Interpretation of the "f_{D_s} puzzle" in the standard model and beyond

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The recent measurement on the decay constant of D_s shows a discrepancy between theory and experiment. We study the leptonic and semileptonic decays of D and D_s simultaneously within the standard model by employing a light-front quark model. There is space by tuning phenomenological parameters which can explain the "f_{D_s} puzzle" and do not contradict other experiments on the semileptonic decays. We also investigate the leptonic decays of D and D_s with a new physics scenario, unparticle physics. The unparticle effects induce a constructive interference with the standard model contribution. The nontrivial phase in unparticle physics could produce direct CP violation which may distinguish it from other new physics scenarios.

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I. INTRODUCTION

Thanks to the recent experimental improvements from B factories, BES, CLEO-c, and hadron colliders, charm physics enters into a "golden time" [1]. Many new charmed resonances are observed, such as D_{sJ} and X, Y, Z states. They open a new window to study nonperturbative QCD. The last neutral meson mixing, D^0-$\bar{D}^0$ mixing, is observed at about 1% level. This is a typical flavor-changing neutral current (FCNC) transition which is loop suppressed in the standard model (SM). It is expected that the FCNC process is sensitive to new physics beyond the SM. On the contrary, the leptonic decays of charm meson, e.g. $D_{(s)} \to l\nu$, are tree dominated. New physics in these processes, if it exists, should be very small. However, recent measurements on the leptonic decays of D_s show that their ratios are larger than expectation. In [2], the authors reviewed the experimental and theoretical status of the decay constants of D and D_s. For most approaches to calculate the decay constant of D_s, theory predictions are smaller than the experiment. In particular, there is a 3 standard deviation between the lattice QCD calculation which claims a precise prediction [3] and the experimental data. A recent updated QCD sum rules analysis provides an upper bound which does not reduce the tension between theory and experiment [4]. The above discrepancy is sometimes called the "f_{D_s} puzzle".

To fully understand the f_{D_s} puzzle, it requires an accurate knowledge of strong interaction which is difficult to obtain at present. In this study, we explore the problem from a phenomenological point of view. Although the treatment has the disadvantage that theoretical errors are not well under control, it permits an analytical treatment and its validity can be tested by many processes. Our method is a light-front approach [5,6]. This is a relativistic quark model and its essential element is hadron's light-front wave function. As a successful phenomenological model, it has been widely applied to calculate many different meson decay constants and form factors. Within this approach, the f_{D_s} puzzle problem really exists. In a previous result [6], the decay constant of D_s is 230 MeV, which is quite different from the experimental value of about 270 MeV. How to explain the puzzle of decay constant within the light-front approach is our first task.

Another possible mechanism to explain the decay constant puzzle is to introduce new physics effects beyond the SM. There have been many scenarios proposed, e.g. charged Higgs and/or leptoquark [7–10]. In order to explain the experiment, the new physics effects must interfere constructively with the dominant SM contribution. The charged Higgs model in [7] is excluded due to its destructive effect. Here, we suggest a new physics scenario, unparticle physics [11,12]. The unparticle is a scale-invariant stuff with no fixed mass. The scale dimension of the unparticle is in general fractional rather than an integral number. Many unusual phenomena caused by unparticles are explored [13,14]. The nontrivial phase in unparticle theory could induce constructive interference and even a sizable CP violation. Some unparticle physics effects in the leptonic decays of $B \to l\nu$ are studied in [15,16]. A detailed analysis of $D_{(s)} \to l\nu$ decays in unparticle physics will be provided in this study.

If there exits new physics in subprocess $c \to sl\nu$ transitions, it should also occur in semileptonic decays of charm mesons, such as $D \to Kl\nu$, $D_s \to \eta$, η', etc. In most literature, the semileptonic decays are either less considered or studied separately. Simultaneously studying the charm leptonic and semileptonic decays may help to discriminate different new physics models. At the same time, to test the validity of the light-front approach, study of the semileptonic decays is necessary.

The paper is organized as follows: In Sec. II, the leptonic and semileptonic decays of D and D_s mesons in SM are

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given. The decay constants and the form factors are calculated within the light-front approach. In Sec. III, we give an analysis of leptonic decays in unparticle physics, concentrating on the unparticle effects on branching ratios and direct CP violation. The last section is devoted to discussions and conclusions.

II. LEPTONIC AND SEMILEPTONIC DECAYS OF D AND D_s MESONS IN SM

A. $D^+_{(s)} \to l^+\nu$ decays in SM

In SM, the purely leptonic decay of $D_{(s)} \to l\nu$ occurs via annihilation of a quark pair to a charged lepton and a neutrino through exchange of a virtual W boson. The lowest order contribution is a tree diagram which is depicted in Fig. 1 for the decay of $D_s^+ \to l^+\nu_l$. The effective Hamiltonian for subprocess $c \to ql\nu$ transition at quark level is

$$\mathcal{H}_{\rm eff}^{\rm SM} = \frac{G_F}{\sqrt{2}} V_{cq}(\bar{q}c)_{V-A}(\bar{\nu}l)_{V-A}, \tag{1}$$

where q denotes d for D^+ and s for D_s^+ meson, respectively, and $V - A = \gamma_\mu(1-\gamma_5)$. The weak radiative corrections are so small that they can be safely neglected. The strong interactions between c quark and antiquark $\bar{q}$ by exchange of gluons are incorporated in the definition of the decay constant as

$$\langle 0|\bar{q}\gamma_\mu\gamma_5 c|D(p)\rangle = if_D p_\mu. \tag{2}$$

Here D represents D^+ or D_s^+ to simplify the representation.

