Extended Friedberg-Lee hidden symmetries, quark masses, and CP violation with four generations

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Motivated in part by the several observed anomalies involving CP asymmetries of B and B_s decays, we consider the standard model with a 4th sequential family (SM4) which seems to offer a rather simple resolution. We initially assume T-invariance by taking the up and down-quark 4×4 mass matrix to be real. Following Friedberg and Lee (FL), we then impose a hidden symmetry on the unobserved (hidden) up and down-quark SU(2) states. The hidden symmetry for four generations ensures the existence of two zero-mass eigenstates, which we take to be the (u, c) and (d, s) states in the up and down-quark sectors, respectively. Then, we simultaneously break T-invariance and the hidden symmetry by introducing two phase factors in each sector. This breaking mechanism generates the small quark masses m_u, m_c and m_d, m_s, which, along with the orientation of the hidden symmetry, determine the size of CP-violation in the SM4. For illustration we choose a specific physical picture for the hidden symmetry and the breaking mechanism that reproduces the observed quark masses, mixing angles and CP-violation, and at the same time allows us to further obtain very interesting relations/predictions for the mixing angles of t and t'. For example, with this choice we get $V_{td} \sim (V_{cb}/V_{cd} - V_{ts}/V_{us}) + \mathcal{O}(\lambda^2)$ and $V_{t'b} \sim V_{t'd} \cdot (V_{cb}/V_{cd})$, $V_{tb'} \sim V_{t'd} \cdot (V_{ts}/V_{us})$, implying that $V_{t'd} > V_{t'b}, V_{tb'}$. We furthermore find that the Cabibbo angle is related to the orientation of the hidden symmetry and that the key CP-violating quantity of our model at high energies, $J_{\mathrm{SM4}} \equiv \mathrm{Im}(V_{tb} V^{\star}_{t'b} V_{t'b'} V^{\star}_{tb'})$, which is the high-energy analogue of the Jarlskog invariant of the SM, is proportional to the light-quark masses and the measured Cabibbo-Kobayashi-Maskawa quark-mixing matrix angles: $|J_{\mathrm{SM4}}| \sim A^3 \lambda^5 \times (\sqrt{m_u/m_t} + \sqrt{m_c/m_{t'}} - \sqrt{m_d/m_b} + \sqrt{m_s/m_{b'}}) \sim 10^{-5}$, where $A \sim 0.81$ and $\lambda = 0.2257$ are the Wolfenstein parameters. Other choices for the orientation of the hidden symmetry and/or the breaking mechanism may lead to different physical outcomes. A general solution, obtained numerically, will be presented in a forthcoming paper.

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I. INTRODUCTION

In spite of the success of the standard model (SM) in explaining almost all of the observed phenomena in particle physics, it does not address some fundamental issues, such as the hierarchy problem, dark matter, the matter antimatter asymmetry in the universe, etc. Also unexplained are the issues in flavor physics, such as the hierarchy of fermion masses and the number of families. There are strong indications, from both the theoretical and experimental points of view, that some of these unresolved questions are related to some new physics, maybe at the near by TeV scale. It is, therefore, hoped that, with the LHC turning on very soon, we will get a first hand glimpse of the new physics at the TeV scale and new hints from nature to some of these issues and, in particular, to the physics of flavor.

In this paper we wish to study some of the fundamental unresolved issues of flavor within a simple extension of the SM, in which a fourth sequential family of fermions is added—the SM4. Indeed, the four generations scenario can play an important role in flavor physics [1], and has recently gained some new interest as it might shed new light on baryogenesis and on CP-violation in K in B, B_s decays [2–6]. This model, which can be regarded as an effective low energy description of some higher energy and more fundamental underlying theory, retains all the features of the SM with three generations (which from here on we will denote as SM3), except that it brings into existence the new heavy fermionic members t' and b', which form the 4th quark doublet and a similar leptonic doublet, where the "neutrino" of the 4th family must also be rather heavy, with mass $\gtrsim M_Z/2$. This may well be an important clue that the underlying nature of the 4th family may be quite different from the 1st three families. This line of thinking may in fact lead to a dark matter candidate [7].

The addition of the fourth generation to the SM3 means that the Cabibbo-Kobayashi-Maskawa quark-mixing matrix (CKM) can now potentially have six independent real parameters/angles and three physical CP-violating phases [8]. The two additional phases (with respect to the SM3) provide new sources of CP-violation and may, thus, give rise to new CP-violating effects. Indeed, in a recent paper [3], it was shown that a fourth family of quarks with $m_{t'}$ in

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the range of ~400–600 GeV provides a simple and perhaps rather natural explanation for the several indications of new physics [9] that have been observed involving *CP* asymmetries in *b*-quark systems, and this in fact forms an important motivation for our work. Such heavy fermionic states point to the interesting possibility that the 4th family may play a role in dynamical electroweak symmetry breaking (EWSB), since the mechanism of dynamical mass generation seems to require such heavy masses [10,11]. In addition, as mentioned above, the new *CP*-violating phases may play an important role in generating the baryon asymmetry in the universe [4,5], which is difficult to address within the SM3.

We note in passing that a 4th generation of quarks (and leptons) with such heavy masses is not ruled out by precision electroweak constraints, but rather requires that correspondingly the Higgs has to be heavier, $\gtrsim 300$ GeV [6].

In recent work [12] that also partly motivated the present work, Friedberg and Lee (FL) suggested a very interesting new approach for the generation of *CP*-violation and quark masses in the SM3: that a weakly broken symmetry which is operational in the SU(2) (weak) fermionic states relates the smallness of *CP*-violation to the smallness of the light-quark masses m_d and m_u. More specifically, they imposed a hidden symmetry on the weak states of the quarks (named henceforward as the hidden frame), which is then weakly broken by small *CP*-phases that generate the nonzero masses for the light-quarks u and d. They found a very interesting relation between *CP*-violation and the light-quark masses:

$$J_{\rm SM} \propto \sqrt{\frac{m_d m_s}{m_b^2}}, \tag{1}$$

where $J_{\rm SM}$ is the Jarlskog invariant responsible for *CP*-violation in the SM3 [8]. While FL have shown that the same mechanism can be also applied to the leptonic sector [12], this idea was further examined and extended by Jarlskog [13] who showed that, when applied only to right-handed neutrinos, the FL hidden symmetry may provide a rather natural explanation for the smallness of neutrino masses.

The main appealing feature of the FL mechanism is that the *CP*-violating phases are the small parameters that control the breaking of the hidden symmetry and are, therefore, the generators of the small masses of the first generation quarks. Unlike the conventional SM3 picture, the FL mechanism gives a physical meaning to the rotations of the quark fields (i.e., from the weak basis to the physical mass eigenstates basis) in the up- and down-quark sector separately, since there is an independent hidden symmetry for each sector.

As we will show in this paper, the idea of FL and their main result in Eq. (1) is extremely interesting when applied to the SM4 case and our extension will lead it to predictive power. In particular, with an appropriate choice of a hidden symmetry, it allows to generate *all four masses* of the u, d, c, and s-quarks in terms of the masses of the four heavy quarks b, t, b', and t' and the new *CP*-phases. It also gives distinct predictions for the 4th generation mixing angles and for the size of *CP*-violation in this theory, subject to the constraints coming from existing data on the SM3's 3×3 CKM matrix and quark masses. Thus, the hidden symmetry framework for the SM4 case can be directly tested in collider experiments. In particular, we give distinct predictions for the new mixing angles and for the size of the new *CP*-violating quantities associated with the dynamics of the 4th generation quarks.

