PHYSICAL REVIEW D **80,** 015016 (2009)

CP violation, single lepton polarization asymmetry, and polarized *CP* asymmetry in $B \to K^* \ell^+ \ell^-$ decay in the four-generation standard model

V. Bashiry,[1,*] N. Shirkhanghah,[2,†] and K. Zeynali[3,‡]

[1]*Engineering Faculty, Cyprus International University, Via Mersin 10, Turkey*
[2]*Islamic Azad University, Khalkhal Branch, Valiasr Street, Khalkhal, Iran*
[3]*Faculty of Physics, University of Tabriz, Tabriz 51664, Iran*

(Received 23 April 2009; published 28 July 2009)

In this paper we present a study of *CP* asymmetry, single lepton polarization asymmetry and polarized *CP* asymmetry in $B \to K^* \ell^+ \ell^-$ decay within the four-generation standard model. Taking $|V^*_{t's} V_{t'b}| = 0.01, 0.02, 0.03$ with phase $\{60°–120°\}$, which is consistent with the $b \to s\ell^+\ell^-$ rate and the Bs mixing parameter $\Delta m_{\rm Bs}$, we find that *CP* asymmetry, single lepton polarization asymmetry and polarized *CP* asymmetry are sensitive to the existence of the fourth generation. This can serve as an indirect method to search for new physics effects, in particular, to search for the fourth-generation quarks(t', b') via their indirect manifestations in loop diagrams.

DOI: 10.1103/PhysRevD.80.015016 PACS numbers: 12.60.−i, 13.30.−a, 14.20.Mr

I. INTRODUCTION

The simple replication of chiral matter is a straightforward extension of the standard model. This fourth generation is often considered and ruled out or disfavored by many researchers. This extension has recently gained more attention (for the most recent studies see [1]) after measurement of direct *CP*-asymmetry of $B \to K\pi$ decays [2] and the nonvanished *CP* phase measured in $b \to s$ transition by CDF [3] and D0 [4]. These issues, in particular, the nonvanished *CP* phase measured in $b \to s$ transition cannot be explained by 3×3 the Cabibbo-Kobayashi-Maskawa (CKM) matrix. A consequential extension of the three-generation standard model (SM3) to the four-generation standard model (SM4) has better solutions to some of the theoretical and experimental problems. Such extension can include extra weak phases in the quark mixing matrix. This might introduce a better solution to the problem related to baryogensis [5]. SM4 can also explain the direct *CP*-asymmetry of $B \to K\pi$ decays [6]. The reasons for discarding the fourth family are based on the interpretation of electroweak precision data and LEP II experiments. Kribs *et al.* indicated that the electroweak precision tests are not in conflict with the existence of the fourth family if the quark mass of the fourth generation satisfy the following relation [7]:

$$m_{t'} - m_{b'} \approx \left(1 + \frac{1}{5} \ln\frac{m_H}{115}\right) \times 55. \quad (1)$$

One of the consequences of their study is extension of the bounds on the Higgs mass [7]. Furthermore, a 4×4 CKM matrix includes more weak phases than the 3×3 CKM which leads to sizable increases in the calculation of *CP* asymmetry [8,9]. The 4×4 CKM matrix clearly affects flavor physics [10–14]. Considering many aspects of the fourth family, some of which are discussed above, the search for the fourth family must be included in the LHC.

In this study, we enlarge the standard model with a complete sequential fourth family of the chiral quark. The fourth quark can contribute in the flavor-changing neutral processes (which are loop induced in the SM) like the other three generations. Thus, the mass and mixing parameters of 4×4 CKM can change the rate of the physical observables, with respect to the three-generation standard model (SM3) calculations. $b \to s\ell^+\ell^-$ is a loop-induced transition in the SM. Like u, c, t quarks, the fourth family t' quark can contribute in this transition. $B \to K^*\ell^+\ell^-$ decay in the quark level is described by $b \to s\ell^+\ell^-$ transition. $B \to K^*\ell^+\ell^-$ decay has been widely studied within SM and beyond [15–23]. The various observables of this decay are going to be measured at the LHCb. Here, we study the *CP* asymmetry, single lepton polarization asymmetry and polarized *CP* asymmetry in $B \to K^*\ell^+\ell^-$ decay in the SM4. Note that single and double lepton polarization asymmetries have been studied in the SM3 and in the model independent approach by Refs. [24,25].

The outline of the paper is as follows: In Sec. II, we calculate the decay amplitude and single lepton polarization *CP* asymmetry of the $B \to K^*\ell^+\ell^-$ decay within SM4. In Sec. III we present the calculation of unpolarized and polarized *CP* asymmetry. Section IV is devoted to the numerical analysis and discussion of the considered transition as well as our conclusions.

II. STRATEGY

In this section, we present the theoretical expressions for the decay widths within SM4. As we mention above, t' can contribute to the $b \to s$ transition as u, c and t quarks do.

*bashiry@ciu.edu.tr
†nasershirkhanghah@yahoo.com
‡k.zeinali@arums.ac.ir

As a result of this contribution the Wilson coefficient of the SM3 is modified. It is easy to see that if we enlarge the standard model with a complete sequential fourth family of the chiral quark the new operators do not appear. In other words, the full operator set for the SM4 is exactly the same as in the SM3.

The Wilson coefficients are modified as follows:

$$\lambda_t C_i \to \lambda_t C_i^{\rm SM} + \lambda_{t'} C_i^{\rm new}, \tag{2}$$

where $\lambda_f = V_{fb}^* V_{fs}$. The unitarity of the 4×4 CKM matrix leads to

$$\lambda_u + \lambda_c + \lambda_t + \lambda_{t'} = 0. \tag{3}$$

One can neglect $\lambda_u = V_{ub}^* V_{us}$ in Eq. (2) which is very small in strength compared to the others ($|\lambda_u| \sim 10^{-3}$). Then, $\lambda_t \approx -\lambda_c - \lambda_{t'}$.

Now, we can rewrite Eq. (1) as

$$\lambda_t C_i^{\rm SM} + \lambda_{t'} C_i^{\rm new} = -\lambda_c C_i^{\rm SM} + \lambda_{t'}(C_i^{\rm new} - C_i^{\rm SM}). \tag{4}$$

It is clear that when $m_{t'} \to m_t$ or $\lambda_{t'} \to 0$, $\lambda_{t'}(C_i^{\rm new} - C_i^{\rm SM})$ vanishes. This is also required by the GIM mechanism.

