PHYSICAL REVIEW D **80,** 014023 (2009)

Vector meson and heavy meson strong interaction

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(Received 8 June 2009; published 29 July 2009)

We calculate the coupling constants between the light vector mesons and heavy mesons within the framework of the light-cone QCD sum rule in the leading order of heavy quark effective theory. The sum rules are very stable with the variations of the Borel parameter and the continuum threshold. The extracted couplings will be useful in the study of the possible heavy meson molecular states. They may also helpful in the interpretation of the proximity of $X(3872)$, $Y(4260)$, and $Z(4430)$ to the threshold of two charmed mesons through the couple-channel mechanism.

DOI: 10.1103/PhysRevD.80.014023 PACS numbers: 12.39.Hg, 12.38.Lg

I. INTRODUCTION

A number of hadronic states, which can not be easily accommodated in the conventional quark model, have been observed experimentally in recent years, such as $X(3872)$ [1], $Y(4260)$ [2], and $Z^+(4430)$ [3]. Their masses are very close to the thresholds of $D\bar{D}^*$, $D^*\bar{D}^*$, and $D^*\bar{D}_1$, respectively. It was speculated that the coupled-channel effect may play an important role because of the attraction between these D mesons. Alternatively, they were considered to be possible candidates of the heavy molecular states composed of two D mesons. These loosely bound states are formed by exchanging light mesons such as π, σ, ρ, and ω, etc. Up until now, the pion heavy meson strong interaction is relatively known due to chiral symmetry. However, the vector meson heavy meson strong interaction has not be extensively studied yet, which accounts for the relatively short distance interaction between two heavy mesons.

Heavy quark effective theory (HQET) [4] is a systematic approach to study the spectra and transition amplitudes of heavy hadrons. In HQET, the expansion is performed in terms of $1/m_Q$, where m_Q is the mass of the heavy quark involved. In the limit $m_Q \to \infty$, heavy hadrons form a series of degenerate doublets due to heavy quark symmetry. The two states in a doublet share the same quantum number j_l, the angular momentum of the light components. The $B(D)$ meson doublets $(0^-, 1^-)$, $(0^+, 1^+)$, and $(1^+, 2^+)$ are conventionally denoted as H, S, and T.

Light-cone QCD sum rules (LCQSR) [5] is a very useful nonperturbative approach to determine various hadronic transition form factors. One considers the T product of the two interpolating currents sandwiched between the vacuum and an hadronic state in this framework. Now the operator product expansion is performed near the light-cone rather than at a small distance as in the conventional QCD sum rules [6]. The double Borel transformation is always invoked to suppress the excited state and the continuum contribution.

The ρ coupling constant between D and D^* was calculated with LCQSR in full QCD in Ref. [7]. The couplings $g_{H^*H^*\rho}$, $f_{H^*H^*\rho}$, $g_{HH\rho}$, and $f_{H^*H\rho}$ were calculated in full QCD in Ref. [8]. Their values in the limit $m_Q \to \infty$ are also discussed in this paper. The ρ coupling between doublets T and H are studied in the leading order of HQET in Ref. [9].

In this work, we use LCQSR to calculate the ρ coupling constants between three doublets H, S, T and within the two doublets H, S. Because of the covariant derivative in the interpolating currents of the T doublet, the contribution from the 3-particle light-cone distribution amplitudes of the ρ meson has to be included when dealing with the ρ decay between doublets T and $H(S)$. We work in HQET to differentiate the two states with the same J^P value yet quite different decay widths. The interpolating currents $J^{\alpha_1\cdots\alpha_j}_{j,P,j_l}$ adopted in our work have been properly constructed in Ref. [10]. They satisfy

$$\langle 0|J^{\alpha_1\cdots\alpha_j}_{j,P,j_l}(0)|j', P', j_l'\rangle = f_{Pj_l}\delta_{jj'}\delta_{PP'}\delta_{j_lj_l'}\eta^{\alpha_1\cdots\alpha_j}, \tag{1}$$

$$\begin{aligned} &i\langle 0|T\{J^{\alpha_1\cdots\alpha_j}_{j,P,j_l}(x)J^{\dagger\beta_1\cdots\beta_{j'}}_{j,P,j_l}(0)\}|0\rangle \\ &= \delta_{jj'}\delta_{PP'}\delta_{j_lj_l'}(-1)^j\mathcal{S}g_t^{\alpha_1\beta_1}\cdots g_t^{\alpha_j\beta_j} \\ &\quad\times\int dt\delta(x-vt)\Pi_{P,j_l}(x), \end{aligned} \tag{2}$$

in the limit $m_Q \to \infty$. Here $\eta^{\alpha_1\cdots\alpha_j}$ is the polarization tensor for the spin j state, v is the velocity of the heavy quark, $g_t^{\alpha\beta} = g^{\alpha\beta} - v^\alpha v^\beta$, $\mathcal{S}$ denotes symmetrizing the indices and subtracting the trace terms separately in the sets $(\alpha_1\cdots\alpha_j)$ and $(\beta_1\cdots\beta_j)$.

II. SUM RULES FOR THE ρ COUPLING CONSTANTS

We shall perform the calculation to the leading order of HQET. According to Ref. [10], the interpolating currents for doublets H, S, and T read as

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$$J^{\dagger}_{0,-,(1/2)} = \sqrt{\tfrac{1}{2}}\bar{h}_v\gamma_5 q, \tag{3}$$

$$J^{\dagger\alpha}_{1,-,(1/2)} = \sqrt{\tfrac{1}{2}}\bar{h}_v\gamma^\alpha_t q, \tag{4}$$

$$J^{\dagger}_{0,+,(1/2)} = \sqrt{\tfrac{1}{2}}\bar{h}_v q, \tag{5}$$

$$J^{\dagger\alpha}_{1,+,(1/2)} = \sqrt{\tfrac{1}{2}}\bar{h}_v\gamma_5\gamma^\alpha_t q, \tag{6}$$

$$J^{\dagger\alpha}_{1,+,(3/2)} = \sqrt{\tfrac{3}{4}}\bar{h}_v\gamma_5(-i)\{\mathcal{D}^\alpha_t - \tfrac{1}{3}\gamma^\alpha_t\hat{\mathcal{D}}_t\}q, \tag{7}$$

$$J^{\dagger\alpha_1\alpha_2}_{2,+,(3/2)} = \sqrt{\tfrac{1}{8}}\bar{h}_v(-i)\{\gamma^{\alpha_1}_t\mathcal{D}^{\alpha_2}_t + \gamma^{\alpha_2}_t\mathcal{D}^{\alpha_1}_t - \tfrac{2}{3}g^{\alpha_1\alpha_2}_t\hat{\mathcal{D}}_t\}q, \tag{8}$$

where h_v is the heavy quark field in HQET, $\gamma^\mu_t \equiv \gamma^\mu - \hat{v}v^\mu$, $\mathcal{D}^\mu_t \equiv \mathcal{D}^\mu - (\mathcal{D}\cdot v)v^\mu$, $g^{\mu\nu}_t \equiv g^{\mu\nu} - v^\mu v^\nu$, and v^μ is the velocity of the heavy quark.

We consider the ρ decay of T_1 to H_1 to illustrate our calculation. Here the subscript of $T(H)$ indicates the spin of the meson involved. Owing to the conservation of the angular momentum of light components in the limit $m_Q \to \infty$, there are three independent ρ coupling constants between doublets T and H. We denote them as $g^{s1}_{TH\rho}$, $g^{d1}_{TH\rho}$, and $g^{d2}_{TH\rho}$, where $s, d,$ and the number following them indicates the orbital and total angular momentum (l, j_h) of the final ρ meson, respectively. All of these three coupling constants appear in the decay process under consideration. The decay amplitude can now be written as

$$\begin{aligned}\mathcal{M}(T_1 \to H_1 + \rho) &= Ii\{\epsilon^{\eta\epsilon^* e^* v}g^{s1}_{T_1H_1\rho} + [\epsilon^{\eta\epsilon^* qv}(e^*\cdot q_t) - \tfrac{1}{3}\epsilon^{\eta\epsilon^* e^* v}q_t^2]g^{d1}_{T_1H_1\rho} \\ &\quad + [\epsilon^{\eta e^* qv}(\epsilon^*\cdot q_t) + \epsilon^{\epsilon^* e^* qv}(\eta\cdot q_t)]g^{d2}_{T_1H_1\rho}\},\end{aligned} \tag{9}$$

where η, ϵ^*, and e^* denote the polarization vector of T_1, H_1, and ρ, respectively, q is the momentum of the ρ meson, $q^2 = m^2_\rho$, and $q^\mu_t \equiv q^\mu - (q\cdot v)v^\mu$. $I = 1, 1/\sqrt{2}$ for the charged and neutral ρ meson, respectively. The vector notations in the Levi-Civita tensor come from an index contraction between the Levi-Civita tensor and the vectors, for example, $\epsilon^{\eta\epsilon^* e^* v} \equiv \epsilon^{\mu\nu\rho\sigma}\eta_\mu\epsilon^*_\nu e^*_\rho v_\sigma$.

To obtain the sum rules for the coupling constants $g^{s1}_{T_1H_1\rho}$, $g^{d1}_{T_1H_1\rho}$, and $g^{d2}_{T_1H_1\rho}$, we consider the correlation functions

$$\begin{aligned}&\int d^4x e^{-ik\cdot x}\langle\rho(q)|T\{J^\beta_{1,-,(1/2)}(0)J^{\dagger\alpha}_{1,+,(3/2)}(x)\}|0\rangle \\ &= Ii\Big\{\epsilon^{\alpha\beta e^* v}G^{s1}_{T_1H_1\rho}(\omega,\omega') \\ &\quad + \Big[\epsilon^{\alpha\beta qv}(e^*\cdot q_t) - \frac{1}{3}\epsilon^{\alpha\beta e^* v}q_t^2\Big]G^{d1}_{T_1H_1\rho}(\omega,\omega') \\ &\quad + [\epsilon^{\alpha e^* qv}q^\beta_t + \epsilon^{\beta e^* qv}q^\alpha_t]G^{d2}_{T_1H_1\rho}(\omega,\omega')\Big\},\end{aligned} \tag{10}$$

where $\omega \equiv 2v\cdot k$, $\omega' \equiv 2v\cdot(k-q)$. In the leading order of HQET, the heavy quark propagator reads as

$$\langle 0|T\{h_v(0)\bar{h}_v(x)\}|0\rangle = \frac{1+\hat{v}}{2}\int dt\delta^4(-x-vt). \tag{11}$$

The correlation function can now be expressed as

$$\begin{aligned}&-\sqrt{\frac{3}{8}}\int dx e^{-ik\cdot x}\int_0^\infty dt\delta(-x-vt) \\ &\times \mathrm{Tr}\Big\{\gamma^\beta_t\frac{1+\hat{v}}{2}(-i\gamma_5)\Big(\mathcal{D}^\alpha_t - \frac{1}{3}\gamma^\alpha_t\hat{\mathcal{D}}_t\Big) \\ &\times\langle\rho(q)|q(x)\bar{q}(0)|0\rangle\Big\}.\end{aligned} \tag{12}$$

It can be further calculated using the light-cone wave functions of the ρ meson. To our approximation, we need the 2- and 3-particle light-cone wave functions. Their definitions are collected in Appendix B.