On the other hand, the construction of a hidden symmetry for the SM4 case, and the generation of the four light-quark masses in conjunction with *T*-violation, is more challenging and rather intricate and analytically involved than in the case of the SM3. This is mainly due to the fact that the phase-space of the hidden symmetry in the SM4 case is much broader and that, as opposed to the FL mechanism for the SM3 where the *CP*-phases generate only the masses of the 1st generation fermions, here we use the new *CP*-phases (of the SM4) as generators of all four light-quark masses m_d, m_u, m_s, m_c, which makes it more difficult to find a physical solution. To put it in another way, our hidden symmetry for the SM4 case defines a plane in which the theory is invariant whereas for three families the symmetry is "one dimensional", i.e., defines a direction/vector.

In order to spell out our notation and the general formalism of the hidden symmetry and its breaking mechanism within the SM4, we first consider the 4×4 up and down-quark Yukawa terms in the SM4 (after EWSB):

$$\mathcal{M}(q^{u,d}) = (q_1^{u,d}, q_2^{u,d}, q_3^{u,d}, q_4^{u,d}) M(q^{u,d}) \begin{pmatrix} q_1^{u,d} \\ q_2^{u,d} \\ q_3^{u,d} \\ q_4^{u,d} \end{pmatrix}, \tag{2}$$

where $q_i^{u,d}$, $i = 1 - 4$, are the hidden SU(2) quark states of the SM4, and $M(q^{u,d})$ are the corresponding mass matrices in the hidden frame basis.

As our zeroth-approximation we assume invariance under time reversal, thus taking $M_0(q^{u,d})$ (the subscript 0 will henceforward denote the zeroth-order quantities) to be real and symmetric. We can then extend FL's idea to the case of the SM4 by "doubling" the hidden symmetry in each quark sector (in the following we drop the indices u and d, where unless stated otherwise, it is understood that the discussion below applies to both up and down sectors):

$$\begin{aligned} &q_1 \to q_1 + \delta_z^1 z + \delta_t^1 t, \qquad q_2 \to q_2 + \delta_z^2 z + \delta_t^2 t, \\ &q_3 \to q_3 + \delta_z^3 z + \delta_t^3 t, \qquad q_4 \to q_4 + \delta_z^4 z + \delta_t^4 t, \end{aligned} \tag{3}$$

where z and t are space-time independent constants of Grassmann algebra anticommuting with the Dirac field operators, and δ_z^i, δ_t^i are c-numbers.

Since $M_0(q)$ is a real symmetric 4×4 matrix, it is characterized in general by 10 real parameters. However, imposing the hidden symmetry in Eq. (3) eliminates 2 of the 10 parameters. The hidden symmetry of Eq. (3) ensures [under the invariance of $\mathcal{M}_0(q^{u,d})$] the existence of two massless quark states in each sector, which we will identify as m_u and m_c (in the up-quark sector) and as m_d and m_s (in the down-quark sector). The corresponding two massless eigenvectors of $M_0(q)$ are thus identified as the zeroth-order u and c states, v_u^0 and v_c^0 (with $m_u^0, m_c^0 = 0$) and in the down-quark sector as the zeroth-order d and s states, v_d^0 and v_s^0 (with $m_d^0, m_s^0 = 0$). That is, since nature proves to have a large hierarchical mass structure in the quark sector, we will consider the SM4 in the chiral limit for the first two generations of quarks—$m_{u,d,c,s} = 0$. Accordingly, the two massive eigenvectors are identified as the zeroth-order t and t' states (or b and b' states) v_t^0 and $v_{t'}^0$, (or v_b^0 and $v_{b'}^0$) with masses (i.e., eigenvalues) m_t^0, $m_{t'}^0$ (or m_b^0, $m_{b'}^0$). In particular, it is easy to show that in the hidden basis $\{q_1, q_2, q_3, q_4\}$ the massless eigenvectors span a 2-dimensional subspace of the form:

$$v_u^0, \quad v_c^0 \in \begin{pmatrix} \delta_z^1 \\ \delta_z^2 \\ \delta_z^3 \\ \delta_z^4 \end{pmatrix}, \begin{pmatrix} \delta_t^1 \\ \delta_t^2 \\ \delta_t^3 \\ \delta_t^4 \end{pmatrix}, \tag{4}$$

and similarly in the down-quark sector.

The next step towards establishing the complete physical picture of quark masses and mixings is to simultaneously break T-invariance and the hidden symmetry by inserting two new phase factors into M_0, in each sector. In the following we will construct a general framework that defines the hidden symmetry in the SM4 scenario in a form that emphasizes the underlying geometrical picture, and, then, give a concrete physical example for the breaking mechanism.

II. HIDDEN SYMMETRY, *T*-INVARIANCE AND THE ZEROTH-ORDER SPECTRUM FOR THE SM4

In a generalization of the FL idea to the case of the SM4, let us assume, at the first stage that the zeroth-order mass matrix M_0 is real and invariant under the following translational symmetry (we will denote this symmetry as hidden symmetry 1, HS1)

$$\begin{aligned} q_1 &\to q_1 + c_\theta z, \qquad q_2 \to q_2 + s_\theta c_\phi z, \\ q_3 &\to q_3 + s_\theta s_\phi c_\omega z, \qquad q_4 \to q_4 + s_\theta s_\phi s_\omega z. \end{aligned} \tag{5}$$

where c_θ, $s_\theta = \cos\theta$, $\sin\theta$ etc., and z is a space-time independent constant of Grassmann algebra anticommuting with the Dirac fields.

This symmetry guarantees that the vector

$$Q_1 = c_\theta q_1 + s_\theta c_\phi q_2 + s_\theta s_\phi c_\omega q_3 + s_\theta s_\phi s_\omega q_4, \tag{6}$$

is a massless eigenstate of the theory, as under the HS1 it transforms as $Q_1 \to Q_1 + z$. On the other hand, the three orthogonal (to Q_1) vectors

$$\begin{aligned} Q_2 &= -s_\theta q_1 + c_\theta c_\phi q_2 + c_\theta s_\phi c_\omega q_3 + c_\theta s_\phi s_\omega q_4 \\ Q_3 &= -s_\phi q_2 + c_\phi c_\omega q_3 + c_\phi s_\omega q_4 \\ Q_4 &= -s_\omega q_3 + c_\omega q_4, \end{aligned} \tag{7}$$

are invariant under the HS1, i.e., $Q_i \to Q_i$ for $i = 2, 3, 4$. The rotation from the hidden frame $\{q_1, q_2, q_3, q_4\}$ to the HS1 frame $\{Q_1, Q_2, Q_3, Q_4\}$ can be written as $Q_i = R_{ij} q_j$, thus defining the real unitary matrix R:

$$R = \begin{pmatrix} c_\theta & s_\theta c_\phi & s_\theta s_\phi c_\omega & s_\theta s_\phi s_\omega \\ -s_\theta & c_\theta c_\phi & c_\theta s_\phi c_\omega & c_\theta s_\phi s_\omega \\ 0 & -s_\phi & c_\phi c_\omega & c_\phi s_\omega \\ 0 & 0 & -s_\omega & c_\omega \end{pmatrix}. \tag{8}$$

Demanding translational invariance under HS1 of Eq. (5), M_0 has only one massless eigenstate (the state Q_1). Thus, in order to enforce the chiral limit for the first two generations, we will demand that the zeroth-order mass matrix is invariant under an additional translation operation, which is operational in the HS1 frame $\{Q_1, Q_2, Q_3, Q_4\}$ and which we will name hidden symmetry 2 (HS2). Without loss of generality, we assume that HS2 is orthogonal to HS1 as follows:

$$\begin{aligned} Q_1 &\to Q_1, \qquad Q_2 \to Q_2 + c_\zeta t, \\ Q_3 &\to Q_3 + s_\zeta c_\eta t, \qquad Q_4 \to Q_4 + s_\zeta s_\eta t. \end{aligned} \tag{9}$$

The additional symmetry HS2 guarantees that the vector

$$P_1 = c_\zeta Q_2 + s_\zeta c_\eta Q_3 + s_\zeta s_\eta Q_4, \tag{10}$$

which is orthogonal to Q_1, is also massless.

