Polarized deep inelastic and elastic scattering from gauge/string duality

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In this paper, we investigate deep inelastic and elastic scattering on a polarized spin-$\frac{1}{2}$ hadron using gauge/string duality. This spin-$\frac{1}{2}$ hadron corresponds to a supergravity mode of the dilatino. The polarized deep inelastic structure functions are computed in the supergravity approximation at large t' Hooft coupling λ and finite x with $\lambda^{-1/2} \ll x < 1$. Furthermore, we discuss the moments of all structure functions, and propose an interesting sum rule $\int_0^1 dx g_2(x, q^2) = 0$ for the g_2 structure function which is known as the Burkhardt-Cottingham sum rule in QCD. In the end, the elastic scattering is studied and elastic form factors of the spin-$\frac{1}{2}$ hadron are calculated within the same framework.

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I. INTRODUCTION

Gauge/string duality [1–3] provides us new insights into gauge theories in the strong coupling regime. According to the gauge/string duality, the dual string theory, which corresponds to conformal gauge theories [e.g., the $\mathcal{N} = 4$ super Yang Mills (SYM) theory], is embedded in $\mathrm{AdS}_5 \times S^5$ space with the metric

$$ds^2 = g_{M,N}\mathrm{d}X^M\mathrm{d}X^N = \left(\frac{r^2}{R^2}\eta_{\mu\nu}\mathrm{d}y^\mu\mathrm{d}y^\nu + \frac{R^2}{r^2}\mathrm{d}r^2\right) + R^2\mathrm{d}\Omega_5^2, \quad (1)$$

where $g_{M,N}$ is the ten-dimensional metric and $\eta_{\mu\nu} = (-,+,+,+)$ is the mostly plus flat space metric. Here we use M, N as indices in ten dimensions, m, n as indices in AdS_5, and μ, ν as those in four-dimensional flat space which lives on the boundary of the AdS_5 space. R, which is the curvature radius of the AdS_5 space, is also equal to the radius of the five-sphere S^5. It is given by the duality

$$R^2 = l_s^2\sqrt{4\pi g_{\mathrm{st}}N}, \quad (2)$$

where the string coupling g_{st} and the string length l_s are given by $4\pi g_{\mathrm{st}} = g_{\mathrm{YM}}^2$ and $l_s^2 = \alpha'$ with α' being the Regge slope parameter, respectively. The t'Hooft coupling is defined as $\lambda = g_{\mathrm{YM}}^2 N = 4\pi g_{\mathrm{st}}N$. One can easily see that the large t'Hooft coupling limit is equivalent to the limit $R^2 \gg l_s^2$. In the limit $g_{st} \ll 1$ and $R^2 \gg l_s^2$, the string theory can be approximated by supergravity. Then the duality reduces to correspondence between gauge theories at large t'Hooft coupling and supergravity, One can investigate the nonperturbative properties of gauge theories at large t'Hooft coupling by studying the corresponding supergravity theory. There are also some interesting connections between the type II-B superstring theory and the $\mathcal{N} = 4$ SYM theory. First, the $SU(4)$ $\mathcal{R}$ symmetry of the $\mathcal{N} = 4$ SYM is the $SO(6)$ isometry of S^5. Furthermore, the $SO(4, 2)$ conformal symmetry of the gauge theory is the isometry of AdS_5. In addition, there is an implication that the radial direction (r) in AdS_5 can be identified with the energy scale in four-dimensional SYM theory, namely, $E \sim \frac{r}{R^2}$.

There has been substantial progress in studying strong coupling gauge theories especially in terms of deep inelastic scattering. A few years ago, Polchinski and Strassler [4,5] studied the deep inelastic scattering on hadrons by using gauge/string duality where the usual structure functions F_1 and F_2 are calculated for both spinless and spin-$\frac{1}{2}$ hadrons when Bjorken-x is finite ($\lambda^{-1/2} \ll x < 1$) where supergravity approximation is valid. The spinless hadron and spin-$\frac{1}{2}$ hadron correspond to supergravity modes of dilaton and dilatino, respectively. Furthermore, they also investigated the case at small-x where the Pomeron contribution with a trajectory of $2 - \mathcal{O}(\frac{1}{\sqrt{\lambda}})$ was found. Since an infrared cutoff Λ is introduced in order to generate confinement, the model is then called hard wall model. There are also some earlier studies [6,7] on high energy scattering in gauge/string duality. There have been a lot of further developments along this direction [8–14]. A saturation picture based on deep inelastic scattering in AdS/CFT is developed [15] afterwards and recently reviewed in Ref. [16]. In addition, the deep inelastic scattering off the finite temperature plasma in gauge/string duality is recently studied in Refs. [17,18].

Our main objective in this paper is to extend the calculation of deep inelastic scattering on a spin-$\frac{1}{2}$ fermion in the hard wall model, and compute the parity-violating structure function F_3 as well as the polarized structure functions g_1, g_2, g_3, g_4, and g_5. Among these five polarized structure

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functions, g_3, g_4, and g_5 are parity-violating structure functions.

Type II-B superstring theory, which lives in a ten-dimensional space (e.g., $\mathrm{AdS}_5 \times S^5$), is a parity-violating theory. It contains massless left-handed Majorana-Weyl gravitinos and massless right-handed Majorana-Weyl dilatinos. The gravitino, which is a spin-$\frac{3}{2}$ fermion, is the superpartner of the graviton. Likewise, the dilatino, which is a spin-$\frac{1}{2}$ fermion, is the superpartner of the dilaton. It has been proved that type II-B superstring theory is anomaly free [19,20] in terms of local (gauge) symmetries. Here we expect that the currents in the dual gauge theory are conserved at finite-x as we will show later in the paper. In this paper, we focus on the spin-$\frac{1}{2}$ dilatino and calculate its structure functions as well as form factors. In order to study the polarized structure functions and form factors, we follow the setup in Ref. [5] and assume the dilatino has a small mass M which eventually can be related to the cutoff scale Λ.

This paper is organized as follows. In Sec. II, we provide the definitions for various structure functions as well as kinematic variables. In Sec. III, we calculate the expectation value of the $\mathcal{R}$ currents in our gedanken experiment of polarized deep inelastic scattering from gauge/string duality. This eventually leads to the structure functions at finite x. Section IV is devoted to the discussions and comments on the structure functions and their sum rules. In Sec. V, we focus on the elastic scattering and derive the form factors for the spin-$\frac{1}{2}$ hadron. Finally, in Sec. VI, we summarize our results.