At the hadron level, the G's in (10) has the following pole terms:

$$\begin{aligned}G_{T_1H_1\rho}(\omega,\omega') &= \frac{f_{-,1/2}f_{+,3/2}g_{T_1H_1\rho}}{(2\bar{\Lambda}_{-,1/2}-\omega')(2\bar{\Lambda}_{+,3/2}-\omega)} \\ &\quad + \frac{c}{2\bar{\Lambda}_{-,1/2}-\omega'} + \frac{c'}{2\bar{\Lambda}_{+,3/2}-\omega},\end{aligned} \tag{13}$$

where $\bar{\Lambda}_{-,1/2} \equiv m_H - m_Q$, $\bar{\Lambda}_{+,3/2} \equiv m_T - m_Q$, $f_{-,1/2}$, etc. are the overlap amplitudes of their interpolating currents with the heavy mesons.

$G_{T_1H_1\rho}(\omega,\omega')$ can now be expressed by the ρ meson light-cone wave functions. After the Wick rotation and the double Borel transformation with ω and ω', the single-pole terms in (13) are eliminated. We arrive at

$$
\begin{aligned}
&\sqrt{6}g^{s1}_{T_1H_1\rho}f_{-,(1/2)}f_{+,(3/2)}e^{-((\bar{\Lambda}_{+,3/2}+\bar{\Lambda}_{-,1/2})/T)}\\
&=\frac{1}{3}f_\rho^T m_\rho^4 h_\parallel^{s[1]}(\bar{u}_0)\frac{1}{T}-\frac{1}{3}f_\rho^T m_\rho^4 (uh_\parallel^s)^{[1]}(\bar{u}_0)\frac{1}{T}-\frac{2}{3}f_\rho^T m_\rho^4\mathcal{S}^{[-1,0]}(u_0)\frac{1}{T}+\frac{1}{24}f_\rho^T m_\rho^4 A_T(\bar{u}_0)\frac{1}{T}-\frac{1}{24}f_\rho^T m_\rho^4 A_T(\bar{u}_0)\bar{u}_0\frac{1}{T}\\
&\quad+\frac{2}{3}f_\rho^T m_\rho^4 B_T^{[3]}(\bar{u}_0)\frac{1}{T}-\frac{1}{4}f_\rho^T m_\rho^2 C_T^{[1]}(\bar{u}_0)Tf_0\left(\frac{\omega_c}{T}\right)-\frac{1}{4}f_\rho^T m_\rho^2 h_\parallel^s(\bar{u}_0)Tf_0\left(\frac{\omega_c}{T}\right)-\frac{1}{12}f_\rho^T m_\rho^2 h_\parallel^{s(1)}(\bar{u}_0)Tf_0\left(\frac{\omega_c}{T}\right)\\
&\quad+\frac{1}{12}f_\rho^T m_\rho^2 (uh_\parallel^s)'(\bar{u}_0)Tf_0\left(\frac{\omega_c}{T}\right)+\frac{1}{3}f_\rho^T m_\rho^2\mathcal{S}^{[1,0]}(u_0)Tf_0\left(\frac{\omega_c}{T}\right)-\frac{1}{6}f_\rho^T m_\rho^2\varphi_\perp(\bar{u}_0)Tf_0\left(\frac{\omega_c}{T}\right)+\frac{1}{6}f_\rho^T m_\rho^2\varphi_\perp(\bar{u}_0)\bar{u}_0Tf_0\left(\frac{\omega_c}{T}\right)\\
&\quad-\frac{1}{96}f_\rho^T m_\rho^2 A_T^{(2)}(\bar{u}_0)Tf_0\left(\frac{\omega_c}{T}\right)+\frac{1}{96}f_\rho^T m_\rho^2 (uA_T)^{(2)}(\bar{u}_0)Tf_0\left(\frac{\omega_c}{T}\right)-\frac{1}{6}f_\rho^T m_\rho^2 B_T^{[1]}(\bar{u}_0)Tf_0\left(\frac{\omega_c}{T}\right)+\frac{1}{24}f_\rho^T\varphi_\perp^{(2)}(\bar{u}_0)T^3f_2\left(\frac{\omega_c}{T}\right)\\
&\quad-\frac{1}{24}f_\rho^T(\varphi_\perp u)^{(2)}(\bar{u}_0)T^3f_2\left(\frac{\omega_c}{T}\right)+\frac{1}{3}f_\rho^T m_\rho^4\mathcal{T}^{[-1,0]}(u_0)\frac{1}{T}-\frac{4}{3}f_\rho^T m_\rho^4\mathcal{T}_2^{[-1,0]}(u_0)\frac{1}{T}+\frac{1}{12}f_\rho^T m_\rho^2\mathcal{T}^{[1,0]}(u_0)Tf_0\left(\frac{\omega_c}{T}\right)\\
&\quad+\frac{1}{6}f_\rho^T m_\rho^2\mathcal{T}_2^{[1,0]}(u_0)Tf_0\left(\frac{\omega_c}{T}\right)-\frac{2}{3}f_\rho^T m_\rho^4\mathcal{T}_3^{[-1,0]}(u_0)\frac{1}{T}-\frac{1}{6}f_\rho^T m_\rho^2\mathcal{T}_3^{[1,0]}(u_0)Tf_0\left(\frac{\omega_c}{T}\right)+\frac{1}{12}f_\rho m_\rho^5 A^{[2]}(\bar{u}_0)\frac{1}{T^2}\\
&\quad-\frac{1}{12}f_\rho m_\rho^5 A^{[1]}(\bar{u}_0)\frac{1}{T^2}+\frac{1}{12}f_\rho m_\rho^5 (uA)^{[1]}(\bar{u}_0)\frac{1}{T^2}+\frac{4}{3}f_\rho m_\rho^5 C^{[4]}(\bar{u}_0)\frac{1}{T^2}-\frac{2}{3}f_\rho m_\rho^5 C^{[3]}(\bar{u}_0)\frac{1}{T^2}+\frac{2}{3}f_\rho m_\rho^5 (uC)^{[3]}(\bar{u}_0)\frac{1}{T^2}\\
&\quad+\frac{1}{24}f_\rho m_\rho^3 A(\bar{u}_0)+\frac{1}{48}f_\rho m_\rho^3 A'(\bar{u}_0)-\frac{1}{48}f_\rho m_\rho^3 (uA)'(\bar{u}_0)-\frac{1}{3}f_\rho m_\rho^3\mathcal{A}^{[0,0]}(u_0)+\frac{1}{6}f_\rho m_\rho^3 C^{[2]}(\bar{u}_0)+\frac{1}{6}f_\rho m_\rho^3 C^{[1]}(\bar{u}_0)\\
&\quad-\frac{1}{6}f_\rho m_\rho^3 (uC)^{[1]}(\bar{u}_0)+\frac{1}{12}f_\rho m_\rho^3 g_\perp^{(a)[1]}(\bar{u}_0)-\frac{1}{12}f_\rho m_\rho^3 g_\perp^{(a)}(\bar{u}_0)+\frac{1}{12}f_\rho m_\rho^3 g_\perp^{(a)}(\bar{u}_0)\bar{u}_0+\frac{1}{3}f_\rho m_\rho^3 g_\perp^{(v)[2]}(\bar{u}_0)\\
&\quad-\frac{1}{12}f_\rho m_\rho\mathcal{A}^{[2,0]}(u_0)T^2f_1\left(\frac{\omega_c}{T}\right)+\frac{1}{2}f_\rho m_\rho^3\mathcal{V}^{[0,0]}(u_0)+\frac{1}{6}f_\rho m_\rho\mathcal{V}^{[2,0]}(u_0)T^2f_1\left(\frac{\omega_c}{T}\right)-\frac{1}{3}f_\rho m_\rho^3\varphi_\parallel^{[2]}(\bar{u}_0)\\
&\quad+\frac{1}{3}f_\rho m_\rho^3\varphi_\parallel^{[1]}(\bar{u}_0)-\frac{1}{3}f_\rho m_\rho^3 (u\varphi_\parallel)^{[1]}(\bar{u}_0)+\frac{4}{3}f_\rho m_\rho^5\Psi^{[-2,0]}(u_0)\frac{1}{T^2}+\frac{1}{6}f_\rho m_\rho^3\mathcal{V}^{[0,0]}(u_0)-\frac{1}{3}f_\rho m_\rho^3\Psi^{[0,0]}(u_0)\\
&\quad+\frac{4}{3}f_\rho m_\rho^5\tilde{\Phi}^{[-2,0]}(u_0)\frac{1}{T^2}-\frac{1}{3}f_\rho m_\rho^3\tilde{\Phi}^{[0,0]}(u_0)+\frac{1}{24}f_\rho m_\rho g_\perp^{(a)(1)}(\bar{u}_0)T^2f_1\left(\frac{\omega_c}{T}\right)+\frac{1}{48}f_\rho m_\rho g_\perp^{(a)(2)}(\bar{u}_0)T^2f_1\left(\frac{\omega_c}{T}\right)\\
&\quad-\frac{1}{48}f_\rho m_\rho (g_\perp^{(a)}u)^{(2)}(\bar{u}_0)T^2f_1\left(\frac{\omega_c}{T}\right)+\frac{1}{6}f_\rho m_\rho g_\perp^{(v)}(\bar{u}_0)T^2f_1\left(\frac{\omega_c}{T}\right)-\frac{1}{6}f_\rho m_\rho\varphi_\parallel(\bar{u}_0)T^2f_1\left(\frac{\omega_c}{T}\right)\\
&\quad-\frac{1}{12}f_\rho m_\rho\varphi_\parallel'(\bar{u}_0)T^2f_1\left(\frac{\omega_c}{T}\right)+\frac{1}{12}f_\rho m_\rho (u\varphi_\parallel)'(\bar{u}_0)T^2f_1\left(\frac{\omega_c}{T}\right), \qquad (14)
\end{aligned}
$$