The most general form of the Yukawa term $\mathcal{M}_0$ that is invariant under the independent translations in both directions HS1 and HS2, can then be written as:

$$\begin{aligned} \mathcal{M}_0 = {}& \alpha |c_\eta Q_4 - s_\eta Q_3|^2 + \beta |c_\zeta Q_4 - s_\zeta s_\eta Q_2|^2 \\ &+ \gamma |c_\zeta Q_3 - s_\zeta c_\eta Q_2|^2, \end{aligned} \tag{11}$$

and this defines the quark mass matrix M_0. Recall that, since M_0 is invariant under HS1 and HS2, two of its four eigenstates, i.e., Q_1 and P_1, are necessarily massless.

Before deriving the full zeroth-order system (i.e., 2 nonzero masses and 4 states), we wish to point out the mapping of our double hidden symmetry (HS1 and HS2) to the generic parametrizations of the hidden symmetry in Eq. (3). In particular, using the definition for HS1 and HS2 in Eqs. (5) and (9), respectively, and the fact that $q = R^{-1} Q$, we obtain the overall hidden symmetry for the SM4 case:

$$\begin{aligned} q_1 &\to q_1 + c_\theta z - s_\theta c_\zeta t, \\ q_2 &\to q_2 + s_\theta c_\phi z + [c_\theta c_\phi c_\zeta - s_\phi s_\zeta c_\eta] t, \\ q_3 &\to q_3 + s_\theta s_\phi c_\omega z + [c_\theta s_\phi c_\omega c_\zeta + c_\phi c_\omega s_\zeta c_\eta - s_\omega s_\zeta s_\eta] t, \\ q_4 &\to q_4 + s_\theta s_\phi s_\omega z + [c_\theta s_\phi s_\omega c_\zeta + c_\phi s_\omega s_\zeta c_\eta + c_\omega s_\zeta s_\eta] t, \end{aligned} \tag{12}$$

from which one can extract the hidden symmetry parameters δ_z^i and δ_t^i of Eq. (3), as a function of the angles which define the orientations of HS1 and HS2 with respect to the hidden frame $\{q_1, q_2, q_3, q_4\}$.

Note that the expression for $\mathcal{M}_0$ in Eq. (11) contains five angles: the two (explicit) angles ζ, η associated with the orientation of HS2 with respect to the HS1 frame $\{Q_1, Q_2, Q_3, Q_4\}$ and the three angles θ, ϕ, ω associated with the orientation of HS1 with respect to the hidden frame $\{q_1, q_2, q_3, q_4\}$, which enter through the rotation $Q = Rq$. Thus, along with the parameters α, β and γ, $\mathcal{M}_0$ in Eq. (11) is parametrized by 8 real parameters (in each sector) as required when imposing the double hidden symmetry (see discussion above). However, there is one nonphysical angle in *each sector* which results from the fact that the two orthogonal states Q_1, P_1 are massless at zeroth-order and are, therefore, indistinguishable. This can be easily understood by considering the geometrical interpretation of the hidden symmetry in the SM4 case. In particular, the double hidden symmetry (HS1 + HS2) defines a plane in the hidden frame $\{q_1, q_2, q_3, q_4\}$ under which the theory is invariant. This is the plane spanned by the two orthogonal vectors Q_1 and P_1. We, therefore, have the freedom to make any unitary transformation in the $Q_1 - P_1$ plane/subspace (in both up and down-quark sectors) without affecting the physical picture. This allows us to eliminate one angle in each of the (v_d^0, v_s^0) and (v_u^0, v_c^0) subspaces. Thus, without loss of generality we find it convenient to choose $\omega = \pi/2$ in both sectors, which sets $Q_4 = q_3$ and Q_1, Q_2, $Q_3 \perp q_3$. This is analogous to a gauge condition in a vector field theory as also identified in [12]. Note that even though at each sector the massless states (v_d^0, v_s^0) and (v_u^0, v_c^0) are indistinguishable at the zeroth-order, as we will see in the next section, after breaking the hidden symmetry this degeneracy is removed, and those (now massive) states become well defined.

We are now ready to derive the mass spectrum and the 4×4 CKM matrix at zeroth-order, i.e., without T-violation. Recall that, by construction, there are two massless states, given by Q_1 and P_1. In order to find the 2 massive states we can apply the original FL formulae for three generations to the $\{Q_2, Q_3, Q_4\}$ subspace. As in [12], we find that the eigensystem of M_0 depends only on two linear combinations of α, β, γ, so that one of these three parameters can be "gauged away". Following the choice of FL in [12], we eliminate the parameter γ using the "gauge" condition (i.e., this has no effect on the physical outcome):

$$\frac{\beta}{\gamma} = 1. \tag{13}$$

Using this condition, we diagonalize the mass matrix M_0 and find that the two massive states are

$$\begin{aligned} P_2 &= -s_\zeta Q_2 + c_\zeta c_\eta Q_3 + c_\zeta s_\eta Q_4, \\ P_3 &= -s_\eta Q_3 + c_\eta Q_4, \end{aligned} \tag{14}$$

with masses:

$$m_{P_2} = \beta, \tag{15}$$

$$m_{P_3} = \alpha + c_\zeta^2 \beta = \alpha + c_\zeta^2 m_{P_2}. \tag{16}$$

Note that, for $m_{P_3} \gg m_{P_2}$ and/or $c_\zeta \to 0$ we have $m_{P_3} \approx \alpha$ and $m_{P_2} \approx \beta$ (see below).

Thus the complete set of eigenstates of M_0 at zeroth-order becomes quite simple, as it is given by $\{Q_1, P_1, P_2, P_3\}$ with masses $\{0, 0, m_{P_2}, m_{P_3}\}$, which we henceforward identify (in each sector) as the zeroth-order quark states:

$$\{v_d^0, v_s^0, v_b^0, v_{b'}^0\} \equiv \{Q_1^d, P_1^d, P_2^d, P_3^d\}, \tag{17}$$

$$\{v_u^0, v_c^0, v_t^0, v_{t'}^0\} \equiv \{Q_1^u, P_1^u, P_2^u, P_3^u\}, \tag{18}$$

with masses $m_d^0 = m_s^0 = m_u^0 = m_c^0 = 0$ and:

$$\begin{aligned} m_b^0 &= \beta_d, \qquad m_{b'}^0 \approx \alpha_d, \\ m_t^0 &= \beta_u, \qquad m_{t'}^0 = \alpha_u + c_{\zeta_u}^2 m_t^0, \end{aligned} \tag{19}$$

where the superscripts d and u distinguish between the parameters in the down-quark and up-quark sectors, respectively. Note that since T-violation is responsible for generating the light-quark masses, it is a small perturbation to the T-invariant zeroth-order spectrum. Thus for all practical purposes we can set $m_b \approx m_b^0$, $m_{b'} \approx m_{b'}^0$, $m_t \approx m_t^0$, and $m_{t'} \approx m_{t'}^0$ (see also below).