The decay width of $D \to l\nu$ is obtained straightforwardly by

$$\Gamma^{\rm SM}(D \to l\nu) = \frac{G_F^2|V_{cq}|^2}{8\pi} f_D^2 M_D m_l^2 \left(1 - \frac{m_l^2}{M_D^2}\right)^2. \tag{3}$$

Because of helicity suppression for spin-0 particle decaying into two spin-1/2 fermions, the decay rate is proportional to the lepton mass squared m_l^2.

It seems that the leptonic decays of $D_{(s)} \to l\nu$ are theoretically simple and clean in SM. The weak interaction is well determined and the information of strong interaction is encoded in terms of decay constants of the meson. If the Cabibbo-Kobayashi-Maskawa (CKM) parameters are known, the decay constants of $f_{D_{(s)}}$ can be extracted from the measured decay ratios by using the above equation. From the recent experimental data ${\rm Br}(D^+ \to \mu^+\nu_\mu) = (4.4 \pm 0.7) \times 10^{-4}$ and ${\rm Br}(D_s^+ \to \mu^+\nu_\mu) = (6.2 \pm 0.6) \times 10^{-3}$ [17], we obtain

$$f_D^{\rm exp} = 221 \pm 17\ {\rm MeV}, \qquad f_{D_s}^{\rm exp} = 270 \pm 13\ {\rm MeV}. \tag{4}$$

where the CKM parameters $V_{cd} = -0.2257$ and $V_{cs} = 0.9737$ are used.

Recently, the HPQCD and UKQCD collaborations improved their techniques of lattice QCD (LQCD). They give a precise prediction for decay constants of D and D_s by [3]

$$f_D^{\rm LQCD} = 207 \pm 4\ {\rm MeV}, \qquad f_{D_s}^{\rm LQCD} = 241 \pm 3\ {\rm MeV}. \tag{5}$$

The most impressive thing about the above results is that the theory error is very small, only 2% or even smaller for f_{D_s}. Comparing Eqs. (4) and (5), the experiment is consistent with theory within the errors for f_D; while the experiment data differ from theory prediction by about 3σ deviations for f_{D_s}. Since the s quark in D_s is heavier than the d quark in D meson, the calculation for f_{D_s} is expected to be more reliable than f_D. The above discrepancy leads to the so-called f_{D_s} puzzle.

At first, we study whether the discrepancy between theory and experiment can be reduced within the framework of SM. Our approach is a covariant light-front quark model (LFQM) [5,6]. This is a relativistic quark model in which a consistent and fully relativistic treatment of quark spins and the center-of-mass motion can be carried out. This model has many advantages. For example, the light-front wave function is manifestly Lorentz invariant as it is expressed in terms of the internal momentum fraction variables which is independent of the total hadron momentum. Some applications of this approach can be found in [18].

In the LFQM, the decay constant of a pseudoscalar meson is represented by

$$f_P = \frac{\sqrt{2N_c}}{8\pi^3}\int dx d^2k_\perp \frac{A}{\sqrt{A^2 + k_\perp^2}}\phi_P(x, k_\perp), \tag{6}$$

where $N_c = 3$ is the color number of QCD, $A = m_1 x + m_2(1-x)$, and $m_{1,2}$ represent masses of the constitute quark and antiquark in the meson. The variable x denotes the light-front momentum fraction and $k_\perp$ denotes the intrinsic transverse momentum of the quark. $\phi_P(x, k_\perp)$ is the hadron light-front wave function. In phenomenology, a Gaussian-type wave function is widely chosen as

$$\phi(x, k_\perp) = N\sqrt{\frac{dk_z}{dx}}\exp\left(-\frac{k_\perp^2 + k_z^2}{2\beta^2}\right), \tag{7}$$

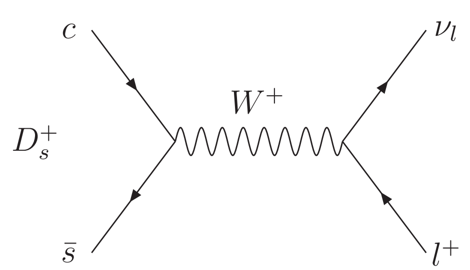
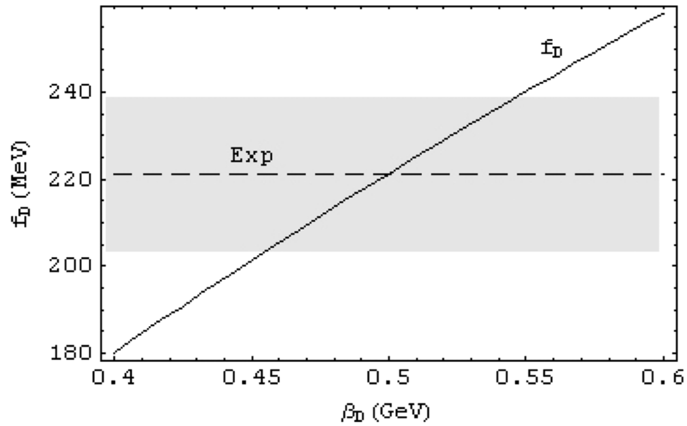


FIG. 1. The lowest order diagram for the decay of $D_s^+ \to l^+\nu_l$ in SM.