II. POLARIZED DEEP INELASTIC SCATTERING

The hadronic tensor $W^{\mu\nu}$ is defined as

$$W^{\mu\nu} = \int d^4\xi e^{iq\cdot\xi}\langle P, \mathcal{Q}, S|[J^\mu(\xi), J^\nu(0)]|P, \mathcal{Q}, S\rangle, \quad (3)$$

with J^μ being the incident current. The hadronic tensor $W_{\mu\nu}$ can be split as

$$W_{\mu\nu} = W^{(S)}_{\mu\nu}(q, P) + iW^{(A)}_{\mu\nu}(q; P, S). \quad (4)$$

According to Lorentz and CP invariance, the symmetrical and antisymmetrical parts can be expressed in terms of eight independent structure functions as [21,22],[1]

$$\begin{aligned} W^{(S)}_{\mu\nu} &= \left(\eta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\left[F_1(x, q^2) + \frac{MS\cdot q}{2P\cdot q}g_5(x, q^2)\right] - \frac{1}{P\cdot q}\left(P_\mu - \frac{P\cdot q}{q^2}q_\mu\right)\left(P_\nu - \frac{P\cdot q}{q^2}q_\nu\right) \\ &\quad \times\left[F_2(x, q^2) + \frac{MS\cdot q}{P\cdot q}g_4(x, q^2)\right] - \frac{M}{2P\cdot q}\left[\left(P_\mu - \frac{P\cdot q}{q^2}q_\mu\right)\left(S_\nu - \frac{S\cdot q}{P\cdot q}P_\nu\right)\right. \\ &\quad \left. + \left(P_\nu - \frac{P\cdot q}{q^2}q_\nu\right)\left(S_\mu - \frac{S\cdot q}{P\cdot q}P_\mu\right)\right]g_3(x, q^2) \\ W^{(A)}_{\mu\nu} &= -\frac{M\varepsilon_{\mu\nu\rho\sigma}q^\rho}{P\cdot q}\left\{S^\sigma g_1(x, q^2) + \left[S^\sigma - \frac{S\cdot q}{P\cdot q}P^\sigma\right]g_2(x, q^2)\right\} - \frac{\varepsilon_{\mu\nu\rho\sigma}q^\rho P^\sigma}{2P\cdot q}F_3(x, q^2), \end{aligned} \quad (5)$$

where M is the mass of the hadron, S is its polarization, q is the momentum carried by the current, and P is the initial momentum of the hadron (see Fig. 1). In deep inelastic scattering, we define the kinematic variables as the following:

$$x = -\frac{q^2}{2P\cdot q} \quad \text{and} \quad P_X^2 = (P + q)^2. \quad (6)$$

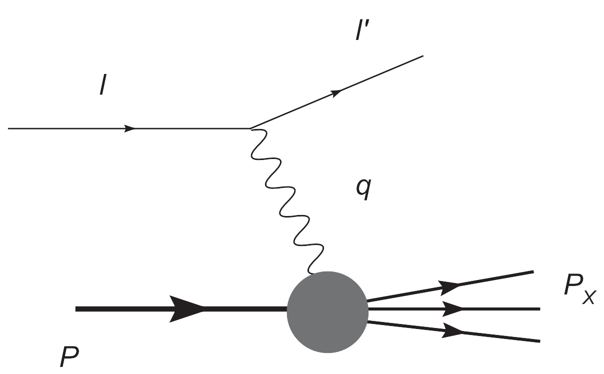

FIG. 1. Illustration of DIS.

The mass of the intermediate state after the scattering is defined as $M_X^2 = s = -P_X^2$. All the structure functions are functions of x and q^2.

III. POLARIZED STRUCTURE FUNCTIONS IN HARD WALL MODEL

In the so-called hard wall model, Polchinski and Strassler impose a confinement scale Λ in the fifth dimension of AdS_5 space. As we will see later in the paper [see Eq. (54)], this scale also provides a mass scale for the hadrons. Following Polchinski and Strassler [5], we perform a gedanken experiment of polarized deep inelastic scattering which occurs between the boundary and the cutoff scale Λ. Here we first summarize their setup before we extend the calculations to the polarized case.

[1]There are some sign changes in our definition comparing to the usual definition in [21,22]. These sign changes arise due to the reason that we use the most plus metric throughout this paper instead of the usual most minus metric.

The incident current is chosen to be the $\mathcal{R}$ current which couples to the hadron as an isometry of S^5. According to the AdS/CFT correspondence, the current excites a non-normalizable mode of a Kaluza-Klein gauge field at the Minkowski boundary of the AdS_5 space,

$$\delta G_{ma} = A_m(y, r)v_a(\Omega), \tag{7}$$

where $v_a(\Omega)$ denotes a Killing vector on S^5 with Ω being the angular coordinates on S^5. $A_m(y, r)$ is the external potential in the gauge theory corresponding to the operator insertion $n_\mu J^\mu(q)$ on the boundary of the fifth dimension of the AdS_5 space with the boundary condition

$$A_\mu(y, \infty) = A_\mu(y)|_{4D} = n_\mu e^{iq\cdot y}. \tag{8}$$

This gauge field fluctuation $A_m(y, r)$ can be viewed as a vector boson field which couples to the $\mathcal{R}$ current J^μ on the Minkowski boundary, and then propagates into the bulk as a gravitational wave, and eventually interacts with the supergravity modes of the dilatino or dilaton. The gauge field satisfies Maxwell's equation in the bulk, $D_m F^{mn} = 0$. With a gauge choice, one can solve this equation for A_μ. Usually people choose the gauge $A_r = 0$. However, the problem is easier in the Lorenz-like gauge, $i\eta^{\mu\nu}q_\mu A_\nu + R^{-4}r\partial_r(r^3A_r) = 0$. With given boundary conditions, one obtains the solution[2]

$$A_\mu = n_\mu e^{iq\cdot y}\frac{qR^2}{r}K_1(qR^2/r), \tag{9}$$

$$A_r = -iq\cdot n e^{iq\cdot y}\frac{R^4}{r^3}K_0(qR^2/r), \tag{10}$$

where $q = \sqrt{q^2}$. (Note that in $- + ++$ metric signature, $Q^2 = q^2 > 0$ for spacelike current.) Since $K_n(qR^2/r) \sim \exp(-qR^2/r)$, the deep inelastic scattering should be localized around $r_{\rm int} \simeq qR^2$ which is far away from the cutoff $r_0 = \Lambda R^2$ for hard scattering when $q^2 \gg \Lambda^2$.

Spin-$\frac{1}{2}$ hadrons correspond to supergravity modes of the dilatino. In the conformal region, one can write the dilatino field as

$$\lambda = \psi(y, r) \otimes \eta(\Omega), \tag{11}$$

where $\psi(y, r)$ is an $SO(4, 1)$ spinor on AdS_5 and $\eta(\Omega)$ is a normalized $SO(5)$ spinor on S^5. The wave function ψ satisfies a five-dimensional Dirac equation[3]

$$-\not{D}\psi = m\psi. \tag{12}$$

[2]Here we have corrected a minus sign typo in the solution of A_r in Ref. [5].
[3]We also noticed that there are some typos in the Dirac equations in Ref. [5] where there is an extra i in Eq. (12) while an i is missing in Eq. (14). The detailed derivation is provided above.

The solution to this Dirac equation is [23]

$$\psi = e^{ip\cdot y}\frac{C'}{r^{5/2}}[J_{mR-1/2}(MR^2/r)P_+ + J_{mR+1/2}(MR^2/r)P_-]u_\sigma, \tag{13}$$

where

$$\not{p}u_\sigma = iMu_\sigma \quad (\sigma = 1, 2), \qquad M^2 = -p^2, \qquad P_\pm = \tfrac{1}{2}(1 \pm \gamma^5). \tag{14}$$

Here we define the γ matrices according to the Dirac algebra in the mostly plus metric signature $- + ++$ (see, e.g., the notation in Ref. [24]),

$$\{\gamma^\mu, \gamma^\nu\} \equiv \gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2\eta^{\mu\nu} \times \mathbf{1}_{4\times 4}, \tag{15}$$

which gives an additional factor of $-i$ in γ_μ ($\mu = 0, 1, 2, 3$). It is straightforward to see that this explains the factor i in front of the fermion mass in Eq. (14) and the γ^5 is the same as the usual definition.