Using the orientation of the HS1 frame Q_i with respect to the hidden frame q_i, i.e., $Q = Rq$ with R given in Eq. (8), and the orientation of the states P_2, P_3 with respect to the $\{Q_2, Q_3, Q_4\}$ subframe [as given in Eq. (14)], we can write the set of four eigenstates in each sector in terms of the weak (hidden) states q_i (as required in order to derive the zeroth-order (real) 4×4 CKM matrix):

$$\begin{pmatrix} v_d^0 \\ v_s^0 \\ v_b^0 \\ v_{b'}^0 \end{pmatrix} = \begin{pmatrix} R_{1i}^d \\ A_i^d \\ B_i^d \\ C_i^d \end{pmatrix} q_i^d, \qquad \begin{pmatrix} v_u^0 \\ v_c^0 \\ v_t^0 \\ v_{t'}^0 \end{pmatrix} = \begin{pmatrix} R_{1i}^u \\ A_i^u \\ B_i^u \\ C_i^u \end{pmatrix} q_i^u, \tag{20}$$

where the superscripts u and d are again added in order to distinguish between the angles associated with the up and down-quark sectors, respectively. Also,

$$A_i^d \equiv \cos\zeta_d \cdot R_{2i}^d + \sin\zeta_d \cdot \cos\eta_d \cdot R_{3i}^d + \sin\zeta_d \cdot \sin\eta_d \cdot R_{4i}^d \tag{21}$$

$$B_i^d \equiv -\sin\zeta_d \cdot R_{2i}^d + \cos\zeta_d \cdot \cos\eta_d \cdot R_{3i}^d + \cos\zeta_d \cdot \sin\eta_d \cdot R_{4i}^d, \tag{22}$$

$$C_i^d \equiv -\sin\eta_d \cdot R_{3i}^d + \cos\eta_d \cdot R_{4i}^d, \tag{23}$$

and similarly for the up-quark sector with A_i^u, B_i^u, C_i^u using R^u and ζ_u, η_u, where ζ_d, η_d (ζ_u, η_u) are the HS2 angles in the down(up)-quark sector and $R^d(R^u)$ is the matrix which defines the rotation from the hidden frame to the HS1 frame in the down(up)-quark sector with the corresponding angles θ_d, ϕ_d, ω_d (θ_u, ϕ_u, ω_u) in Eq. (8).

Then denoting by $D_0 = (v_d^0, v_s^0, v_b^0, v_{b'}^0)$ and $U_0 = (v_u^0, v_c^0, v_t^0, v_{t'}^0)$ the unitary matrices that diagonalize the real and symmetric mass matrices in the down and up-quark sectors, respectively:

$$D_0^\dagger M_0(q^d) D_0 = \mathrm{diag}(0, 0, m_b^0, m_{b'}^0), \tag{24}$$

$$U_0^\dagger M_0(q^u) U_0 = \mathrm{diag}(0, 0, m_t^0, m_{t'}^0), \tag{25}$$

we can obtain the 4×4 zeroth-order CKM matrix of the SM4 (i.e., without T-violation):

$$V^0(\mathrm{CKM}) = U_0^\dagger D_0. \tag{26}$$

The general expression for V^0(CKM) in terms of the angles that define the hidden symmetry in the up and down-quark sectors is rather complicated to be written here. Let us, therefore, choose a specific physical orientation of the hidden symmetry, where the direction of HS2 is partly fixed by the angle ζ with the choice $\zeta = \omega = \pi/2$ in each sector (recall that we have fixed the angle $\omega = \pi/2$ in a manner similar to choosing a gauge). This orientation is physically viable in the sense that it reproduces the observed light-quark masses and the measured CKM mixing angles. It will be used in the next sections to demonstrate the general mechanism for breaking the hidden symmetry and T-invariance and the corresponding generation of the light-quark masses.

In particular, using Eqs. (17)–(26) with $\zeta = \omega = \pi/2$ we obtain

$$\begin{aligned}
V_{ud}^0 &= c_{\theta_u} c_{\theta_d} + s_{\theta_u} s_{\theta_d} \cos(\phi_u - \phi_d),\\
V_{us}^0 &= s_{\theta_u} c_{\eta_d} \sin(\phi_u - \phi_d),\\
V_{ub}^0 &= c_{\theta_u} s_{\theta_d} - s_{\theta_u} c_{\theta_d} \cos(\phi_u - \phi_d),\\
V_{ub'}^0 &= -s_{\theta_u} s_{\eta_d} \sin(\phi_u - \phi_d),\\
V_{cd}^0 &= -s_{\theta_d} c_{\eta_u} \sin(\phi_u - \phi_d),\\
V_{cs}^0 &= s_{\eta_u} s_{\eta_d} + c_{\eta_u} c_{\eta_d} \cos(\phi_u - \phi_d),\\
V_{cb}^0 &= c_{\eta_u} c_{\theta_d} \sin(\phi_u - \phi_d),\\
V_{cb'}^0 &= s_{\eta_u} c_{\eta_d} - c_{\eta_u} s_{\eta_d} \cos(\phi_u - \phi_d),\\
V_{td}^0 &= c_{\theta_d} s_{\theta_u} - s_{\theta_d} c_{\theta_u} \cos(\phi_u - \phi_d),\\
V_{ts}^0 &= -c_{\eta_d} c_{\theta_u} \sin(\phi_u - \phi_d),\\
V_{tb}^0 &= s_{\theta_u} s_{\theta_d} + c_{\theta_u} c_{\theta_d} \cos(\phi_u - \phi_d),\\
V_{tb'}^0 &= s_{\eta_d} c_{\theta_u} \sin(\phi_u - \phi_d),\\
V_{t'd}^0 &= s_{\theta_d} s_{\eta_u} \sin(\phi_u - \phi_d),\\
V_{t's}^0 &= s_{\eta_d} c_{\eta_u} - c_{\eta_d} s_{\eta_u} \cos(\phi_u - \phi_d),\\
V_{t'b}^0 &= -s_{\eta_u} c_{\theta_d} \sin(\phi_u - \phi_d),\\
V_{t'b'}^0 &= c_{\eta_d} c_{\eta_u} + s_{\eta_d} s_{\eta_u} \cos(\phi_u - \phi_d).
\end{aligned} \tag{27}$$

From these expressions we can find the size of some of the hidden symmetry angles in terms of the observed 3×3 CKM elements and, also, several interesting and surprising relations/predictions for the mixing angles of the 4th generation quarks with the first 3 generations:

$$-\tan\theta_u = \frac{V_{us}}{V_{ts}} = \frac{V_{ub'}}{V_{tb'}}, \tag{28}$$

$$-\tan\theta_d = \frac{V_{cd}}{V_{cb}} = \frac{V_{t'd}}{V_{t'b}}, \tag{29}$$

$$-\tan\eta_u = \frac{V_{t'd}}{V_{cd}}, \tag{30}$$

$$-\tan\eta_d = \frac{V_{ub'}}{V_{us}}, \tag{31}$$

implying $V_{t'd} > V_{t'b}$ and $V_{ub'} > V_{tb'}$—opposite to the hierarchical pattern as observed in the SM3's 3×3 block.

In addition, taking $V_{ts}^2/V_{us}^2 \sim V_{cb}^2/V_{cd}^2 \ll 1$, $V_{ud} \sim 1 - \lambda^2/2$ and $V_{cs} \sim 1 - \lambda^2/2$, where $\lambda \sim 0.2257$ is the Wolfenstein parameter [14], we find that $\phi_u - \phi_d$ is the Cabibbo angle (i.e., the Wolfenstein parameter) with

$$\sin(\phi_u - \phi_d) \sim \lambda \sim 0.2257, \tag{32}$$

$$\cos(\phi_u - \phi_d) \sim V_{ud} - \mathcal{O}(\lambda^2), \tag{33}$$

and

$$c_{\theta_d} \sim \frac{V_{cb}}{V_{cd}} \sim \mathcal{O}(\lambda) \tag{34}$$

$$c_{\theta_u} \sim \frac{V_{ts}}{V_{us}} \sim \mathcal{O}(\lambda) \tag{35}$$

$$\cos(\eta_u - \eta_d) \sim V_{cs} - \mathcal{O}(\lambda^2), \tag{36}$$

also implying that $\eta_u \sim \eta_d$. This in turn gives

$$V_{t'b'} \sim V_{cs} \tag{37}$$

$$V_{ub'} \sim V_{t'd}. \tag{38}$$

Furthermore, for the top-quark mixing angles we get

$$V_{tb} \sim 1 - \mathcal{O}(\lambda^2), \tag{39}$$

$$V_{td} \sim \left(\frac{V_{cb}}{V_{cd}} - \frac{V_{ts}}{V_{us}}\right) + \mathcal{O}(\lambda^2), \tag{40}$$

In the next sections we will use this physical setup to break T-invariance and derive the CP-violating parameters of the model.