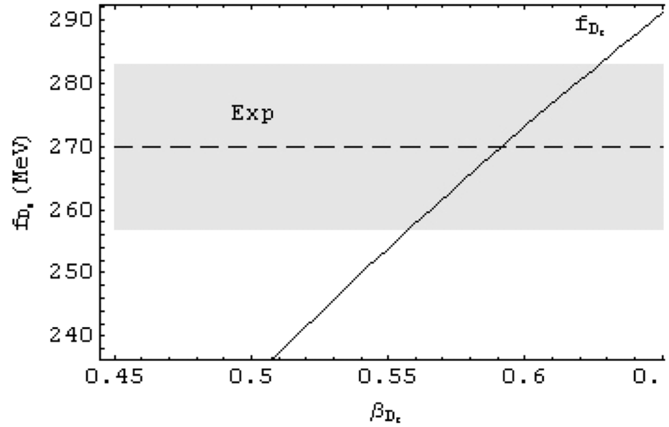


FIG. 2. The decay constants f_D and f_{D_s} vs the parameter β. The dashed line represents the central value and the gray region the error of the experimental data.

where the normalization constant $N = 4(\frac{\pi}{\beta^2})^{3/4}$ and $\frac{dk_z}{dx} = \frac{e_1 e_2}{x(1-x)M_0}$ with $e_i = \sqrt{m_i^2 + k_\perp^2 + k_z^2}$ and $M_0 = \sqrt{\frac{k_\perp^2+m_1^2}{1-x} + \frac{k_\perp^2+m_2^2}{x}}$. The parameter β in the wave function determines the confinement scale and is expected to be of order $\Lambda_{\rm QCD}$. The quark masses and β are the only required phenomenological parameters which makes LFQM predictive.

In this study, the constitute quark masses are chosen as in [6]: $m_d = 0.26$ GeV and $m_c = 1.40$ GeV. So the decay constant depends only on parameter β. Figure 2 plots the decay constants of f_D and f_{D_s} that vary with β. We can see that the decay constant increases as β increases, and the relation is nearly a linear function. The slopes $\partial f/\partial\beta$ for D and D_s are the same to a good accuracy. It is noted that we observe this linear relation for the first time although the reason for the relation is unknown. By tuning the parameter β, it is easy to make theory be consistent with experiment. The β is within a reasonable parameter region and its variation is small, less than 10%. For example, $\beta_{D_s} = 0.59 \pm 0.03$ GeV. In fact, as we will show for the semileptonic decays, the recent accurate data provide better predictions and determinations of parameters than before.

Here, we present two different results for decay constants and branching ratios for semileptonic decays in Table I and II. Model I refers to choosing parameters as in [6]. The decay constants are chosen as $f_D = 200$ MeV and $f_{D_s} = 230$ MeV. The corresponding parameters are fixed to $\beta_D = 0.448$ GeV, $\beta_{D_s} = 0.492$ GeV. Obviously, the decay constants in this model are smaller than experimental data. In model II, we make the decay constants fit the experiment. In order to fulfill this, the parameters change to $\beta_D = 0.499$ GeV, $\beta_{D_s} = 0.592$ GeV. The experimental data are taken from the Particle Data Group [17]. The ratio of $D \to \tau\nu$ is predicted to be 1.2×10^{-3} in model II, which is close to the present experimental upper limit. This process should be observed soon. For the decays to electrons, the ratios are predicted to be of order 10^{-7} or 10^{-8} which makes them difficult to observe.

TABLE I. The decay constants of D and D_s in the light-front quark model (in units of MeV).

	Model I	Model II	Exp.
f_D (MeV)	200	221	221 ± 17
f_{D_s} (MeV)	230	270	270 ± 13

B. Semileptonic decays of $D \to K^{(*)} l\nu$ and $D_s \to \phi(\eta, \eta')$ in SM

The semileptonic decays are more complicated in strong dynamics than leptonic processes because more hadrons are participating. They play an important role in testing consistency of the LFQM approach and new physics scenarios. After determining parameters β from the leptonic decays, we are able to give predictions for semileptonic decays. In general, the variations of β change the charm meson wave function and modify the hadron transitions. The chosen processes have the same subprocess $c \to sl\nu$ transition as the leptonic decays $D_s \to l\nu$ since we are interested in the f_{D_s} puzzle. Thus, the processes to be considered include $D \to K^{(*)} l\nu$ and $D_s \to \phi(\eta, \eta')$ decays. They are classified into two categories: $D \to Pl\nu$

TABLE II. The branching ratios for the leptonic decays of D and D_s in the light-front quark model.

Decay mode	Model I	Model II	Exp.
$D_s \to \mu\nu$	4.5×10^{-3}	6.2×10^{-3}	$(6.2 \pm 0.6) \times 10^{-3}$
$D_s \to \tau\nu$	4.4×10^{-2}	6.0×10^{-2}	$(6.6 \pm 0.6) \times 10^{-2}$
$D_s \to e\nu$	1.1×10^{-7}	1.5×10^{-7}	$<1.3 \times 10^{-4}$
$D \to \mu\nu$	3.6×10^{-4}	4.4×10^{-4}	$(4.4 \pm 0.7) \times 10^{-4}$
$D \to \tau\nu$	9.6×10^{-4}	1.2×10^{-3}	$<2.1 \times 10^{-3}$
$D \to e\nu$	8.5×10^{-9}	1.0×10^{-8}	$<2.4 \times 10^{-5}$

and $D \to V l \nu$ depending on whether the final meson is pseudoscalar or vector.

For semileptonic decay of D meson to a pseudoscalar, i.e. $D(P) \to P(P')l\nu$, the differential partial width is given by [19]

$$\frac{d\Gamma}{dq^2}(D \to P l\nu) = \frac{G_F^2 |V_{cs}|^2 p^3}{24\pi^3} |F_1^{DP}(q^2)|^2, \tag{8}$$

where $q = P - P'$ is the momentum transfer and q^2 is the invariant mass of the lepton-neutrino pair; p is the final meson momentum in the D rest frame with

$$p = |\vec{P}'| = \frac{\sqrt{(M_D^2 - (M - \sqrt{q^2})^2)(M_D^2 - (M + \sqrt{q^2})^2)}}{2M_D}, \tag{9}$$

where M denotes the final meson mass. Neglecting the light lepton mass, the differential partial width is governed by one form factor $F_1(q^2)$. The $D \to P$ transition form factors are defined by

$$\begin{aligned} &\langle P(P')|\bar{s}\gamma_\mu c|D(P)\rangle \\ &= F_1(q^2)\left[(P + P')_\mu - \frac{M_D^2 - M^2}{q^2} q_\mu\right] \\ &\quad + \frac{M_D^2 - M^2}{q^2} F_0(q^2) q_\mu. \end{aligned} \tag{10}$$