$$
\begin{aligned}
&\sqrt{6}g^{d1}_{T_1H_1\rho}f_{-,(1/2)}f_{+,(3/2)}e^{-((\bar{\Lambda}_{+,3/2}+\bar{\Lambda}_{-,1/2})/T)}\\
&=-f_\rho^T m_\rho^2 h_\parallel^{s[1]}(\bar{u}_0)\frac{1}{T}+f_\rho^T m_\rho^2 (uh_\parallel^s)^{[1]}(\bar{u}_0)\frac{1}{T}+2f_\rho^T m_\rho^2\mathcal{S}^{[-1,0]}(u_0)\frac{1}{T}+\frac{1}{16}f_\rho^T m_\rho^2 A_T(\bar{u}_0)\frac{1}{T}-\frac{1}{16}f_\rho^T m_\rho^2 A_T(\bar{u}_0)\bar{u}_0\frac{1}{T}\\
&\quad-2f_\rho^T m_\rho^2 B_T^{[3]}(\bar{u}_0)\frac{1}{T}-\frac{1}{4}f_\rho^T\varphi_\perp(\bar{u}_0)Tf_0\left(\frac{\omega_c}{T}\right)+\frac{1}{4}f_\rho^T\varphi_\perp(\bar{u}_0)\bar{u}_0Tf_0\left(\frac{\omega_c}{T}\right)-f_\rho^T m_\rho^2\mathcal{T}^{[-1,0]}(u_0)\frac{1}{T}+f_\rho^T m_\rho^2\mathcal{T}_2^{[-1,0]}(u_0)\frac{1}{T}\\
&\quad-f_\rho^T m_\rho^2\mathcal{T}_3^{[-1,0]}(u_0)\frac{1}{T}-\frac{1}{4}f_\rho m_\rho^3 A^{[2]}(\bar{u}_0)\frac{1}{T^2}+\frac{1}{4}f_\rho m_\rho^3 A^{[1]}(\bar{u}_0)\frac{1}{T^2}-\frac{1}{4}f_\rho m_\rho^3 (uA)^{[1]}(\bar{u}_0)\frac{1}{T^2}-4f_\rho m_\rho^3 C^{[4]}(\bar{u}_0)\frac{1}{T^2}\\
&\quad+2f_\rho m_\rho^3 C^{[3]}(\bar{u}_0)\frac{1}{T^2}-2f_\rho m_\rho^3 (uC)^{[3]}(\bar{u}_0)\frac{1}{T^2}-\frac{1}{2}f_\rho m_\rho\mathcal{A}^{[0,0]}(u_0)-\frac{1}{4}f_\rho m_\rho g_\perp^{(a)[1]}(\bar{u}_0)-\frac{1}{8}f_\rho m_\rho g_\perp^{(a)}(\bar{u}_0)\\
&\quad+\frac{1}{8}f_\rho m_\rho g_\perp^{(a)}(\bar{u}_0)\bar{u}_0-f_\rho m_\rho g_\perp^{(v)[2]}(\bar{u}_0)+f_\rho m_\rho\mathcal{V}^{[0,0]}(u_0)+f_\rho m_\rho\varphi_\parallel^{[2]}(\bar{u}_0)-f_\rho m_\rho\varphi_\parallel^{[1]}(\bar{u}_0)+f_\rho m_\rho (u\varphi_\parallel)^{[1]}(\bar{u}_0)\\
&\quad-4f_\rho m_\rho^3\Psi^{[-2,0]}(u_0)\frac{1}{T^2}-4f_\rho m_\rho^3\tilde{\Phi}^{[-2,0]}(u_0)\frac{1}{T^2}, \qquad (15)
\end{aligned}
$$

$$\sqrt{6}g^{d2}_{\mathcal{T}_1H_1\rho}f_{-,(1/2)}f_{+,(3/2)}e^{-((\bar{\Lambda}_{+,3/2}+\bar{\Lambda}_{-,1/2})/T)}$$
$$= \frac{3}{16}f^T_\rho m^2_\rho A_T(\bar{u}_0)\frac{1}{T} - \frac{3}{16}f^T_\rho m^2_\rho A_T(\bar{u}_0)\bar{u}_0\frac{1}{T} - \frac{3}{4}f^T_\rho \varphi_\perp(\bar{u}_0)Tf_0\left(\frac{\omega_c}{T}\right) + \frac{3}{4}f^T_\rho\varphi_\perp(\bar{u}_0)\bar{u}_0Tf_0\left(\frac{\omega_c}{T}\right)$$
$$- 3f^T_\rho m^2_\rho \mathcal{T}^{[-1,0]}_2(u_0)\frac{1}{T} - 3f^T_\rho m^2_\rho\mathcal{T}^{[-1,0]}_3(u_0)\frac{1}{T} + \frac{3}{2}f_\rho m_\rho\mathcal{A}^{[0,0]}(u_0) - \frac{3}{8}f_\rho m_\rho g^{(a)}_\perp(\bar{u}_0) + \frac{3}{8}f_\rho m_\rho g^{(a)}_\perp(\bar{u}_0)\bar{u}_0, \quad (16)$$

where $f_n(x) = 1 - e^{-x}\sum_{k=0}^n \frac{x^k}{k!}$ is the continuum subtraction factor, and ω_c is the continuum threshold, $u_0 = \frac{T_1}{T_1+T_2}$, $T = \frac{T_1T_2}{T_1+T_2}$, and $\bar{u}_0 = 1 - u_0$. T_1 and T_2 are the two Borel parameters. We have used the Borel transformation $\tilde{\mathcal{B}}^T_\omega e^{\alpha\omega} = \delta(\alpha - \frac{1}{T})$ to obtain (14)–(16). In the above expressions, we have used the following functions: $\mathcal{F}^{[a]}(\bar{u}_0)$ and $\mathcal{F}^{[a,b]}(u_0)$. They are defined as

$$\mathcal{F}^{[n]}(\bar{u}_0) \equiv \int_0^{\bar{u}_0}\cdots\int_0^{x_3}\int_0^{x_2}\mathcal{F}(x_1)dx_1dx_2\cdots dx_n, \quad (17)$$

$$\mathcal{F}^{[0,0]}(u_0) \equiv \int_0^{u_0}\int_{u_0-\alpha_2}^{1-\alpha_2}\frac{\mathcal{F}(1-\alpha_2-\alpha_3,\alpha_2,\alpha_3)}{\alpha_3}d\alpha_3d\alpha_2, \quad (18)$$

$$\mathcal{F}^{[1,0]}(u_0) \equiv \int_0^{u_0}\frac{\mathcal{F}(1-u_0,\alpha_2,u_0-\alpha_2)}{u_0-\alpha_2}d\alpha_2 - \int_0^{1-u_0}\frac{\mathcal{F}(u_0,1-u_0-\alpha_3,\alpha_3)}{\alpha_3}d\alpha_3, \quad (19)$$

$$\mathcal{F}^{[2,0]}(u_0) \equiv \int_0^{u_0}d\alpha_2\frac{\partial[\mathcal{F}(1-\alpha_2-\alpha_3,\alpha_2,\alpha_3)]/\partial\alpha_2}{\alpha_3}\Bigg|_{\alpha_3=u_0-\alpha_2} - \int_0^{1-u_0}d\alpha_3\frac{\partial[\mathcal{F}(1-\alpha_2-\alpha_3,\alpha_2,\alpha_3)]/\partial\alpha_2}{\alpha_3}\Bigg|_{\alpha_2=u_0}, \quad (20)$$

$$\mathcal{F}^{[-1,0]}(u_0) \equiv \int_0^{u_0}\int_0^{u_0-\alpha_2}\mathcal{F}(1-\alpha_2-\alpha_3,\alpha_2,\alpha_3)d\alpha_3d\alpha_2 + \int_0^{u_0}\int_{u_0-\alpha_2}^{1-\alpha_2}\frac{(u_0-\alpha_2)\mathcal{F}(1-\alpha_2-\alpha_3,\alpha_2,\alpha_3)}{\alpha_3}d\alpha_3d\alpha_2, \quad (21)$$

$$\mathcal{F}^{[-2,0]}(u_0) \equiv \int_0^{u_0}\int_0^{u_0-\alpha_2}\int_0^{\alpha_3}\mathcal{F}(1-\alpha_2-x,\alpha_2,x)dxd\alpha_3d\alpha_2 + \frac{1}{2}\int_0^{u_0}\int_0^{u_0-\alpha_2}\alpha_3\mathcal{F}(1-\alpha_2-\alpha_3,\alpha_2,\alpha_3)d\alpha_3d\alpha_2$$
$$+ \frac{1}{2}\int_0^{u_0}\int_{u_0-\alpha_2}^{1-\alpha_2}\frac{(u_0-\alpha_2)^2}{\alpha_3}\mathcal{F}(1-\alpha_2-\alpha_3,\alpha_2,\alpha_3)d\alpha_3d\alpha_2, \quad (22)$$

$$\mathcal{F}^{[-3,0]}(u_0) \equiv \int_0^{u_0}\int_0^{u_0-\alpha_2}\int_0^{\alpha_3}\int_0^{x_2}\mathcal{F}(1-\alpha_2-x_1,\alpha_2,x_1)dx_1dx_2d\alpha_3d\alpha_2$$
$$+ \frac{1}{2}\int_0^{u_0}\int_0^{u_0-\alpha_2}\int_0^{\alpha_3}x\mathcal{F}(1-\alpha_2-x,\alpha_2,x)dxd\alpha_3d\alpha_2 + \frac{1}{6}\int_0^{u_0}\int_0^{u_0-\alpha_2}\alpha_3^2\mathcal{F}(1-\alpha_2-\alpha_3,\alpha_2,\alpha_3)d\alpha_3d\alpha_2$$
$$+ \frac{1}{6}\int_0^{u_0}\int_{u_0-\alpha_2}^{1-\alpha_2}\frac{(u_0-\alpha_2)^3}{\alpha_3}\mathcal{F}(1-\alpha_2-\alpha_3,\alpha_2,\alpha_3)d\alpha_3d\alpha_2. \quad (23)$$

