Revisiting $B \to \phi\pi$ decays in the standard model

Ying Li*
Department of Physics, Yantai University, Yantai 264005, People's Republic of China

Cai-Dian Lü and Wei Wang
Institute of High Energy Physics and Theoretical Physics Center for Science Facilities, CAS, P.O. Box 918(4), 100049, People's Republic of China
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In the standard model, we reinvestigate the rare decay $B \to \phi\pi$, which is viewed as an ideal probe to detect the new physics signals. We find that the tiny branching ratio in the naive factorization can be dramatically enhanced by the radiative corrections and the $\omega - \phi$ mixing effect, while the long-distance contributions are negligibly small. Assuming the Cabibbo-Kobayashi-Maskawa angle $\gamma = (58.6 \pm 10)°$ and the mixing angle $\theta = -(3.0 \pm 1.0)°$, we obtain the branching ratios of $B \to \phi\pi$ as $\mathrm{Br}(B^\pm \to \phi\pi^\pm) = (3.2^{+0.8-1.2}_{-0.7+1.8}) \times 10^{-8}$ and $\mathrm{Br}(B^0 \to \phi\pi^0) = (6.8^{+0.3-0.7}_{-0.3+1.0}) \times 10^{-9}$. If the future experiment reports a branching ratio of $(0.2–0.5) \times 10^{-7}$ for $B^- \to \phi\pi^-$ decay, it may not be a clear signal for any new physics scenario. In order to discriminate the large new physics contributions from those due to the $\omega - \phi$ mixing, we propose to measure the ratio of branching fractions of the charged and neutral B decay channel. We also study the direct CP asymmetries of these two channels: $(-8.0^{+0.9+1.5}_{-1.0-0.1})\%$ and $(-6.3^{-0.5+2.5}_{+0.7-2.5})\%$ for $B^\pm \to \phi\pi^\pm$ and $B^0 \to \phi\pi^0$, respectively. These asymmetries are dominated by the mixing effect.

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B meson decays provide valuable information on the flavor structure of the weak interactions so that they always are used to precisely test the standard model (SM) and to search for the possible signals of the new physics beyond the SM. Charmless two-body nonleptonic decay processes, such as $B \to \phi\pi$, are of great interests, since the branching ratios are very tiny in the SM. The experimentalists have reported the following measurements [1]:

$$\begin{aligned} \mathrm{BR}(B^- \to \phi\pi^-) &= (-0.04 \pm 0.17) \times 10^{-6}, \\ \mathrm{BR}(\bar{B}^0 \to \phi\pi^0) &= (0.12 \pm 0.13) \times 10^{-6}, \end{aligned} \tag{1}$$

while the upper bounds at 90% probability are given as

$$\mathrm{BR}\,(B^- \to \phi\pi^-) < 2.4 \times 10^{-7}, \tag{2}$$

$$\mathrm{BR}\,(\bar{B}^0 \to \phi\pi^0) < 2.8 \times 10^{-7}. \tag{3}$$

On the theoretical side, since these decay modes are absent from any annihilation diagram contribution, calculations of hadronic matrix elements are quite reliable, and these decays have been analyzed in the SM by different groups [2,3]. In the SM, these channels are highly suppressed for several reasons listed as follows. First, at the quark level, these decays proceed via $b \to d\bar{s}s$, which is a flavor changing neutral current process. The flavor changing neutral current transition is induced by the loop effects and the relevant Wilson coefficients are very small. Second, the Cabibbo-Kobayashi-Maskawa (CKM) matrix element for this transition $V_{tb}V_{td}^*$ is tiny. Finally, in order to produce a ϕ meson from the vacuum, at least three gluons are required which suppresses these channels further. Feynmann diagrams for these decays are often referred to as the hairpin diagram (the last reference in Ref. [2]), which is shown in Fig. 1. Because of the tiny branching ratio in the SM, $B \to \phi\pi$ decay is usually considered as an ideal place to search for the possible new physics scenarios [4].

However, before we turn to the new physics scenario, it is logical to investigate all possible contributions in the SM: contributions in the naive factorization, radiative corrections (vertex corrections and the hard spectator diagram), long-distance contributions such as rescattering from $B \to KK^*$ decays, and contributions due to the ω-ϕ mixing. The motif of this paper is to investigate the possibility of the enhancement of $B \to \phi\pi$ decays in the SM.