III. T-VIOLATION AND HIDDEN SYMMETRY BREAKING MECHANISM

There are, of course, several ways to break the hidden symmetry without breaking T-invariance (we will further comment on that below). Here we wish to extend the attractive mechanism for the simultaneous breaking of both the hidden symmetry and T-invariance, that was suggested by Friedberg and Lee in [12] in the SM3 case, and formulate the general breaking mechanism for the SM4 case.

In particular, when the hidden symmetry and T-invariance are broken simultaneously, the massless states v_d^0, v_s^0, v_u^0, v_c^0 (which were protected by the hidden symmetry) acquire a mass which is directly related to the size of the phases responsible for T-violation: two CP-violating phases in the up-quark sector are needed to generate the masses m_u and m_c, while two CP-violating phases in the down-quark sector generate the masses m_d and m_s. Since we know that $m_{u,c} \ll m_{t,t'}$ and $m_{d,s} \ll m_{b,b'}$, we can treat the effect of T-violation as a perturbation to the zeroth-order (T-invariant) approximation in both the down and up-quark sectors.

In what follows we will describe the breaking mechanism using the generic notation outlined in the previous section, which holds for both down and up-quark sectors. The application of the results below to a specific sector is straightforward.

In order to break the hidden symmetry we rewrite the zeroth-order Yukawa term $\mathcal{M}_0$ in terms of its eigenstates:

$$\mathcal{M}_0 = \sum_i m_i^0 |v_i^0|^2 = m_{P_2}|P_2|^2 + m_{P_3}|P_3|^2, \tag{41}$$

where we have used the fact that $m_{Q_1} = m_{P_1} = 0$. This gives [see Eqs. (17), (18), and (20)]:

$$(M_0)_{ij} = m_{P_2} B_i B_j + m_{P_3} C_i C_j, \tag{42}$$

where we have dropped the superscripts d or u in the coefficients $B_i^{d,u}$ and $C_i^{d,u}$ [as defined in Eqs. (22) and (23)], so that the expression above applies to both down and up sectors. Following the original Friedberg and Lee proposal in [12], in the minimal setup, T-invariance and the hidden symmetry can then be broken by inserting a phase in any one of the nondiagonal entries of $(M_0)_{ij}$ as follows:

$$(\Delta M)_{ij} = (m_{P_2} B_i B_j + m_{P_3} C_i C_j) \cdot (e^{i\delta_{ij}} - 1), \qquad j > i;$$
$$(\Delta M)_{ji} = (\Delta M)^\star_{ij}, \tag{43}$$

such that

$$M = M_0 + \Delta M. \tag{44}$$

and $\delta_{ij} \ll 1$, hence, $\Delta M \ll M_0$ so that ΔM can be treated as a perturbation. As we shall demonstrate in the next section, in the minimal setup, two such phase insertions (in each sector) are required in different locations in M_0 in order to break both HS1 and HS2 and to generate the observable masses of the first 2 light generations of quarks.

Before studying the breaking pattern described in Eq. (43), we wish to briefly comment here on other possible frameworks for the breaking mechanism that may lead to richer phenomenological implications for flavor physics. In particular, as mentioned above, in general the hidden symmetry can be broken without affecting T-invariance in the theory. Thus, one can envision a more general breaking term that can allow the generation of the light-quark masses in different limits of the T-invariance breaking. For example, in Eq. (43) we could replace the breaking term $(e^{i\delta} - 1)$ with $(e^{\Delta} e^{i\delta} - 1)$, in which case T-invariance and the chiral limit in the light-quarks sector can be restored in different limits of Δ, $\delta \to 0$.[1] In particular, with such a term the chiral limit can still be broken with $\delta \to 0$ as long as $\Delta \neq 0$. Indeed, such a more general breaking term can give rise to a richer phenomenology for flavor physics and CP-violation—for example, it may allow keeping the light-quark masses small even with a large CP-phase (δ), if Δ is adjusted accordingly. Such issues and a more elaborate study of the hidden symmetry breaking mechanism will be presented in [15].

Coming back to the minimal setup of Eq. (43), let us write the overall T-violating term as

$$\Delta M \equiv \Delta M_z + \Delta M_t, \tag{45}$$

where ΔM_z and ΔM_t contain the new phases that break HS1 and HS2, respectively, each given by the generic form

[1]We thank the referee for raising this point and for suggesting the modified breaking term $(e^{\Delta} e^{i\delta} - 1)$, which effectively demonstrates the importance and the impact of the breaking mechanism on the phenomenology of flavor with hidden symmetries.

in Eq. (43). The T-violating mass term ΔM then shifts the zeroth-order masses and states. Using perturbation theory, these shifts are given in the general case without degeneracies by

$$\Delta m_q \equiv m_q - m_q^0 = (v_q^0)^\dagger \Delta M v_q^0, \tag{46}$$

$$\Delta v_q \equiv v_q - v_q^0 = \sum_{q\neq q'} \frac{(v_{q'}^0)^\dagger \Delta M v_q^0}{m_q^0 - m_{q'}^0} v_{q'}^0, \tag{47}$$

where m_q^0 and v_q^0 are the zeroth-order masses and states (i.e., v_q^0 and $v_{q'}^0$ stands for any one of the vectors Q_1, P_1, P_2, P_3 in either the up- or down-sectors), Δm_q are the mass shifts due to the breaking of the hidden symmetry and Δv_q contains the imaginary terms which are $\propto i\sin\delta_{ij}$ from which the physical T-violating elements of the 4×4 CKM matrix are constructed.

In our case, however, the states Q_1 and P_1 are degenerate. Thus, in order to find the physical masses ($m_\pm$) and their corresponding physical states ($v_\pm$) in the $Q_1 - P_1$ subspace, we need to diagonalize the following 2×2 perturbation mass matrix in the $Q_1 - P_1$ subspace:

$$\begin{aligned} \Delta m(Q_1, P_1) &= \begin{pmatrix} Q_1^\dagger \Delta M Q_1 & Q_1^\dagger \Delta M P_1 \\ P_1^\dagger \Delta M Q_1 & P_1^\dagger \Delta M P_1 \end{pmatrix} \\ &\equiv \begin{pmatrix} \Delta m_{QQ} & \Delta m_{QP} \\ \Delta m_{PQ} & \Delta m_{PP} \end{pmatrix}, \end{aligned} \tag{48}$$

where $\Delta m_{QP} = (\Delta m_{PQ})^\dagger$ and Δm_{QQ}, Δm_{PP} are real. That is, after breaking T-invariance, the physical masses and states of the first two generations are given by

$$\begin{aligned} m_\pm = &\frac{\Delta m_{QQ} + \Delta m_{PP}}{2} \\ &\times \left[1 \pm \sqrt{1 - \frac{4(\Delta m_{QQ}\Delta m_{PP} - \Delta m_{QP}\Delta m_{PQ})}{(\Delta m_{QQ} + \Delta m_{PP})^2}}\right], \end{aligned} \tag{49}$$

and

$$\begin{aligned} v_+ = &\frac{1}{\sqrt{|\Delta m_{QP}|^2 + (m_+ - \Delta m_{QQ})^2}} \\ &\times [|\Delta m_{QP}| Q_1 + (m_+ - \Delta m_{QQ}) P_1] \\ v_- = &\frac{1}{\sqrt{|\Delta m_{PQ}|^2 + (m_- - \Delta m_{PP})^2}} \\ &\times [(m_- - \Delta m_{PP}) Q_1 + |\Delta m_{PQ}| P_1]. \end{aligned} \tag{50}$$