III. NUMERICAL ANALYSIS

In our numerical analysis, we need the mass parameters $\bar{\Lambda}$'s and f's, the overlapping amplitudes of these interpolating currents. We adopt $\bar{\Lambda}_{-,1/2}$ from Ref. [11]: $\bar{\Lambda}_{-,1/2} = 0.5$ GeV, $f_{-,1/2} = 0.25 \pm 0.04$ GeV$^{3/2}$. $\bar{\Lambda}_{+,1/2}$, $f_{+,1/2}$, $\bar{\Lambda}_{+,3/2}$, and $f_{+,3/2}$ are given in Ref. [12]:

$$\bar{\Lambda}_{+,1/2} = 1.15 \text{ GeV},$$
$$f_{+,1/2} = -0.40 \pm 0.06 \text{ GeV}^{3/2},$$
$$\bar{\Lambda}_{+,3/2} = 0.82 \text{ GeV},$$
$$f_{+,3/2} = 0.19 \pm 0.03 \text{ GeV}^{5/2}.$$

The parameters that appear in the distribution amplitudes of the ρ meson take the values from Ref. [13]. We use the values at the scale $\mu = 1$ GeV in our calculation under the consideration that the heavy quark behaves almost as a spectator of the decay processes in our discussion in the leading order of HQET:

$f_\rho^\parallel$ [MeV]	$f_\rho^\perp$ [MeV]	$a_2^\parallel$	$a_2^\perp$	$\zeta_{3\rho}^\parallel$	$\tilde{\omega}_{3\rho}^\parallel$	$\omega_{3\rho}^\parallel$	$\omega_{3\rho}^\perp$	$\zeta_4^\parallel$	$\tilde{\omega}_4^\parallel$	$\zeta_4^\perp$	$\tilde{\zeta}_4^\perp$
216(3)	165(9)	0.15(7)	0.14(6)	0.030(10)	−0.09(3)	0.15(5)	0.55(25)	0.07(3)	−0.03(1)	−0.03(5)	−0.08(5)

We will work at the symmetry point, i.e., $T_1 = T_2 = 2T$, $u_0 = 1/2$. This comes from the observation that the mass differences between H, S, and T are less than 0.4 GeV in the leading order of HQET. They are much smaller than the Borel parameter $T_1, T_2 \sim 3$ GeV used below. On the other hand, every reliable sum rule has a working window of the Borel parameter T within which the sum rule is insensitive to the variation of T. So it is reasonable to choose a common point $T_1 = T_2$ at the overlapping region of T_1 and T_2. Furthermore, choosing $T_1 = T_2$ will enable us to subtract the continuum contribution cleanly, while the asymmetric choice will lead to the very difficult continuum substraction [14].

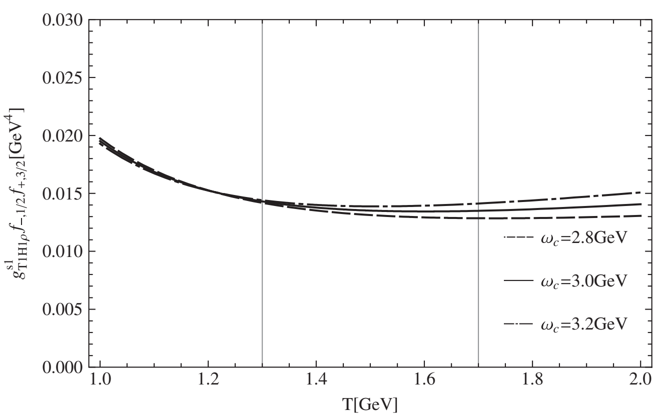

FIG. 1. The sum rule for $g^{s1}_{T_1H_1\rho}f_{-,1/2}f_{+,3/2}$ with $\omega_c = 2.8$, 3.0, 3.2 GeV.

From the requirement that the pole contribution is larger than 60%, we get the upper bound of the Borel parameter. This leads to $T < 1.7$ GeV. The convergence requirement of the operator product expansion leads to the lower bound of the Borel parameter $T = 1.3$ GeV, starting from which the stability of the sum rule develops. The resulting sum rules are plotted in Figs. 1–3 in the working interval $1.3\ \text{GeV} < T < 1.7$ GeV and $\omega_c = 2.8, 3.0, 3.2$ GeV.

Other ρ coupling constants between H, S, and T doublets can be calculated in the same way as $g_{T_1H_1\rho}$. Their definitions and the relevant correlators are given in Appendix A. Here we simply present the sum rules for these coupling constants:

$$g^{p0}_{H_1H_1\rho}f^2_{-,(1/2)} = \frac{1}{4}e^{((2\bar{\Lambda}_{-,1/2})/T)}\bigg\{-f_\rho m_\rho^3 A^{[1]}(\bar{u}_0)\frac{1}{T^2} - 8f_\rho m_\rho^3 C^{[3]}(\bar{u}_0)\frac{1}{T^2} + 4f_\rho^T m_\rho^2 h_\parallel^{s[1]}(\bar{u}_0)\frac{1}{T} + 4f_\rho m_\rho \varphi_\parallel^{[1]}(\bar{u}_0)\bigg\}, \tag{24}$$

$$g^{p1}_{H_1H_1\rho}f^2_{-,(1/2)} = \frac{1}{8}e^{((2\bar{\Lambda}_{-,1/2})/T)}\bigg\{f_\rho^T m_\rho^2 A_T(\bar{u}_0)\frac{1}{T} - 2f_\rho m_\rho g_\perp^{(a)}(\bar{u}_0) - 4f_\rho^T\varphi_\perp(\bar{u}_0)Tf_0\bigg(\frac{\omega_c}{T}\bigg)\bigg\}, \tag{25}$$

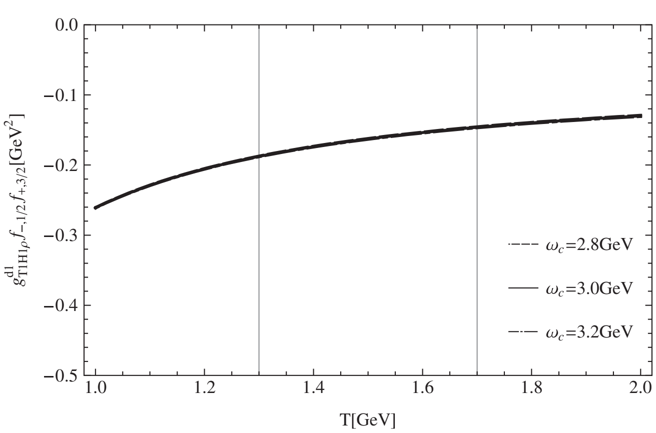

FIG. 2. The sum rule for $g^{d1}_{T_1H_1\rho}f_{-,1/2}f_{+,3/2}$ with $\omega_c = 2.8$, 3.0, 3.2 GeV.

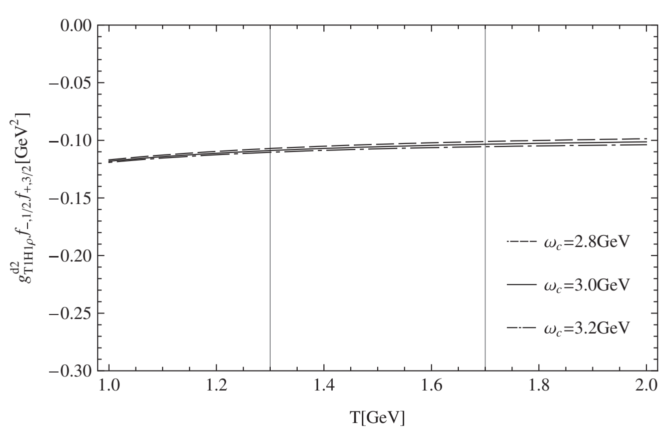

FIG. 3. The sum rule for $g^{d2}_{T_1H_1\rho}f_{-,1/2}f_{+,3/2}$ with $\omega_c = 2.8$, 3.0, 3.2 GeV.

$$g^{p0}_{S_1S_1\rho}f^2_{+,(1/2)} = \frac{1}{4}e^{((2\bar{\Lambda}_{+,1/2})/T)}\bigg\{-4f^T_\rho m^2_\rho h^{s[1]}_\parallel(\bar{u}_0)\frac{1}{T} - f_\rho m^3_\rho A^{[1]}(\bar{u}_0)\frac{1}{T^2} - 8f_\rho m^3_\rho C^{[3]}(\bar{u}_0)\frac{1}{T^2} + 4f_\rho m_\rho \varphi^{[1]}_\parallel(\bar{u}_0)\bigg\}, \quad (26)$$

$$g^{p1}_{S_1S_1\rho}f^2_{+,(1/2)} = \frac{1}{8}e^{((2\bar{\Lambda}_{+,1/2})/T)}\bigg\{-f^T_\rho m^2_\rho A_T(\bar{u}_0)\frac{1}{T} + 4f^T_\rho \varphi_\perp(\bar{u}_0)Tf_0\bigg(\frac{\omega_c}{T}\bigg) - 2f_\rho m_\rho g^{(a)}_\perp(\bar{u}_0)\bigg\}, \quad (27)$$

$$g^{s1}_{S_0H_1\rho}f_{-,(1/2)}f_{+,(1/2)} = \frac{1}{16}e^{((\bar{\Lambda}_{+,1/2}+\bar{\Lambda}_{-,1/2})/T)}\bigg\{-8f^T_\rho m^2_\rho C^{[1]}_T(\bar{u}_0) + f^T_\rho m^2_\rho A'_T(\bar{u}_0) - 4f^T_\rho \varphi'_\perp(\bar{u}_0)T^2 f_1\bigg(\frac{\omega_c}{T}\bigg) - 8f_\rho m_\rho g^{(v)}_\perp(\bar{u}_0)Tf_0\bigg(\frac{\omega_c}{T}\bigg)\bigg\}, \quad (28)$$