Then, the total width is

$$\Gamma(D \to P l\nu) = \int_0^{(M_D - M)^2} dq^2 \frac{d\Gamma}{dq^2}. \tag{11}$$

For $D(P) \to V(P')l^+\nu$ decays where V represents a vector meson, the differential partial width is given by

$$\frac{d\Gamma}{dq^2}(D \to V l\nu) = \frac{G_F^2 |V_{cs}|^2}{96\pi^3} \frac{pq^2}{M_D^2} \sum_{i=+,-,0} |H_i(q^2)|^2, \tag{12}$$

where p is the vector meson momentum in the D rest frame; the helicity amplitudes $H_i(q^2)$ are given by the combinations of form factors

$$\begin{aligned} H_\pm(q^2) &= (M_D + M_V)A_1(q^2) \mp \frac{M_D p}{M_D + M_V} V(q^2), \\ H_0(q^2) &= \frac{1}{2M_V\sqrt{q^2}}\left[(M_D^2 - M_V^2 - q^2)(M_D + M_V)A_1(q^2) \right. \\ &\quad \left. - 2\frac{p^2 M_D^2}{M_D + M_V} A_2(q^2)\right]. \end{aligned} \tag{13}$$

The $D \to V$ form factors are defined by

$$\begin{aligned} &\langle V(P', \epsilon)|\bar{s}\gamma_\mu c|D(P)\rangle \\ &= -\frac{1}{M_D + M_V} \epsilon_{\mu\nu\alpha\beta} \epsilon^{*\nu} (P' + P)^\alpha q^\beta V(q^2), \\ &\langle V(P', \epsilon)|\bar{s}\gamma_\mu \gamma_5 c|D(P)\rangle \\ &= i\left\{(M_D + M_V)\epsilon_\mu^* A_1(q^2) \right. \\ &\quad - \frac{\epsilon^* \cdot (P + P')}{M_D + M_V}(P + P')_\mu A_2(q^2) \\ &\quad \left. - 2M_V \frac{\epsilon^* \cdot (P + P')}{q^2} q_\mu [A_3(q^2) - A_0(q^2)]\right\}, \end{aligned} \tag{14}$$

where ϵ_μ is the polarization vector of the vector meson V and it satisfies $\epsilon \cdot P' = 0$.

For $P \to P$ and $P \to V$ transition form factors, the detailed formulas in the covariant light-front approach are given in [5,6]. We will not display their explicit forms here for simplicity. Besides some parameters mentioned in the previous subsection, other necessary parameters are: $m_s = 0.37$ GeV, $\beta_K = 0.3864$ GeV, $\beta_{K^*} = 0.2727$ GeV, and $\beta_\phi = 0.3070$ GeV. They are all taken from [6].

For η and η' mesons, the case is complicated by their mixing, i.e., the flavor eigenstates are not the physical states. Following [20], the η-η' mixing is given by

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} \eta_q \\ \eta_s \end{pmatrix}, \tag{15}$$

where ϕ is the mixing angle, $\eta_q = \frac{u\bar{u}+d\bar{d}}{\sqrt{2}}$, and $\eta_s = s\bar{s}$. From the Feynman diagram, we know that only $s\bar{s}$ contributes to the final meson $\eta(\eta')$ in $D_s \to \eta(\eta')$. We need to know β_η^s and $\beta_{\eta'}^s$ in order to calculate the relevant form factors. In the light-front quark model, the parameter β is extracted from the decay constant. The decay constants are taken to be $f_\eta^s = -113$ MeV and $f_{\eta'}^s = 141$ MeV [21]. The mixing angle is chosen as an averaged value: $\phi = 39.3°$ [20]. From these above parameters, we obtain $\beta_\eta^s = 0.4059$ GeV and $\beta_{\eta'}^s = 0.4256$ GeV.

In the covariant light-front approach, the formulas of form factors are derived in the frame $q^+ = 0$ with $q^2 = -q_\perp^2 \leq 0$, only values of the form factors in the spacelike momentum region can be obtained. The advantage of this choice is that the so-called Z-graph contribution arising from the nonvalence quarks vanishes. In order to obtain the physical form factors, an analytical extension from the spacelike region to the timelike region is required. The form factors in the spacelike region can be parametrized in a three-parameter form as

$$F(q^2) = \frac{F(0)}{1 - a(\frac{q^2}{M_D^2}) + b(\frac{q^2}{M_D^2})^2}, \tag{16}$$

where F represents the form factors F_1, A_1, A_2, and V; $F(0)$ represents form factor at $q^2 = 0$. The parameters $F(0)$ and

a, b are fixed by performing a three-parameter fit to the form factors in the spacelike region which can be calculated. We then use these parameters to determine the physical form factors in the timelike region. The parameters of a, b, and $F(0)$ are fitted from the form factors at momentum region $-15\ \mathrm{GeV}^2 \leq q^2 \leq 0$.

The fitted values of $F(0)$ and a, b for different form factors F_1, A_1, A_2, and V are given in Table III. Because A_0 and A_3 do not appear in Eqs. (12) and (13), we will not include them in Table III. Our results in model I are consistent with those in [6]. The form factors for F_1, A_1, A_2 in model I and model II are nearly equal. There is only a 5–10% difference for form factors $V(q^2)$ in models I and II.

