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Astrophysical constraints on unparticle-inspired models of gravity

O. Bertolami,[*,†] J. Páramos,[*,‡] and P. Santos[*,§]

Departamento de Física, Instituto Superior Técnico, Avenida Rovisco Pais 1, 1049-001 Lisboa, Portugal
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We use stellar dynamics arguments to constrain the relevant parameters of unparticle-inspired models of gravity. We show that resulting bounds do constrain the parameters of the theory of unparticles, as far as its energy scale satisfies the condition $\Lambda_U \geq 1$ TeV and d_U is close to unity.

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I. INTRODUCTION

It has been remarked that the standard model (SM) is likely to be incomplete due to the apparent lack of scale invariant objects, unparticles [1], besides its well-known shortcomings. Implementing scale invariance requires considering an additional set of fields with a nontrivial IR fixed point, the Banks-Zacks (BZ) fields. The interaction between the SM and BZ fields occurs through the exchange of particles with a large mass scale, M_*, written as

$$\mathcal{L}_{\rm BZ} = \frac{1}{M_*^k} O_{\rm SM} O_{\rm BZ}, \tag{1}$$

where $O_{\rm SM}$ is an operator with mass dimension $d_{\rm SM}$ built out of SM fields and $O_{\rm BZ}$ is an operator with mass dimension $d_{\rm BZ}$ built out of BZ fields.

At an energy scale Λ_U, the BZ operators match onto unparticles operators (O_U) and Eq. (1) matches onto

$$\mathcal{L}_U = \frac{C_U \Lambda_U^{d_{\rm BZ}-d_U}}{M_*^k} O_{\rm SM} O_U, \tag{2}$$

where d_U is the scaling dimension of O_U, which can be fractional, and C_U is a coefficient function.

Considering tensor-type unparticle interactions with the stress-energy tensor of SM states leads to a modification to the Newtonian potential $\Phi(r)$, usually referred to as ungravity—a gravitational potential with a power-law addition [2],

$$V(r) = -\frac{G_U M}{r}\left[1 + \left(\frac{R_G}{r}\right)^{2d_u-2}\right], \tag{3}$$

where R_G is the characteristic length scale of ungravity,

$$R_G = \frac{1}{\pi \Lambda_U}\left(\frac{M_{\rm Pl}}{M_*}\right)^{1/(d_U-1)}\left[\frac{2(2-\alpha)}{\pi} \times \frac{\Gamma(d_U+\frac{1}{2})\Gamma(d_U-\frac{1}{2})}{\Gamma(2d_U)}\right]^{1/(2d_U-2)}, \tag{4}$$

and $\Lambda_U \geq 1$ TeV is the energy scale of the unparticle interaction (the lower bound reflects the lack of detection of these interactions within the available energy range), $M_{\rm Pl}$ is the Planck mass, and α is a constant dependent on the type of propagator (unity in the case of a graviton).

The Newtonian potential is recovered for $d_U = 1$, $R_G = 0$ (if $d_U > 1$), or $R_G \to \infty$ (if $d_U < 1$), so that

$$G_U = \frac{G}{1 + (\frac{R_G}{R_0})^{2d_U-2}}, \tag{5}$$

where R_0 is the distance where the gravitational potential matches the Newtonian one, $V(R_0) = \Phi(R_0)$. Unfortunately, the value of R_0 is unknown; this may be circumvented by considering only values of d_U near unity [2], so that Eq. (3) is approximately given by

$$V(r) = -\frac{GM}{2r}\left[1 + \left(\frac{R_G}{r}\right)^{2d_u-2}\right]. \tag{6}$$

Notice that corrections of this type also arise in the context of a gravity model with vector-induced spontaneous Lorentz symmetry breaking [3].

II. POLYTROPIC STELLAR MODEL

In what follows, we examine the bounds on parameters R_G and d_U in Eq. (6) arising from astrophysical considerations about stellar equilibrium. In order to do so, we shall extend considerably the range of ungravity corrections. Before discussing these bounds in detail, we point out that astrophysical and cosmological constraints on unparticles have been discussed in Refs. [4–10], and the ones arising from nucleosynthesis have been studied in Ref. [11]. We also mention that the technique to be employed has been developed to constrain Yukawa-type corrections to the Newtonian potential [12] as well as to examine alternative gravity models with nonminimal coupling between curvature and matter [13].