The dilatino is taken to be in a charge eigenstate with charge $\mathcal{Q}$ under the $U(1)$ symmetry, which yields $v_a\partial^a\eta(\Omega) = i\mathcal{Q}\eta(\Omega)$. This $U(1)$ symmetry arises from the $U(1)$ subgroup of the $SU(4)$ $\mathcal{R}$ symmetry.

For the initial hadron, by assuming $MR^2/r \ll 1$ in the interaction region and expanding the Bessel functions in Eq. (13) up to linear term in M, one gets

$$\psi_{\rm i} \approx e^{iP\cdot y}\frac{c'_{\rm i}}{\Lambda^{3/2}R^{9/2}}\left(\frac{r_0}{r}\right)^{mR+2}\left[P_+u_{{\rm i}\sigma} + \frac{Mr_0}{2(mR+1/2)\Lambda r}P_-u_{{\rm i}\sigma}\right]. \tag{16}$$

$$\bar{\psi}_{\rm i} \approx e^{-iP\cdot y}\frac{c'_{\rm i}}{\Lambda^{3/2}R^{9/2}}\left(\frac{r_0}{r}\right)^{mR+2}\left[\bar{u}_{{\rm i}\sigma}P_- + \frac{Mr_0}{2(mR+1/2)\Lambda r}\bar{u}_{{\rm i}\sigma}P_+\right]. \tag{17}$$

In order to obtain a polarized contribution of the structure function, we have kept the next leading order M of the initial hadron. The conformal dimension Δ of the state is found to be $mR + 2$. For the intermediate hadron, $M_X \gg \Lambda$ and

$$\psi_X \approx e^{i(P+q)\cdot y}\frac{c'_X s^{1/4}\Lambda^{1/2}R^{1/2}}{r^{5/2}}[J_{mR-1/2}(M_XR^2/r)P_+ + J_{mR+1/2}(M_XR^2/r)P_-]u_{X\sigma}. \tag{18}$$

$$\bar{\psi}_X \approx e^{-i(P+q)\cdot y}\frac{c'_X s^{1/4}\Lambda^{1/2}R^{1/2}}{r^{5/2}}\bar{u}_{X\sigma}[P_-J_{mR-1/2}(M_XR^2/r) + P_+J_{mR+1/2}(M_XR^2/r)]. \tag{19}$$

Before getting into the detailed calculation, let us look into the center of mass square of the intermediate states in ten dimensions. It is easy to see that

$$\tilde{s} = -g^{M,N} P_{X,M} P_{X,N} \leq \frac{R^2}{r_{\rm int}^2} q^2 \left(\frac{1}{x} - 1\right) \quad \text{with } r_{\rm int} = qR^2. \tag{20}$$

Thus we know that $\alpha' \tilde{s} = \frac{1}{\sqrt{\lambda}}(\frac{1}{x} - 1) \ll 1$ when $\frac{1}{\sqrt{\lambda}} \ll x < 1$. In this range of x, only massless string states are the relevant intermediate states produced during the interaction, and the supergravity calculation should be valid and reliable to obtain the structure functions. When x gets smaller, the massive string modes are excited and string scattering amplitude should be taken into account. This has been calculated thoroughly for F_1 and F_2. Unfortunately, we leave this part of the calculation for polarized structure functions for future studies.

Therefore, it is straightforward to compute the matrix element and obtain

$$n_\mu \langle P_X, X, \sigma' | J^\mu(0) | P, \mathcal{Q}, \sigma\rangle = i\mathcal{Q} \int d^6x_\perp \sqrt{-g} A_m \bar{\lambda}_X \gamma^m \lambda_{\rm i} \tag{21}$$

$$= i\mathcal{Q} \int d^6x_\perp \sqrt{-g}(A_\mu \bar{\lambda}_X e^\mu{}_{\hat{\mu}} \gamma^{\hat{\mu}} \lambda_{\rm i} + A_r \bar{\lambda}_X e^r{}_{\hat{r}} \gamma^{\hat{r}} \lambda_{\rm i}), \tag{22}$$

where n_μ is the polarization of the current J^μ, $\hat{\mu}$ and $\hat{r}$ are the tangent space index, and the vielbein $e^\mu{}_{\hat{\mu}}$ and $e^r{}_{\hat{r}}$ are given by

$$e^\mu{}_{\hat{\mu}} = \frac{R}{r}\eta^\mu{}_{\hat{\mu}} \quad \text{and} \quad e^r{}_{\hat{r}} = \frac{r}{R}. \tag{23}$$

Here the vielbein is used to make the product Lorentz invariant due to the fact that the gamma matrices are defined in the flat spacetime. It then follows that

$$\begin{aligned} n_\mu \langle P_X, X, \sigma' | J^\mu(0) | P, \mathcal{Q}, \sigma\rangle = {}& i\mathcal{Q} \int dr \frac{r^3}{R^3} R^5 \frac{c_{\rm i}'}{\Lambda^{3/2} R^{9/2}} \frac{(\Lambda R^2)^{mR+2}}{r^{mR+2}} \frac{c_X' s^{1/4} \Lambda^{1/2} R^{1/2}}{r^{5/2}} \\ & \times \bigg(\frac{qR^2}{r} K_1(qR^2/r) J_{mR-1/2}(M_X R^2/r) \frac{R}{r} \bar{u}_{X\sigma'} \not{n} P_+ u_{{\rm i}\sigma} - iq\cdot n \frac{R^4}{r^3} K_0(qR^2/r) \\ & \times J_{mR+1/2}(M_X R^2/r) \frac{r}{R} \bar{u}_{X\sigma'} \gamma^5 P_+ u_{{\rm i}\sigma} + \frac{qR^2}{r} K_1(qR^2/r) J_{mR+1/2}(M_X R^2/r) \frac{R}{r} \frac{MR^2}{(2mR+1)r} \\ & \times \bar{u}_{X\sigma'} \not{n} P_- u_{{\rm i}\sigma} - iq\cdot n \frac{R^4}{r^3} K_0(qR^2/r) J_{mR-1/2}(M_X R^2/r) \frac{r}{R} \frac{MR^2}{(2mR+1)r} \bar{u}_{X\sigma'} \gamma^5 P_- u_{{\rm i}\sigma} \bigg). \end{aligned} \tag{24}$$

After changing variables to $z = \frac{R^2}{r}$, one finds

$$\begin{aligned} n_\mu \langle P_X, X, \sigma' | J^\mu(0) | P, \mathcal{Q}, \sigma\rangle = {}& i\mathcal{Q} c_{\rm i}' c_X' s^{1/4} \Lambda^{\tau - 1/2} \int_0^{1/\Lambda} {\rm d}z z^\tau (qK_1(qz) J_{\tau-2}(M_X z) \bar{u}_{X\sigma'} \not{n} P_+ u_{{\rm i}\sigma} - iq\cdot n K_0(qz) \\ & \times J_{\tau-1}(M_X z) \bar{u}_{X\sigma'} \gamma^5 P_+ u_{{\rm i}\sigma} + qz K_1(qz) J_{\tau-1}(M_X z) \frac{M \bar{u}_{X\sigma'} \not{n} P_- u_{{\rm i}\sigma}}{2(\tau-1)} - i(q\cdot n) z K_0(qz) \\ & \times J_{\tau-2}(M_X z) \frac{M \bar{u}_{X\sigma'} \gamma^5 P_- u_{{\rm i}\sigma}}{2(\tau-1)} \bigg), \end{aligned} \tag{25}$$

where

$$r_0 = \Lambda R^2 \quad \text{and} \quad \tau = \Delta - 1/2 = mR + \tfrac{3}{2}. \tag{26}$$