Now, we give predictions for branching ratios of semileptonic decays of D and D_s. The results are displayed in Table IV. The experimental data about D_s decays are taken from a most recent measurement from the CLEO Collaboration [22]. About the numerical results, some comments are given below:

(1) The predictions in model II are in general smaller than those in model I. For most processes, the results in model II are closer to the experimental data. In other words, the semileptonic decays prefer larger decay constants of D and D_s, which is indicated by leptonic processes.

(2) For $D_s^+ \to \eta(\eta')e^+\nu_e$, $D_s^+ \to \phi e^+\nu_e$, and $D^0 \to K^{*-}e^+\nu_e$ decays, the theory is in good agreement with the experiment.

(3) It is difficult to understand the decay $D^+ \to \bar{K}^{*0}e^+\nu_e$ where the theory prediction is larger than the experiment. Under the isospin symmetry, $\frac{\mathrm{Br}(D^+\to\bar{K}^{*0}e^+\nu_e)}{\mathrm{Br}(D^0\to K^{*-}e^+\nu_e)} = \frac{\tau_{D^+}}{\tau_{D^0}} = 2.54$. But the experiment result is $\frac{\mathrm{Br}(D^+\to\bar{K}^{*0}e^+\nu_e)}{\mathrm{Br}(D^0\to K^{*-}e^+\nu_e)} = \frac{\tau_{D^+}}{\tau_{D^0}} = 1.54$. The discrepancy between the theory and experiment may be related to the old puzzle about lifetime difference between D^+ and D^0.

(4) Our results favor a large η-η' mixing angle $\phi = 39°$. This may be due to our neglecting the glue component in η'.

TABLE III. The form factors of D and D_s in the light-front quark model.

	Model I			Model II		
F	$F(0)$	a	b	$F(0)$	a	b
F_1^{DK}	0.79	1.18	0.27	0.78	1.15	0.24
$A_1^{DK^*}$	0.65	0.55	0.03	0.64	0.49	0.02
$A_2^{DK^*}$	0.57	1.07	0.33	0.57	1.05	0.27
V^{DK^*}	0.95	1.35	0.49	0.90	1.28	0.40
$F_1^{D_sK}$	0.72	1.27	0.37	0.70	1.18	0.28
$A_1^{D_sK^*}$	0.56	0.67	0.09	0.53	0.55	0.04
$A_2^{D_sK^*}$	0.49	1.14	0.50	0.50	1.08	0.35
$V^{D_sK^*}$	0.89	1.49	0.76	0.81	1.34	0.51
$F_1^{D_s\eta}$	0.50	1.17	0.34	0.48	1.11	0.25
$F_1^{D_s\eta'}$	0.62	1.14	0.31	0.60	1.08	0.23
$A_1^{D_s\phi}$	0.65	0.60	0.05	0.62	0.49	0.02
$A_2^{D_s\phi}$	0.57	1.04	0.37	0.58	0.99	0.26
$V^{D_s\phi}$	1.03	1.35	0.57	0.94	1.22	0.39

TABLE IV. The branching ratios of semileptonic decays of D and D_s in the light-front quark model (in units of %).

Decay mode	Model I	Model II	Exp.
$D^0 \to K^-e^+\nu_e$	3.90	3.81	3.58 ± 0.06 [17]
$D^0 \to K^{*-}e^+\nu_e$	2.57	2.38	2.38 ± 0.16 [17]
$D^+ \to \bar{K}^0e^+\nu_e$	9.96	9.74	8.6 ± 0.5 [17]
$D^+ \to \bar{K}^{*0}e^+\nu_e$	6.50	6.02	3.66 ± 0.21 [17]
$D_s^+ \to \eta e^+\nu_e$	2.42	2.25	$2.48 \pm 0.29 \pm 0.13$ [22]
$D_s^+ \to \eta' e^+\nu_e$	0.95	0.91	$0.91 \pm 0.33 \pm 0.05$ [22]
$D_s^+ \to \phi e^+\nu_e$	2.95	2.58	$2.29 \pm 0.37 \pm 0.11$ [22]

III. LEPTONIC AND SEMILEPTONIC DECAYS OF D AND D_s MESONS IN UNPARTICLE PHYSICS

A. Leptonic decays in unparticle physics

As discussed in the introduction, new physics effects must interfere constructively with the SM contribution and enhance the rate of leptonic decay. Unparticle physics can provide such interference due to the nontrivial phase effects. The scale dimension $d_{\mathcal{U}}$ of the unparticle is in general fractional rather than an integral number. The fractional dimension induces a phase factor $e^{-id_{\mathcal{U}}\pi}$ in the propagator of the unparticle field.

The scale invariant unparticle fields emerge below an energy scale $\Lambda_{\mathcal{U}}$ which is at the order of TeV. The unparticle has some peculiar characteristics that make it different from the ordinary particle. The interactions between the unparticle and the SM particles are described in the framework of low energy effective theory. For our purpose, the coupling of a scalar unparticle to two SM fermions (quarks or leptons) is given by an effective interaction as

$$\mathcal{L}_{\mathrm{eff}}^{\mathcal{U}} = \frac{C_{f'f}}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}}} \bar{f}'\gamma_\mu(1-\gamma_5)f\partial^\mu O_{\mathcal{U}} + \mathrm{H.c.}, \qquad (17)$$

where $O_{\mathcal{U}}$ denotes the scalar unparticle fields. The $C_{f'f}$ are dimensionless coefficients and they depend on different flavors in general. There are two reasons that we do not consider the vector unparticle. (1) The transverse vector unparticle does not contribute to the leptonic decay of a pseudoscalar meson [15]. (2) Even if there is a nontransverse vector contribution, another constraint will suppress it significantly. For the vector unparticle, it is pointed out that conformal symmetry puts a lower bound on its scale dimension $d_{\mathcal{U}} \geq 3$ [23]. If we take this constraint seriously, the vector unparticle effects in most processes will be very small and are negligible.