$$g^{d1}_{S_0H_1\rho}f_{-,(1/2)}f_{+,(1/2)} = \frac{1}{4}e^{((\bar{\Lambda}_{+,1/2}+\bar{\Lambda}_{-,1/2})/T)}\bigg\{-4f^T_\rho \varphi^{[1]}_\perp(\bar{u}_0) + f^T_\rho m^2_\rho A^{[1]}_T(\bar{u}_0)\frac{1}{T^2} + 16f^T_\rho m^2_\rho B^{[3]}_T(\bar{u}_0)\frac{1}{T^2} - 2A^{[2]}(\bar{u}_0)f_\rho m^3_\rho\frac{1}{T^3} - 8f_\rho m_\rho g^{(v)[2]}_\perp(\bar{u}_0)\frac{1}{T} + 8f_\rho m_\rho \varphi^{[2]}_\parallel(\bar{u}_0)\frac{1}{T}\bigg\}, \quad (29)$$

$$\begin{aligned}
&\sqrt{6}g^{p1}_{T_1S_1\rho}f_{+,(1/2)}f_{+,(3/2)}e^{-((\bar{\Lambda}_{+,3/2}+\bar{\Lambda}_{+,1/2})/T)}\\
&= -\frac{1}{4}f^T_\rho m^2_\rho C^{[2]}_T(\bar{u}_0) + \frac{1}{4}f^T_\rho m^2_\rho C^{[1]}_T(\bar{u}_0) - \frac{1}{4}f^T_\rho m^2_\rho (uC_T)^{[1]}(\bar{u}_0) - f^T_\rho m^2_\rho \tilde{\mathcal{S}}^{[0,0]}(u_0) - \frac{1}{32}f^T_\rho m^2_\rho A'_T(\bar{u}_0) + \frac{1}{32}f^T_\rho m^2_\rho (uA_T)'(\bar{u}_0)\\
&\quad - \frac{1}{2}f^T_\rho m^2_\rho B^{[2]}_T(\bar{u}_0) + \frac{1}{8}f^T_\rho \varphi'_\perp(\bar{u}_0)T^2 f_1\bigg(\frac{\omega_c}{T}\bigg) - \frac{1}{8}f^T_\rho (u\varphi_\perp)'(\bar{u}_0)T^2 f_1\bigg(\frac{\omega_c}{T}\bigg) - \frac{1}{2}f^T_\rho m^2_\rho \mathcal{T}^{[0,0]}_3(u_0) + \frac{1}{2}f^T_\rho m^2_\rho \mathcal{T}^{[0,0]}_4(u_0)\\
&\quad - \frac{3}{16}f_\rho m^3_\rho A^{[1]}(\bar{u}_0)\frac{1}{T} - f_\rho m^3_\rho \mathcal{A}^{[-1,0]}(u_0)\frac{1}{T} + \frac{1}{2}f_\rho m^3_\rho \mathcal{V}^{[-1,0]}(u_0)\frac{1}{T} - \frac{1}{2}f_\rho m_\rho \mathcal{A}^{[1,0]}(u_0)Tf_0\bigg(\frac{\omega_c}{T}\bigg)\\
&\quad - \frac{1}{8}f_\rho m_\rho g^{(a)}_\perp(\bar{u}_0)Tf_0\bigg(\frac{\omega_c}{T}\bigg) - \frac{1}{4}f_\rho m_\rho g^{(v)[1]}_\perp(\bar{u}_0)Tf_0\bigg(\frac{\omega_c}{T}\bigg) - \frac{1}{4}f_\rho m_\rho g^{(v)}_\perp(\bar{u}_0)Tf_0\bigg(\frac{\omega_c}{T}\bigg) + \frac{1}{4}f_\rho m_\rho g^{(v)}_\perp(\bar{u}_0)\bar{u}_0 Tf_0\bigg(\frac{\omega_c}{T}\bigg)\\
&\quad + \frac{1}{4}f_\rho m_\rho \mathcal{V}^{[1,0]}(u_0)Tf_0\bigg(\frac{\omega_c}{T}\bigg) + \frac{1}{4}f_\rho m_\rho \varphi^{[1]}_\parallel(\bar{u}_0)Tf_0\bigg(\frac{\omega_c}{T}\bigg),
\end{aligned} \quad (30)$$

$$\begin{aligned}
&\sqrt{6}g^{p2}_{T_1S_1\rho}f_{+,(1/2)}f_{+,(3/2)}e^{-((\bar{\Lambda}_{+,3/2}+\bar{\Lambda}_{+,1/2})/T)}\\
&= \frac{3}{4}f^T_\rho m^2_\rho C^{[2]}_T(\bar{u}_0) - \frac{3}{4}f^T_\rho m^2_\rho C^{[1]}_T(\bar{u}_0) + \frac{3}{4}f^T_\rho m^2_\rho (uC_T)^{[1]}(\bar{u}_0) - \frac{3}{5}f^T_\rho m^2_\rho \varphi^{[1]}_\perp(\bar{u}_0) + \frac{3}{5}f^T_\rho m^2_\rho (u\varphi_\perp)^{[1]}(\bar{u}_0)\\
&\quad + \frac{3}{20}f^T_\rho m^4_\rho A^{[1]}_T(\bar{u}_0)\frac{1}{T^2} - \frac{3}{20}f^T_\rho m^4_\rho (uA_T)^{[1]}(\bar{u}_0)\frac{1}{T^2} + \frac{9}{160}f^T_\rho m^2_\rho A'_T(\bar{u}_0) - \frac{9}{160}f^T_\rho m^2_\rho (uA_T)'(\bar{u}_0) - \frac{24}{5}f^T_\rho m^4_\rho B^{[4]}_T(\bar{u}_0)\frac{1}{T^2}\\
&\quad + \frac{12}{5}f^T_\rho m^4_\rho B^{[3]}_T(\bar{u}_0)\frac{1}{T^2} - \frac{12}{5}f^T_\rho m^4_\rho (uB_T)^{[3]}(\bar{u}_0)\frac{1}{T^2} - \frac{3}{10}f^T_\rho m^2_\rho B^{[2]}_T(\bar{u}_0) - \frac{3}{5}f^T_\rho m^2_\rho B^{[1]}_T(\bar{u}_0) + \frac{3}{5}f^T_\rho m^2_\rho (uB_T)^{[1]}(\bar{u}_0)\\
&\quad - \frac{9}{40}f^T_\rho \varphi'_\perp(\bar{u}_0)T^2 f_1\bigg(\frac{\omega_c}{T}\bigg) + \frac{9}{40}f^T_\rho (u\varphi_\perp)'(\bar{u}_0)T^2 f_1\bigg(\frac{\omega_c}{T}\bigg) + \frac{6}{5}f^T_\rho m^4_\rho \mathcal{T}^{[-2,0]}(u_0)\frac{1}{T^2} - \frac{3}{10}f^T_\rho m^2_\rho \mathcal{T}^{[0,0]}(u_0)\\
&\quad + \frac{12}{5}f^T_\rho m^4_\rho \mathcal{T}^{[-2,0]}_3(u_0)\frac{1}{T^2} + \frac{9}{10}f^T_\rho m^2_\rho \mathcal{T}^{[0,0]}_3(u_0) + \frac{12}{5}f^T_\rho m^4_\rho \mathcal{T}^{[-2,0]}_4(u_0)\frac{1}{T^2} + \frac{9}{10}f^T_\rho m^2_\rho \mathcal{T}^{[0,0]}_4(u_0) - \frac{3}{10}f_\rho m^5_\rho A^{[3]}(\bar{u}_0)\frac{1}{T^3}\\
&\quad + \frac{3}{10}f_\rho m^5_\rho A^{[2]}(\bar{u}_0)\frac{1}{T^3} - \frac{3}{10}f_\rho m^5_\rho (uA)^{[2]}(\bar{u}_0)\frac{1}{T^3} - \frac{9}{80}f_\rho m^3_\rho A^{[1]}(\bar{u}_0)\frac{1}{T} - \frac{3}{40}f_\rho m^3_\rho A(\bar{u}_0)\frac{1}{T} + \frac{3}{40}f_\rho m^3_\rho A(\bar{u}_0)\bar{u}_0\frac{1}{T}\\
&\quad - \frac{6}{5}f_\rho m^3_\rho g^{(v)[3]}_\perp(\bar{u}_0)\frac{1}{T} + \frac{6}{5}f_\rho m^3_\rho g^{(v)[2]}_\perp(\bar{u}_0)\frac{1}{T} - \frac{6}{5}f_\rho m^3_\rho (ug^{(v)}_\perp)^{[2]}(\bar{u}_0)\frac{1}{T} + \frac{9}{10}f_\rho m^3_\rho \mathcal{V}^{[-1,0]}(u_0)\frac{1}{T} + \frac{6}{5}f_\rho m^3_\rho \varphi^{[3]}_\parallel(\bar{u}_0)\frac{1}{T}\\
&\quad - \frac{6}{5}f_\rho m^3_\rho \varphi^{[2]}_\parallel(\bar{u}_0)\frac{1}{T} + \frac{6}{5}f_\rho m^3_\rho (u\varphi_\parallel)^{[2]}(\bar{u}_0)\frac{1}{T} - \frac{12}{5}f_\rho m^5_\rho \mathcal{V}^{[-3,0]}(u_0)\frac{1}{T^3} - \frac{24}{5}f_\rho m^5_\rho \Phi^{[-3,0]}(u_0)\frac{1}{T^3}\\
&\quad + \frac{24}{5}f_\rho m^5_\rho \Psi^{[-3,0]}(u_0)\frac{1}{T^3} + \frac{6}{5}f_\rho m^3_\rho \Phi^{[-1,0]}(u_0)\frac{1}{T} - \frac{6}{5}f_\rho m^3_\rho \Psi^{[-1,0]}(u_0)\frac{1}{T} - \frac{9}{20}f_\rho m_\rho g^{(v)[1]}_\perp(\bar{u}_0)Tf_0\bigg(\frac{\omega_c}{T}\bigg)\\
&\quad + \frac{9}{20}f_\rho m_\rho g^{(v)}_\perp(\bar{u}_0)Tf_0\bigg(\frac{\omega_c}{T}\bigg) - \frac{9}{20}f_\rho m_\rho g^{(v)}_\perp(\bar{u}_0)\bar{u}_0 Tf_0\bigg(\frac{\omega_c}{T}\bigg) + \frac{9}{20}f_\rho m_\rho \mathcal{V}^{[1,0]}(u_0)Tf_0\bigg(\frac{\omega_c}{T}\bigg)\\
&\quad + \frac{9}{20}f_\rho m_\rho \varphi^{[1]}_\parallel(\bar{u}_0)Tf_0\bigg(\frac{\omega_c}{T}\bigg) + \frac{3}{10}f_\rho m_\rho \varphi_\parallel(\bar{u}_0)Tf_0\bigg(\frac{\omega_c}{T}\bigg) - \frac{3}{10}f_\rho m_\rho \varphi_\parallel(\bar{u}_0)\bar{u}_0 Tf_0\bigg(\frac{\omega_c}{T}\bigg),
\end{aligned} \quad (31)$$

