(\eta) = a_{\mathrm{RH}}\eta^2 \sim \tfrac{1}{4}H_0^2R_0^3\eta^2, \quad (2.15)$$

where $R_0 \sim 10^{26}h_0^{-1}$ m is the present Hubble radius of the Universe, and $H_0 \sim 100h_0$ km/s-Mpc is the Hubble parameter today. The solution (2.11) is of the form

$$F \sim a^{\beta_{\mathrm{RH}}}, \quad (2.16)$$

where

$$\beta_{\mathrm{RH}} \equiv q_\pm = \frac{9}{4} - 2\delta \pm \sqrt{4\delta^2 - \delta - \frac{47}{16}}. \quad (2.17)$$

C. Radiation-Dominated (RD) phase ($\alpha = 1$)

In this last case, the scale factor of the Universe is

$$a = a_{\mathrm{RD}}\eta \sim H_0R_0^2\eta. \quad (2.18)$$

The solution for F is

$$F \sim a^{\beta_{\mathrm{RD}}}, \quad (2.19)$$

where

$$\beta_{\mathrm{RD}} = \frac{3}{2} - 2\delta \pm \sqrt{4\delta^2 - 2\delta - \frac{7}{4}}. \quad (2.20)$$

These solutions have been determined for $\mathcal{F} \sim 1$. By using (2.11) one infers that the regime we are concerned with applies for amplitudes of the magnetic field such that

$$|\mathbf{B}(\eta)| \gg B_0, \qquad B_0 \equiv \sqrt{2}\left(\frac{C}{\gamma|\delta|}\right)^{1/(2(\delta-1))},$$

or equivalently, in terms of the conformal time, it applies for conformal time η larger than η_*,

$$\eta \gg \eta_*, \qquad \eta_* \equiv \frac{1}{a_{(\alpha)}}\left[\frac{\sqrt{2}}{F_{(\beta)}}\left(\frac{C}{\gamma|\delta|}\right)^{1/2(\delta-1)}\right]^{1/(\alpha(\beta-2))},$$

where $a_{(\alpha)} = a_{\rm dS}$, $a_{\rm RH}$, $a_{\rm RD}$ are defined in Eqs. (2.12), (2.15), and (2.18), and $\beta = \beta_{\rm dS}$, $\beta_{\rm RH}$, $\beta_{\rm RD}$ are given by Eqs. (2.14), (2.17), and (2.20).

The above solutions for $F = F_k(a)$ allow one to estimate the strength of the primordial magnetic field. According to Turner-Widrow's model [12], if one assumes that the Universe had gone through a period of inflation at the grand unified theories (GUT) scale ($M_{\rm GUT} \sim 10^{16} \div 10^{17}$ GeV) and that fluctuations of the electromagnetic field have come out from the horizon where the Universe had gone through about 55 e-foldings of inflation, then [12]

$$r \approx (7\times 10^{25})^{-2(p+2)}\left(\frac{M_{\rm GUT}}{m_{Pl}}\right)^{4(q-p)/3} \times \left(\frac{T_{\rm RH}}{m_{Pl}}\right)^{2(2q-p)/3}\left(\frac{T_*}{m_{Pl}}\right)^{-8q/3}\lambda_{\rm Mpc}^{-2(p+2)}, \qquad (2.21)$$

where $T_{\rm RH}$ is the reheating temperature, T_* is the temperature at which plasma effects become dominant (i.e. the Universe first becomes a good conductor), and $m_{Pl} \sim 10^{19}$ GeV is the Planck mass. Finally, $p = p_\pm$ and $q = q_\pm$ are the exponents of the scale factor $a(\eta)$ during the dS and RH epochs [see Eqs. (2.13) and (2.16)]. Results are independent on the parameter γ.

The temperature T_* can be estimated via reheating processes [12] $T_* = \min\{(T_{\rm RH}M_{\rm GUT})^{1/2}; (T_{\rm RH}^2 m_{Pl})^{1/3}\}$, and for $T < T_*$ ρ_B evolves as $\rho_B \sim a^{-4}$. Notice that the reheating temperature $T_{\rm RH}$ is given by $T_{\rm RH} = \{10^9 \text{ GeV}, M_{\rm GUT}\}$ [12]. Imposing that $r \sim 10^{-37}$, we infer the values for the parameter δ yielding the observed strength of the cosmological magnetic field. Results are reported in Tables I and II.