In this study, we are only interested in the effects of the virtual unparticle, thus it only appears as a propagator with momentum P and scale dimension $d_{\mathcal{U}}$. The propagator for the scalar unparticle field in the timelike momentum region with $P^2 \geq 0$ is given by [12,13]

$$\int d^4x e^{iP\cdot x}\langle 0|TO_{\mathcal{U}}(x)O_{\mathcal{U}}(0)|0\rangle = i\frac{A_{d_{\mathcal{U}}}}{2\sin(d_{\mathcal{U}}\pi)}\frac{1}{(P^2+i\epsilon)^{2-d_{\mathcal{U}}}}e^{-id_{\mathcal{U}}\pi}, \quad (18)$$

where

$$A_{d_{\mathcal{U}}} = \frac{16\pi^{5/2}}{(2\pi)^{2d_{\mathcal{U}}}}\frac{\Gamma(d_{\mathcal{U}}+1/2)}{\Gamma(d_{\mathcal{U}}-1)\Gamma(2d_{\mathcal{U}})}. \quad (19)$$

The function $\sin(d_{\mathcal{U}}\pi)$ in the denominator implies that the scale dimension $d_{\mathcal{U}}$ cannot be integral for $d_{\mathcal{U}} > 1$ in order to avoid singularity [for $d_{\mathcal{U}} = 1$ the singularity is canceled by $\Gamma(d_{\mathcal{U}} - 1)$ term in $A_{d_{\mathcal{U}}}$]. The phase factor $e^{-id_{\mathcal{U}}\pi}$ provides a CP conserving phase which produces peculiar interference effects in high energy scattering processes, CP violation in B and D decays, etc. There may be scale symmetry violation after the spontaneous breaking due to the coupling of the unparticle to the Higgs [24]. The estimate of the violation is difficult, so we neglect it in this study.

The Feynman diagram of unparticle contribution is obtained by replacing the W boson to unparticle field. Figure 3 depicts the lowest order diagram for the decay of $D_s^+ \to l^+\nu_l$ in unparticle physics. The amplitude for quark level transition of $c \to ql\nu$ is

$$A^{\mathcal{U}} = -i\frac{A_{d_{\mathcal{U}}}}{2\sin(d_{\mathcal{U}}\pi)}\frac{C_q}{\Lambda_{\mathcal{U}}^{2d_{\mathcal{U}}}}\frac{P^\mu P^\nu}{(P^2)^{2-d_{\mathcal{U}}}}e^{-id_{\mathcal{U}}\pi}\bar{q}\gamma_\mu(1-\gamma_5) \times c\bar{\nu}\gamma_\nu(1-\gamma_5)l, \quad (20)$$

where $P^2 = M_D^2$ in $D \to l\nu$ decays and $C_q \equiv C_{cq}C_{l\nu}$. In the SM, the W boson can be integrated out and the interaction of four fermions becomes a local interaction at low energy. Because an unparticle is different from a heavy particle with a fixed mass, the unparticle propagation is a nonlocal interaction. But P^2 is a constant, and we can still consider the $c \to ql\nu$ transition as an effective interaction.

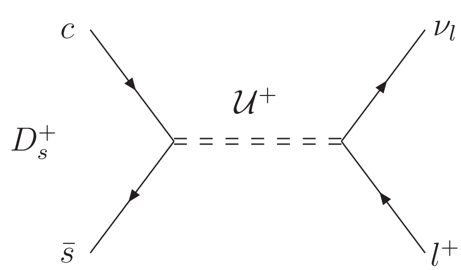


FIG. 3. The lowest order diagram for the decay of $D_s^+ \to l^+\nu_l$ in unparticle physics. The double dashed lines represent the unparticle.

Considering Eq. (6), the $P^\mu P^\nu$ term in Eq. (20) can be replaced by $P^2 g^{\mu\nu}$ (note that they are not equal) in the final result. The $c \to ql\nu$ transition is rewritten by

$$\mathcal{H}_{\rm eff}^{\mathcal{U}} = \frac{A_{d_{\mathcal{U}}}}{2\sin(d_{\mathcal{U}}\pi)}\frac{C_q}{M_D^2}\left(\frac{M_D^2}{\Lambda_{\mathcal{U}}^2}\right)^{d_{\mathcal{U}}}e^{-id_{\mathcal{U}}\pi}(\bar{q}c)_{V-A}(\bar{\nu}l)_{V-A} = re^{-i\phi_{\mathcal{U}}}\mathcal{H}_{\rm eff}^{\rm SM}, \quad (21)$$

where r and $\phi_{\mathcal{U}}$ are

$$r = \frac{A_{d_{\mathcal{U}}}}{2\sin(d_{\mathcal{U}}\pi)}\frac{C_q}{M_D^2}\left(\frac{M_D^2}{\Lambda_{\mathcal{U}}^2}\right)^{d_{\mathcal{U}}}\frac{\sqrt{2}}{G_F V_{cq}}, \qquad \phi_{\mathcal{U}} = d_{\mathcal{U}}\pi. \quad (22)$$

Equation (21) shows a clear physical meaning: the unparticle effects are equivalent to multiplying a constant factor with a CP-conserving phase to the SM contribution.

Combining the SM and unparticle contributions, we obtain the decay rate of $D \to l\nu$ decays as

$$\Gamma(D \to l\nu) = \Gamma^{\rm SM}(D \to l\nu)|1 + re^{-i\phi_{\mathcal{U}}}|^2. \quad (23)$$

Our result is principally consistent with the formulation for B decays in [15,19].