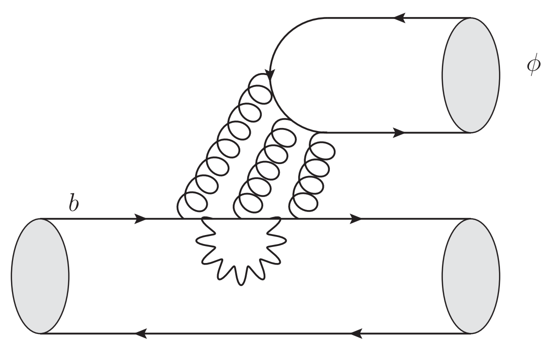

FIG. 1. Hairpin diagrams for $B \to \phi\pi$ decays.

*liying@ytu.edu.cn.

The $\Delta B = 1$ effective weak Hamiltonian in SM is given by [5]:

$$H_{\rm eff} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \Big(C_1 Q_1^p + C_2 Q_2^p + \sum_{i=3}^{10,7\gamma,8g} C_i Q_i \Big) + {\rm H.c.}, \tag{4}$$

where $\lambda_p = V_{pb} V_{pd}^*$. $Q_{1,2}^p$ are the left-handed current-current operators arising from the W-boson exchange, $Q_{3,\ldots,6}$ and $Q_{7,\ldots,10}$ are QCD and electroweak penguin operators, and $Q_{7\gamma}$ and Q_{8g} are the electromagnetic and chromomagnetic dipole operators, respectively. Their explicit expressions can be found in Ref. [5].

The physics above the scale m_b in the B meson weak decays have been incorporated into the Wilson coefficients of the effective Hamiltonian. The remanent task is to evaluate the matrix element of each four-quark operator. The simplest way is to decompose it into two simpler parts: one is the decay constant of the emitted meson; the other part is the B-to-light meson form factor. Both of these two parts can be directly extracted from the experimental data, or evaluated from some nonperturbative method such as the Lattice QCD and QCD sum rules. In the naive factorization [6], the decay amplitudes can be written as

$$A^{NF}_{B^-\to\pi^-\phi} = \sqrt{2} A^{NF}_{\bar{B}^0\to\pi^0\phi} = A_{\pi\phi} \sum_{p=u,c} \lambda_p \Big(a_3 + a_5 - \frac{1}{2} a_7 - \frac{1}{2} a_9 \Big), \tag{5}$$

where

$$A_{\pi\phi} = -i\sqrt{2} G_F m_\phi f_\phi (\epsilon_\phi^* \cdot p_B) F_+^{B\pi}(m_\phi^2), \tag{6}$$

and a_i is the Wilson coefficient combination as defined in Ref. [3]. In the naive factorization, the branching ratios are given as

$$\begin{aligned} {\rm Br}(B^\pm \to \phi\pi^\pm) &= 9.0 \times 10^{-10}, \\ {\rm Br}(B^0 \to \phi\pi^0) &= 4.1 \times 10^{-10}. \end{aligned} \tag{7}$$

In the calculation, we have used $f_\phi = 0.22$ GeV. The meson masses and the B meson lifetime are taken from [7]. Since the emitted ϕ meson is very light compared with the B meson, as a safe approximation we will use the value at the zero recoil point as $F_+^{B\pi} = 0.25$. The value of the CKM matrix elements used are taken from [7]

$$\begin{aligned} |V_{ub}| &= 0.0039, \qquad |V_{ud}| = 0.974, \\ |V_{cb}| &= 0.0422, \qquad |V_{cd}| = 0.226, \end{aligned} \tag{8}$$

and the phase γ associated with V_{ub} is 58.6°. Compared with Eqs. (2) and (3), we can see that the results in the naive factorization are far below the experimental upper bound. The tiny branching ratios are due to the cancellation of the Wilson coefficients C_3, C_4, C_5, C_6. This cancellation also reflects the fact that ϕ can only be produced by at least three gluons. The renormalization group evolved Wilson coefficients at the scale m_b contains the multiloop contributions above the scale m_b, which lead to small branching fractions of $B \to \phi\pi$ decays. Below this scale, the radiative corrections may provide sizable contributions. In the QCD factorization (QCDF) approach [8,9], the hadronic matrix elements of local operators Q_i can be written as

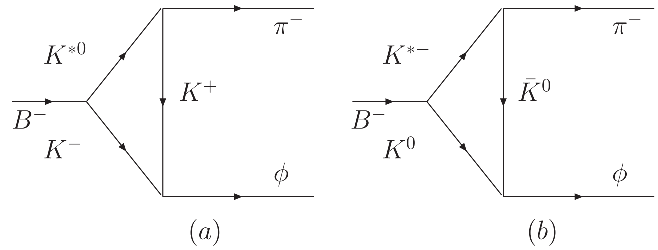

FIG. 2. Feynman diagrams of the final state interactions.