The most important operators for $B \to X_s \ell^+ \ell^-$ are

$$\begin{aligned} O_7 &= \frac{e}{16\pi^2} \bar{m}_b(\mu)(\bar{s}_L \sigma_{\mu\nu} b_R) F^{\mu\nu}, \\ O_9 &= \frac{\alpha_{\rm em}}{4\pi}(\bar{s}_L \gamma_\mu b_L)(\bar{\ell}\gamma^\mu \ell), \\ O_{10} &= \frac{\alpha_{\rm em}}{4\pi}(\bar{s}_L \gamma_\mu b_L)(\bar{\ell}\gamma^\mu \gamma_5 \ell). \end{aligned} \tag{5}$$

The large q^2 region is usually considered less favorable, because it has a smaller rate and suffers from large nonperturbative corrections. However, the experimental efficiency is better there [26]. For small values of q^2, the operator O_7 becomes dominant due to the $1/q^2$ pole in the photon propagator. At high q^2 region the O_7 contribution is rather small [27]. The rate in the high q^2 region has a smaller renormalization scale dependence and m_c dependence [28]. Despite the experimental advantages, the large q^2 region is less favored, because it has a large hadronic uncertainty [29]. The $1/m_b^3$ corrections are not much smaller than the $1/m_b^2$ ones [30] when the operator product expansion becomes an expansion in $\Lambda_{\rm QCD}/(m_b - \sqrt{q^2})$ [31] instead of $\Lambda_{\rm QCD}/m_b$.

The QCD corrected effective Hamiltonian for the $b \to s\ell^+\ell^-$ transitions leads to the following free quark decay amplitude:

$$\begin{aligned} M = \frac{G_F V_{tb} V_{ts}^*}{\sqrt{2}\pi} \alpha_{\rm em} \Big[& C_9^{\rm tot}(\bar{s}\gamma_\mu P_L b)\bar{\ell}\gamma_\mu \ell \\ & + C_{10}^{\rm tot}(\bar{s}\gamma_\mu P_L b)\bar{\ell}\gamma_\mu \gamma^5 \ell \\ & - 2C_7^{\rm tot} \bar{s} i\sigma_{\mu\nu}\frac{q^\nu}{q^2}(m_b P_R + m_s P_L) b \bar{\ell}\gamma_\mu \ell \Big], \end{aligned} \tag{6}$$

where $q^2 = (p_1 + p_2)^2$ and p_1 and p_2 are the final leptons four-momenta and the chiral projection operators P_L and P_R are defined as

$$P_L = \frac{1-\gamma_5}{2}, \qquad P_R = \frac{1+\gamma_5}{2},$$

and $C_i^{\rm tot}$'s are as follows:

$$C_i^{\rm tot}(\mu) = C_i^{\rm eff}(\mu) + \frac{\lambda_{t'}}{\lambda_t} C_i^{\rm new}(\mu), \tag{7}$$

where the last terms in these expressions describe the contributions of the t' quark to the Wilson coefficients. $\lambda_{t'}$ can be parametrized as

$$\lambda_{t'} = V_{t'b}^* V_{t's} = r_{sb} e^{i\phi_{sb}}. \tag{8}$$

Neglecting the terms of $O(m_q^2/m_W^2)$, $q = u, d, c$, the analytic expressions for all Wilson coefficients in the SM in the leading order (LO) and in the next-to-leading logarithmic approximation (NLO) can be found in [32–42].

The explicit forms of the $C_i^{\rm new}$ can be obtained from the corresponding expression of the Wilson coefficients in the SM by substituting $m_t \to m_{t'}$.

We neglect long-distance resonance effects, which have their origin in real intermediate $c\bar{c}$ family, in the amplitudes (or in the OPE) for simplicity.

The matrix element for the exclusive decay can be obtain by sandwiching Eq. (6) between initial hadron state $B(p_B)$ and final hadron state K^* in terms of form factors as:

$$\begin{aligned} &\langle K^*(p_{K^*}, \varepsilon)|\bar{s}\gamma_\mu(1 \pm \gamma^5)b|B(p_B)\rangle \\ &\quad = -\epsilon_{\mu\nu\rho\sigma}\varepsilon^{*\nu} p_{K^*}^\rho q^\sigma \frac{2V(q^2)}{m_B + m_{K^*}} \\ &\qquad \pm i\varepsilon_\mu (m_B + m_{K^*}) A_1(q^2) \\ &\qquad \mp i(p_B + p_{K^*})_\mu (\varepsilon^* q)\frac{A_2(q^2)}{m_B + m_{K^*}} \\ &\qquad \mp i q_\mu \frac{2m_{K^*}}{q^2}(\varepsilon^* q)[A_3(q^2) - A_0(q^2)]. \end{aligned} \tag{9}$$

$$\begin{aligned} &\langle K^*(p_{K^*}, \varepsilon)|\bar{s} i\sigma_{\mu\nu} q^\nu (1 \pm \gamma^5)b|B(p_B)\rangle \\ &\quad = 4\epsilon_{\mu\nu\rho\sigma}\varepsilon^{*\nu} p^\rho q^\sigma T_1(q^2) \pm 2i[\varepsilon_\mu^*(m_B^2 - m_{K^*}^2) \\ &\qquad - (p_B + p_{K^*})_\mu (\varepsilon^* q)]T_2(q^2) \\ &\qquad \pm 2i(\varepsilon^* q)\Big[q_\mu - (p_B + p_{K^*})_\mu \frac{q^2}{m_B^2 - m_{K^*}^2}\Big] T_3(q^2). \end{aligned} \tag{10}$$

Using the equation of motion, the form factor $A_3(q^2)$ can be written in terms of the form factors $A_1(q^2)$, $A_2(q^2)$ as follows:

$$A_3 = \frac{m_B + m_{K^*}}{2m_{K^*}} A_1 - \frac{m_B - m_{K^*}}{2m_{K^*}} A_2. \tag{11}$$

In order to guarantee the finiteness of Eq. (9) at the $q^2 = 0$, we demand that $A_3(q^2 = 0) = A_0(q^2 = 0)$.

Using the above definitions the transition operator is calculated as

$$\mathcal{M} = \frac{G_F\alpha}{4\sqrt{2}\pi} V_{tb}V_{ts}^*\{\bar{\ell}\gamma^\mu(1-\gamma^5)\ell[-2B_0\epsilon_{\mu\nu\lambda\sigma}\varepsilon^{*\nu}p_{K^*}^\lambda q^\sigma - iB_1\varepsilon_\mu^* + iB_2(\varepsilon^* q)(p_B + P_{K^*})_\mu + iB_3(\varepsilon^* q)q\varepsilon_\mu] + \bar{\ell}\gamma^\mu(1+\gamma^5)\ell[-2C_1\epsilon_{\mu\nu\lambda\sigma}\varepsilon^{*\nu}p_{K^*}^\lambda q^\sigma - iD_1\varepsilon_\mu^* + iD_2(\varepsilon^* q)(p_B + P_{K^*})_\mu + iD_3(\varepsilon^* q)q\varepsilon_\mu]\} \tag{12}$$

$$B_0 = C_{LL}^{\rm tot}\frac{V}{m_B + m_{K^*}} - 2(C_{BR}^{\rm tot} + C_{SL}^{\rm tot})\frac{T_1}{q^2}$$