Using the following integral results

$$\int_0^\infty dz z^\tau K_1(qz) J_{\tau-2}(M_X z) = \frac{2^{\tau-1} M_X^{\tau-2} q}{(M_X^2 + q^2)^\tau} \Gamma(\tau) \tag{27}$$

$$\int_0^\infty dz z^\tau K_0(qz) J_{\tau-1}(M_X z) = \frac{2^{\tau-1} M_X^{\tau-1}}{(M_X^2 + q^2)^\tau} \Gamma(\tau) \tag{28}$$

$$\int_0^\infty dz z^{\tau+1} K_1(qz) J_{\tau-1}(M_X z) = \frac{2^\tau M_X^{\tau-1} q}{(M_X^2 + q^2)^{\tau+1}} \Gamma(\tau+1) \tag{29}$$

$$\int_0^\infty dz z^{\tau+1} K_0(qz) J_{\tau-2}(M_X z) = \frac{2^\tau M_X^{\tau-2}}{(M_X^2 + q^2)^{\tau+1}} [q^2 \Gamma(\tau+1) - (M_X^2 + q^2)\Gamma(\tau)], \tag{30}$$

where the upper limits are approximately set to be ∞, we have

$$\langle P_X, X, \sigma'|J^\mu(0)|P, \mathcal{Q}, \sigma\rangle = i\mathcal{Q} c_i' c_X' s^{1/4}\Lambda^{\tau-1/2}2^{\tau-1}M_X^{\tau-2}(M_X^2+q^2)^{-\tau}\Gamma(\tau)\bigg(q^2\bar{u}_{X\sigma'}\gamma^\mu P_+ u_{i\sigma} - iM_X q^\mu \bar{u}_{X\sigma'}P_+u_{i\sigma} + \frac{\tau}{\tau-1}\frac{MM_X}{M_X^2+q^2}q^2\bar{u}_{X\sigma'}\gamma^\mu P_- u_{i\sigma} + i\frac{\tau}{\tau-1}\frac{Mq^\mu q^2}{M_X^2+q^2}\bar{u}_{X\sigma'}P_-u_{i\sigma} - i\frac{Mq^\mu}{\tau-1}\bar{u}_{X\sigma'}P_-u_{i\sigma}\bigg) \tag{31}$$

or its complex conjugate,[4]

$$\langle P, \mathcal{Q}, \sigma|J^\mu(0)|P_X, X, \sigma'\rangle = -i\mathcal{Q} c_i' c_X' s^{1/4}\Lambda^{\tau-1/2}2^{\tau-1}M_X^{\tau-2}(M_X^2+q^2)^{-\tau}\Gamma(\tau)\bigg(q^2\bar{u}_{i\sigma}\gamma^\mu P_+ u_{X\sigma'} - iM_X q^\mu \bar{u}_{i\sigma}P_-u_{X\sigma'} + \frac{\tau}{\tau-1}\frac{MM_Xq^2}{M_X^2+q^2}\bar{u}_{i\sigma}\gamma^\mu P_- u_{X\sigma'} + i\frac{\tau}{\tau-1}\frac{Mq^\mu q^2}{M_X^2+q^2}\bar{u}_{i\sigma}P_+u_{X\sigma'} - i\frac{Mq^\mu}{\tau-1}\bar{u}_{i\sigma}P_+u_{X\sigma'}\bigg). \tag{32}$$

With the help of Eq. (14), it is easy to see that $q_\mu\langle P_X, X, \sigma'|J^\mu(0)|P, \mathcal{Q}, \sigma\rangle = 0$ and $q_\nu\langle P, \mathcal{Q}, \sigma|J^\nu(0)|P_X, X, \sigma'\rangle = 0$ as a result of current conservation. In fact, with the present next-leading-order approximation, we can only show that $q_\mu\langle P_X, X, \sigma'|J^\mu(0)|P, \mathcal{Q}, \sigma\rangle \sim M^2/q^2$. Nevertheless, we can expand the initial wave function up to next-to-next-to-leading order (NNLO) (M^2 order), and find that M^2/q^2 contributions are canceled by NNLO terms in the initial wave function. If one continues to do this to higher orders, one can show that the current conservation is true for all orders of M^2/q^2. Moreover, using the recursion relations of Bessel functions, integrating the dz integral by parts and requiring the M/Λ and M_X/Λ to be the zeros of Bessel functions as we use in the later elastic calculation, one can show that $q_\nu\langle P, \mathcal{Q}, \sigma|J^\nu(0)|P_X, X, \sigma'\rangle = 0$ vanishes exactly.

Following Polchinski and Strassler, we also define $T^{\mu\nu}$ as

$$T^{\mu\nu} = i\langle P, \mathcal{Q}, S|T(J^\mu(q)J^\nu(0))|P, \mathcal{Q}, S\rangle. \tag{33}$$

Its imaginary part can be written as

$$\mathrm{Im}T^{\mu\nu} = 2\pi^2\sum_X \delta(M_X^2 + (p+q)^2) \times\langle P, \mathcal{Q}, S|J^\nu(0)|P+q, X\rangle \times\langle P+q, X|J^\mu(0)|P, \mathcal{Q}, S\rangle. \tag{34}$$

In the large q^2 limit, we approximately write $\sum_X \delta(M_X^2 + (p+q)^2) \simeq \frac{1}{2\pi M_X\Lambda}$.

Summing over radial excitations and the final state spin, but keeping the initial spin, along with the relation $\frac{1}{2\pi}W_{\mu\nu}^{S,A} = 2\,\mathrm{Im}T_{\mu\nu}^{S,A}$ derived from the optical theorem, yields

$$W_{\mu\nu}^{(S)} = \pi A'\mathcal{Q}^2(\Lambda^2/q^2)^{\tau-1}x^{\tau+1}(1-x)^{\tau-2}\bigg\{\bigg(\eta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\bigg)\bigg(\frac{1}{2} + \frac{q\cdot S}{2P\cdot q}M\bigg) - \frac{1}{P\cdot q}\bigg(P_\mu - \frac{P\cdot q}{q^2}q_\mu\bigg)\bigg(P_\nu - \frac{P\cdot q}{q^2}q_\nu\bigg) - \frac{M}{2P\cdot q}\bigg[\bigg(P_\mu - \frac{P\cdot q}{q^2}q_\mu\bigg)\bigg(S_\nu - \frac{S\cdot q}{q^2}q_\nu\bigg) + \bigg(P_\nu - \frac{P\cdot q}{q^2}q_\nu\bigg)\bigg(S_\mu - \frac{S\cdot q}{q^2}q_\mu\bigg)\bigg]\bigg\} \tag{35}$$

$$= \pi A_0'\mathcal{Q}^2(\Lambda^2/q^2)^{\tau-1}x^{\tau+1}(1-x)^{\tau-2}\bigg\{\bigg(\eta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\bigg)\bigg(\frac{1}{2} + \frac{q\cdot S}{2P\cdot q}M\bigg) - \frac{1}{P\cdot q}\bigg(P_\mu - \frac{P\cdot q}{q^2}q_\mu\bigg)\bigg(P_\nu - \frac{P\cdot q}{q^2}q_\nu\bigg)\times\bigg(1 + \frac{q\cdot S}{P\cdot q}M\bigg) - \frac{M}{2P\cdot q}\bigg[\bigg(P_\mu - \frac{P\cdot q}{q^2}q_\mu\bigg)\bigg(S_\nu - \frac{S\cdot q}{q^2}q_\nu\bigg) + \bigg(P_\nu - \frac{P\cdot q}{q^2}q_\nu\bigg)\bigg(S_\mu - \frac{S\cdot q}{q^2}q_\mu\bigg)\bigg]\bigg\} \tag{36}$$

and

[4]Note that terms like $iM_Xq^\mu\bar{u}_{i\sigma}P_-u_{X\sigma'}$ do not change sign due to the fact that γ^0 is imaginary in the notation that we are working with.