$$\begin{aligned} \langle \pi(p)\phi(q)|Q_i|\bar{B}(p)\rangle = {}& F_+^{B\to\pi} \int_0^1 dv T^I(v)\Phi_\phi(v) \\ & + \int_0^1 d\xi du dv T^{II}(\xi, u, v) \\ & \times \Phi_B(\xi)\Phi_\pi(u)\Phi_\phi(v), \end{aligned} \tag{9}$$

where ϕ_M ($M = \phi, \pi, B$) are light-cone distribution amplitudes of the meson M; T_i^I and T_i^{II} are hard scattering kernels. To be more specific, the Wilson coefficient combination $a_3 + a_5$ is replaced by the α_3^p, while $a_7 + a_9$ is replaced by the $\alpha_{3{\rm EW}}^p$, which has been defined in Ref. [9]. For the numerical evaluation, we use the input parameters as given in the QCD factorization approach [9]. In the first term on the right-hand side of Eq. (9), the form factor cannot be factorized and the perturbative coefficients are calculated at the scale $\mu \sim m_b$. The second term, which involves the hard scattering diagram and the annihilation diagram, can be factorized into the convolution of light-cone distribution amplitudes and the hard kernels. The perturbative coefficients are evaluated at the scale $\mu_h = \sqrt{\mu\Lambda_{\rm QCD}}$. In the vertex corrections, the factorization scale is chosen as $\mu = m_b/2 \sim 2.1$ GeV. The corresponding factorization scale in hard scattering and annihilation terms are $\mu_h \sim 1$ GeV. With these input parameters, branching ratios are obtained as

$$\begin{aligned} {\rm Br}(B^\pm \to \phi\pi^\pm) &= 1.1 \times 10^{-8}, \\ {\rm Br}(B^0 \to \phi\pi^0) &= 5.2 \times 10^{-9}. \end{aligned} \tag{10}$$

Compared with results in the naive factorization approach, we find that the branching ratios are enhanced by a factor of about 10. We should point out that decay amplitudes strongly depend on the factorization scale. For example, if we chose the factorization scale as $\mu = 2.1$ GeV for the hard scattering diagram, the branching ratio will be reduced by a factor of 3. The difference caused by the factorization scale characterizes the size of the subleading corrections for the hard scattering diagrams. Our results are larger by a factor of 2 than the results given in Ref. [9], where the central values of the scales are chosen as $\mu = m_b = 4.2$ GeV and $\mu_h = 1.45$ GeV. The sensitivity of branching ratios to choices of the Wilson coefficients can be eliminated by including the subleading order corrections in the future.

Apart from the perturbative contributions, $B \to \phi\pi$ decays also receive some nonperturbative corrections: $B \to K^{(*)}K^{(*)}$ then $K^{(*)}K^{(*)} \to \phi\pi$ through exchanging a $K^{(*)}$ meson, which is also called final state interaction (FSI). The diagrams of the FSI are shown in Fig. 2. In the $m_b \to \infty$ limit, the FSI is power suppressed and believed to be vanished. Since the b quark mass is limited, the FSI is not zero and the t-channel FSI has been modeled as the one-particle-exchange picture [10]. As an example, we will study the FSI effects from the $B^- \to K^{*-}K^0$ decays. The short distance contribution to the $B^- \to K^{*-}K^0$ is given as

$$A(B^- \to K^{*-}K^0) = -i\frac{G_F}{\sqrt{2}} f_K A_0^{BK^*}(2m_{K^*}\epsilon^*_{K^*}\cdot p_B) \times \sum_p \lambda_p\left[\alpha_4^p - \frac{1}{2}\alpha_{4,\mathrm{EW}}^p\right]. \tag{11}$$

The absorptive part of long-distance contribution to $B^- \to \phi\pi^-$ is given as

$$\begin{aligned} A_{\mathrm{abs}} = &-i\frac{G_F}{\sqrt{2}} f_K A_0^{BK^*}\sum_p \lambda_p\left[\alpha_4^p - \frac{1}{2}\alpha_{4,\mathrm{EW}}^p\right] \\ &\times \int_{-1}^{1}\frac{|\vec{p}_1| d\cos\theta}{16\pi m_B} 4m_{K^*} g_{K^*K\pi} g_{\phi KK} \\ &\times\left(-p_B\cdot p_3 + \frac{p_B\cdot p_1 p_1\cdot p_3}{m_{K^*}^2}\right) \\ &\times\frac{E_2|\vec{p}_4| - E_4|\vec{p}_2|\cos\theta}{m_\phi}\times\frac{F(t, m^2)}{t - m^2}, \end{aligned} \tag{12}$$