The corresponding corrections/shifts to the physical states are still calculated from Eq. (47), where now v_q^0, $v_{q'}^0 \in \{v_-, v_+, P_2, P_3\}$. In particular, let us further define the "perturbation matrix":

$$(v_q^0)^\dagger \Delta M v_{q'}^0 \equiv i P_{qq'}, \tag{51}$$

where $q \neq q'$ and, to $\mathcal{O}(\delta)$, $P_{qq'}$ are real and $P_{q'q} = -P_{qq'}$. That is, $(v_{q'}^0)^\dagger \Delta M v_q^0 = [(v_q^0)^\dagger \Delta M v_{q'}^0]^\dagger = -iP_{qq'}$, where $q, q' \in d, s, b, b'$ in the down-quark sector and $q, q' \in u, c, t, t'$ in the up-quark sector. Also note that the perturbation matrix is diagonal in the $(v_- - v_+)$ subspace to $\mathcal{O}(\delta)$ [i.e., $P_{ds} = P_{sd} \approx \mathcal{O}(\delta^2)$ and $P_{uc} = P_{cu} \approx \mathcal{O}(\delta^2)$].

In the next section, for simplicity we will consider the case where the perturbation is diagonal in the $Q_1 - P_1$ subspace, i.e., $\Delta m_{QP} = 0$ in Eq. (48), so that $v_- = Q_1$ and $v_+ = P_1$. In this simple case we can use Eqs. (47) and (51) to obtain the $\mathcal{O}(\delta)$ shifts, Δv_q, to the zeroth-order states $(v_d^0, v_s^0, v_b^0, v_{b'}^0)$ and $(v_u^0, v_c^0, v_t^0, v_{t'}^0)$ [as defined in Eqs. (20)–(23)]:

$$\begin{aligned} \Delta v_d &= i\left(\frac{P_{db}}{m_b} v_b^0 + \frac{P_{db'}}{m_{b'}} v_{b'}^0\right) \\ \Delta v_u &= i\left(\frac{P_{ut}}{m_t} v_t^0 + \frac{P_{ut'}}{m_{t'}} v_{t'}^0\right) \\ \Delta v_s &= i\left(\frac{P_{sb}}{m_b} v_b^0 + \frac{P_{sb'}}{m_{b'}} v_{b'}^0\right) \\ \Delta v_c &= i\left(\frac{P_{ct}}{m_t} v_t^0 + \frac{P_{ct'}}{m_{t'}} v_{t'}^0\right) \\ \Delta v_b &= i\left(\frac{P_{db}}{m_b} v_d^0 + \frac{P_{sb}}{m_b} v_s^0 + \frac{P_{bb'}}{m_{b'} - m_b} v_{b'}^0\right) \\ \Delta v_t &= i\left(\frac{P_{ut}}{m_t} v_u^0 + \frac{P_{ct}}{m_t} v_c^0 + \frac{P_{tt'}}{m_{t'} - m_t} v_{t'}^0\right) \\ \Delta v_{b'} &= i\left(\frac{P_{db'}}{m_{b'}} v_d^0 + \frac{P_{sb'}}{m_{b'}} v_s^0 + \frac{P_{bb'}}{m_{b'} - m_b} v_b^0\right) \\ \Delta v_{t'} &= i\left(\frac{P_{ut'}}{m_{t'}} v_u^0 + \frac{P_{ct'}}{m_{t'}} v_c^0 + \frac{P_{tt'}}{m_{t'} - m_t} v_t^0\right) \end{aligned} \tag{52}$$

such that, to $\mathcal{O}(\delta)$, the physical states are given by $v_q = v_q^0 + \Delta v_q$. The corresponding $\mathcal{O}(\delta)$ corrections to v_q^0 in the general case where the perturbation is not diagonal in the $Q_1 - P_1$ subspace, can be easily derived from the expressions for $v_\pm$ in Eq. (50) and the shifts Δv_q in Eq. (52) above. For example,

$$\begin{aligned} \Delta v_- = &\frac{1}{\sqrt{|\Delta m_{PQ}|^2 + (m_- - \Delta m_{PP})^2}} \\ &\times [(m_- - \Delta m_{PP}) \cdot \Delta v_d + |\Delta m_{PQ}| \cdot \Delta v_s] \end{aligned} \tag{53}$$

where $\Delta v_{d,s}$ are given in Eq. (52).

The physical (T-violating) 4×4 CKM matrix elements are, therefore, given symbolically by (u and d stand for any of the up- and down-quark states, respectively):

$$V_{ud} = (v_u)^\dagger \cdot v_d = V_{ud}^0 + (\Delta v_u)^\dagger \cdot v_d^0 + v_u^0 \cdot \Delta v_d, \tag{54}$$

where $V_{ud}^0 = (v_u^0)^T \cdot v_d^0$ is the zeroth-order CKM matrix elements and the terms $[(\Delta v_u)^\dagger \cdot v_d^0]$, $[v_u^0 \cdot \Delta v_d]$ which are also functions of the zeroth-order CKM elements, are readily obtained from Eq. (52) above. For example, in the simple case where $v_- = Q_1$ and $v_+ = P_1$, V_{ud} [i.e., now the (11) elements of V] is given by

$$V_{ud} = V_{ud}^0 + i\left[\frac{P_{db}}{m_b} V_{ub}^0 + \frac{P_{db'}}{m_{b'}} V_{ub'}^0 - \frac{P_{ut}}{m_t} V_{td}^0 - \frac{P_{ut'}}{m_{t'}} V_{t'd}^0\right] + \mathcal{O}(\delta^2). \quad (55)$$

Note that the zeroth-order elements V_{ud}^0, given in Eq. (27), are a good approximation to the magnitude of the physical CKM angles [i.e., up to corrections of $\mathcal{O}(\delta^2)$, where δ is any one of the CP-violating phases].

IV. A PHYSICAL FRAMEWORK FOR T-VIOLATION

In the previous two sections we have described the general features of the hidden symmetry and the generic mechanism of breaking T-invariance and generating the corresponding light-quark masses in coincidence with the breaking of the hidden symmetry in the case of SM4. In this section we would like to give a concrete physical example (i.e., compatible with all relevant known data) which is relatively simple analytically, therefore, providing insight for the physical picture. Our chosen setup below illustrates the power of this mechanism in predicting the new mixing angles and phases associated with the 4th generation of quarks and the size of CP-violation of the theory.