The simplest model available for stellar structure involves the polytropic gas model, which assumes the state equation $P = K\rho^{(n+1)/n}$, where P is the pressure, ρ is the density, n is the so-called polytropic index, and K is the polytropic constant. The above equation of state allows one to write the relevant thermodynamical quantities as

[*]Also at Instituto de Plasmas e Fusão Nuclear, Av. Rovisco Pais 1, 1049-001 Lisboa, Portugal.
[†]orfeu@cosmos.ist.utl.pt
[‡]paramos@ist.edu
[§]paulo_santos@nfist.ist.utl.pt

$$\rho = \rho_c \theta^n(\xi), \qquad T = T_c \theta(\xi), \qquad P = P_c \theta^{n+1}(\xi), \tag{7}$$

where ρ_c, T_c, and P_c correspond to the values of density, temperature, and pressure at the core of the star, respectively. The dimensionless function $\theta(\xi)$ depends on the dimensionless variable ξ, related to the radial coordinate through $r = \beta\xi$, where

$$\beta = \sqrt{\frac{(n+1)K}{2\pi G}\rho_c^{(1-n)/n}}. \tag{8}$$

Using Eq. (7), the hydrostatic equilibrium condition

$$\frac{d}{dr}\left(\frac{r^2}{\rho}\frac{dP}{dr}\right) = -G\frac{dM(r)}{dr} \tag{9}$$

may be rewritten as

$$\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2\frac{d\theta}{d\xi}\right) = -\theta^n, \tag{10}$$

the Lane-Emden equation [14]. This differential equation is subjected to the initial conditions $\theta(0) = 1$ and $\theta'(0) = 0$. A solution to the Lane-Emden equation allows for the determination of the thermodynamical quantities of a star in terms of their values at its center. The profile of $\theta(\xi)$ depends only on the choice of the polytropic index n, not on the size of the star, manifesting the homology symmetry of this equation.

III. MODIFIED LANE-EMDEN EQUATION

In this section, we develop a method similar to that presented in Ref. [12], in order to extract the relevant bounds on d_U and R_G. We consider the modified potential Eq. (6) and assume the validity of the Newtonian regime (low density and small velocities) to obtain the modified hydrostatic equilibrium equation:

$$\frac{r^2}{\rho}\frac{dP(r)}{dr} = -\frac{GM(r)}{2}\left[1 + (2d_U - 1)\left(\frac{R_G}{r}\right)^{2d_U-2}\right]. \tag{11}$$

After some algebraic manipulation, this can be cast as

$$\frac{1}{r^2}\frac{d}{dr}\left(\frac{r^2}{\rho}\frac{dP(r)}{dr}\right) = -2\pi\rho G\left[1 + (2d_U - 1)\left(\frac{R_G}{r}\right)^{2d_U-2}\right] + \frac{GM(r)}{2R_G^3}(2d_U - 1)(2d_U - 2) \times \left(\frac{R_G}{r}\right)^{2d_U+1}. \tag{12}$$

Performing the substitutions $r = \beta\xi$ and $\rho = \rho_c\theta^n$, we obtain the perturbed Lane-Emden equation

$$\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2\frac{d\theta}{d\xi}\right) = -\frac{\theta^n}{2}\left[1 + (2d_U - 1)\left(\frac{\xi_G}{\xi}\right)^{2d_U-2}\right] + \frac{M(\xi)}{4\pi\rho_c^3\xi_G^3}(2d_U - 1)(d_U - 1) \times \left(\frac{\xi_G}{\xi}\right)^{2d_U+1}, \tag{13}$$

where $\xi_G = R_G/\beta$ has been defined, for convenience. Using relation $dM(r)/dr = 4\pi\rho(r)r^2$, together with the definitions of Eqs. (7) and (8), we obtain

$$M(\xi) = -4\pi\left(\frac{(n+1)K}{2\pi G}\right)^{3/2}\rho_c^{(3-n)/2n}\xi^2\frac{d\theta}{d\xi}, \tag{14}$$

which can be used to simplify the second term on the right-hand side of Eq. (13), which now reads

$$\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2\frac{d\theta}{d\xi}\right) = -\frac{\theta^n}{2}\left[1 + (2d_U - 1)\left(\frac{\xi_G}{\xi}\right)^{2d_U-2}\right] - (2d_U - 1)(d_U - 1)\frac{1}{\xi}\frac{d\theta}{d\xi}\left(\frac{\xi_G}{\xi}\right)^{2d_U-2}. \tag{15}$$