Some comments are in order. First, during the radiation-dominated era, the plasma effects induce a rapid decay of the electric field, whereas the magnetic field remains [12].

TABLE I. Values of δ for $r \sim 10^{-37}$ at 1 Mpc and for $M_{\rm GUT} \sim 10^{17}$ GeV and $T_{RH} \sim 10^{15} - 10^{17}$ GeV. The cases p_+, q_+, and p_+, q_- do not admit solutions.

p, q	$M_{\rm GUT}$ (GeV)	$T_{\rm RH}$ (GeV)	T_* (GeV)	$\delta\sim$
p_-, q_+	10^{17}	10^{15}	10^{15}	1.280
		10^{16}	10^{16}	1.278
		10^{17}	10^{16}	1.265
p_-, q_-	10^{17}	10^{15}	10^{15}	1.315
		10^{16}	10^{16}	1.295
		10^{17}	10^{16}	1.297

TABLE II. Values of δ for $r \sim 10^{-37}$ at 1 Mpc and for $M_{\rm GUT} \sim 10^{16}$, 10^{17} GeV and $T_{\rm RH} \sim 10^9$ GeV, $T_* \sim 10^{12}$ GeV. The cases p_+, q_+ and p_+, q_- do not admit solutions.

p, q	$M_{\rm GUT}$ (GeV)	$T_{\rm RH}$ (GeV)	T_* (GeV)	$\delta\sim$
p_-, q_+	10^{17}	10^9	10^{12}	1.331
	10^{16}			1.360
p_-, q_-	10^{17}	10^9	10^{12}	1.375
	10^{16}			1.382

Moreover, the functions $F(\eta)$ have been obtained for a cosmological background which evolves according to standard cosmology. In particular, during the radiation-dominated era the scale factor evolves according to the power law $a \sim \eta \sim t^{1/2}$ (t is the cosmic time). The "magnetic" component of the energy density, therefore, is assumed negligible with respect to the radiation energy density: $\rho_{\rm total} = \rho_{\rm rad} + \rho_B \simeq \rho_{\rm rad}(= \frac{\pi^2 g_*}{30}T^4)$. The validity of the condition $\rho_B < \rho_{\rm rad}$, that will be discussed in the next section when we will study the origin of baryon asymmetry, yields a constraint on the temperature at which the nonlinear effects are active. Yet, in order that predictions of the standard cosmology (such as BBN, CMB, and large scale structure formation) remain unaltered, we assume that after the conformal time $\tilde\eta$ (or after the cosmic time $\tilde t$ or the temperature $\tilde T$) corrections to the standard linear electrodynamics vanish, i.e. $\gamma = 0$ for $\eta > \tilde\eta$, and $\gamma \neq 0$ for $\eta_{\rm RD} < \eta < \tilde\eta$, where $\eta_{\rm RD}$ is the time when radiation-dominated era starts (that, in our model, it does coincide with the end of reheating). Of course, $\tilde\eta \ll \eta_{\rm end}$, where $\eta_{\rm end}$ corresponds to the end of the radiation-dominated era. As we have seen, $F = a^2 B$ evolves as $F \sim F_0 a^{\beta_{(\pm)}}$, where $\beta_{\rm RD}^{(\pm)}$ are the two solutions of $\beta_{\rm RD}$ in (2.20). Using the range of values for δ reported in Tables I and II one can show that $\beta_{\rm RD}^{(+)} \in (0.42, 0.50)$ and $\beta_{\rm RD}^{(-)} \in (-3, -2.4)$. All these features will play a relevant role in the framework of the gravitational baryogenesis (next section).