where p_1, p_2, p_3, p_4 denotes the momentum of the K^{*-}, K^0, π^-, ϕ mesons, respectively. θ is the angle between the momenta $\vec{p}_1$ and $\vec{p}_3$. The coupling constants $g_{\phi KK}$ and $g_{K^*K\pi}$ can be determined through the experimental data on $\phi \to KK$ and $K^* \to K\pi$ decays [7], and we get $g_{\phi KK} = 4.51$ and $g_{K^*K\pi} = 4.86$. Because of the small branching ratios (of order 10^{-7}) of $B \to KK^*$ decays [11], the long-distance contributions to $B \to \pi\phi$ decays are not expected to give sizable corrections. The numerical results also show that these contributions are negligibly small.

All the above investigations are based on the hypothesis that $\omega - \phi$ are ideally mixing: $\omega = \frac{u\bar{u}+d\bar{d}}{\sqrt{2}}$ and $\phi = s\bar{s}$. But generally, ω and ϕ can mix with each other via strong interactions. With the aid of a mixing angle θ, one can parametrize the $\omega - \phi$ mixing, so that the physical ω and ϕ are related to the two states $n\bar{n} = \frac{u\bar{u}+d\bar{d}}{\sqrt{2}}$ and $s\bar{s}$

$$\begin{pmatrix}\omega\\ \phi\end{pmatrix} = \begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}n\bar{n}\\ s\bar{s}\end{pmatrix}. \tag{13}$$

Recent studies within the chiral perturbative theory imply a mixing angle of $\theta = -(3.4 \pm 0.3)^\circ$ [12], while the most recent treatment implies an energy-dependent mixing which varies from -0.45° at the ω mass to -4.64° at the ϕ mass [13]. Although the $n\bar{n}$ component in the ϕ meson is tiny, it may sizably contribute to the branching ratio and direct CP violation parameters of the rare decays $B \to \phi\pi$ [14].

For the $n\bar{n}$ component of ϕ, both the emission and annihilation topologies contribute to the $B \to \phi\pi$ decays. Therefore, not only penguin operators but also tree operators should be taken into account. For the $n\bar{n}$ part, the decay amplitudes are given:

$$\begin{aligned}\sqrt{2}A^{n\bar{n}}_{B^-\to\pi^-\phi} = &A_{\pi\phi}\sum_{p=u,c}\lambda_p\left[\delta_{pu}(\alpha_2+\beta_2) + 2\alpha_3^p + \alpha_4^p + \frac{1}{2}\alpha_{3,\mathrm{EW}}^p - \frac{1}{2}\alpha_{4,\mathrm{EW}}^p + \beta_3^p + \beta_{3,\mathrm{EW}}^p\right] \\ &+ A_{\phi\pi}\sum_{p=u,c}\lambda_p\left[\delta_{pu}(\alpha_1+\beta_2) + \alpha_4^p + \alpha_{4,\mathrm{EW}}^p + \beta_3^p + \beta_{3,\mathrm{EW}}^p\right],\end{aligned} \tag{14}$$

$$\begin{aligned}-2A^{n\bar{n}}_{\bar{B}^0\to\pi^0\phi} = &A_{\pi\phi}\sum_{p=u,c}\lambda_p\left[\delta_{pu}(\alpha_2-\beta_1) + 2\alpha_3^p + \alpha_4^p + \frac{1}{2}\alpha_{3,\mathrm{EW}}^p - \frac{1}{2}\alpha_{4,\mathrm{EW}}^p + \beta_3^p - \frac{1}{2}\beta_{3,\mathrm{EW}}^p - \frac{3}{2}\beta_{4,\mathrm{EW}}^p\right] \\ &+ A_{\phi\pi}\sum_{p=u,c}\lambda_p\left[\delta_{pu}(-\alpha_2-\beta_1) + \alpha_4^p - \frac{3}{2}\alpha_{3,\mathrm{EW}}^p - \frac{1}{2}\alpha_{4,\mathrm{EW}}^p + \beta_3^p - \frac{1}{2}\beta_{3,\mathrm{EW}}^p - \frac{3}{2}\beta_{4,\mathrm{EW}}^p\right],\end{aligned} \tag{15}$$

where $A_{\phi\pi}$ and $A_{\phi\pi}$ are defined by:

$$\begin{aligned} A_{\pi\phi} &= -i\sqrt{2}G_F m_\phi F_+^{B\to\pi} f_\phi(\epsilon_\phi^* \cdot p_B); \\ A_{\phi\pi} &= -i\sqrt{2}G_F m_\phi A_0^{B\to\phi} f_\pi(\epsilon_\phi^* \cdot p_B), \end{aligned} \tag{16}$$

with the $B \to \phi$ form factor $A_0^{B\to\phi} = 0.28$ for the $\bar{n}n$ component. The Wilson coefficients α_i come from vertex corrections and hard spectator corrections, and β_i represent of contribution of annihilation diagrams, which can be found in Ref. [9].