It is interesting to point out that this modified Lane-Emden equation, unlike Eq. (10), has no homology symmetry, due to the presence of ξ_G in Eq. (15)—and hence the stability of the star will depend on its radius. The unperturbed central temperature T_c of a star is obtained from the solution $\theta_0(\xi)$ of Eq. (10) [14],

$$T_{c0} \propto \left[\xi_{10}\left(\frac{d\theta_0}{d\xi}\right)_{\xi=\xi_{10}}\right]^{-1}, \tag{16}$$

where ξ_{10} signals the surface of the star, defined as $\theta_0(\xi_{10}) = 0$. Considering $n = 3$, which describes fairly well the overall features of the Sun, one finds $\xi_{10} \approx 6.90$ [14]. In the presence of the ungravity perturbation into the gravitational potential Eq. (6), the central temperature T_c will be shifted from T_{c0}, the value obtained by using the solution θ_0, to the unperturbed Lane-Emden equation, Eq. (10), yielding the ratio

$$T_r \equiv \frac{T_c}{T_{c0}} = \frac{\xi_{10}}{\xi_1}\frac{\frac{d\theta_0}{d\xi}}{\frac{d\theta}{d\xi}}. \tag{17}$$

We now seek a numerical solution of Eq. (15) that allows us to estimate the ratio Eq. (17) to obtain a contour plot of the relative shift $T_r - 1$ for different values of d_U and R_G (for $n = 3$).

We consider two ranges of values for d_U and R_G: $d_U \gtrsim 1$ for $R_G < R_S$ and $d_U \lesssim 1$ for $R_G > R_S$, $R_S \approx 7 \times 10^8$ m being the Sun's radius. For $d_U \gtrsim 1$, we also assume that the length scale R_G is larger than the Schwarzschild radius of the Sun, $R_M = 2GM_\odot/c^2 \approx 1.5$ km, so that no relativistic corrections of the form R_M/r have to be considered. In what concerns the modified Lane-Emden equation, Eq. (15), the following ranges are considered: $1.0 < d_U < 1.06$ for $0 < \xi_G < 1$ and $0.94 < d_U < 1$ for $10 < \xi_G < 10^4$.

IV. RESULTS

Numerical solutions of Eq. (15) allow for obtaining contour plots for $T_r - 1$ as a function of d_U and ξ_G. The results are depicted in Figs. 1 and 2 for $|T_r - 1| \leq 0.06$, the uncertainty in the Sun's central temperature [14].

Designating the line in Fig. 1 that indicates a 6% change of the Sun's central temperature as $R_-(d_U)$, one sees that $R_G(d_U) > R_-(d_U)$ for $d_U \lesssim 1$. Thus, Eq. (4) leads to

$$\frac{M_*}{M_{\rm Pl}} > [\pi \Lambda_U R_-(d_U)]^{1-d_U} f(d_U, \alpha), \tag{18}$$

where

$$f(d_U, \alpha) = \sqrt{\frac{2(2-\alpha)}{\pi} \frac{\Gamma(d_U + \frac{1}{2})\Gamma(d_U - \frac{1}{2})}{\Gamma(2d_U)}} \tag{19}$$

is defined, for convenience. One might plot the lower bound obtained above as a function of d_U, fixing the model parameters α and Λ_U. This is depicted in Fig. 3, for $\alpha = 0$, $2/3$, 1, 1.9, and suitable values for Λ_U; all lines converge to the trivial point $d_U = 1$, $M_* \geq 0$.

Similarly, one obtains from Fig. 2 the upper bound $R_G(d_U) < R_+(d_U)$, where the latter denotes the line corresponding to the 6% change in the Sun's central temperature. Resorting again to Eq. (4), this again yields

$$\frac{M_*}{M_{\rm Pl}} > [\pi \Lambda_U R_+(d_U)]^{1-d_U} f(d_U, \alpha). \tag{20}$$

The obtained lower bound is depicted in Fig. 4 (as before, the lines converge to the point $d_U = 1$, $M_* \geq 0$).

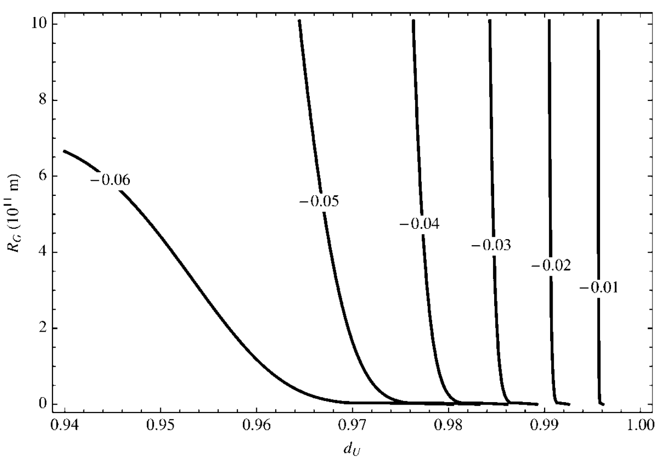

FIG. 1. Contour plot of $T_r - 1$ in function of R_G and d_U.