III. GRAVITATIONAL BARYOGENESIS

To begin with, we shortly recall the main topics of the gravitational baryogenesis. The latter, as already pointed out, is a mechanism for generating the baryon number asymmetry during the expansion of the Universe by means of a dynamical breaking of *CPT* (and *CP*) [38]. In this approach the thermal equilibrium is preserved. The interaction responsible for *CPT* violation is given by a coupling between the derivative of the Ricci scalar curvature R and the baryon current J^μ [41]

$$\frac{1}{M_*^2}\int d^4x\sqrt{-g}J^\mu\partial_\mu R, \qquad (3.1)$$

where M_* is the cutoff scale characterizing the effective theory. If there exist interactions that violate the baryon number B in thermal equilibrium, then a net baryon asymmetry can be generated and gets frozen-in below the decoupling temperature T_D.

From (3.1) it follows:

$$M_*^{-2}(\partial_\mu R)J^\mu = M_*^{-2}\dot{R}(n_B - n_{\bar{B}}),$$

where $\dot{R} = dR/dt$. Therefore the effective chemical potential for baryons and antibaryons is $\mu_B = \dot{R}/M_*^2 = -\mu_{\bar{B}}$, and the net baryon number density at the equilibrium turns out to be (as $T \gg m_B$, where m_B is the baryon mass) $n_B = g_b\mu_B T^2/6$. $g_b \sim \mathcal{O}(1)$ is the number of intrinsic degrees of freedom of baryons. The baryon number to entropy ratio, that defines the baryon asymmetry, is therefore [38]

$$\eta_B = \frac{n_B}{s} \simeq -\frac{15g_b}{4\pi^2 g_*}\frac{\dot{R}}{M_*^2 T}\bigg|_{T_D}, \tag{3.2}$$

where $s = 2\pi^2 g_{*s}T^3/45$, and g_{*s} counts the total degrees of freedom for particles that contribute to the entropy of the Universe. g_{*s} takes values very close to the total degrees of freedom of effective massless particles g_*, i.e. $g_{*s} \simeq g_* \sim 106$. η_B does not vanish provided that the time derivative of the Ricci scalar is nonvanishing.

In the context of general relativity, the Ricci scalar and the trace T_g of the energy-momentum tensor ($T_g^{\mu\nu}$) are related by the relation

$$R = -8\pi G T_g = -8\pi G(1-3w)\rho,$$

where ρ is the matter density, $w = p/\rho$ is the adiabatic parameter, p the pressure, and $T_g = T^\mu_{g\mu}$. $\dot{R}$ is zero in the radiation-dominated epoch of the standard Friedman-Robertson-Walker cosmology, because (in the limit of *exact* conformal invariance) $w = 1/3$. However, deviations from the standard electrodynamics prevent the Ricci curvature and its first time derivative from vanishing (as seen from the point of view of the new structure of the energy-momentum tensor). Therefore a net baryon asymmetry may be generated also during the radiation-dominated era (for other applications and scenarios see [38,42–44]).

A. Gravitational Baryogenesis in nonlinear Electrodynamics

We wish now discuss the origin of the baryon asymmetry in the framework of the nonlinear electrodynamics. The epoch of the Universe we refer is the radiation-dominated era. As pointed out at the end of Sec. II, we assume that from the beginning of the radiation-dominated era to time $\tilde{t}$ the nonlinear terms of electromagnetism are non zero. The latter may break the conformal invariance and therefore $1 - 3w \neq 0$, or equivalently, the trace of the energy-momentum tensor does not vanishes. As a consequence, R and $\dot{R}$ are different from zero. In fact, by making use of the expression for the energy-momentum tensor

$$T_{g\mu\nu} = \frac{1}{4\pi}\left[\frac{\partial L}{\partial X}F_{\mu\alpha}F^\alpha{}_\nu + g_{\mu\nu}L\right], \tag{3.3}$$

we infer that the trace T_g is given by

$$T_g = -\frac{\gamma(\delta-1)}{\pi}X^\delta.$$

Equation (2.19) implies that $\dot{X} = (\beta_{\rm RD} - 2)HB^2$, where $H = \dot{a}/a$.