The total amplitudes are given as

$$A_{B^-\to\pi^-\phi} = [A^{\rm QCDF}_{B^-\to\pi^-\phi} + iAbs(B^- \to \pi^-\phi)]\cos\theta + A^{n\bar{n}}_{B^-\to\pi^-\phi}\sin\theta, \tag{17}$$

$$A_{\bar{B}^0\to\pi^0\phi} = [A^{\rm QCDF}_{\bar{B}^0\to\pi^0\phi} + iAbs(\bar{B}^0 \to \pi^0\phi)]\cos\theta + A^{n\bar{n}}_{\bar{B}^0\to\pi^0\phi}\sin\theta. \tag{18}$$

If one adopts the mixing angle $\theta = -3°$, the branching ratios of $B \to \phi\pi$ are

$$\begin{aligned} {\rm Br}(B^- \to \phi\pi^-) &= 3.2 \times 10^{-8}, \\ {\rm Br}(B^0 \to \phi\pi^0) &= 6.8 \times 10^{-9}. \end{aligned} \tag{19}$$

Comparing with the results in Eq. (10), we found that the branching ratio of charged channel $B^- \to \phi\pi^-$ is enhanced remarkably. But the branching ratio of the neutral channel $B^0 \to \phi\pi^0$ is changed little. It can be understood as follows. $B^\pm \to \pi^\pm\phi(n\bar{n})$ is a color-allowed channel which has large decay amplitude. The experimentalists have measured the branching fraction of $B^\pm \to \pi^\pm\omega$ as

$${\rm Br}\,(B^\pm \to \pi^\pm\omega) = (6.9 \pm 0.5) \times 10^{-6}. \tag{20}$$

Thus the $B^- \to \phi\pi^-$ branching ratio can be enhanced about 1.7×10^{-8} purely from the mixing effect. On the contrary, $B^0 \to \pi^0\phi(n\bar{n})$ is a color-suppressed process, whose decay amplitude is much smaller than the color-allowed mode $B^\pm \to \pi^\pm\phi(n\bar{n})$. The mixing effect will not change the branching ratio of $B^0 \to \pi^0\phi$ remarkably. We give the dependence of branching ratios on the mixing angle θ and the CKM angle γ in Fig. 3. In the left diagram of Fig. 3, we set $\gamma = 58.6°$ and change θ from $-5°$ to zero; in the right part, $\theta = -3°$ and $\gamma \in (50°, 90°)$. Indicated in this diagram, the branching ratio of $B^- \to \phi\pi^-$ is sensitive to both θ and γ, whereas the $B^0 \to \phi\pi^0$ does not have this character.

Our results are below the experimental bound, but the branching ratio of $B^- \to \phi\pi^-$ can be enhanced to roughly 0.6×10^{-7} if the mixing angle is taken as $-4.64°$. This value is smaller than the upper bound only by a factor of 4. The branching ratio of order 10^{-7} is the signal for the new physics scenarios. If the future experiments report a branching ratio of $(0.2–0.5) \times 10^{-7}$, it may not be the signal for any new physics at all but may be caused by the mixing between ω and ϕ.

Since both the new physics effect and the mixing effect may give relatively large branching ratios, it is necessary to find a way to discriminate them. We propose a ratio R of branching fractions, which is defined as

$$R = \frac{{\rm Br}(B^- \to \phi\pi^-)}{{\rm Br}(B^0 \to \phi\pi^0)}\frac{\tau_{B^0}}{\tau_{B^-}} = \left|\frac{A_{B^-\to\phi\pi^-}}{A_{B^0\to\phi\pi^0}}\right|^2. \tag{21}$$