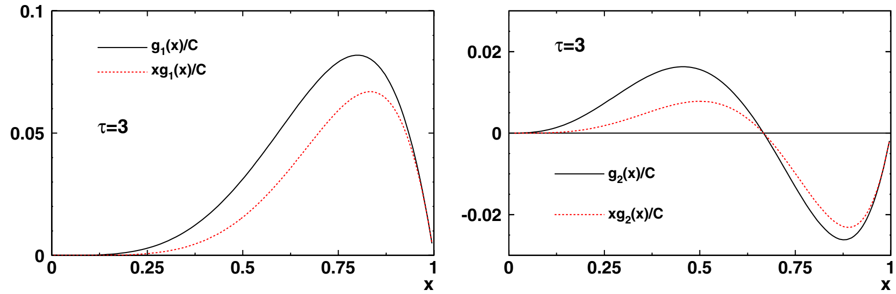

FIG. 2 (color online). Illustration of the g_1 and g_2 structure functions, where $C = \frac{1}{2}\pi A' Q^2(\Lambda^2/q^2)^{\tau-1}$ and $\tau = 3$.

$$W_{\mu\nu}^{(A)} = \pi A_0' Q^2(\Lambda^2/q^2)^{\tau-1}x^{\tau+1}(1-x)^{\tau-2}\bigg\{-\frac{\epsilon_{\mu\nu\alpha\beta}q^\alpha P^\beta}{2P\cdot q} - \frac{M\epsilon_{\mu\nu\alpha\beta}q^\alpha S^\beta}{2P\cdot q} - \frac{M\epsilon_{\mu\nu\alpha\beta}q^\alpha}{2P\cdot q}\bigg(S^\beta - \frac{q\cdot S}{P\cdot q}P^\beta\bigg) \times\bigg(\frac{1}{2x}\frac{\tau+1}{\tau-1} - \frac{\tau}{\tau-1}\bigg)\bigg\}, \tag{37}$$

where $A' = \pi|c_i'|^2|c_X'|^2 2^{2\tau}\Gamma^2(\tau)$. To obtain $W_{\mu\nu}^{(A)}$, we have used the identity,

$$\epsilon^{\mu\nu\alpha\beta}q_\alpha[(q\cdot S)P_\beta - (P\cdot q)S_\beta] = q^\mu\epsilon^{\nu\alpha\beta\gamma}P_\alpha q_\beta S_\gamma - q^\nu\epsilon^{\mu\alpha\beta\gamma}P_\alpha q_\beta S_\gamma - q^2\epsilon^{\mu\nu\alpha\beta}P_\alpha S_\beta. \tag{38}$$

Comparing with Eq. (5), we arrive at the final results:

$$2F_1 = F_2 = F_3 = 2g_1 = g_3 = g_4 = g_5 = \pi A' Q^2(\Lambda^2/q^2)^{\tau-1}x^{\tau+1}(1-x)^{\tau-2} \tag{39}$$

$$2g_2 = \bigg(\frac{1}{2x}\frac{\tau+1}{\tau-1} - \frac{\tau}{\tau-1}\bigg)\pi A' Q^2(\Lambda^2/q^2)^{\tau-1}x^{\tau+1}(1-x)^{\tau-2}. \tag{40}$$

The F_1 and F_2 are exactly the same as the results found in Ref. [5] by Polchinski and Strassler. The results for F_3 and all of the polarized structure functions are new. These structure functions are essentially calculated from the so-called double trace operators with their twist $\tau_p \geq 2$. In Fig. 2, we illustrate the x dependence of the g_1 and g_2 structure functions. The F_3, g_3, g_4, and g_5 structure functions are just twice of the g_1. The g_2 structure functions are especially interesting; it is negative at the large x region and positive at the relatively small x region which shares the same feature as seen in the proton g_2 experiment data.

IV. DISCUSSIONS

In this section, we focus on the interpretation of the structure functions that we obtained from the last section using gauge/string duality. We also compare the results with the structure functions obtained in QCD for nucleons. (For a review in QCD, see e.g., Refs. [21,22,25,26].)

(i) Since only the linear term in M is kept in the initial wave function and throughout the calculation, the results shown above are from the leading order calculation. The corrections are of order $\frac{M^2}{q^2}$ and $\frac{\Lambda^2}{q^2}$.

(ii) In QCD, there is an interesting inequality $F_1 \geq g_1$ which is derived from the positivity of the cross section [25]. Here we see that $F_1 = g_1$, and the bound is saturated. This indicates that the initial hadron is completely polarized. In terms of string theory language, this implies that the struck dilatino just tunnels or shrinks to smaller size of the order of the inverse momentum transfer during the scattering. As a result, the structure function exhibits a power law behavior in terms of the q^2 dependence which comes from the tunneling probability [4,5].

(iii) It is now straightforward to compute the moments of all the structure functions when the contributions from $x \ll \lambda^{-1/2}$ are negligible. Typically there are just two different kinds of moments, e.g.,

$$\int_0^1 2g_1(x,q^2)x^{n-1}\mathrm{d}x = \pi A' Q^2(\Lambda^2/q^2)^{\tau-1}\frac{\Gamma(\tau-1)\Gamma(\tau+n+1)}{\Gamma(2\tau+n)} \tag{41}$$

$$\int_0^1 2g_2(x,q^2)x^{n-1}\mathrm{d}x = \pi A' Q^2(\Lambda^2/q^2)^{\tau-1}\frac{\Gamma(\tau-1)\Gamma(\tau+n)}{\Gamma(2\tau+n)}\frac{1-n}{2}. \tag{42}$$

We expect that the moments are correct at least for $n > 2$ where the low-x contributions are negligible.

(iv) In addition, when one sets $n = 1$ for g_2, one finds an interesting sum rule

$$\int_0^1 dx g_2(x, q^2) = 0, \tag{43}$$

which is completely independent of τ and q^2. In QCD, this sum rule is known as the Burkhardt-Cottingham sum rule [27] in the large Q^2 limit. However, this sum rule can be invalidated by non-Regge divergence at low x.

(v) Now let us take a closer look at the $n = 1$ moment of the g_1 structure functions:

$$\int_0^1 2g_1(x, q^2)dx = \pi A' \mathcal{Q}^2(\Lambda^2/q^2)^{\tau-1} \times \frac{\Gamma(\tau-1)\Gamma(\tau+2)}{\Gamma(2\tau+1)}. \tag{44}$$

For sufficiently large $q^2 \to \infty$, this integral vanishes. This contradicts with the naive expectation that $\int_0^1 g_1(x, q^2)dx$ should remain finite as $q^2 \to \infty$ since the dilatino has spin-$\frac{1}{2}$.