By making use of the Einstein field equations

$$H = \frac{\pi}{3m_P}\sqrt{\frac{4\pi g_*}{5}}T^2, \tag{3.4}$$

the parameter η_B characterizing the baryon asymmetry [see Eq. (3.2)] can be cast in the form

$$\eta_B = 8g_b\sqrt{\frac{5}{\pi g_*}}(\beta_{\rm RD}-2)\delta(\delta-1)\gamma\left(\frac{B^2}{2}\right)^\delta\frac{T_D}{M_*^2 m_P^3}. \tag{3.5}$$

Equation (3.5) expresses the baryon asymmetry in terms of parameters characterizing the nonlinear electrodynamics. In the standard case, i.e. $\gamma = 0$, η_B vanishes and no net baryon asymmetry can be generated, as expected.

Introducing the dimensionless parameter $\Gamma \equiv \gamma[{\rm GeV}]^{4(1-\delta)}$, Eq. (3.5) can be rewritten as

$$\Gamma\left(\frac{B}{{\rm GeV}^2}\right)^{2\delta} = N\eta_B\frac{\rm GeV}{T_D}\left(\frac{M_*}{\rm GeV}\right)^2\left(\frac{m_P}{\rm GeV}\right)^3, \tag{3.6}$$

where

$$N \equiv \sqrt{\frac{\pi g_*}{5}}\frac{2^\delta}{8g_b(\beta_{\rm RD}-2)\delta(\delta-1)}.$$

The bound $\eta_B \lesssim 9\times 10^{-11}$ and Eq. (3.6) give a constraint (upper bound) on the free parameter γ for fixed magnetic field strengths. For our estimations, we use the following values of parameters: As pointed out in [38], a possible choice of the cutoff scale M_* is $M_* = m_{Pl}/\sqrt{8\pi}$ if $T_D = M_I$, where $M_I \sim 2 10^{16}$ GeV is the upper bound on the tensor mode fluctuation constraints in inflationary scale [45]. For T_D, we use the decoupling temperature at the GUT scale, $T_D \sim 10^{16}$ GeV (a decoupling temperature at the GUT scale is phenomenologically acceptable if the unwanted relics like gravitinos are decoupled at the Planck scale so that they will be diluted away during inflation and will not be regenerated at reheating at the GUT scale). By using the range of values for δ reported in Tables I and II, and setting $\beta^{(+)}_{\rm RD} \sim 0.5$ and $\beta^{(-)}_{\rm RD} \sim -3$, one may obtain an estimation on Γ: $\Gamma \sim 10^{146}$ for $B \sim 10^{-10}$ G, and $\Gamma \sim 10^{173}$ for $B \sim 10^{-20}$ G.

As a final comment, we analyze the validity of our approximation $\rho_B < \rho_{\rm rad}$. In the regime we worked, $\gamma X^{\delta-1} \gg 1$, see Sec. II, the energy density of the electromagnetic field reads $\rho_B \sim \gamma(B^2/2)^\delta$. By making use of

Eq. (3.6) and $\Gamma \equiv \gamma[\mathrm{GeV}]^{4(1-\delta)}$, we get

$$\Gamma\left(\frac{B}{\mathrm{GeV}^2}\right)^{2\delta} = N\eta_B\left(\frac{M_*}{\mathrm{GeV}}\right)^2\left(\frac{m_P}{\mathrm{GeV}}\right)^3\frac{\mathrm{GeV}}{T_D}. \qquad (3.7)$$

The condition $\rho_{\rm rad} > \rho_B$ gives the lower bound on the temperature T

$$T > 1.1 \times 10^{15}\left(\frac{10^{16}\ \mathrm{GeV}}{T_D}\right)^{1/4}\ \mathrm{GeV}, \qquad (3.8)$$

where we have used $\frac{30N}{2^\delta \pi^2 g_*} \sim \mathcal{O}(10^{-2})$. As Eq. (3.8) shows, our assumptions are consistent for temperatures of the Universe varying in the range $T_{\rm RH} > T > \tilde{T}$, i.e. the nonlinear electrodynamics effects are active at GUT scales. In this regime, nonlinear electrodynamics allows one to account for both the amplification of the primordial magnetic fields and the origin of the baryon asymmetry.