With some new physics effects, the $B \to \phi\pi$ decays can be enhanced, either by larger Wilson coefficients of the SM operators or through introducing new effective operators beyond. They will contribute to both $B^- \to \phi\pi^-$ and $\bar{B}^0 \to \phi\pi^0$ decays. In this case, the ratio R is identically 2. Considering the $\omega - \phi$ mixing, the ratio R is a function of the mixing angle θ. As stated above, the neutral channel is not changed sizably, whereas the charged decay $B^\pm \to \pi^\pm\phi$ is enhanced by the mixing effect, so the ratio R deviates from 2. Using $\gamma = (58.6 \pm 10)°$, we give the dependence of R on θ in Fig. 4, through which one can determine the mixing angle θ with the observable R. From

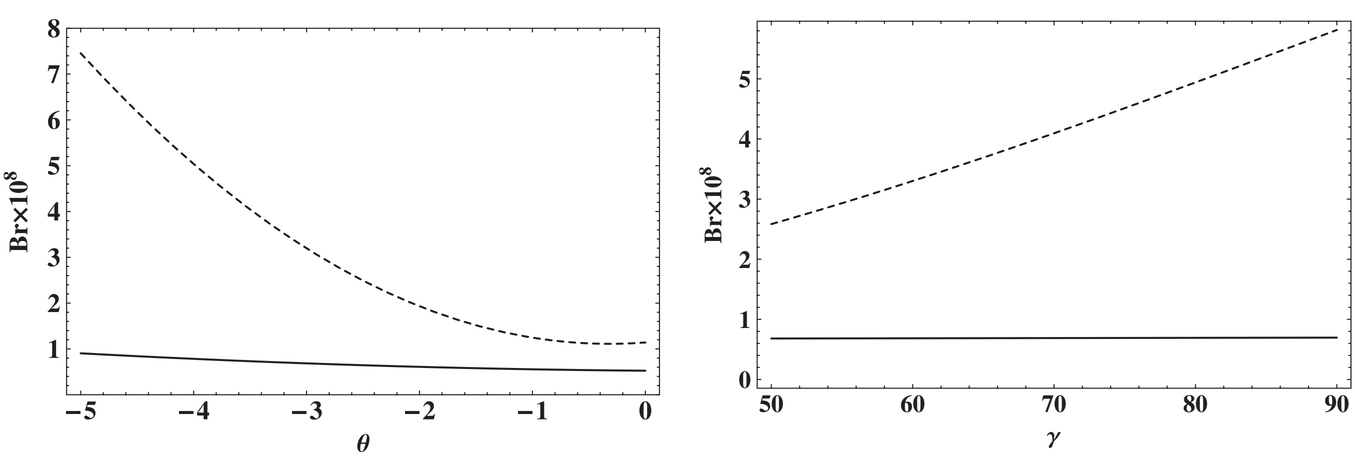

FIG. 3. Dependence of the CP averaged branching ratios on the mixing angle θ (left panel) and the CKM phase angle γ (right panel), where the dashed and solid lines correspond to the charged channel and neutral channel, respectively.

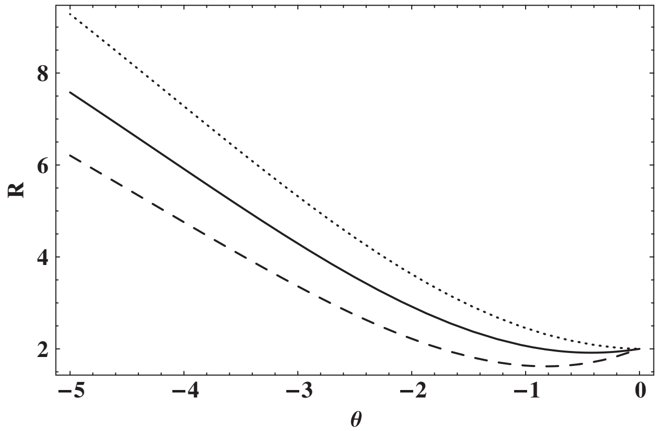

FIG. 4. Dependence of the ratio R on the mixing angle θ with $\gamma = (58.6 \pm 10)^\circ$. The solid line is the central value of γ, while the short dashed line and the long dashed line correspond to the upper limit and the lower limit, respectively.

this diagram, we can obtain $R = 4.3$ when $\theta = -3^\circ$ and $\gamma = 58.6^\circ$.