$$B_1 = C_{LL}^{\rm tot}(m_B + m_{K^*})A_1 - 2(C_{BR}^{\rm tot} - C_{SL}^{\rm tot})(m_B^2 - m_{K^*}^2)\frac{T_2}{q^2}$$

$$B_2 = C_{LL}^{\rm tot}\frac{A_2}{m_B + m_{K^*}} - 2(C_{BR}^{\rm tot} - C_{SL}^{\rm tot})\frac{1}{q^2}\times\left[T_2 + \frac{q^2}{m_B^2 - m_{K^*}^2}T_3\right]$$

$$B_3 = 2m_{K^*}C_{LL}^{\rm tot}\frac{A_3 - A_0}{q^2} + 2(C_{BR}^{\rm tot} - C_{SL}^{\rm tot})\frac{T_3}{q^2}$$

$$C_1 = B_0(C_{LL}^{\rm tot} \to C_{LR}^{\rm tot}) \qquad D_1 = B_1(C_{LL}^{\rm tot} \to C_{LR}^{\rm tot})$$

$$D_2 = B_2(C_{LL}^{\rm tot} \to C_{LR}^{\rm tot}) \qquad D_3 = B_3(C_{LL}^{\rm tot} \to C_{LR}^{\rm tot}) \tag{13}$$

where $C_{SL}^{\rm tot} = -2m_sC_7^{\rm tot}$, $C_{BR}^{\rm tot} = -2m_bC_7^{\rm tot}$, $C_{LL}^{\rm tot} = C_9^{\rm tot} - C_{10}^{\rm tot}$ and $C_{LL}^{\rm tot} = C_9^{\rm tot} + C_{10}^{\rm tot}$.

From matrix element Eq. (12) it is easy to derive the invariant dilepton mass spectrum for the $B \to K^*\ell^+\ell^-$ decay corresponding to the transversally, normally and longitudinally polarized lepton ℓ^-:

$$\frac{d\Gamma(\vec{n})}{dq^2} = \frac{1}{2}\left(\frac{d\Gamma}{dq^2}\right)_0[1 + (P_L\vec{e}_L + P_N\vec{e}_N + P_T\vec{e}_T).\vec{n}] \tag{14}$$

where single lepton polarizations are defined as

$$P_i(q^2) = \frac{\frac{d\Gamma(\vec{n}=\vec{e}_i)}{dq^2} - \frac{d\Gamma(\vec{n}=-\vec{e}_i)}{dq^2}}{\frac{d\Gamma(\vec{n}=\vec{e}_i)}{dq^2} + \frac{d\Gamma(\vec{n}=-\vec{e}_i)}{dq^2}}$$

the expression for unpolarized decay spectrum is calculated as

$$\left(\frac{d\Gamma}{dq^2}\right)_0 = \frac{G_F^2\alpha^2}{2^{14}\pi^5 m_B}|V_{tb}V_{ts}^*|^2 v\sqrt{\lambda(1,r,s)}\Delta, \tag{15}$$

with $\lambda(r,s) = 1 + s^2 + r^2 - 2r - 2s - 2rs$ and

$$\Delta = \frac{32}{3}m_B^4\lambda[(m_B^2 s - m_\ell^2)(|B_0|^2 + |C_1|^2) + 6m_\ell^2\,{\rm Re}(B_0C_1^*)] + 96m_\ell^2\,{\rm Re}(B_1D_1^*) + \frac{8}{r}m_B^2m_\ell^2\lambda({\rm Re}[B_1(-B_3^* + D_2^* + D_3^*)] + {\rm Re}[D_1(-D_3^* + B_2^* + B_3^*)]) + \frac{8}{r}m_B^4m_\ell^2(1-r)\lambda\,{\rm Re}[(B_2 - D_2)(B_3^* - D_3^*)] - \frac{8}{r}m_B^4m_\ell^2(2 + 2r - s)\lambda\,{\rm Re}(B_2D_2^*) + \frac{4}{r}m_B^4m_\ell^2 s\lambda|B_3 - D_3|^2 - \frac{8}{3rs}m_B^2\lambda[m_\ell^2(2 - 2r + s) + m_B^2 s(1 - r - s)][{\rm Re}(B_1B_2^*) + {\rm Re}(D_1D_2^*)] + \frac{4}{3rs}[2m_\ell^2(\lambda - 6rs) + m_B^2 s(\lambda + 12rs)](|B_1|^2 + |D_1|^2) + \frac{4}{3rs}m_B^4\lambda(m_B^2 s\lambda + m_\ell^2[2\lambda + 3s(2 + 2r - s)](|B_2|^2) + |D_2|^2). \tag{16}$$

Using the definition of lepton polarization we obtain the expression for P_L, P_N and P_T:

$$P_L = \frac{4}{\Delta}m_B^2 v\left\{\frac{1}{3r}\lambda^2 m_B^4[|B_2|^2 - |D_2|^2] + \frac{8}{3}\lambda m_B^4 s[|B_0|^2 - |C_1|^2] - \frac{2}{3r}m_B^2\lambda(1 - r - s)[{\rm Re}(B_1B_2^*) - {\rm Re}(D_1D_2^*)] + \frac{1}{3r}(\lambda + 12rs)[|B_1|^2 - |D_1|^2]\right\} \tag{17}$$

$$P_T = \frac{\pi}{\Delta}m_B\sqrt{s\lambda}\left\{-8m_B^2m_\ell\,{\rm Re}[(B_0 + C_1)(B_1^* + D_1^*)] + \frac{1}{r}m_B^2m_\ell(1 + 3r - s)[{\rm Re}(B_1D_2^*) - {\rm Re}(B_2D_1^*)] + \frac{1}{rs}m_\ell(1 - r - s)\times[|B_1|^2 - |D_1|^2] - \frac{1}{r}m_B^2m_\ell(1 - r - s)\,{\rm Re}[(B_1 + D_1)(B_3^* - D_3^*)] + \frac{1}{rs}m_B^4m_\ell(1 - r)\lambda[|B_2|^2 - |D_2|^2] + \frac{1}{r}m_B^4m_\ell\lambda\,{\rm Re}[(B_2 + D_2)(B_3^* - D_3^*)] - \frac{1}{rs}m_B^2m_\ell[\lambda + (1 - r - s)(1 - r)][{\rm Re}(B_1B_2^*) - {\rm Re}(D_1D_2^*)]\right\} \tag{18}$$

$$P_N = \frac{\pi}{\Delta} \upsilon m_B^3 \sqrt{s\lambda}\{8m_\ell \,\mathrm{Im}[(B_1^* C_1) + (B_0^* D_1)] + \frac{1}{r} m_B^2 m_\ell \lambda \,\mathrm{Im}[(B_2 - D_2)(B_3^* - D_3^*)] - \frac{1}{r} m_\ell (1 + 3r - s)\,\mathrm{Im}[(B_1 - D_1)(B_2^* - D_2^*)] - \frac{1}{r} m_\ell (1 - r - s)\,\mathrm{Im}[(B_1 - D_1)(B_3^* - D_3^*)]. \quad (19)$$