$$\sqrt{6}g^{f2}_{T_1S_1\rho}f_{+,(1,2)}f_{+,(3/2)}e^{-((\bar{\Lambda}_{+,3/2}+\bar{\Lambda}_{+,1/2})/T)}$$
$$= -3f^T_\rho\varphi^{[1]}_\perp(\bar{u}_0) + 3f^T_\rho(u\varphi_\perp)^{[1]}(\bar{u}_0) + \frac{3}{4}f^T_\rho m^2_\rho A^{[1]}_T(\bar{u}_0)\frac{1}{T^2} - \frac{3}{4}m^2_\rho(uA_T)^{[1]}(\bar{u}_0)f^T_\rho\frac{1}{T^2} - 24f^T_\rho m^2_\rho B^{[4]}_T(\bar{u}_0)\frac{1}{T^2}$$
$$+ 12f^T_\rho m^2_\rho B^{[3]}_T(\bar{u}_0)\frac{1}{T^2} - 12f^T_\rho m^2_\rho(uB_T)^{[3]}(\bar{u}_0)\frac{1}{T^2} + 6f^T_\rho m^2_\rho \mathcal{T}^{[-2,0]}(u_0)\frac{1}{T^2} + 12f^T_\rho m^2_\rho \mathcal{T}^{[-2,0]}_3(u_0)\frac{1}{T^2}$$
$$+ 12f^T_\rho m^2_\rho \mathcal{T}^{[-2,0]}_4(u_0)\frac{1}{T^2} - \frac{3}{2}f_\rho m^3_\rho A^{[3]}(\bar{u}_0)\frac{1}{T^3} + \frac{3}{2}f_\rho m^3_\rho A^{[2]}(\bar{u}_0)\frac{1}{T^3} - \frac{3}{2}f_\rho m^3_\rho(uA)^{[2]}(\bar{u}_0)\frac{1}{T^3} - 6f_\rho m_\rho g^{(v)[3]}_\perp(\bar{u}_0)\frac{1}{T}$$
$$+ 6f_\rho m_\rho g^{(v)[2]}_\perp(\bar{u}_0)\frac{1}{T} - 6f_\rho m_\rho(ug^{(v)}_\perp)^{[2]}(\bar{u}_0)\frac{1}{T} - 6f_\rho m_\rho \mathcal{V}^{[-1,0]}(u_0)\frac{1}{T} + 6f_\rho m_\rho\varphi^{[3]}_\parallel(\bar{u}_0)\frac{1}{T} - 6f_\rho m_\rho\varphi^{[2]}_\parallel(\bar{u}_0)\frac{1}{T}$$
$$+ 6f_\rho m_\rho(u\varphi_\parallel)^{[2]}(\bar{u}_0)\frac{1}{T} - 12f_\rho m^3_\rho \mathcal{V}^{[-3,0]}(u_0)\frac{1}{T^3} - 24f_\rho m^3_\rho\Phi^{[-3,0]}(u_0)\frac{1}{T^3} + 24f_\rho m^3_\rho\Psi^{[-3,0]}(u_0)\frac{1}{T^3}. \quad (32)$$

Because of heavy quark symmetry, the ρ coupling constants with the same (l, j_h) between two doublets are not independent in the leading order of HQET. The values of these coupling constants multiplied by the decay constants of the initial and the final heavy mesons are

$$\tilde{g}^{p_0}_{H_0H_0\rho} = -\tilde{g}^{p_0}_{H_1H_1\rho} = -0.23 \pm 0.03\ \text{GeV}^2,$$
$$\tilde{g}^{p_1}_{H_1H_0\rho} = \tilde{g}^{p_1}_{H_1H_1\rho} = -0.33 \pm 0.01\ \text{GeV}^2,$$
$$\tilde{g}^{s_1}_{S_0H_1\rho} = -\tilde{g}^{s_1}_{S_1H_0\rho} = -\tilde{g}^{s_1}_{S_1H_1\rho} = -0.28 \pm 0.02\ \text{GeV}^3,$$
$$\tilde{g}^{d_1}_{S_0H_1\rho} = -\tilde{g}^{d_1}_{S_1H_0\rho} = -\tilde{g}^{d_1}_{S_1H_1\rho} = -0.27 \pm 0.04\ \text{GeV},$$
$$\tilde{g}^{p_0}_{S_0S_0\rho} = -\tilde{g}^{p_0}_{S_1S_1\rho} = -0.23 \pm 0.02\ \text{GeV}^2,$$
$$\tilde{g}^{p_1}_{S_1S_0\rho} = -\tilde{g}^{p_1}_{S_1S_1\rho} = -0.32 \pm 0.02\ \text{GeV}^2,$$
$$\tilde{g}^{s_1}_{T_1H_0\rho} = -2\tilde{g}^{s_1}_{T_1H_1\rho} = -2\sqrt{\frac{2}{3}}\tilde{g}^{s_1}_{T_2H_1\rho} = -0.03 \pm 0.002\ \text{GeV}^4,$$
$$\tilde{g}^{d_1}_{T_1H_0\rho} = 2\tilde{g}^{d_1}_{T_1H_1\rho} = -2\sqrt{\frac{2}{3}}\tilde{g}^{d_1}_{T_2H_1\rho} = -0.33 \pm 0.04\ \text{GeV}^2,$$
$$\tilde{g}^{d_2}_{T_1H_1\rho} = \sqrt{\frac{3}{2}}\tilde{g}^{d_2}_{T_2H_0\rho} = \sqrt{6}\tilde{g}^{d_2}_{T_2H_1\rho} = -0.11 \pm 0.01\ \text{GeV}^2,$$
$$\tilde{g}^{p_1}_{T_1S_0\rho} = 2\tilde{g}^{p_1}_{T_1S_1\rho} = 2\sqrt{\frac{2}{3}}\tilde{g}^{p_1}_{T_2S_1\rho} = 0.19 \pm 0.02\ \text{GeV}^3,$$
$$\tilde{g}^{p_2}_{T_1S_1\rho} = -\sqrt{\frac{3}{2}}\tilde{g}^{p_2}_{T_2S_0\rho} = -\sqrt{6}\tilde{g}^{p_2}_{T_2S_1\rho} = -0.20 \pm 0.02\ \text{GeV}^3,$$
$$\tilde{g}^{f_2}_{T_1S_1\rho} = -\sqrt{\frac{3}{2}}\tilde{g}^{f_2}_{T_2S_0\rho} = -\sqrt{6}\tilde{g}^{f_2}_{T_2S_1\rho} = 0.22 \pm 0.03\ \text{GeV}, \quad (33)$$

where $\tilde{g}^{p_0}_{H_0H_0} \equiv g^{p_0}_{H_0H_0}f^2_{-,1/2}$, etc. The errors come from the variations of T and ω_c in the working region and the central value corresponds to $T = 1.5$ GeV and $\omega_c = 3.0$ GeV. The g's with their errors are

$$g^{p_0}_{H_0H_0\rho} = -g^{p_0}_{H_1H_1\rho} = -3.6 \pm 0.4 \pm 0.9\ \text{GeV}^{-1},$$
$$g^{p_1}_{H_1H_0\rho} = g^{p_1}_{H_1H_1\rho} = -5.2 \pm 0.2 \pm 1.3\ \text{GeV}^{-1},$$
$$g^{s_1}_{S_0H_1\rho} = -g^{s_1}_{S_1H_0\rho} = -g^{s_1}_{S_1H_1\rho} = 2.8 \pm 0.2 \pm 0.7,$$
$$g^{d_1}_{S_0H_1\rho} = -g^{d_1}_{S_1H_0\rho} = -g^{d_1}_{S_1H_1\rho} = 2.7 \pm 0.4 \pm 0.6\ \text{GeV}^{-2},$$
$$g^{p_0}_{S_0S_0\rho} = -g^{p_0}_{S_1S_1\rho} = -1.4 \pm 0.2 \pm 0.4\ \text{GeV}^{-1},$$
$$g^{p_1}_{S_1S_0\rho} = -g^{p_1}_{S_1S_1\rho} = -2.0 \pm 0.2 \pm 0.5\ \text{GeV}^{-1},$$
$$g^{s_1}_{T_1H_0\rho} = -2g^{s_1}_{T_1H_1\rho} = -2\sqrt{\frac{2}{3}}g^{s_1}_{T_2H_1\rho} = -0.6 \pm 0.04 \pm 0.2,$$
$$g^{d_1}_{T_1H_0\rho} = 2g^{d_1}_{T_1H_1\rho} = -2\sqrt{\frac{2}{3}}g^{d_1}_{T_2H_1\rho} = -7.0 \pm 0.9 \pm 1.8\ \text{GeV}^{-2},$$
$$g^{d_2}_{T_1H_1\rho} = \sqrt{\frac{3}{2}}g^{d_2}_{T_2H_0\rho} = \sqrt{6}g^{d_2}_{T_2H_1\rho} = -2.2 \pm 0.1 \pm 0.6\ \text{GeV}^{-2},$$
$$g^{p_1}_{T_1S_0\rho} = 2g^{p_1}_{T_1S_1\rho} = 2\sqrt{\frac{2}{3}}g^{p_1}_{T_2S_1\rho} = 2.5 \pm 0.2 \pm 0.6\ \text{GeV}^{-1},$$
$$g^{p_2}_{T_1S_1\rho} = -\sqrt{\frac{3}{2}}g^{p_2}_{T_2S_0\rho} = -\sqrt{6}g^{p_2}_{T_2S_1\rho} = -2.6 \pm 0.2 \pm 0.6\ \text{GeV}^{-1},$$
$$g^{f_2}_{T_1S_1\rho} = -\sqrt{\frac{3}{2}}g^{f_2}_{T_2S_0\rho} = -\sqrt{6}g^{f_2}_{T_2S_1\rho} = 2.9 \pm 0.4 \pm 0.7\ \text{GeV}^{-3}. \quad (34)$$

The second error comes from the uncertainty of f's. The above relations between coupling constants are consistent with the HQET leading order expectation.