As in the previous section, here also we consider a specific orientation for the hidden symmetry, where the direction of HS2 is partly fixed by setting $\zeta = \pi/2$ in each sector. The hidden symmetry is then broken by inserting the phases in the 12 and 34 elements of the mass matrix M_0, such that

$$\Delta M_z = (\Delta M)_{12}, \qquad \Delta M_t = (\Delta M)_{34}, \quad (56)$$

where $(\Delta M)_{ij}$ is defined in Eq. (43). Note that with $\omega = \zeta = \pi/2$ we have $B_1 = s_\theta$, $B_2 = -c_\theta c_\phi$, $B_3 = 0$, $B_4 = -c_\theta s_\phi$, $C_1 = 0$, $C_2 = s_\eta s_\phi$, $C_3 = -c_\eta$, and $C_4 = -s_\eta c_\phi$ [see Eqs. (22) and (23)]. Thus, the overall T-violating term, $\Delta M = \Delta M_z + \Delta M_t$, is given by:

$$\Delta M = \begin{pmatrix} 0 & -m_{P_2} s_\theta c_\theta c_\phi (e^{i\delta_{12}} - 1) & 0 & 0 \\ -m_{P_2} s_\theta c_\theta c_\phi (e^{-i\delta_{12}} - 1) & 0 & 0 & 0 \\ 0 & 0 & 0 & m_{P_3} s_\eta c_\eta c_\phi (e^{i\delta_{34}} - 1) \\ 0 & 0 & m_{P_3} s_\eta c_\eta c_\phi (e^{-i\delta_{34}} - 1) & 0 \end{pmatrix}. \quad (57)$$

For simplicity and without loss of generality, we will further take $s_\phi \ll 1$ for $\phi = \phi_d \sim \phi_u$ [recall that $\cos(\phi_u - \phi_d) \sim V_{ud} \sim 1$ implying $\phi_u \sim \phi_d$, see previous section], which allows us to obtain a relatively compact analytical picture. In particular, one simplification that arises with this choice, is that the perturbation in the $Q_1 - P_1$ subspace, $\Delta m(Q_1, P_1)$ in Eq. (48), is approximately diagonal so that $m_- \approx \Delta m_{QQ}$, $m_+ \approx \Delta m_{PP}$ and the corresponding states are $v_- \approx Q_1$, $v_+ \approx P_1$ in each sector. In particular, ΔM in Eq. (57) generates the following light-quark masses (we now add the superscripts d and u to distinguish between the angles in the down and up-quark sectors):

$$m_d \approx 2m_b s_{\theta_d}^2 c_{\theta_d}^2 (1 - \cos\delta_{12}^d), \quad (58)$$

$$m_s \approx 2m_{b'} s_{\eta_d}^2 c_{\eta_d}^2 (1 - \cos\delta_{34}^d), \quad (59)$$

$$m_u \approx 2m_t s_{\theta_u}^2 c_{\theta_u}^2 (1 - \cos\delta_{12}^u), \quad (60)$$

$$m_c \approx 2m_{t'} s_{\eta_u}^2 c_{\eta_u}^2 (1 - \cos\delta_{34}^u). \quad (61)$$

where [see Eq. (19) and set $\zeta = \pi/2$]:

$$m_b \approx \beta_d, \qquad m_{b'} \approx \alpha_d, \qquad m_t \approx \beta_u, \qquad m_{t'} \approx \alpha_u. \quad (62)$$

As expected, we cannot reproduce the physical light-quark mass spectrum if any of the phases δ_{ij} above vanishes. Note also that, since $\eta_u \sim \eta_d$ and $\theta_u \sim \theta_d$ [see Eqs. (34) and (36)], we can also use the expressions in Eqs. (58)–(61) for the light-quark mass terms to relate the phases in one sector to the phases in the other sector:

$$\frac{\delta_{12}^d}{\delta_{12}^u} \sim \sqrt{\frac{m_d m_t}{m_u m_b}} \sim 10, \quad (63)$$

$$\frac{\delta_{34}^d}{\delta_{34}^u} \sim \sqrt{\frac{m_s m_{t'}}{m_c m_{b'}}} \sim 0.3, \quad (64)$$

where we have taken $m_{t'}/m_{b'} \sim 1$.

Finally, for our chosen orientation with $\zeta = \pi/2$ and $\phi \ll 1$, the $P_{qq'}$ elements required to calculate the imaginary terms of the 4×4 CKM elements [see Eqs. (51)–(55)] are given by (to first order in δ_{ij}):

$$P_{db} = m_b s_{\theta_d} c_{\theta_d} \sin\delta_{12}^d, \qquad P_{sb'} = m_{b'} s_{\eta_d} c_{\eta_d} \sin\delta_{34}^d,$$
$$P_{ut} = m_t s_{\theta_u} c_{\theta_u} \sin\delta_{12}^u, \qquad P_{ct'} = m_{t'} s_{\eta_u} c_{\eta_u} \sin\delta_{34}^u, \quad (65)$$

and all other $P_{qq'}$ elements vanish. Using the expressions for the light-quark masses in Eqs. (58)–(61), we can reexpress the elements of the perturbation matrix $P_{qq'}$ in

Eq. (65) above in terms of the CP-phases and the light-quark masses:

$$P_{db} \approx \sqrt{m_d m_b}\cos(\tfrac{1}{2}\delta_{12}^d), \qquad P_{sb'} \approx \sqrt{m_s m_{b'}}\cos(\tfrac{1}{2}\delta_{34}^d),$$
$$P_{ut} \approx \sqrt{m_u m_t}\cos(\tfrac{1}{2}\delta_{12}^u), \qquad P_{ct'} \approx \sqrt{m_c m_{t'}}\cos(\tfrac{1}{2}\delta_{34}^u), \tag{66}$$

V. CP-INVARIANTS WITH FOUR GENERATIONS

As in the SM3, CP-violation in the SM4 can also be parametrized using CP-invariants a la the Jarlskog invariant $J_{\rm SM}$ of the SM3 [8]. Indeed, as was shown in [8], the invariant CP-violation measure in the four quark families case can be expressed in terms of four "copies" similar to $J_{\rm SM}$ (out of which only three are independent): J_{123}, J_{124}, J_{134}, and J_{234}, where the indices indicate the generation number, i.e., in this language one identifies $J_{\rm SM}$ with J_{123} even though these two CP-invariants are not quite the same as $J_{\rm SM}$ is no longer a valid CP-quantity in the SM4.

A generic derivation of the four J_{ijk} copies in terms of the quark masses and CKM mixing angles is quite complicated and we are unable to give it in a compact analytical format. There are several useful general formulations in the literature for the parametrization of CP-violation in the SM4 [8,16], but none is at the level of simplification required for an analytical study of CP-violation in our model. A numerical calculation/study of the CP-violating quantities in our model is, however, straight forward following the prescription of the previous sections. This will be presented elsewhere [15].

On the other hand, as was observed more than 10 years ago [17] and noted again recently in [4], in the chiral limit $m_{u,d,s,c} \to 0$, CP-violation in the SM4 effectively "shrinks" to the CP-violation picture of a three generation model involving the 4th generation heavy quarks. This chiral limit, which is in the spirit of our current study, is clearly applicable at high energies of the EW-scale and above. Moreover, it allows us to derive a compact analytical estimate for the expected size of CP-violation in our model.[2]

As was shown in [17], in the chiral limit there is no CP-violation within the three families SM3 and so all CP-violating effects are attributed to the new physics—in our case, to the fourth generation of quarks. The key CP-violating quantity in this limit can be written as [17]

$$J_{\rm SM4} = {\rm Im}(V_{tb}V_{t'b}^\star V_{t'b'}V_{tb'}^\star), . \tag{67}$$

since this is the only CP-violating quantity that survives when one takes the limit $m_{u,d,s,c} \to 0$.

Thus, in order to get some insight for the expected size of CP-violation in our model, it is sufficient to derive an estimate for $J_{\rm SM4}$. In particular, we will calculate $J_{\rm SM4}$ for the specific orientation used in the previous section, i.e., for the case $\zeta = \pi/2$ and $\phi \ll 1$.