Before we explain this problem, let us review the case of the F_1 and F_2 structure function [5]. According to energy momentum conservation, the second moment of F_1 and the first moment of F_2 should have a nonzero limit as $q^2 \to \infty$. This is known to be determined by the operator product expansion coefficients of $J^\mu J^\nu \sim T^{\mu\nu}$. However, it is not true for the result that we found above. This indicates that some contributions to F_1 and F_2 which peak around $x = 0$ are missing in the above calculation. The missing contributions are Pomeron exchanges. At large t'Hooft coupling, the Pomeron exchange is a graviton exchange which yields

$$xF_1 \sim F_2 \propto x^{-1+\mathcal{O}(1/\sqrt{\lambda})} \tag{45}$$

at small x, where the correction to the Pomeron intercept arises from the curvature of AdS_5. The Pomeron contribution will survive in the large q^2 limit and give us a nonvanishing second moment of F_1 [5,15].

Therefore, there should be a similar contribution to g_1 at small x. Usually, the physical scattering amplitudes, which can be written in terms of $F_1 + g_1$ and $F_1 - g_1$, have the same leading order $1/x$ singularity. In other words, g_1 should always be less singular than F_1. In terms of the Regge theory, there should be an axial vector Regge exchange contribution which yields a singular [28]

$$g_1 \sim \frac{1}{x^{\alpha_{R1}}}, \tag{46}$$

with $\alpha_{R1} = 1 - \mathcal{O}(\frac{1}{\sqrt{\lambda}})$ when x is extremely small. This contribution will also survive in the large q^2 limit and yield a finite first moment. This may indicate that most of the hardon spin is carried by the small-x constituents inside the hadron:

(i) Normally in QCD, the g_1 structure function contains two parts, namely, the singlet part and the nonsinglet part. The singlet part contains the polarized singlet quark and gluon spin contributions, while the nonsinglet part can be cast into the Bjorken sum rule. One can subtract off the singlet part and derive the Bjorken sum rule by calculating $\int_0^1 dx[g_1^p(x, q^2) - g_1^n(x, q^2)]$ at the large Q^2 limit. Here $g_1^p(x, q^2)$, $g_1^n(x, q^2)$ stand for the g_1 structure functions of proton and neutron, respectively. Since in our above AdS/CFT calculation we only use the $U(1)$ subgroup of the $SU(4)$ $\mathcal{R}$-flavor-symmetry group and calculate the contributions from double trace operators, we cannot distinguish the singlet part from the nonsinglet part. Both parts are not included in the calculation. However, if one includes the contribution from the axial currents and uses the full $SU(4)$ group, then one gets an additional flavor factor $\langle \mathcal{Q}|T^a T^b|\mathcal{Q}\rangle$, where T^a are the $SU(4)$ flavor matrices and the flavor indices a, b are set equal. It is straightforward to see that this flavor factor also contains both singlet and nonsinglet parts. According to our calculation at finite x, both of them are small at the large q^2 limit. The detailed discussions on the small-x limit of the g_1 structure function will be available in Ref. [28].

(ii) The parity-violating structure functions F_3, g_3, g_4 and g_5 are as large as the F_2 structure function due to the reason that the dilatino is right-handed fermion in massless limit. They are tightly related to the peculiar wave-function of the dilatino. However, we expect that g_1 and g_2 may exhibit some common features of the polarized structure functions of spin-$\frac{1}{2}$ hadrons in the nonperturbative region when the coupling is large.

V. ELASTIC FORM FACTORS

In this section, we focus on elastic scattering off a spin-$\frac{1}{2}$ fermion in gauge/string duality in the hard wall model framework. In the case of the elastic scattering, the final state is the same as the initial state which allows us to set $M_X^2 = M^2$ and $x = 1$. Thus, the only variable is q^2. In AdS/QCD model, the meson form factors have been extensively studied in Refs. [29–38]. Furthermore, the nucleon (spin-$\frac{1}{2}$ hadron) form factors are then computed in Refs. [39–41]. Here in this section, we would like to follow the formalism that we developed for the deep inelastic scattering, and use it in the elastic scattering, then calculate all possible form factors for spin-$\frac{1}{2}$ hadrons. Here in this section, we need to keep the full dilatino wave function since q^2/M^2 is no longer a large parameter.

To compute the form factors, one can first write down the most general definition for elastic form factors

$$\langle P_X, \mathcal{Q}, \sigma'|J^\mu(0)|P, \mathcal{Q}, \sigma\rangle = i\mathcal{Q}\bar{u}_{X\sigma'}\Gamma^\mu u_{i\sigma} \tag{47}$$

with

$$\Gamma^\mu = \gamma^\mu \mathcal{F}_1(q^2) + \frac{\sigma^{\mu\nu}q_\nu}{2M}\mathcal{F}_2(q^2) - iq^\mu \mathcal{F}_3(q^2) + \gamma^\mu\gamma^5\mathcal{F}_1^5(q^2) - i\frac{q^\mu}{M}\gamma^5\mathcal{F}_3^5(q^2), \tag{48}$$

where we have used the fact that 1, γ^μ, $\sigma^{\mu\nu}$, $\gamma^\mu\gamma^5$ and γ^5 form the complete sets of 4×4 γ matrices. Among all these form factors, $\mathcal{F}_1(q^2)$ and $\mathcal{F}_2(q^2)$ are the Dirac and Pauli form factors, respectively. They are related to the vector current exchange. $\mathcal{F}_1^5(q^2)$ and $\mathcal{F}_3^5(q^2)$ are the axial form factors related to axial vector current. $\mathcal{F}_3(q^2)$ usually vanishes if the current is conserved. It is easy to see that in our present framework, the $\sigma^{\mu\nu}$ component is missing, and thus the $\mathcal{F}_2$ is zero. In Ref. [41], where nonvanishing Pauli form factor $\mathcal{F}_2$ is obtained, a new $\sigma^{\mu\nu}$ term has to be introduced into the action.

Before we calculate the form factors from the current expectation value, let us take a look at how current conservation is satisfied. The current conservation condition can be written as

$$q_\mu\langle P_X, \mathcal{Q}, \sigma'|J^\mu(0)|P, \mathcal{Q}, \sigma\rangle \sim \int_0^{1/\Lambda} dz z^2[qK_1(qz)J_{\tau-2}(Mz)J_{\tau-2}(Mz)\bar{u}_{X\sigma'}\not{q}P_+u_{i\sigma} - iq^2K_0(qz)J_{\tau-2}(Mz)J_{\tau-1}(Mz)\bar{u}_{X\sigma'}P_+u_{i\sigma} + qK_1(qz)J_{\tau-1}(Mz)J_{\tau-1}(Mz)\bar{u}_{X\sigma'}\not{q}P_-u_{i\sigma} + iq^2K_0(qz)J_{\tau-1}(Mz)J_{\tau-2}(Mz)\bar{u}_{X\sigma'}P_-u_{i\sigma}]. \tag{49}$$