Replacing the ρ meson parameters by those for the ω meson, one obtains the ω meson couplings with the heavy mesons:

$$
\begin{aligned}
&\tilde{g}^{p_0}_{H_0H_0\omega} = -\tilde{g}^{p_0}_{H_1H_1\omega} = -0.21 \pm 0.03\,\mathrm{GeV}^2,\\
&\tilde{g}^{p_1}_{H_1H_0\omega} = \tilde{g}^{p_1}_{H_1H_1\omega} = -0.29 \pm 0.01\,\mathrm{GeV}^2,\\
&\tilde{g}^{s_1}_{S_0H_1\omega} = -\tilde{g}^{s_1}_{S_1H_0\omega} = -\tilde{g}^{s_1}_{S_1H_1\omega} = -0.25 \pm 0.02\,\mathrm{GeV}^3,\\
&\tilde{g}^{d_1}_{S_0H_1\omega} = -\tilde{g}^{d_1}_{S_1H_0\omega} = -\tilde{g}^{d_1}_{S_1H_1\omega} = -0.23 \pm 0.04\,\mathrm{GeV},\\
&\tilde{g}^{p_0}_{S_0S_0\omega} = -\tilde{g}^{p_0}_{S_1S_1\omega} = -0.21 \pm 0.02\,\mathrm{GeV}^2,\\
&\tilde{g}^{p_1}_{S_1S_0\omega} = -\tilde{g}^{p_1}_{S_1S_1\omega} = -0.27 \pm 0.02\,\mathrm{GeV}^2,\\
&\tilde{g}^{s_1}_{T_1H_0\omega} = -2\tilde{g}^{s_1}_{T_1H_1\omega} = -2\sqrt{\frac{2}{3}}\tilde{g}^{s_1}_{T_2H_1\omega} = -0.03 \pm 0.002\,\mathrm{GeV}^4,\\
&\tilde{g}^{d_1}_{T_1H_0\omega} = 2\tilde{g}^{d_1}_{T_1H_1\omega} = -2\sqrt{\frac{2}{3}}\tilde{g}^{d_1}_{T_2H_1\omega} = -0.31 \pm 0.04\,\mathrm{GeV}^2,\\
&\tilde{g}^{d_2}_{T_1H_1\omega} = \sqrt{\frac{3}{2}}\tilde{g}^{d_2}_{T_2H_0\omega} = \sqrt{6}\tilde{g}^{d_2}_{T_2H_1\omega} = -0.09 \pm 0.01\,\mathrm{GeV}^2,\\
&\tilde{g}^{p_1}_{T_1S_0\omega} = 2\tilde{g}^{p_1}_{T_1S_1\omega} = 2\sqrt{\frac{2}{3}}\tilde{g}^{p_1}_{T_2S_1\omega} = 0.11 \pm 0.01\,\mathrm{GeV}^3,\\
&\tilde{g}^{p_2}_{T_1S_1\omega} = -\sqrt{\frac{3}{2}}\tilde{g}^{p_2}_{T_2S_0\omega} = -\sqrt{6}\tilde{g}^{p_2}_{T_2S_1\omega} = -0.11 \pm 0.01\,\mathrm{GeV}^3,\\
&\tilde{g}^{f_2}_{T_1S_1\omega} = -\sqrt{\frac{3}{2}}\tilde{g}^{f_2}_{T_2S_0\omega} = -\sqrt{6}\tilde{g}^{f_2}_{T_2S_1\omega} = 0.12 \pm 0.02\,\mathrm{GeV},
\end{aligned}
\tag{35}
$$

$$
\begin{aligned}
&g^{p_0}_{H_0H_0\omega} = -g^{p_0}_{H_1H_1\omega} = -3.3 \pm 0.4 \pm 0.9\,\mathrm{GeV}^{-1},\\
&g^{p_1}_{H_1H_0\omega} = g^{p_1}_{H_1H_1\omega} = -4.7 \pm 0.2 \pm 1.1\,\mathrm{GeV}^{-1},\\
&g^{s_1}_{S_0H_1\omega} = -g^{s_1}_{S_1H_0\omega} = -g^{s_1}_{S_1H_1\omega} = 2.5 \pm 0.2 \pm 0.6,\\
&g^{d_1}_{S_0H_1\omega} = -g^{d_1}_{S_1H_0\omega} = -g^{d_1}_{S_1H_1\omega} = 2.3 \pm 0.4 \pm 0.6\,\mathrm{GeV}^{-2},\\
&g^{p_0}_{S_0S_0\omega} = -g^{p_0}_{S_1S_1\omega} = -1.3 \pm 0.2 \pm 0.4\,\mathrm{GeV}^{-1},\\
&g^{p_1}_{S_1S_0\omega} = -g^{p_1}_{S_1S_1\omega} = -1.7 \pm 0.1 \pm 0.4\,\mathrm{GeV}^{-1},\\
&g^{s_1}_{T_1H_0\omega} = -2g^{s_1}_{T_1H_1\omega} = -2\sqrt{\frac{2}{3}}g^{s_1}_{T_2H_1\omega} = -0.6 \pm 0.03 \pm 0.2,\\
&g^{d_1}_{T_1H_0\omega} = 2g^{d_1}_{T_1H_1\omega} = -2\sqrt{\frac{2}{3}}g^{d_1}_{T_2H_1\omega} = -6.4 \pm 0.9 \pm 1.6\,\mathrm{GeV}^{-2},\\
&g^{d_2}_{T_1H_1\omega} = \sqrt{\frac{3}{2}}g^{d_2}_{T_2H_0\omega} = \sqrt{6}g^{d_2}_{T_2H_1\omega} = -2.0 \pm 0.1 \pm 0.5\,\mathrm{GeV}^{-2},\\
&g^{p_1}_{T_1S_0\omega} = 2g^{p_1}_{T_1S_1\omega} = 2\sqrt{\frac{2}{3}}g^{p_1}_{T_2S_1\omega} = 2.3 \pm 0.2 \pm 0.6\,\mathrm{GeV}^{-1},\\
&g^{p_2}_{T_1S_1\omega} = -\sqrt{\frac{3}{2}}g^{p_2}_{T_2S_0\omega} = -\sqrt{6}g^{p_2}_{T_2S_1\omega} = -2.3 \pm 0.2 \pm 0.6\,\mathrm{GeV}^{-1},\\
&g^{f_2}_{T_1S_1\omega} = -\sqrt{\frac{3}{2}}g^{f_2}_{T_2S_0\omega} = -\sqrt{6}g^{f_2}_{T_2S_1\omega} = 2.5 \pm 0.3 \pm 0.6\,\mathrm{GeV}^{-3}.
\end{aligned}
\tag{36}
$$

Here we take the following values for the parameters f_ω, f^T_ω [15], and m_ω: $f_\omega = 0.195$ GeV, $f^T_\omega = 0.145$ GeV, and $m_\omega = 0.78$ GeV.

IV. CONCLUSION

We have calculated the light vector meson couplings with heavy mesons in the leading order of HQET within the framework of LCQSR. The sum rules are stable with the variations of the Borel parameter and the continuum threshold. Some possible sources of the errors in our calculation include the inherent inaccuracy of LCQSR: the omission of the higher order terms in operator product expansion, the choice of ω_c, the variation of the coupling constant with the Borel parameter T in the working interval, and the approximation in the light-cone distribution amplitudes of the ρ meson. The uncertainty in f's and $\bar{\Lambda}$'s also leads to errors.

The 3-particle light-cone distribution amplitudes of the ρ meson are not as well known as the 2-particle ones. This may lead to additional systematical errors in our calculation of the ρ couplings involving the T doublet. However, the covariant derivative D_μ has to be introduced to construct the appropriate interpolating currents for the T doublet. This demands the inclusion of these 3-particle light-cone distribution amplitudes for the completeness of our calculation, which is another error source.

The ρ coupling constants between H_1 and H_0 have been calculated with the LCQSR approach in full QCD in Ref. [8]. We quote the numerical results below [8]:

$$\begin{aligned} f_{B^*B^*\rho} &= 0.82\ \text{GeV}^{-1}, & f_{B^*B\rho} &= 0.85\ \text{GeV}^{-1}, \\ f_{D^*D^*\rho} &= 0.78\ \text{GeV}^{-1}, & f_{D^*D\rho} &= 0.81\ \text{GeV}^{-1}. \end{aligned} \tag{37}$$

To the leading order of HQET, these four coupling constants share one common asymptotic value $-g^{p1}_{H_1H_0\rho}/4\sqrt{2} = 0.93\ \text{GeV}^{-1}$ in Eq. (34), where the factor $4\sqrt{2}$ arises from different definitions of the coupling constants in the present work and Ref. [8]. In other words, the $1/m_Q$ correction to our calculation is expected to be within 20% as far as the final numerical results are concerned.

The extracted vector meson heavy meson coupling constants may be helpful in the study of the interaction between two $B(D)$ mesons. They may play an important role in the formation of these possible molecular candidates composed of two $B(D)$ mesons. They may also play a role in the interpretation of the proximity of $X(3872)$, $Y(4260)$, and $Z(4430)$ to the threshold of two charmed mesons through the couple-channel mechanism.

ACKNOWLEDGMENTS

P. Z. Huang thanks Z. G. Luo for helpful discussions. This project is supported by the National Natural Science Foundation of China under Grants No. 10625521, No. 10721063, and the Ministry of Science and Technology of China (2009CB825200).