IV. THE NOVELLO-BERGLIAFFA-SALIM MODEL OF NONLINEAR ELECTRODYNAMICS

In the framework of nonlinear electrodynamics, we shall now analyze the Novello-Bergliaffa-Salim (NBS) model [46]. This model is particularly interesting because the nonlinear terms of the electromagnetic field give rise to a ‘‘fluid’’ with an asymptotically negative equation of state. Therefore, the accelerated expansion of the Universe can be attributed to these nonlinear corrections to the standard electromagnetic Lagrangian.

The Lagrangian of the nonlinear electrodynamics of the NBS model is [47]

$$L_\mu = -X - \frac{\mu^8}{X}, \qquad (4.1)$$

where $[\mu] = (\mathrm{energy})^2$. It corresponds to $C = 1$, $\delta = -1$, and $\gamma = \mu^8$ in (2.1).

To derive an upper bound on the parameter μ, NBS assume that dark energy can be described by the non linear term, and using the current value for $\Omega_{\rm de} = \rho_{\rm de}/\rho_{\rm cr}$, where $\rho_{\rm cr} = 3H_0^2/8\pi G$ is the critical energy density, they find [46]

$$\mu^4 \lesssim 3.74 \times 10^{-28}\ \frac{\mathrm{gr}}{\mathrm{cm}^3} = 1.683 \times 10^{-45}\ \mathrm{GeV}^4. \qquad (4.2)$$

The extremely small value of μ implies a negligible correction to Maxwell’s electromagnetism. Nonetheless, one should keep in mind that for extremely low magnetic field strength it is the $1/F$ term of the NBS Lagrangian that dominates.

A. Primordial Magnetic Field

In studying the amplification of the magnetic fields, we follow Sec. II. In order to obtain the required value $r \sim 10^{-37}$ corresponding to the observed values of the galactic magnetic field, we assume that the NBS nonlinear electromagnetism is turned off during the de Sitter era, and turns on at the reheating era, until the time $\tilde{\eta}$ of the radiation-dominated era. The wave equation for F is given by (2.10) with $\delta = -1$. As before, we assume that the (F^2/a^4) term is dominant.

1. Inflationary de Sitter (dS) phase

If during this era the nonlinear electrodynamics effects are absent, then the wave equation for F is $\Box_\eta F = 0$, whose solution is $F \sim \sin k\eta$ (the solution is independent whether $k\eta \gtrless 1$ as a consequence of the conformal invariance of the minimally coupled electromagnetic field [12]). In the long wavelength approximation, one obtains

$$F \sim \eta \sim a^{-1}. \qquad (4.3)$$

2. Reheating (RH) phase

In this phase of the evolution of the Universe, the wave equation (2.10) admits the solution

$$F \sim a^{(19\pm\sqrt{105})/4}. \qquad (4.4)$$

3. Radiation-dominated (RD) phase

During the RD era, finally, the solution for F is given by

$$F \sim a^{(9\pm\sqrt{17})/2}. \qquad (4.5)$$

Values of the parameter r are obtained using Eq. (2.21) with the exponents p and q given by Eqs. (4.3) and (4.4), $p = -1$ and $q = (19 \pm \sqrt{105})/4$. Results are reported in Table III.

It is interesting to note that the NBS model allows for an amplification of the magnetic fields. In particular, we can see that the required amplification (leading to $r \sim 10^{-37}$) may occur for the set of values

$$\{M_{\rm GUT}, T_{\rm RH}, T_*\} = \{(10^{17}, 10^{15}, 10^{15}), (10^{17}, 10^{16}, 10^{15.5}), (10^{17}, 10^{17}, 10^{16})\}\ \mathrm{GeV}.$$

B. Baryon Asymmetry

Let us now investigate the baryon asymmetry in the framework of the model of NBS.

TABLE III. Values of r at 1 Mpc and for different $\{M_{\rm GUT}, T_{\rm RH}, T_*\}$.