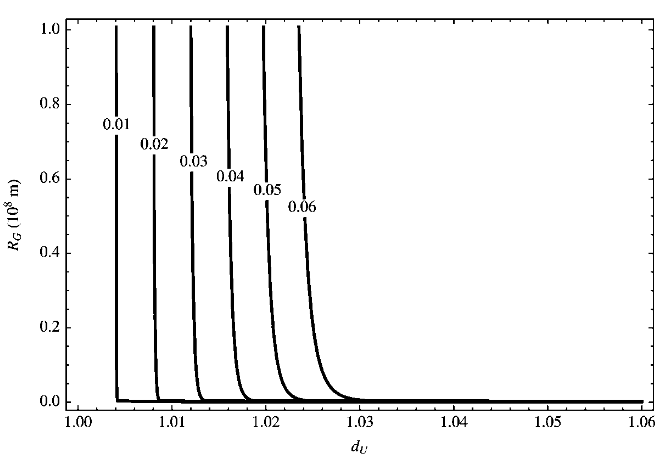

FIG. 2. Contour plot of $T_r - 1$ in function of $\log R_G$ and d_U.

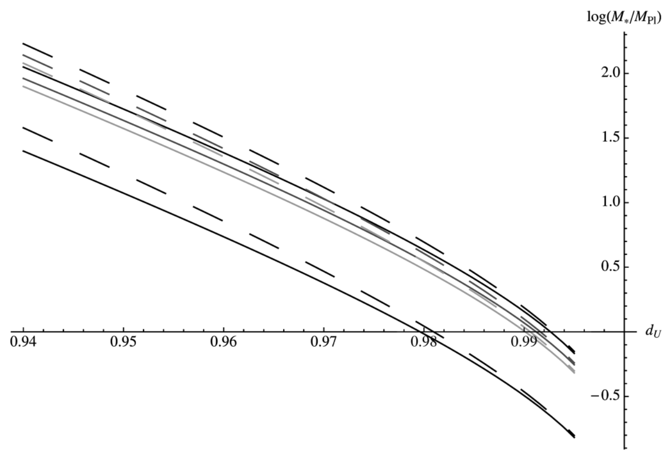

FIG. 3. Lower bound of $\log(M_*/M_{\rm Pl})$ for $\alpha = 0$ (black curve), $\alpha = 2/3$ (dark grey curve), $\alpha = 1$ (light grey curve), and $\alpha = 1.9$ (black, lower curve), and $\Lambda_U = 1$ TeV (solid curve) and $\Lambda_U = 10^3$ TeV (dashed curve).

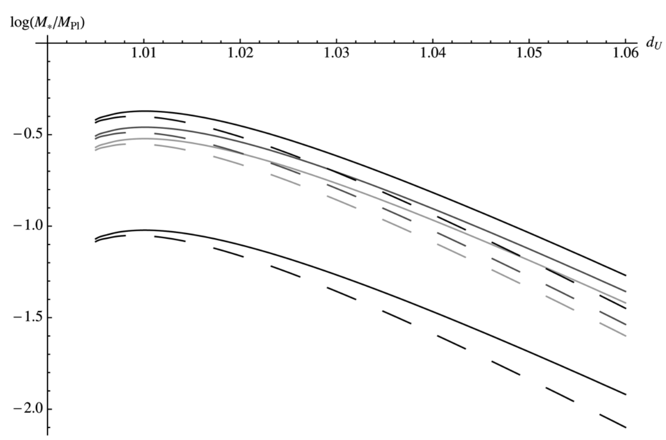

FIG. 4. Lower bound of $\log(M_*/M_{\rm Pl})$ for $\alpha = 0$ (black curve), $\alpha = 2/3$ (dark grey curve), $\alpha = 1$ (light grey curve), and $\alpha = 1.9$ (black, lower curve), and $\Lambda_U = 1$ TeV (solid curve) and $\Lambda_U = 10^3$ TeV (dashed curve).