III. UNPOLARIZED AND POLARIZED *CP* ASYMMETRY

The normalized *CP* asymmetry defined in terms of the difference of decay rate of particle and antiparticle channels are as follows [43–45]:

$$A_{CP}(\vec{n} = \pm \vec{e}_i) = \frac{\frac{d\Gamma(\hat{s},\vec{n})}{d\hat{s}} - \frac{d\bar{\Gamma}(\hat{s},\vec{\bar{n}})}{d\hat{s}}}{(\frac{d\Gamma(\hat{s})}{d\hat{s}})_0 + (\frac{d\bar{\Gamma}(\hat{s})}{d\hat{s}})_0}, \quad (20)$$

where

$$\frac{d\Gamma(\hat{s},\vec{n})}{d\hat{s}} = \frac{d\Gamma(B \to K^* \ell^+ \ell^-(\vec{n}))}{d\hat{s}}, \qquad \frac{d\bar{\Gamma}(\hat{s},\vec{\bar{n}})}{d\hat{s}} = \frac{d\Gamma(\bar{B} \to \bar{K}^* \ell^+(\vec{\bar{n}}) \ell^-)}{d\hat{s}}, \quad (21)$$

here, $\vec{n}$ and $\vec{\bar{n}}$ are the spin directions for ℓ^- and ℓ^+ for B-decay and $\bar{B}$-decay, respectively, and $i = L, N, T$. Taking into account the fact that $\vec{\bar{e}}_{L,N} = -\vec{e}_{L,N}$, and $\vec{\bar{e}}_T = \vec{e}_T$, we obtain

$$A_{CP}(\vec{n} = \pm \vec{e}_i) = \frac{1}{2}\left\{\frac{(\frac{d\Gamma}{d\hat{s}})_0 - (\frac{d\bar{\Gamma}}{d\hat{s}})_0}{(\frac{d\Gamma}{d\hat{s}})_0 + (\frac{d\bar{\Gamma}}{d\hat{s}})_0} \pm \frac{(\frac{d\Gamma}{d\hat{s}})_0 P_i - ((\frac{d\Gamma}{d\hat{s}})_0 P_i)_{|(\lambda_{t'}/\lambda_t)\to(\lambda_{t'}/\lambda_t)^*}}{(\frac{d\Gamma}{d\hat{s}})_0 + (\frac{d\bar{\Gamma}}{d\hat{s}})_0}\right\}. \quad (22)$$

Using Eq. (16), we get from Eq. (22),

$$A_{CP}(\vec{n} = \pm \vec{e}_i) \approx \frac{1}{2}\left\{\frac{\Delta - \bar{\Delta}}{\Delta + \bar{\Delta}} \pm \frac{\Delta_i - \bar{\Delta}_i}{\Delta + \bar{\Delta}}\right\} = \frac{1}{2}\{A_{CP}(\hat{s}) \pm A^i_{CP}(\hat{s})\}, \quad (23)$$

where the upper sign in the definition of A^i_{CP} corresponds to L and N polarizations, while the lower sign corresponds to T polarization.

The $A^i_{CP}(\hat{s})$ terms in Eq. (23) describe an additional contribution to the unpolarized *CP* asymmetry. We calculate the $A_{CP}(\hat{s})$ and $A^i_{CP}(\hat{s})$. The results are given as

$$A_{CP}(s) = \frac{-4\,\mathrm{Im}(\frac{\lambda_{t'}}{\lambda_t})\Sigma}{2\Delta + 4\,\mathrm{Im}(\frac{\lambda_{t'}}{\lambda_t})\Sigma}, \qquad A^i_{CP}(s) = \frac{-4\,\mathrm{Im}(\frac{\lambda_{t'}}{\lambda_t})\Sigma^i}{2\Delta + 4\,\mathrm{Im}(\frac{\lambda_{t'}}{\lambda_t})\Sigma}, \quad i = L, T, N \quad (24)$$

where the explicit expressions for $\Sigma(\hat{s})$ and $\Sigma^i(\hat{s})$, where $i = L, N, T$, are as follows:

$$\Sigma(s) = V_1\{\mathrm{Im}(C_7^{\mathrm{SM}^*} C_9^{\mathrm{new}}) + \mathrm{Im}(C_7^{\mathrm{new}} \xi_1^*)\} + V_2\,\mathrm{Im}(C_7^{\mathrm{SM}^*} C_7^{\mathrm{new}}) + V_3\,\mathrm{Im}(C_9^{\mathrm{new}} \xi_1^*)$$

$$\Sigma^L(s) = -\frac{m_B^2 s \upsilon}{2m_\ell^2 + m_B^2 s}(V_3\{\mathrm{Im}(C_{10}^{\mathrm{SM}^*} C_9^{\mathrm{new}}) + \mathrm{Im}(C_{10}^{\mathrm{new}} \xi_1^*)\} + V_1 \mathrm{Im}(C_{10}^{\mathrm{SM}^*} C_7^{\mathrm{new}}))$$

$$\Sigma^T(s) = \pi m_B \sqrt{s\lambda}(V_4\{\mathrm{Im}(C_7^{\mathrm{SM}^*} C_9^{\mathrm{new}}) + \mathrm{Im}(C_7^{\mathrm{new}} \xi_1^*)\} + V_5\{\mathrm{Im}(C_{10}^{\mathrm{SM}^*} C_9^{\mathrm{new}}) + \mathrm{Im}(C_{10}^{\mathrm{new}} \xi_1^*)\} + V_6\,\mathrm{Im}(C_{10}^{\mathrm{SM}^*} C_7^{\mathrm{new}}) - V_7\,\mathrm{Im}(C_7^{\mathrm{SM}^*} C_7^{\mathrm{new}}) + V_8\,\mathrm{Im}(C_9^{\mathrm{new}} \xi_1^*))$$

$$\Sigma^N(s) = \frac{\pi}{2} \upsilon m_B \sqrt{s\lambda}(V_8\{\mathrm{Re}(C_{10}^{SM^*} C_9^{\mathrm{new}}) + \mathrm{Re}(C_{10}^{\mathrm{new}} \xi_1^*)\} + V_4\{\mathrm{Re}(C_{10}^{\mathrm{SM}^*} C_7^{\mathrm{new}}) + \mathrm{Re}(C_7^{\mathrm{SM}} C_{10}^{\mathrm{new}*})\}) \quad (25)$$