In [15,19], the authors point out a novel CP asymmetry in the leptonic decays caused by the CP-even phase of the unparticle. The phase $\phi_{\mathcal{U}}$ mimics the strong interaction phase caused by final state interactions. This phenomenon distinguishes the unparticle model from other new physics scenarios. If there is CP asymmetry in $D \to l\nu$ decays observed in experiment, it would be a clear signal of unparticle physics. Unlike the B^+ decay where the CKM parameter V_{ub} contains a CP violating weak phase, the V_{cd} and V_{cs} in D decays have no weak phase in SM. In order to produce CP asymmetry which requires both CP-even and CP-odd phase differences, we make an assumption that the coupling coefficient $C_{cq}C_{l\nu}$ is complex and contains a CP-odd phase ϕ_w even if its origin is unknown, and then $C_{cq}C_{l\nu} \to C_{cq}C_{l\nu}e^{-i\phi_w}$. After this assumption, the direct CP asymmetry in $D_s \to l\nu$ decay is

$$A_{CP}(D_s \to l\nu) \equiv \frac{\Gamma(D_s^- \to l^-\bar{\nu}) - \Gamma(D_s^+ \to l^+\nu)}{\Gamma(D_s^- \to l^-\bar{\nu}) + \Gamma(D_s^+ \to l^+\nu)} = \frac{2r\sin\phi_{\mathcal{U}}\sin\phi_w}{1 + r^2 + 2r\cos\phi_{\mathcal{U}}\cos\phi_w}. \quad (24)$$

The CP violation is caused by the interference between SM and unparticle contributions.

Now, we discuss the numerical results. We fix the scale parameter $\Lambda_{\mathcal{U}} = 1$ TeV. The coupling coefficients have been defined to be $C_s = C_{cs}C_{l\nu}$ and $C_d = C_{cd}C_{l\nu}$. The SM parameters are taken from model I: $f_D = 200$ MeV, $f_{D_s} = 230$ MeV. We do not use model II since new physics is not necessary in it. At first, we give the results for $D_s \to \mu\nu_\mu$ where a new physics effect is expected to be most important. Figure 4 plots the dependence of branching ratio on the scale dimension $d_{\mathcal{U}}$ at different values of

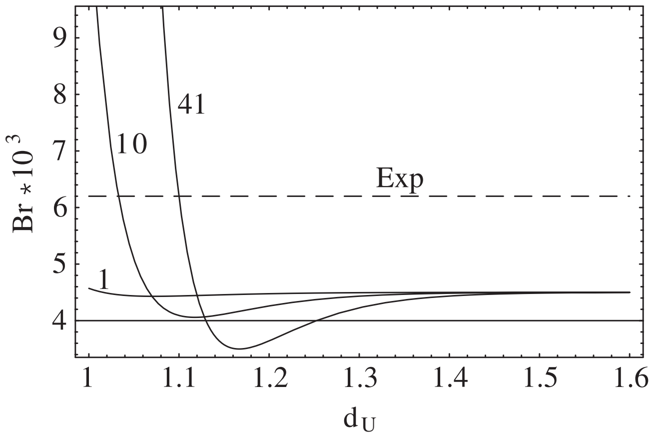


FIG. 4. The branching ratio of $D_s \to \mu\nu_\mu$ vs the scale dimension $d_{\mathcal{U}}$. The C_s are taken to be three values: 1, 10, 41.

$C_s = 1, 10, 41$. We give the results in the range $1 < d_{\mathcal{U}} < 1.6$. In order to explain the experiment, the C_s needs to be larger than 10. For the CP asymmetry, we consider a maximal case, i.e. the CP-odd phase $\phi_w = \pi/2$. Figure 5 plots the dependence of direct CP asymmetry on the scale dimension $d_{\mathcal{U}}$ also at values of $C_s = 1, 10, 41$. It is seen that the maximal A_{CP} can reach 35%. Considering the experimental constraint for branching ratio, A_{CP} reaches 10% for $C_s = 10$ and 30% for $C_s = 44$. Thus, the order 10% CP asymmetry is possible in unparticle physics. Our predictions for direct CP violation seems large. This is because we adopt a maximal case for the CP-odd phase. This phase is unknown and other choices will decrease the predictions. A recent measurement from CLEO Collaboration shows no CP violation, $A_{CP} = (4.8 \pm 6.1)\%$ [25]. But it does not exclude the unparticle scenario because the experimental errors are large and we consider the maximal CP violation in theory. In fact, even a 1% CP violation is a support for unparticle theory. Similar results for $D \to \mu\nu_\mu$ decay are given in Figs. 6 and 7.

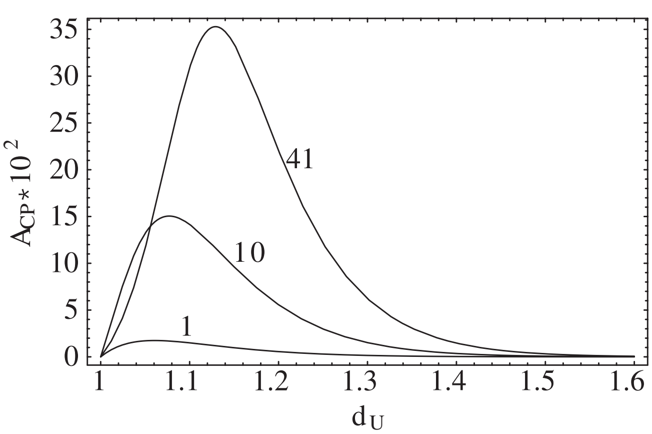


FIG. 5. The direct CP asymmetry of $D_s \to \mu\nu_\mu$ vs the scale dimension $d_{\mathcal{U}}$. The C_s are taken to be: 1, 10, 41.