Using the Dirac equation, one can simplify the above expression and obtain

$$q_\mu\langle P_X, \mathcal{Q}, \sigma'|J^\mu(0)|P, \mathcal{Q}, \sigma\rangle \sim i\int_0^{1/\Lambda} dz z^2[qMK_1(qz)J_{\tau-2}(Mz)J_{\tau-2}(Mz)\bar{u}_{X\sigma'}P_+u_{i\sigma} - qMK_1(qz)J_{\tau-2}(Mz)J_{\tau-2}(Mz)\bar{u}_{X\sigma'}P_-u_{i\sigma} - q^2K_0(qz)J_{\tau-2}(Mz)J_{\tau-1}(Mz)\bar{u}_{X\sigma'}P_+u_{i\sigma} + qMK_1(qz)J_{\tau-1}(Mz)J_{\tau-1}(Mz)\bar{u}_{X\sigma'}P_-u_{i\sigma} - qMK_1(qz)J_{\tau-1}(Mz)J_{\tau-1}(Mz)\bar{u}_{X\sigma'}P_+u_{i\sigma} + q^2K_0(qz)J_{\tau-1}(Mz)J_{\tau-2}(Mz)\bar{u}_{X\sigma'}P_-u_{i\sigma}]. \tag{50}$$

Using the following identities,

$$\frac{d}{dx}[x^\nu K_\nu(x)] = -x^\nu K_{\nu-1}(x),\quad \frac{d}{dx}[x^\nu J_\nu(x)] = x^\nu J_{\nu-1}(x),\quad \frac{d}{dx}[x^{-\nu} J_\nu(x)] = -x^{-\nu} J_{\nu+1}(x), \tag{51}$$

one can easily show that

$$\int_0^{1/\Lambda} z^2K_0(qz)J_{\tau-2}(Mz)J_{\tau-1}(Mz)dz = -\frac{1}{q\Lambda^2}K_1(q/\Lambda)J_{\tau-2}(M/\Lambda)J_{\tau-1}(M/\Lambda) + \frac{1}{q}\int_0^{1/\Lambda} z^2K_1(qz) \times [MJ_{\tau-2}(Mz)J_{\tau-2}(Mz) - MJ_{\tau-1}(Mz)J_{\tau-1}(Mz)]dz \tag{52}$$

and eventually

$$q_\mu\langle P_X, \mathcal{Q}, \sigma'|J^\mu(0)|P, \mathcal{Q}, \sigma\rangle \sim K_1\left(\frac{q}{\Lambda}\right)J_{\tau-1}\left(\frac{M}{\Lambda}\right)J_{\tau-2}\left(\frac{M}{\Lambda}\right). \tag{53}$$

This indicates that the current is conserved when

$$M = \beta_{\tau-2,k}\Lambda \quad \text{or} \quad M = \beta_{\tau-1,k}\Lambda, \tag{54}$$

where $\beta_{\tau-2,k}$ and $\beta_{\tau-1,k}$ are kth zeros of $J_{\tau-2}(\frac{M}{\Lambda})$ and $J_{\tau-1}(\frac{M}{\Lambda})$, respectively. This is essentially equivalent to the mass spectrum found in Ref. [42] by requiring the vanishing chiral spinor wave function on the hard wall located at $r_0 = \Lambda R^2$.

Furthermore, we would like to comment that in the large q^2 limit, the current conservation is trivially satisfied when one sets the upper limit of the z integral as ∞, where we find

$$q_\mu\langle P_X, \mathcal{Q}, \sigma'|J^\mu(0)|P, \mathcal{Q}, \sigma\rangle \sim i(\bar{u}_{X\sigma'}P_+u_{i\sigma} - \bar{u}_{X\sigma'}P_-u_{i\sigma})\mathcal{I}, \tag{55}$$

where $\mathcal{I}$ is found to be

$$\mathcal{I} = \frac{2(\tau-1)}{M}\left(\frac{M^2}{q^2}\right)^{\tau-1}\left[{}_2F_1\left(\tau-\frac{3}{2}, \tau; 2\tau-3; \frac{-4M^2}{q^2}\right) - {}_2F_1\left(\tau-\frac{1}{2}, \tau; 2\tau-2; \frac{-4M^2}{q^2}\right)\right] - \frac{2\tau}{M}\left(\frac{M^2}{q^2}\right)^{\tau}{}_2F_1\left(\tau-\frac{1}{2}, \tau+1; 2\tau-1; \frac{-4M^2}{q^2}\right). \tag{56}$$

Using Taylor expansions of the hypergeometric functions, one can easily show that $\mathcal{I} = 0$.

A. Elastic form factors in the large q^2 limit

Assuming $q \gg \Lambda$, one can set the upper limit of the dz integral as ∞ and thus obtain

$$\mathcal{F}_1(q^2) = |c'|^2 \frac{\Lambda}{M}(\tau-1)\left(\frac{M^2}{q^2}\right)^{\tau-1} \times {}_2F_1\left(\tau-\frac{3}{2}, \tau; 2\tau-3; \frac{-4M^2}{q^2}\right) + |c'|^2 \frac{\Lambda}{M}\tau\left(\frac{M^2}{q^2}\right)^{\tau} \times {}_2F_1\left(\tau-\frac{1}{2}, \tau+1; 2\tau-1; \frac{-4M^2}{q^2}\right) \quad (57)$$

$$\mathcal{F}_2(q^2) = 0 \quad \text{and} \quad \mathcal{F}_3(q^2) = 0 \quad (58)$$

and

$$\mathcal{F}_1^5(q^2) = |c'|^2 \frac{\Lambda}{M}(\tau-1)\left(\frac{M^2}{q^2}\right)^{\tau-1} \times {}_2F_1\left(\tau-\frac{3}{2}, \tau; 2\tau-3; \frac{-4M^2}{q^2}\right) - |c'|^2 \frac{\Lambda}{M}\tau\left(\frac{M^2}{q^2}\right)^{\tau} \times {}_2F_1\left(\tau-\frac{1}{2}, \tau+1; 2\tau-1; \frac{-4M^2}{q^2}\right) \quad (59)$$

$$\mathcal{F}_3^5(q^2) = 2|c'|^2 \frac{\Lambda}{M}(\tau-1)\left(\frac{M^2}{q^2}\right)^{\tau} \times {}_2F_1\left(\tau-\frac{1}{2}, \tau; 2\tau-2; \frac{-4M^2}{q^2}\right). \quad (60)$$

At the large q^2 limit, we find that $\mathcal{F}_1(q^2) \simeq \mathcal{F}_1^5(q^2) \simeq |c'|^2 \frac{\Lambda}{M}(\tau-1)(\frac{M^2}{q^2})^{\tau-1}$ and $\mathcal{F}_3^5(q^2) \simeq 2|c'|^2 \frac{\Lambda}{M}(\tau-1)(\frac{M^2}{q^2})^{\tau}$.