with

$$V_1 = \frac{32(2m_\ell^2 + m_B^2 s)}{3m_B^2(m_B - m_{K^*})(m_B + m_{K^*})^2 rs^2}(A_2\lambda m_B^2(m_b - m_s)\{m_{K^*}^4(-1+r+s)T_2 - m_B^2 m_{K^*}^2(\lambda + 2(-1+r+s))T_2 + m_B^4((-1+\lambda+r+s)T_2 + \lambda s T_3)\} + (m_B + m_{K^*})\{A_1(m_B+m_{K^*})(m_b - m_s)[12(m_B^2 - m_{K^*}^2)^2 rsT_2 + \lambda(m_{K^*}^4 T_2 - m_B^2 m_{K^*}^2(1+r+s)T_2 + m_B^4(s(T_2 + (-1+s)T_3) + r(T_2 + sT_3)))] + 8\lambda m_B^4(m_B - m_{K^*})(m_b + m_s) rsT_1 V\})$$

$$V_2 = \frac{128(2m_\ell^2 + m_B^2 s)}{3m_B^2(m_B^2 - m_{K^*}^2)^2 rs^3}(12(m_B^2 - m_{K^*}^2)^4(m_b - m_s)^2 rsT_2^2 + \lambda^2 m_B^4(m_b - m_s)^2(m_{K^*}^2 T_2 - m_B^2(T_2 + sT_3))^2 + \lambda(m_B^2 - m_{K^*}^2)^2\{2m_b m_s[-m_{K^*}^4 T_2 + 2m_B^2 m_{K^*}^2(r+s)T_2^2 + m_B^4(T_2(T_2 - 2sT_2 - 2(-1+s)sT_3) + r(8sT_1^2 - 2T_2^2 - 2sT_2T_3))] + (m_b^2 + m_s^2)[m_{K^*}^4 T_2^2 - 2m_B^2 m_{K^*}^2(r+s)T_2^2 + m_B^4(T_2((-1+2s)T_2 + 2(-1+s)sT_3) + 2r(4sT_1^2 + T_2^2 + sT_2T_3))]\})$$

$$V_3 = \frac{8(2m_\ell^2 + m_B^2 s)}{3(m_B + m_{K^*})^2 rs}(2A_1A_2\lambda m_B^2(m_B + m_{K^*})^2(-1+r+s) + A_1^2(m_B + m_{K^*})^4(\lambda + 12rs) + \lambda m_B^4(A_2^2\lambda + 8rsV^2))$$

$$V_4 = \frac{-64m_\ell}{s}(A_1(m_B + m_{K^*})(m_b + m_s)T_1 + (m_B - m_{K^*})(m_b - m_s)T_2 V)$$

$$V_5 = \frac{2m_\ell}{(m_B + m_{K^*})^2 rs}(\{2(A_0 - A_3)m_{K^*}(m_B + m_{K^*}) + A_1(m_B + m_{K^*})^2 + A_2 m_B^2(-1+r)\} \times \{A_2\lambda m_B^2 + A_1(m_B + m_{K^*})^2(-1+r+s)\})$$

$$V_6 = \frac{4m_\ell^2(m_b - m_s)}{m_B^2(m_B - m_{K^*})(m_B + m_{K^*})^2 rs^2}(4(A_0 - A_3)m_{K^*}(m_B + m_{K^*})\{m_{K^*}^4(-1+r+s)T_2 - m_B^2 m_{K^*}^2(\lambda + 2(-1+r+s))T_2 + m_B^4((-1+\lambda+r+s)T_2 + \lambda sT_3)\} + A_2 m_B^2\{m_{K^*}^4(1+\lambda-2r+r^2+4rs-s^2)T_2 - 2m_B^2 m_{K^*}^2(1+r^2-s^2+r(-2+\lambda+4s))T_2 + m_B^4((1-\lambda-2r+2\lambda r+r^2+4rs-s^2)T_2 + 2\lambda(-1+r)sT_3)\} + A_1(m_B + m_{K^*})^2\{2m_{K^*}^4(-1+r+s)T_2 - m_B^2 m_{K^*}^2[-3+\lambda+r^2-2r(-1+s)+2s+s^2]T_2 + m_B^4[(-1+\lambda+r^2-2rs+s^2)T_2 + s(\lambda+r^2+(-1+s)^2-2r(1+s))T_3]\})$$

$$V_7 = \frac{512m_\ell}{m_B^2 s^2}(m_B^2 - m_{K^*}^2)(m_b^2 - m_s^2)T_1T_2 V_8 = -32m_\ell m_B^2 A_1 V.$$

IV. NUMERICAL ANALYSIS

We have tried to analyze the dependency of the various asymmetries on the fourth-generation quark mass ($m_{t'}$) and the product of quark mixing matrix elements ($V_{t'b}^* V_{t's} = r_{sb}e^{i\phi_{sb}}$). For numerical evaluation, we use the various particle masses and lifetimes of the B meson from [46]. The quark masses (in GeV) we used are $m_b = 4.8$, $m_c = 1.35$, $m_t = 175$ GeV, $m_W = 80.41$ GeV the CKM matrix elements as $|V_{cb}V_{cs}^*| = 0.041$, $\alpha = 1/128$ and the weak mixing angle $\sin^2\theta_W = 0.23$.

The input parameters we used for Wilson coefficients in the SM are shown in Table I.

In order to perform quantitative analysis of the physical observables, numerical values for the new parameters ($m_{t'}$, r_{sb}, ϕ_{sb}) are necessary. Using the experimental values of $B \to X_s\gamma$ and $B \to X_s\ell^+\ell^-$, the bounds on $r_{sb} \sim \{0.01 - 0.03\}$, $\phi_{sb} \sim \{\pi/3 - 2\pi/3\}$ and $m_{t'} \sim \{200, 600\}$ (GeV) have been obtained [8,47]. Also considering Δm_{B_s}, ϕ_{sb} receives a strong restriction ($\phi_{sb} \sim \pi/2$) [48].

For the values of the form factors, we have used the results of [49], where the radiative corrections to the leading twist contribution and $SU(3)$ breaking effects are also taken into account. The q^2 dependence of the form factors can be represented in terms of three parameters as

$$F(q^2) = \frac{F(0)}{1 - a_F\frac{q^2}{m_B^2} + b_F(\frac{q^2}{m_B^2})^2},$$

TABLE I. The Wilson coefficients $c_i(\mu)$ at the scale $\mu = m_{b,\mathrm{pole}}$ in the SM. Here $c_7^{\mathrm{eff}} \equiv c_7 - \frac{1}{3}c_5 - c_6$.

$\bar{C}_1$	$\bar{C}_2$	$\bar{C}_3$	$\bar{C}_4$	$\bar{C}_5$	$\bar{C}_6$	C_7^{eff}	C_9	C_{10}
+1.107	−0.248	−0.011	−0.026	−0.007	−0.031	−0.313	4.344	−4.669

TABLE II. B meson decay form factors in a three-parameter fit, where the radiative corrections to the leading twist contribution and SU(3) breaking effects are taken into account.