Another observable in B decays is the direct CP violation parameter, which is defined as

$$A_{CP} = \frac{\Gamma(B^- \to \phi\pi^-) - \Gamma(B^+ \to \phi\pi^+)}{\Gamma(B^- \to \phi\pi^-) + \Gamma(B^+ \to \phi\pi^+)}. \quad (22)$$

In order to have nonzero direct CP asymmetry, the decay amplitude needs to contain at least two interfering contributions with different strong and weak phases. Since only penguin operators contribute to this decay mode in the absence of $\omega - \phi$ mixing, the direct CP asymmetry turns out to be identically zero. In the mixing scenario, there is a small portion of the $u\bar{u}$ component in the ϕ meson, and tree operators contribute so that the direct CP asymmetries are nonzero. If $\theta < -3^\circ$, the contribution from the $n\bar{n}$ part dominate the $\phi\pi^-$ progressively, and the direct CP violation becomes stable as the magnitude of θ increases. Because $B^0 \to \pi^0\phi(n\bar{n})$ has a small amplitude, the direct CP of this decay mode comes from interference between tree contribution of $n\bar{n}$ and penguin from both $n\bar{n}$ and $s\bar{s}$, which makes the CP violation sensitive to mixing angle θ. With the definition in Eq. (22) and the mixing angle $\theta = -3^\circ$, the direct CP violation parameters are predicted in the QCDF approach as

$$A_{CP}(B^- \to \phi\pi^-) = -8.0\%,$$
$$A_{CP}(\bar{B}^0 \to \phi\pi^0) = -6.3\%. \quad (23)$$

In the left part of Fig. 5, we illustrate the dependence of A_{CP} on the mixing angle θ. In the right part of the Fig. 5, we set $\theta = -3^\circ$, and draw the relation between A_{CP} and the CKM angle γ.

In the following, we will briefly analyze the potential uncertainties to the calculation of $B \to \phi\pi$. If ϕ is purely made of the $\bar{s}s$ component, the decay $B \to \phi\pi$ is free from annihilation diagrams and the CKM matrix element $V_{tb}V_{ts}$ is well constrained. Then the dominant uncertainties are from the form factors and Wilson coefficients. The form factors can be well constrained by various nonleptonic channels and semileptonic B decays, while the uncertainties in Wilson coefficients (different factorization scales) have been shown to be large in the above.

Tree operators give dominant contributions to the decay amplitude of $B^- \to \pi^-\phi(\bar{n}n)$, which are comparable with the penguins in decay amplitudes of $B \to \pi\phi(\bar{s}s)$. Thus the major uncertainties to the branching ratio of $B^- \to \pi^-\phi$ are not very large, since contributions from tree operators are also under control. If the mixing angle is very close to 0, the direct CP asymmetry is induced by the interference between the tree amplitudes in $B^- \to \pi^-\phi(\bar{n}n)$ and the penguins in $B^- \to \pi^-\phi(\bar{s}s)$. Penguins in $B \to \pi\phi(\bar{s}s)$ will not give any difference to the two terms proportional to V_u and V_c. Thus the direct CP asymmetry will not have large uncertainties, which is about 10%. If the mixing

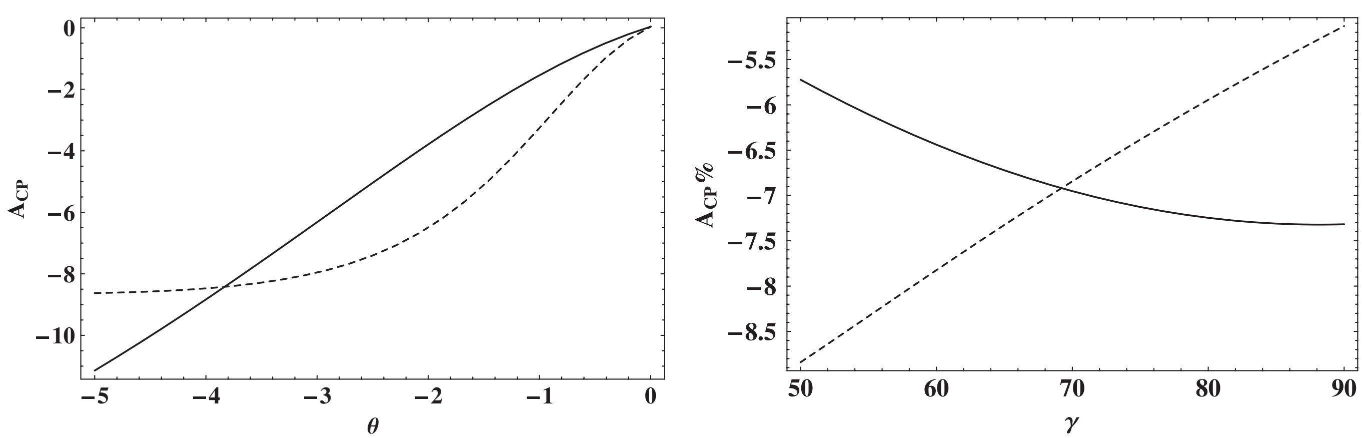

FIG. 5. Dependence of the direct CP asymmetries (in units of percent) on the mixing angle θ (left panel) and the CKM phase angle γ (right panel), where dashed lines and the solid lines correspond to charged channel and neutral channel, respectively.

angle sizably deviates from 0, all observables in $B^- \to \pi^- \phi$ decay will be directly constrained by the $B^- \to \pi^- \omega$ channel. In both cases, the direct CP asymmetry in $B^- \to \pi^- \phi$ can be well predicted.