B. Elastic form factors in the small q^2 limit

In the small q^2 limit, we expand the Bessel functions $K_{0,1}(q^2)$ up to $q^2 \log q^2$ but neglect q^2 terms. It is then straightforward to evaluate the dz integral which yields

$$\mathcal{F}_1(q^2) = |c'|^2 \frac{M}{2\Lambda}\left[J_{\tau-2}\left(\frac{M}{\Lambda}\right)J'_{\tau-1}\left(\frac{M}{\Lambda}\right) - J_{\tau-1}\left(\frac{M}{\Lambda}\right)J'_{\tau-2}\left(\frac{M}{\Lambda}\right)\right] + 2|c'|^2\left(\frac{M}{2\Lambda}\right)^{2\tau-1}\frac{q^2}{M^2}\ln\left(\frac{q}{\Lambda}\right) \times \frac{{}_2F_3(\tau-\frac{3}{2}, \tau; \tau-1, \tau+1, 2\tau-3; -M^2/\Lambda^2)}{2\tau\Gamma(\tau-1)^2} + \frac{1}{2}|c'|^2\left(\frac{M}{2\Lambda}\right)^{2\tau+1}\frac{q^2}{M^2}\ln\left(\frac{q}{\Lambda}\right) \times \frac{{}_2F_3(\tau-\frac{1}{2}, \tau+1; \tau, \tau+2, 2\tau-1; -M^2/\Lambda^2)}{2(\tau+1)\Gamma(\tau)^2} \quad (61)$$

$$\mathcal{F}_2(q^2) = 0 \quad \text{and} \quad \mathcal{F}_3(q^2) = 0 \quad (62)$$

and

$$\mathcal{F}_1^5(q^2) = \frac{1}{2}|c'|^2 J_{\tau-2}\left(\frac{M}{\Lambda}\right)J_{\tau-1}\left(\frac{M}{\Lambda}\right) + 2|c'|^2\left(\frac{M}{2\Lambda}\right)^{2\tau-1}\frac{q^2}{M^2}\ln(q/\Lambda)\frac{{}_2F_3(\tau-\frac{3}{2}, \tau; \tau-1, \tau+1, 2\tau-3; -M^2/\Lambda^2)}{2\tau\Gamma(\tau-1)^2} - \frac{1}{2}|c'|^2\left(\frac{M}{2\Lambda}\right)^{2\tau+1}\frac{q^2}{M^2}\ln(q/\Lambda)\frac{{}_2F_3(\tau-\frac{1}{2}, \tau+1; \tau, \tau+2, 2\tau-1; -M^2/\Lambda^2)}{2(\tau+1)\Gamma(\tau)^2} \quad (63)$$

together with

$$\mathcal{F}_3^5(q^2) = -|c'|^2\left(\frac{M}{\Lambda}\right)^{2\tau-1}2^{2-2\tau}\ln(q/\Lambda) \times \frac{{}_1F_2(\tau-\frac{1}{2}; \tau+1, 2\tau-2; -M^2/\Lambda^2)}{\Gamma(\tau-1)\Gamma(\tau+1)}. \quad (64)$$

In the end, one can use numerical methods and evaluate all these form factors with the chosen τ and ratio M/Λ, then plot them in terms of functions of q^2/M^2 (see Fig. 3). According to the power counting rule, we set $\tau = 3$ for now. It is easy to see that the above form factors give rise to logarithmic divergent charge radii for the charged hadron. This is peculiar in the hard wall model and will be cured in our follow-up phenomenological studies [43]. Besides, we

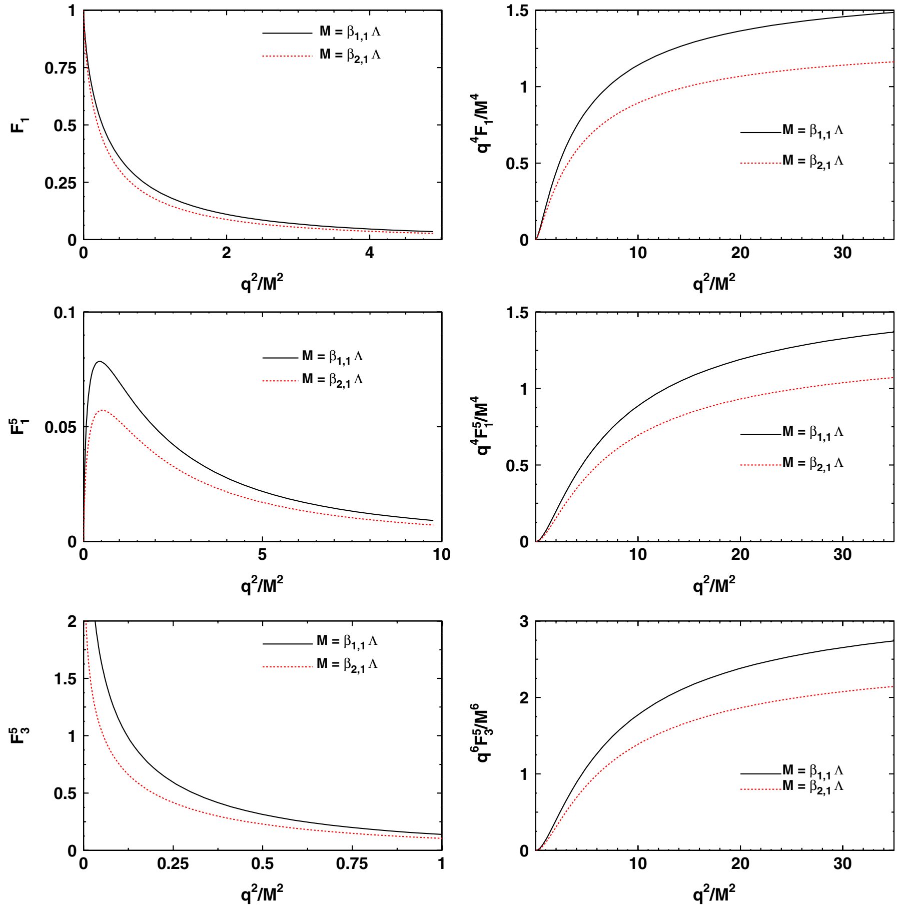

FIG. 3 (color online). Illustration of the $\mathcal{F}_1(q^2)$, $\mathcal{F}_1^5(q^2)$, and $\mathcal{F}_3^5(q^2)$, where we have normalized $\mathcal{F}_1(0) = 1$. We also set $\tau = 3$ and $M = \beta_{1,1}\Lambda$ or $M = \beta_{2,1}\Lambda$, where $\beta_{1,1}$ and $\beta_{2,1}$ are the first zero root of $J_1(\beta)$ and $J_2(\beta)$, respectively.

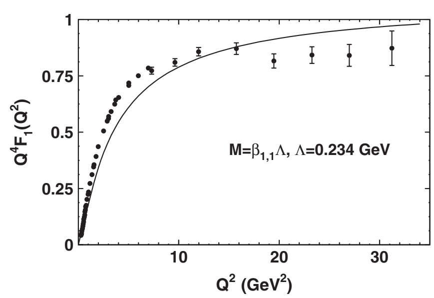

FIG. 4. Predictions for $Q^4 F_1^p(Q^2)$ in the hard wall model. The data points are taken from [44].

also compare our results of $Q^4 F_1^p(Q^2)$ with experimental data, which are shown in Fig. 4).

VI. CONCLUSION

Using gauge/string duality, we have calculated the structure functions as well as the form factors of a spin-$\frac{1}{2}$ hadron. Especially the polarized structure functions and parity-violating structure functions are new. We find that the Burkhardt-Cottingham sum rule is also true in our present calculation when the small-x contribution to g_2 is negligible. However, the situation for the g_1 structure function is more subtle and complicated. We conjecture that there should be an axial Regge contribution to g_1 at small x which may indicate that most of the hadron spin is carried by small-x partons. The phenomenological application of the above calculation is very appealing and will be available soon [43].

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