	$F(0)$	a_F	b_F
$A_1^{B\to K^*}$	0.34 ± 0.05	0.60	-0.023
$A_2^{B\to K^*}$	0.28 ± 0.04	1.18	0.281
$V^{B\to K^*}$	0.46 ± 0.07	1.55	0.575
$T_1^{B\to K^*}$	0.19 ± 0.03	1.59	0.615
$T_2^{B\to K^*}$	0.19 ± 0.03	0.49	-0.241
$T_3^{B\to K^*}$	0.13 ± 0.02	1.20	0.098

where the values of parameters $F(0)$, a_F and b_F for the $B \to K^*$ decay are listed in Table II.

From explicit expressions of the lepton polarizations and other observables one can easily see that they depend on both q^2 and the new parameters $(m_{t'}, r_{sb})$. We should eliminate the dependence of the lepton polarization on one of the variables. We eliminate the variable q^2 by performing integration over q^2 in the allowed kinematical region. The averaged lepton polarizations and CP asymmetries are defined as

$$\langle \mathcal{P}(\mathcal{A}) \rangle = \frac{\int_{4\hat{m}_\ell^2}^{(1-\sqrt{\hat{r}_K})^2} \mathcal{P}(\mathcal{A}) \frac{d\mathcal{B}}{d\hat{s}} d\hat{s}}{\int_{4\hat{m}_\ell^2}^{(1-\sqrt{\hat{r}_K})^2} \frac{d\mathcal{B}}{d\hat{s}} d\hat{s}}.$$

Figures 1–5 depict the dependency of the single lepton polarization asymmetries in terms of mixing angle ϕ_{sb} for fixed value of the fourth-generation quark mass ($m_{t'} =$ 400 GeV) and three different values of the r_{sb}. Figures 6–8 show the dependency of unpolarized and polarized CP asymmetry on the ϕ_{sb} for three different values of $r_{sb} = 0.01, 0.02, 0.03$ when the fourth-generation quark mass ($m_{t'} = 400$ GeV). It is worth mentioning that though we show figures for fixed value of up-type quark ($m_{t'} =$ 400 GeV), a full analysis of the t' mass dependence would

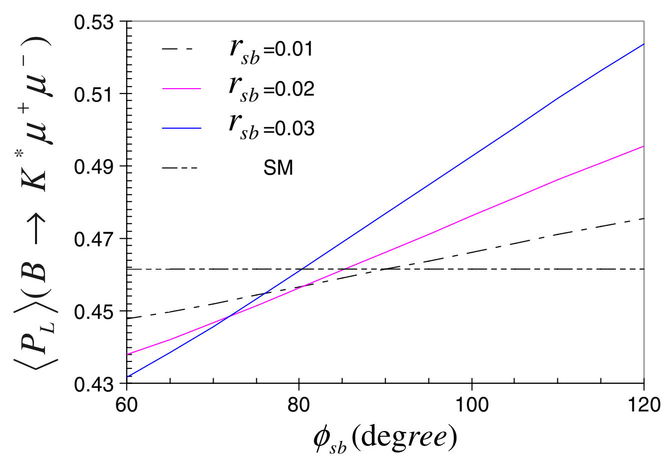

FIG. 1 (color online). The dependence of the $\langle P_L \rangle$ for the $B \to K^* \mu^+ \mu^-$ decay on mixing angle ϕ for three different values of $r_{sb} = 0.01, 0.02, 0.03$ when the fourth-generation quark mass $m_{t'} = 400$ GeV.

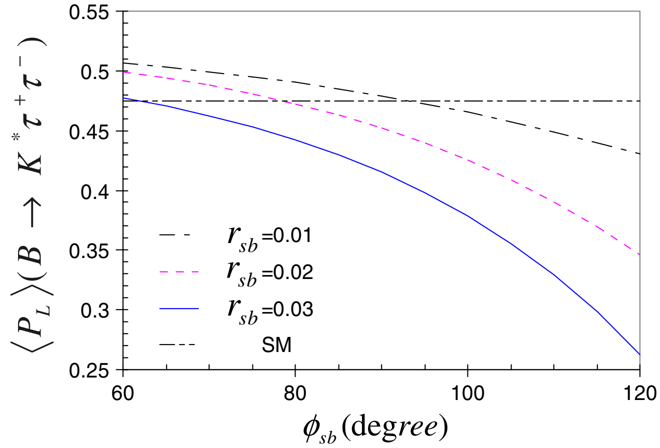

FIG. 2 (color online). The same as in Fig. 1, but for the tau lepton channel.

require a phenomenological analysis that goes beyond the purpose of the present paper. Note also that $\langle P_N \rangle$, $\langle A_{CP} \rangle$, $\langle A_{CP}^L \rangle$, $\langle A_{CP}^T \rangle$ and $\langle A_{CP}^N \rangle$, for the μ channel do not deviate from the SM3 values significantly. Hence, we do not present their predictions in the figures.

(i) $\langle P_L \rangle$ depicts strong dependency on mixing angle ϕ_{sb}. The magnitude of $\langle P_L \rangle$ is suppressed when ϕ_{sb} takes small values (less than about 80°) for the μ channel (see Fig. 1). For high values of ϕ_{sb}, $\langle P_L \rangle$ is enhanced for the muon channel. While $\langle P_L \rangle$ for the tau channel is enhanced at low values of ϕ_{sb}, it is suppressed at high values (see Fig. 2).

(ii) $\langle P_T \rangle$ is increased by the parameters of fourth generations for muon lepton (see Fig. 3). For the tau channel the dependency is both increasing an decreasing depending on the values of ϕ_{sb} (see Fig. 4). The discrepancy with respect to the SM3 values almost vanishes when $\phi_{sb} \approx 90°$ for the tau channel.

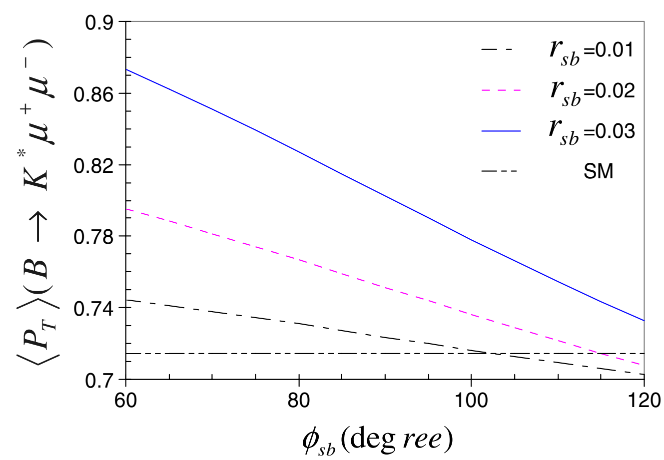

FIG. 3 (color online). The dependence of the $\langle P_T \rangle$ for the $B \to K^* \mu^+ \mu^-$ decay on mixing angle ϕ for three different values of $r_{sb} = 0.01, 0.02, 0.03$ when the fourth-generation quark mass $m_{t'} = 400$ GeV.