Using the $P_{qq'}$ factors of Eq. (66) and based on Eq. (55), we can calculate (to $\mathcal{O}(\delta)$) the relevant complex CKM elements which enter $J_{\rm SM4}$ in Eq. (67):

$$V_{tb} \approx V_{tb}^0 + i\bigg[V_{td}^0\sqrt{\frac{m_d}{m_b}}\cos\bigg(\frac{1}{2}\delta_{12}^d\bigg) - V_{ub}^0\sqrt{\frac{m_u}{m_t}}\cos\bigg(\frac{1}{2}\delta_{12}^u\bigg)\bigg], \tag{68}$$

$$V_{t'b} \approx V_{t'b}^0 + i\bigg[V_{t'd}^0\sqrt{\frac{m_d}{m_b}}\cos\bigg(\frac{1}{2}\delta_{12}^d\bigg) - V_{cb}^0\sqrt{\frac{m_c}{m_{t'}}}\cos\bigg(\frac{1}{2}\delta_{34}^u\bigg)\bigg], \tag{69}$$

$$V_{tb'} \approx V_{tb'}^0 + i\bigg[V_{ts}^0\sqrt{\frac{m_s}{m_{b'}}}\cos\bigg(\frac{1}{2}\delta_{34}^d\bigg) - V_{ub'}^0\sqrt{\frac{m_u}{m_t}}\cos\bigg(\frac{1}{2}\delta_{12}^u\bigg)\bigg], \tag{70}$$

$$V_{t'b'} \approx V_{t'b'}^0 + i\bigg[V_{t's}^0\sqrt{\frac{m_s}{m_{b'}}}\cos\bigg(\frac{1}{2}\delta_{34}^d\bigg) - V_{cb'}^0\sqrt{\frac{m_c}{m_{t'}}}\cos\bigg(\frac{1}{2}\delta_{34}^u\bigg)\bigg]. \tag{71}$$

We can now estimate the size of CP-violation in our model, which can emanate in high-energy processes involving t' and b' exchanges. In particular, since the zeroth-order CKM elements are a good approximation for the magnitude of physical elements, we set $V_{ij}^0 \sim V_{ij}$ and use the results and relations obtained for the CKM elements in the previous sections [see Eqs. (28)–(40)]: $V_{tb} \sim 1$, $V_{t'b'} \sim V_{cs} \sim 1$, $V_{t'b} \sim V_{ub'} \times (V_{cb}/V_{cd})$ and $V_{tb'} \sim V_{ub'} \times (V_{ts}/V_{us})$. We then obtain

$$J_{\rm SM4} \approx V_{ub'}\frac{V_{ts}}{V_{us}}\bigg[V_{cb}\sqrt{\frac{m_c}{m_{t'}}}\cos\bigg(\frac{1}{2}\delta_{34}^u\bigg) - V_{ub'}\sqrt{\frac{m_d}{m_b}}\cos\bigg(\frac{1}{2}\delta_{12}^d\bigg)\bigg] + V_{ub'}\frac{V_{cb}}{V_{cd}} \times\bigg[V_{ub'}\sqrt{\frac{m_u}{m_t}}\cos\bigg(\frac{1}{2}\delta_{12}^u\bigg) - V_{ts}\sqrt{\frac{m_s}{m_{b'}}}\cos\bigg(\frac{1}{2}\delta_{34}^d\bigg)\bigg]. \tag{72}$$

[2]Note that, although their is no CP-violation in our model in the chiral limit $m_{u,d,s,c} \to 0$ (which is our zeroth-order approximation), we can use the CP-violating quantities obtained in [17] in this limit, since those are given in terms of the physical mixing angles. In our model, the imaginary parts of these mixing angles are proportional to the very small light-quark masses.

Setting $V_{cb} \sim -V_{ts} \sim A\lambda^2$ and $V_{ts}/V_{us} \sim V_{cb}/V_{cd} \sim -A\lambda$ and (consistent with their measured values [14], where $A \sim 0.81$ and $\lambda = 0.2257$ is the Wolfenstein parameter), and taking $V_{ub'} \sim V_{cb} \sim A\lambda^2$ and $m_{t'} \sim 2m_t$, $m_{b'} \sim m_{t'} - 55$ GeV, consistent with the electroweak precision tests [6,18], we obtain

$$|J_{\rm SM4}| \sim A^3\lambda^5 \times \left[\sqrt{\frac{m_u}{m_t}} + \sqrt{\frac{m_c}{m_{t'}}} - \sqrt{\frac{m_d}{m_b}} + \sqrt{\frac{m_s}{m_{b'}}}\right] \sim 10^{-5}, \tag{73}$$

where we have used $\cos(\delta^d_{12}/2) \sim \cos(\delta^d_{34}/2) \sim \cos(\delta^u_{12}/2) \sim \cos(\delta^u_{34}/2) \sim 1$ for the numerical estimate (see below). Indeed, with the above chosen values for the CKM elements and the 4th generation quark masses, all the four phases are fixed by the requirement that they reproduce the corresponding light-quark masses as given in Eqs. (58)–(61). In particular, according to Eqs. (58)–(61) and the relations between the hidden symmetry angles and the CKM elements as given by Eqs. (28)–(31), we have

$$\cos(\delta^u_{34}) \sim 1 - \frac{m_c}{2m_{t'}\frac{V^2_{ub'}}{V^2_{cd}}} \sim 0.945, \tag{74}$$

$$\cos(\delta^d_{12}) \sim 1 - \frac{m_d}{2m_b\frac{V^2_{cb}}{V^2_{cd}}} \sim 0.98, \tag{75}$$

$$\cos(\delta^d_{34}) \sim 1 - \frac{m_s}{2m_{b'}\frac{V^2_{ub'}}{V^2_{us}}} \sim 0.995, \tag{76}$$

$$\cos(\delta^u_{12}) \sim 1 - \frac{m_u}{2m_t\frac{V^2_{ts}}{V^2_{us}}} \sim 0.9998, \tag{77}$$

consistent with our perturbative description of CP-violation.

From Eq. (73) we see that as the CP-violating phases δ^d_{12}, $\delta^u_{34} \rightarrow 0$, both m_d and m_c approach zero and, therefore, also $J_{\rm SM4} \rightarrow 0$. Note also that, for our chosen orientation of the hidden symmetry, we have $J_{\rm SM4} \sim 10^{-5} \sim J_{\rm SM}$, i.e., the SM4 analogue of the SM3's Jarlskog invariant at high energies and the original measured SM3's Jarlskog invariant are of similar size. These results demonstrate the highly predictive power of our model for the description of CP-violation and the generation of the light-quark masses in the SM4. In particular, once the magnitude of the mixing angles and the masses of the 4th generation quarks are measured, our model gives a very distinct prediction for the expected size of CP-violation in the SM4, which can be directly confirmed at high-energy collider experiments. In a forthcoming paper [15], we will perform a full numerical study and scan the complete range of the free parameter space of our model, subject to the relevant existing data. We will also suggest ways to test our model in the upcoming LHC and the future machines such as a Super-B factory and the International Linear Collider.

VI. SUMMARY

Motivated by the recent hints of CP anomalies in the B-system and by the idea of Friedberg and Lee in [12], we have presented a new framework for CP-violation and the generation of the light-quark masses in the SM with four families—the SM4.

We have applied the basic ingredients of the FL mechanism to the SM4 case, by constructing an extended (double) hidden symmetry suitable for four families which defines the zeroth-order states in the up and down-quarks sectors and which ensures T-invariance. We then outlined the breaking mechanism of both the hidden symmetry and T-invariance in the SM4 case, from which we obtained the CP-violating measure and the physical states in this model. We have shown that this mechanism, when applied to the SM4, can be highly predictive and can be tested in future experiments. In particular, we gave one physically relevant example for the predictive power of our model by choosing a specific orientation of the hidden symmetry. This allowed us to analytically derive the physical (observed) quark states, and to give a prediction for the size of the mixing angles between the 4th generation and the 1st three generations of the SM3 and for the size of CP-violation associated with the 4th generation quarks.

A complete numerical study of our model, which explores the full phase-space of viable hidden symmetries for the SM4 and the corresponding range of the expected size of CP-violation and of the 4th generation mixing angles, is in preparation and will be presented in [15].

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