$M_{\rm GUT}$ (GeV)	$T_{\rm RH}$ (GeV)	T_* (GeV)	r
10^{17}	10^{15}	10^{15}	10^{-38}
	10^{16}	$10^{15.5}$	10^{-37}
	10^{16}	10^{16}	10^{-47}
	10^{17}	10^{16}	10^{-37}
10^{17}	10^{9}	10^{12}	10^{-42}
10^{16}	10^{9}	10^{12}	10^{-53}

The trace of the energy-momentum tensor for the NBS nonlinear electrodynamics Lagrangian (4.1) reads

$$T^{(\mathrm{NBS})} = \rho - 3p = \frac{8\mu^8}{X}, \tag{4.6}$$

which is obtained by averaging the magnetic (and electric) field on a sufficiently large time-dependent three volume

$$\begin{aligned} \bar{E}_i &= 0, \qquad \bar{B}_i = 0, \qquad \overline{E_i B_j} = 0, \\ \overline{E_i E_j} &= -\frac{\mathbf{E}^2}{3}\delta_{ij}, \qquad \overline{B_i B_j} = -\frac{\mathbf{B}^2}{3}\delta_{ij}. \end{aligned} \tag{4.7}$$

As in the case discussed in Sec. III, we assume the background evolves as in the standard cosmology, which means that the energy density of the magnetic field is lesser than the energy density of radiation. The time derivative of the Ricci scalar is given by

$$\dot{R} = -\frac{128(5 \pm \sqrt{17})}{2}\frac{\mu^8}{B^2}\frac{H}{m_P^2}, \tag{4.8}$$

where $H = \dot{a}/a$. By using again the Einstein field equations (3.4), the net baryon asymmetry generated by nonlinear electrodynamics turns out to be

$$\eta_B = N' \frac{\mu^8}{B^2}\frac{T_D}{M_*^2 m_P^3}, \tag{4.9}$$

where $N' = N|_{\delta=-1}$. η_B vanishes as $\mu = 0$. The observed baryon asymmetry can be generated provided that the temperature at which the NBS nonlinear electrodynamics is active satisfies the constraint (3.8), that is at GUT scales.

If we consider μ as a free parameter, which does not satisfy Eq. (4.2), then bounds on μ from (4.9) follow by using the previous values of the parameters $M_* \sim 10^{16}$ GeV, $T_D \sim 10^{16}$ GeV, and a fixed magnetic field strength. For example, for $B \sim 10^{-20}$ G, one obtains $\mu^4 \lesssim 10^{-12}$ GeV^4. On the other hand, if one assumes that the bound (4.2) holds for conformal time η such that $\eta_{\mathrm{RD}} < \eta < \tilde{\eta}$, then to obtain the observed baryon asymmetry the magnetic field strength must be of the order $\gtrsim 10^{-54}$ G, which seems not to be cosmologically interesting.

V. CONCLUSION

In this paper we have studied the amplification of the magnetic field and the origin of the baryon asymmetry in the framework of the nonlinear electrodynamics. In particular we have analyzed Lagrangian densities of the form $\mathcal{L} \sim X + \gamma X^\delta$ and $\mathcal{L} \sim X + \mu^8/X$. The baryon asymmetry is generated by means of the (gravitational) coupling between baryon current and curvature of the background, which is non vanishing during the radiation-dominated era owing the nonlinear effects in the electromagnetism.

For the Lagrangian of the form $X + \gamma X^\delta$, which we have studied in the regime in which the nonlinear term dominates the standard X term, and for the de Sitter, reheating, and radiation-dominated eras, we have found that the amplification of the primordial magnetic field occurs provided that the parameter δ falls in the range [1.26; 1.38]. Moreover, the analysis has been performed also for the origin of the baryon asymmetry occurring during the radiation-dominated era.

As concerns the model proposed by Novello-Bergliaffa-Salim, with $\mathcal{L} \sim X + \mu^8/X$, the analysis of the amplification of the primordial magnetic fields shows that the required values $r \sim 10^{-37}$, necessary for explaining the observed galactic magnetic fields, is obtained provided that the electromagnetic nonlinear terms turn on at the reheating era, but are zero at the de Sitter epoch.

In conclusion, the nonlinear electrodynamics, which is the reduction in the Abelian sector of an effective model of the low energy (3 + 1) QCD [49], seems a promising candidate for studying cosmological scenarios which go beyond the standard cosmology and particle physics.

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