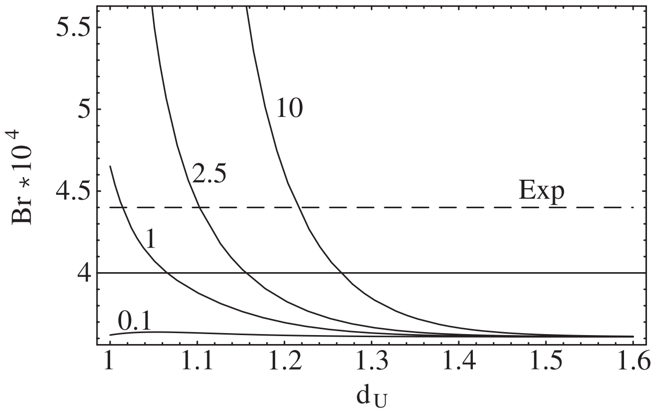


FIG. 6. The branching ratio of $D \to \mu\nu_\mu$ vs the scale dimension $d_{\mathcal{U}}$. The C_d are taken to be four values: 0.1, 1, 2.5, 10.

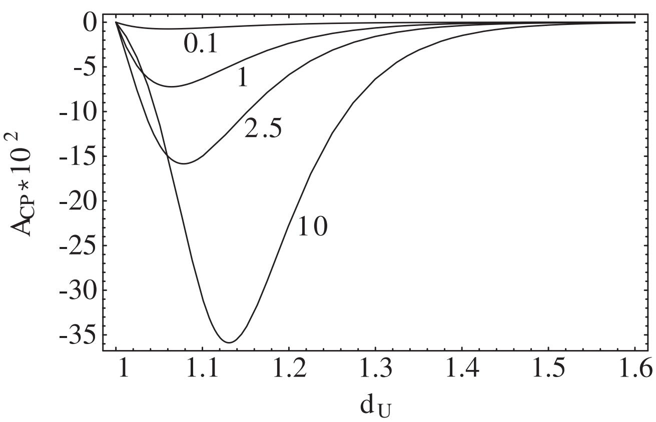


FIG. 7. The direct CP asymmetry of $D \to \mu\nu_\mu$ vs the scale dimension $d_{\mathcal{U}}$. The C_d are taken to be: 0.1, 1, 2.5, 10.

The experiments provide stringent constraints on the unparticle parameters. Without loss of generality, scale dimension is chosen to be $d_{\mathcal{U}} = 1.1$. Then the observed branching ratios of $D_s(D) \to l\nu$ can be used to constrain the coupling coefficients C_d and C_s. Table V lists the constraints on the C_s and C_d from $D_s(D) \to l\nu$ decays.

B. Semileptonic decays in unparticle physics

For the semileptonic decays, the quark level transition of $c \to ql\nu$ is nearly the same as that in the leptonic decay except the momentum of the unparticle is not equal to that of D meson. The unparticle momentum is equal to the

TABLE V. The constraints on the C_s and C_d from $D_s(D) \to l\nu$ decays with $d_{\mathcal{U}} = 1.1$.

	$D_s \to \mu\nu$	$D_s \to \tau\nu$	$D_s \to e\nu$
C_s	41	46	
	$D \to \mu\nu$	$D \to \tau\nu$	$D \to e\nu$
C_d	2.5		

lepton pair momentum q, and the amplitude of the subprocess is

$$A^{\mathcal{U}} = -i\frac{A_{d_{\mathcal{U}}}}{2\sin(d_{\mathcal{U}}\pi)}\frac{C_q}{\Lambda_{\mathcal{U}}^{2d_{\mathcal{U}}}}\frac{q^\mu q^\nu}{(q^2)^{2-d_{\mathcal{U}}}}e^{-id_{\mathcal{U}}\pi}\bar{q}\gamma_\mu(1-\gamma_5) \times c\bar{\nu}\gamma_\nu(1-\gamma_5)l. \quad (25)$$

By using the equation of motion, $\bar{\nu}\gamma_\nu(1-\gamma_5)lq^\nu = 0$ in the zero lepton mass limit. Thus, a conclusion is obtained: the scalar unparticle contribution is helicity suppressed and vanishes for semileptonically decaying to the light lepton (e, μ). In the leptonic decay case, because the leading SM contribution suffers the helicity suppression, the unparticle effects, although suppressed, play an important role. While, for the semileptonic decay, the leading SM contribution is not suppressed. So the unpaticle effects are negligible due to helicity suppression and the weak coupling with SM particles. The vector unparticle may contribute to the semileptonic decays, but it does not enhance the ratio of the leptonic decay and cannot solve the f_{D_s} puzzle.

IV. DISCUSSIONS AND CONCLUSIONS

We have studied the leptonic and semileptonic decays of D and D_s within SM utilizing a light-front quark model. We find that it is not difficult to solve the f_{D_s} puzzle by adjusting parameters reasonably. The predictions for semileptonic decays are consistent with experiment. Although the numerical results depend on the model and the theory uncertainties are not under control, the conclusion may be general and model independent. There is a sufficient space due to strong interaction uncertainties which can explain the discrepancy between theory and experiment. This conclusion is different from claims from lattice QCD and QCD sum rules.

We also study the leptonic decays in unparticle physics. The unparticle induces constructive interference effects which can enhance the theory to be consistent with the experiment. Production of CP asymmetry at percent level is possible. This would be a clear signal for unparticle physics. We hope the future experiment can test its validity.

The discrepancy between the theory and experiment becomes smaller if we use the most recent measurement from the CLEO Collaboration with $f_{D_s} = 259.5 \pm 6.6 \pm 3.1$ MeV [25]. The solution of the f_{D_s} puzzle requires more accurate measurements on the leptonic and semileptonic decays of charm mesons. The future BESIII will provide precise determination of D_s decay constant with errors to 1% [26].

In conclusion, there is space in SM to interpret the f_{D_s} puzzle without contradictions with the other experiments. The observation of CP violation at percent level may be an ideal test of the unparticle scenario.

ACKNOWLEDGMENTS

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