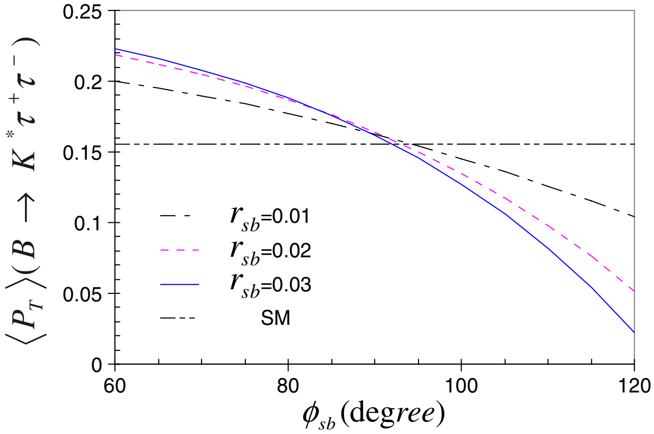

FIG. 4 (color online). The same as in Fig. 3, but for the tau lepton channel.

(iii) $\langle P_N \rangle$ changes its sign when compare to the corresponding SM3 values for the tau channel. Also the magnitude of $\langle P_N \rangle$ shows sizable discrepancy with respect to the SM3 value (see Fig. 5).

(iv) While $\langle A_{CP} \rangle$ and $\langle A_{CP}^L \rangle$ is enhanced by the different values of ϕ_{sb} and r_{sb} for the tau channel, $\langle A_{CP}^N \rangle$ is suppressed for the same values of ϕ_{sb} and r_{sb}.

Finally, a quantitative estimation about the ability to measure the various physical observables is in order. An observation of a 3σ signal for an asymmetry of the order of the 1% for the branching ratio in order of $\sim 10^{-6}$ requires about $\sim 10^{10}$ B.

The number of $b\bar{b}$ pairs that are produced at B–factories and LHC are about $\sim 5 \times 10^8$ and 10^{12} respectively. Although these statistics indicate that the lepton polarization and unpolarized and polarized *CP* asymmetries in the $B \to K^* \ell^+ \ell^-$ decay may be detectable at LHC, the various technical problems and the efficiency of detecting lepton polarization, particularly tau leptons, makes the proposed

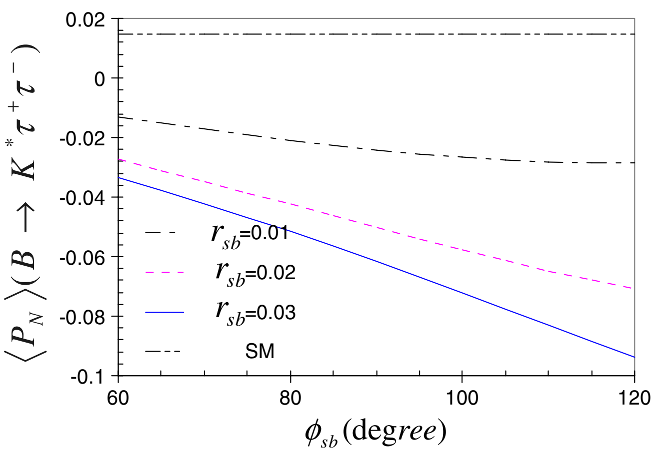

FIG. 5 (color online). The dependence of the $\langle P_L \rangle$ for the $B \to K^* \tau^+ \tau^-$ decay on mixing angle ϕ for three different values of $r_{sb} = 0.01$, 0.02, 0.03 when the fourth-generation quark mass $m_{t'} = 400$ GeV.

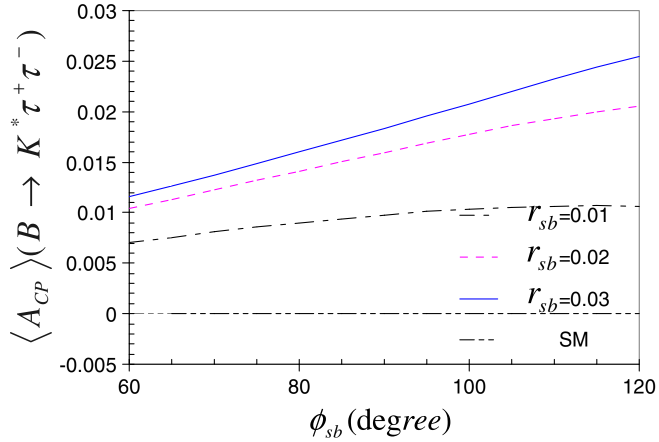

FIG. 6 (color online). The dependence of the $\langle A_{CP} \rangle$ for the $B \to K^* \tau^+ \tau^-$ decay on mixing angle ϕ for three different values of $r_{sb} = 0.01, 0.02, 0.03$ when the fourth-generation quark mass $m_{t'} = 400$ GeV.

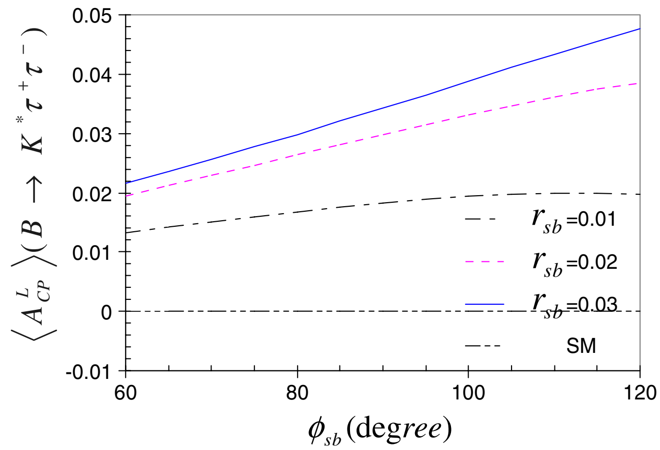

FIG. 7 (color online). The same as in Fig. 6, but for the longitudinally polarized *CP* asymmetry $\langle A_{CP}^L \rangle$.

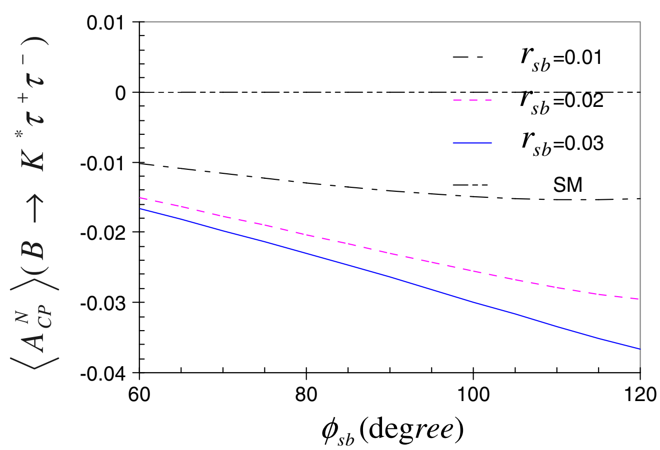

FIG. 8 (color online). The same as in Fig. 6, but for the normally polarized *CP* asymmetry $\langle A_{CP}^N \rangle$.

observables a highly challenging task for future hadronic collider experiments like LHCb, ATLAS or CMS. If we put this challenging issue aside, it is worth mentioning that the ratio of physical observables (for instance, *CP* asymmetry) suffers less from the uncertainty among the form factors where large parts of the uncertainties partially cancel out.