For $B^0 \to \pi^0 \phi(\bar{n}n)$, tree operators are color suppressed, thus the whole evaluation suffers from large potential uncertainties. But fortunately, we can use the available experimental data [11]

$$\mathrm{Br}\,(B^0 \to \pi^0 \omega) < 5 \times 10^{-7} \tag{24}$$

to constrain the uncertainty. Even if one assumes the branching fraction of $B^0 \to \pi^0 \omega$ is reported as 5×10^{-7} in the future, the $B^0 \to \pi^0 \phi$ will receive a branching ratio of 15×10^{-10} purely from the $\omega - \phi$ mixing effect. As we can see, it is much smaller than the one from the emission diagram. The resultant branching ratio will not be changed too much. But the direct CP asymmetry, which is related to the size of color-suppressed tree contribution, receives large uncertainties, as the color-suppressed tree operators may induce large strong phases inferred from the $B \to \pi\pi$ data [11]. These uncertainties to the direct CP asymmetry may be constrained by the precise data for nonleptonic $B \to VP$ decays in the future.

Using the CKM angle $\gamma = (58.6 \pm 10)^\circ$ and the mixing angle $\theta = -(3.0 \pm 1.0)^\circ$, we get the results

$$\begin{aligned}\mathrm{Br}(B^\pm \to \phi\pi^\pm) &= (3.2^{+0.8-1.2}_{-0.7+1.8}) \times 10^{-8},\\ \mathrm{Br}(B^0 \to \phi\pi^0) &= (6.8^{+0.3-0.7}_{-0.3+1.0}) \times 10^{-9};\end{aligned} \tag{25}$$

$$\begin{aligned}A_{CP}(B^\pm \to \phi\pi^\pm) &= (-8.0^{+0.9+1.5}_{-1.0-0.1})\%,\\ A_{CP}(B^0 \to \phi\pi^0) &= (-6.3^{-0.5+2.5}_{+0.7-2.5})\%;\end{aligned} \tag{26}$$

$$R = 4.3^{+1.0-1.4}_{-0.9+1.6}. \tag{27}$$

The first uncertainties are from the γ angle and the second uncertainties are from the mixing angle θ. It is worthwhile to point out that the direct CP asymmetry of $B^0 \to \phi\pi^0$ may receive potentially larger uncertainties.

Our analysis can be directly generalized to other similar channels such as $B \to \phi\rho$ decays, though there are several differences between $B \to \phi\rho$ and $B \to \phi\pi$ decays. The contributions from the mixing mechanism are larger, since the branching ratio of $B^- \to \omega\rho^-$ is larger than that of $B^- \to \omega\pi^-$ (in units of 10^{-6}): $\mathrm{Br}(B^- \to \omega\rho^-) = (10.6^{+2.6}_{-2.3}) > \mathrm{Br}(B^- \to \omega\pi^-) = (6.9 \pm 0.5)$ [11]. The transverse polarization of $B \to \phi\rho$ also receives sizable contributions from the dipole operator $O_{7\gamma}$ [15].

Because of tiny branching ratios in the SM, the authors in Refs. [4] argued that the decay mode $B \to \phi\pi$ is a good place for probing the new physics effect. In the present paper, we have studied several contributions to $B \to \pi\phi$ decays in the SM. We find that the small branching fraction, expected in the naive factorization approach, can be remarkably enhanced by the radiative corrections and the $\omega - \phi$ mixing mechanism. The final results for the branching ratio of $B^- \to \pi^-\phi$ are smaller than the present upper limit by a factor of 4–20. We conclude that the observation of this channel with the branching ratio of roughly 0.5×10^{-7} may not be a clear signal for the new physics effects. On the contrary, that may be induced by the $\omega - \phi$ mixing. In order to discriminate the two different contributions, we propose to measure the ratio R of the branching fractions in the future. The contributions from the $\omega - \phi$ mixing effect also provide nontrivial strong phases, which potentially results in large direct CP asymmetries. These results can be tested on future experiments.

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