Discussion

As stated in Ref. [2], corrections to the Newtonian potential $\Phi_N(r)$ with $d_U < 1$ might appear unfeasible, since these will overcome the $1/r$ dependence of $\Phi_N(r)$ for $r > R_g$ and lead to long-range deviations. This might be alleviated by letting R_G be so large that this crossover occurs well beyond the relevant astrophysical range, and one can no longer assume a static, spherically symmetric ansatz for the metric.

Alternatively, one may consider values so close to unity, $d_U \lesssim 1$, that the perturbation to the Newtonian potential, Eq. (6), may be expanded as $(r/R_G)^{2-2d_U} \approx 1 + 2(d_U - 1)\log(r/R_g)$, and the logarithmic dependence is attenuated by the small $d_U - 1$ term: for instance, for $r = 100$ AU, the typical dimension of the Solar System, and $R_G \sim 10$ AU, the maximum value considered here, this yields $(r/R_G)^{2-2d_U} \approx 1 + 4(d_U - 1)$; assuming the same value for R_G and instead, if $r \sim 100$ kpc $\sim 10^{10}$ AU, the size of a galaxy, one still gets $(r/R_G)^{2-2d_U} \approx 1 + 20(d_U - 1)$.

With these considerations in mind, the method developed here shows that one can successfully constrain the range of M_U for $d_U \lesssim 1$: in particular, assuming $\Lambda_U \gtrsim 1$ TeV, one achieves lower bounds in the range $M_* \gtrsim (10^{-1}\text{–}10^{2})M_{\text{Pl}}$ (even lower bounds can be obtained for values of d_U closer to unity).

For the case $d_U \gtrsim 1$, one obtains a lower bound exhibiting a peak around $d_U = 1.01$, with typical values $M_* \gtrsim (10^{-2}\text{–}10^{-1})M_{\text{Pl}}$. Reference [2] presents the lower bounds for M_* as a function of Λ_U, for $d_U = 2, 3, 4$—values which are beyond the reach of this study. In a subsequent study, the cases $d_U = 1.1$, 1.5, 2 were considered, with the first case closer to the range considered here [10].

By solving Eq. (15) for $d_U = 1.1$ and finding the value of R_G that yields $T_r - 1 = 0.06$, one obtains a lower bound of about $M_* > (10^{-4}\text{–}10^{-2})M_{\text{Pl}} \approx (10^{12}\text{–}10^{14})$ TeV, depending on Λ_U and M_* (this may be checked by extrapolating the plot in Fig. 2). This limit is much greater than the one found in Ref. [10], where a result $M_* \gtrsim (10^{2}\text{–}10^{6})$ TeV is reported (for $\Lambda_U = 10^6$ TeV). This indicates that the developed method hints at a much more stringent bound for M_*, for $d_U \gtrsim 1$.

V. CONCLUSIONS

In this work we have set up a formalism to constrain ungravity-inspired deviations from the Newtonian hydrostatic equilibrium conditions within a star. This leads to a perturbed Lane-Emden problem that is then examined for the polytropic index $n = 3$. From the resulting change in the star's central temperature, we obtain constraints on the ungravity parameters R_G and d_U. Given that the overall properties of the Sun are well described by the $n = 3$ case, we allow for the ungravity correction to affect this up to the upper bound on the Sun's central temperature, $\Delta T_c/T_c \approx 0.06$.

We find that, for $d_U \gtrsim 1$ and $\Lambda_U \gtrsim 1$ TeV, lower bounds on M_* are in the range $(10^{-2}\text{–}10^{-1})M_{\text{Pl}}$. For $d_U \lesssim 1$ and $\Lambda_U \gtrsim 1$ TeV, M_* must lie in the range above $(10^{-1}\text{–}10^{2})M_{\text{Pl}}$. Of course, our bounds are complementary to the ones obtained from torsion balance experiments, which test a much smaller range of R_G [15], actually about 80 μm.

The reported results for $d_U \gtrsim 1$ are either more stringent [4,5,7–10] or similar [6] to those previously available. The lower bound derived for $d_U \lesssim 1$ is more relevant, since it has been not obtained so far. In our opinion, this arises from misconception that a negative exponent in Eq. (6) is disallowed by long-range experiments [15]: while this is true for large values of $1 - d_U$, a range closer to unity, $d_U \lesssim 1$, yields an approximately logarithmic correction, with large deviations from the Newtonian potential suppressed by the smallness of the $d_U - 1$ term.

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