To sum up, we studied single lepton polarization and polarized and unpolarized *CP* asymmetry in $B \to K^* \ell^+ \ell^-$ decay with the four-generation standard model (SM4). We found that these physical observables are sensitive to the values of new quarks' mixing parameters and fourth-quark mass $m_{t'}$. If we overcome the technical challenges, the measurement of magnitude and sign of some of these physical observables, which are less sensitive to the hadronic uncertainties than the branching ratio, can be used to search for new physics effects indirectly.

[1] M. Bobrowski, A. Lenz, J. Riedl, and J. Rohrwild, Phys. Rev. D **79**, 113006 (2009).
[2] E. Barberio (Heavy Flavor Averaging Group), arXiv:hep-ex/0603003.
[3] T. Aaltonen *et al.* (CDF Collaboration), Phys. Rev. Lett. **100**, 161802 (2008).
[4] V. M. Abazov *et al.* (D0 Collaboration), Phys. Rev. Lett. **101**, 241801 (2008).
[5] W. S. Hou, Chin. J. Phys. (Taipei) **47**, 134 (2009).
[6] W. S. Hou, M. Nagashima, and A. Soddu, Phys. Rev. D **76**, 016004 (2007).
[7] G. D. Kribs, T. Plehn, M. Spannowsky, and T. M. P. Tait, Phys. Rev. D **76**, 075016 (2007).
[8] K. Zeynali and V. Bashiry, Phys. Rev. D **78**, 033001 (2008).
[9] W. S. Hou, M. Nagashima, and A. Soddu, Phys. Rev. Lett. **95**, 141601 (2005).
[10] W. S. Hou, M. Nagashima, and A. Soddu, Phys. Rev. D **76**, 016004 (2007).
[11] A. Arhrib and W. S. Hou, J. High Energy Phys. 07 (2006) 009.
[12] W. S. Hou, M. Nagashima, and A. Soddu, Phys. Rev. Lett. **95**, 141601 (2005).
[13] V. Bashiry, S. M. Zebarjad, F. Falahati, and K. Azizi, J. Phys. G **35**, 065005 (2008).
[14] V. Bashiry and K. Zeynali, J. High Energy Phys. 12 (2007) 055.
[15] A. Ali, P. Ball, L. T. Handoko, and G. Hiller, Phys. Rev. D **61**, 074024 (2000).
[16] A. Ali, E. Lunghi, C. Greub, and G. Hiller, Phys. Rev. D **66**, 034002 (2002).
[17] M. Beneke, T. Feldmann, and D. Seidel, Nucl. Phys. **B612**, 25 (2001).
[18] T. Feldmann and J. Matias, J. High Energy Phys. 01 (2003) 074.
[19] F. Kruger and J. Matias, Phys. Rev. D **71**, 094009 (2005).
[20] C. Bobeth, G. Hiller, and G. Piranishvili, J. High Energy Phys. 07 (2008) 106.
[21] U. Egede, T. Hurth, J. Matias, M. Ramon, and W. Reece, J. High Energy Phys. 11 (2008) 032.
[22] C. H. Chen, C. Q. Geng, and L. Li, Phys. Lett. B **670**, 374 (2009).
[23] W. Altmannshofer, P. Ball, A. Bharucha, A. J. Buras, D. M. Straub, and M. Wick, J. High Energy Phys. 01 (2009) 019.
[24] T. M. Aliev, M. K. Cakmak, and M. Savci, Nucl. Phys. **B607**, 305 (2001).
[25] T. M. Aliev, V. Bashiry, and M. Savci, J. High Energy Phys. 05 (2004) 037.
[26] K. S. M. Lee and F. J. Tackmann, Phys. Rev. D **79**, 114021 (2009).
[27] Z. Ligeti and F. J. Tackmann, Phys. Lett. B **653**, 404 (2007); Tobias Hurth, arXiv:hep-ph/0703226; C. Greub, V. Pilipp, and C. Schüpbach, J. High Energy Phys. 12 (2008) 040.
[28] A. Ghinculov, T. Hurth, G. Isidori, and Y. P. Yao, Nucl. Phys. **B685**, 351 (2004).
[29] For a recent review, see: T. Hurth, Int. J. Mod. Phys. A **22**, 1781 (2007); and references therein.
[30] C. W. Bauer and C. N. Burrell, Phys. Lett. B **469**, 248 (1999).
[31] M. Neubert, J. High Energy Phys. 07 (2000) 022.
[32] G. Buchalla, A. J. Buras, and M. E. Lautenbacher, Rev. Mod. Phys. **68**, 1125 (1996).
[33] A. J. Buras and M. Munz, Phys. Rev. D **52**, 186 (1995).
[34] A. J. Buras, M. Misiak, M. Munz, and S. Pokorski, Nucl. Phys. **B424**, 374 (1994).
[35] C. S. Lim, T. Morozumi, and A. I. Sanda, Phys. Lett. B **218**, 343 (1989).
[36] A. Ali, T. Mannel, and T. Morozumi, Phys. Lett. B **273**, 505 (1991).
[37] F. Kruger and L. M. Sehgal, Phys. Lett. B **380**, 199 (1996).
[38] B. Grinstein, M. J. Savage, and M. B. Wise, Nucl. Phys. **B319**, 271 (1989).
[39] G. Cella, G. Ricciardi, and A. Viceré, Phys. Lett. B **258**, 212 (1991).
[40] C. Bobeth, M. Misiak, and J. Urban, Nucl. Phys. **B574**, 291 (2000).
[41] H. H. Asatrian, H. M. Asatrian, C. Greub, and M. Walker, Phys. Lett. B **507**, 162 (2001).
[42] M. Misiak, Nucl. Phys. **B393**, 23 (1993); **B439**, 461(E) (1995).
[43] K. S. Babu, K. R. S. Balaji, and I. Schienbein, Phys. Rev. D **68**, 014021 (2003).
[44] T. M. Aliev, V. Bashiry, and M. Savci, Eur. Phys. J. C **31**, 511 (2003).
[45] V. Bashiry, Phys. Rev. D **77**, 096005 (2008).
[46] C. Amsler *et al.*, Phys. Lett. B **667**, 1 (2008).
[47] A. Arhrib and W. S. Hou, Eur. Phys. J. C **27**, 555 (2003).
[48] W. S. Hou, H-n. Li, S. Mishima, and M. Nagashima, Phys. Rev. Lett. **98**, 131801 (2007).
[49] P. Ball and V. M. Braun, Phys. Rev. D **58**, 094016 (1998).