APPENDIX A: THE ρ DECAY AMPLITUDES OF HEAVY MESONS

The definitions of the ρ coupling constants not presented in the text are

$$\mathcal{M}(H_0 \to H_0 + \rho) = I(e^* \cdot q_t) g^{p0}_{H_0H_0\rho}, \tag{A1}$$

$$\mathcal{M}(H_1 \to H_0 + \rho) = I\epsilon^{\eta e^* qv} g^{p1}_{H_1H_0\rho}, \tag{A2}$$

$$\begin{aligned} \mathcal{M}(H_1 \to H_1 + \rho) = {}& I(e^* \cdot q_t)(\epsilon^* \cdot \eta_t) g^{p0}_{H_1H_1\rho} \\ &+ I[(e^* \cdot \eta_t)(\epsilon^* \cdot q_t) \\ &- (e^* \cdot \epsilon^*_t)(\eta \cdot q_t)] g^{p1}_{H_1H_1\rho}, \end{aligned} \tag{A3}$$

$$\mathcal{M}(S_0 \to S_0 + \rho) = I(e^* \cdot q_t) g^{p0}_{S_0S_0\rho}, \tag{A4}$$

$$\mathcal{M}(S_1 \to S_0 + \rho) = I\epsilon^{\eta e^* qv} g^{p1}_{S_1S_0\rho}, \tag{A5}$$

$$\begin{aligned} \mathcal{M}(S_1 \to S_1 + \rho) = {}& I(e^* \cdot q_t)(\epsilon^* \cdot \eta_t) g^{p0}_{S_1S_1\rho} \\ &+ I[(e^* \cdot \eta_t)(\epsilon^* \cdot q_t) \\ &- (e^* \cdot \epsilon^*_t)(\eta \cdot q_t)] g^{p1}_{S_1S_1\rho}, \end{aligned} \tag{A6}$$

$$\begin{aligned} \mathcal{M}(S_0 \to H_1 + \rho) = {}& I(e^* \cdot \epsilon^*_t) g^{s1}_{S_0H_1\rho} \\ &+ I[(\epsilon^* \cdot q_t)(e^* \cdot q_t) \\ &- \tfrac{1}{3}(e^* \cdot \epsilon^*_t) q_t^2] g^{d1}_{S_0H_1\rho}, \end{aligned} \tag{A7}$$

$$\begin{aligned} \mathcal{M}(S_1 \to H_0 + \rho) = {}& I(e^* \cdot \eta_t) g^{s1}_{S_1H_0\rho} \\ &+ I[(\eta \cdot q_t)(e^* \cdot q_t) \\ &- \tfrac{1}{3}(e^* \cdot \eta_t) q_t^2] g^{d1}_{S_1H_0\rho}, \end{aligned} \tag{A8}$$

$$\begin{aligned} \mathcal{M}(S_1 \to H_1 + \rho) = {}& I\epsilon^{\eta\epsilon^* e^* v} g^{s1}_{S_1H_1\rho} \\ &+ I[\epsilon^{\eta\epsilon^* qv}(e^* \cdot q_t) \\ &- \tfrac{1}{3}\epsilon^{\eta\epsilon^* e^* v} q_t^2] g^{d1}_{S_1H_1\rho}, \end{aligned} \tag{A9}$$

$$\begin{aligned} \mathcal{M}(T_1 \to H_0 + \rho) = {}& I(e^* \cdot \eta_t) g^{s1}_{T_1H_0\rho} \\ &+ I[(\eta \cdot q_t)(e^* \cdot q_t) \\ &- \tfrac{1}{3}(e^* \cdot \eta_t) q_t^2] g^{d1}_{T_1H_0\rho}, \end{aligned} \tag{A10}$$

$$\begin{aligned} \mathcal{M}(T_1 \to H_1 + \rho) = {}& I\epsilon^{\eta\epsilon^* e^* v} g^{s1}_{T_1H_1\rho} \\ &+ I[\epsilon^{\eta\epsilon^* qv}(e^* \cdot q_t) \\ &- \tfrac{1}{3}\epsilon^{\eta\epsilon^* e^* v} q_t^2] g^{d1}_{T_1H_1\rho} \\ &+ I[\epsilon^{\eta e qv}(q_t \cdot \epsilon^*) \\ &+ \epsilon^{\epsilon^* e qv}(q_t \cdot \eta)] g^{d2}_{T_1H_1\rho}, \end{aligned} \tag{A11}$$

$$\mathcal{M}(T_1 \to S_0 + \rho) = I\epsilon^{\eta e^* q v} g^{p1}_{T_1 S_0 \rho}, \tag{A12}$$

$$\begin{aligned}\mathcal{M}(T_1 \to S_1 + \rho) = {}& I[(e^* \cdot \eta_t)(\epsilon^* \cdot q_t) - (e^* \cdot \epsilon_t^*)(\eta \cdot q_t)] g^{p1}_{T_1 S_1 \rho} \\ &+ I\left[(e_t^* \cdot \eta)(q_t \cdot \epsilon^*) + (q_t \cdot \eta)(e_t^* \cdot \epsilon^*) - \frac{2}{3}(e_t^* \cdot \eta)(e^* \cdot q_t)\right] g^{p2}_{T_1 S_1 \rho} \\ &+ I\left\{(q_t \cdot \eta)(q_t \cdot \epsilon^*)(e^* \cdot q_t) - \frac{q_t^2}{5}[(\eta_t \cdot \epsilon^*)(e^* \cdot q_t) + (e_t^* \cdot \eta)(q_t \cdot \epsilon^*) + (q_t \cdot \eta)(e_t^* \cdot \epsilon^*)]\right\} g^{f2}_{T_1 S_1 \rho},\end{aligned} \tag{A13}$$

$$\mathcal{M}(T_2 \to H_0 + \rho) = I\eta_{\alpha_1 \alpha_2}(\epsilon^{\alpha_1 e^* q v} q_t^{\alpha_2} + \epsilon^{\alpha_2 e^* q v} q_t^{\alpha_1}) g^{d2}_{T_2 H_0 \rho}, \tag{A14}$$

$$\begin{aligned}\mathcal{M}(T_2 \to H_1 + \rho) = {}& I\eta_{\alpha_1\alpha_2}[\epsilon^{*\alpha_1} e_t^{*\alpha_2} + \epsilon^{*\alpha_2} e_t^{*\alpha_1} - \tfrac{2}{3} g_t^{\alpha_1\alpha_2}(e_t^* \cdot \epsilon^*)] g^{s1}_{T_2 H_1 \rho} \\ &+ I\eta_{\alpha_1\alpha_2}\{[\epsilon^{*\alpha_1} q_t^{\alpha_2} + \epsilon^{*\alpha_2} q_t^{\alpha_1} - \tfrac{2}{3} g_t^{\alpha_1\alpha_2}(q_t \cdot \epsilon^*)](e^* \cdot q_t) \\ &- \tfrac{1}{3}[\epsilon^{*\alpha_1} e_t^{*\alpha_2} + \epsilon^{*\alpha_2} e_t^{*\alpha_1} - \tfrac{2}{3} g_t^{\alpha_1\alpha_2}(e_t^* \cdot \epsilon^*)] q_t^2\} g^{d1}_{T_2 H_1 \rho} \\ &+ I\eta_{\alpha_1\alpha_2}\{2[e_t^{*\alpha_1} q_t^{\alpha_2}(q_t \cdot \epsilon^*) + q_t^{\alpha_1} e_t^{*\alpha_2}(q_t \cdot \epsilon^*) - 2 q_t^{\alpha_1} q_t^{\alpha_2}(e_t^* \cdot \epsilon^*)] \\ &+ [\epsilon_t^{*\alpha_1} q_t^{\alpha_2} + \epsilon_t^{*\alpha_2} q_t^{\alpha_1} - 2 g_t^{\alpha_1\alpha_2}(q_t \cdot \epsilon^*)](e^* \cdot q_t) - [\epsilon_t^{*\alpha_1} e_t^{*\alpha_2} + \epsilon_t^{*\alpha_2} e_t^{*\alpha_1} - 2 g_t^{\alpha_1\alpha_2}(e_t^* \cdot \epsilon^*)] q_t^2\} g^{d2}_{T_2 H_1 \rho},\end{aligned} \tag{A15}$$

$$\begin{aligned}\mathcal{M}(T_2 \to S_0 + \rho) = {}& I\eta_{\alpha_1\alpha_2}\left[e_t^{*\alpha_1} q_t^{\alpha_2} + q_t^{\alpha_1} e_t^{*\alpha_2} - \frac{2}{3} g_t^{\alpha_1\alpha_2}(e^* \cdot q_t)\right] g^{p2}_{T_2 S_0 \rho} \\ &+ I\eta_{\alpha_1\alpha_2}\left\{q_t^{\alpha_1} q_t^{\alpha_2}(e^* \cdot q_t) - \frac{q_t^2}{5}[g_t^{\alpha_1\alpha_2}(e^* \cdot q_t) + e_t^{*\alpha_1} q_t^{\alpha_2} + q_t^{\alpha_1} e_t^{*\alpha_2}]\right\} g^{f2}_{T_2 S_0 \rho},\end{aligned} \tag{A16}$$

$$\begin{aligned}\mathcal{M}(T_2 \to S_1 + \rho) = {}& I\eta_{\alpha_1\alpha_2}\left[-\epsilon^{\alpha_1 e^* q v}\epsilon_t^{*\alpha_2} - \epsilon^{\alpha_2 e^* q v}\epsilon_t^{*\alpha_1} + \frac{2}{3} g_t^{\alpha_1\alpha_2}\epsilon^{\epsilon^* e^* q v}\right] g^{p1}_{T_2 S_1 \rho} \\ &+ I\eta_{\alpha_1\alpha_2}[\epsilon^{\alpha_1 \epsilon^* e^* v} q_t^{\alpha_2} + \epsilon^{\alpha_2 \epsilon^* e^* v} q_t^{\alpha_1} + \epsilon^{\alpha_1 \epsilon^* q v} e_t^{\alpha_2} + \epsilon^{\alpha_2 \epsilon^* q v} e_t^{*\alpha_1}] g^{p2}_{T_2 S_1 \rho} \\ &+ I\eta_{\alpha_1\alpha_2}\left\{\epsilon^{\alpha_1 \epsilon^* q v} q_t^{\alpha_2}(e^* \cdot q_t) + \epsilon^{\alpha_2 \epsilon^* q v} q_t^{\alpha_1}(e^* \cdot q_t)\right. \\ &\left.- \frac{q_t^2}{5}[\epsilon^{\alpha_1 \epsilon^* q v} e_t^{*\alpha_2} + \epsilon^{\alpha_2 \epsilon^* q v} e_t^{*\alpha_1} + \epsilon^{\alpha_1 \epsilon^* e^* v} q_t^{\alpha_2} + \epsilon^{\alpha_2 \epsilon^* e^* v} q_t^{\alpha_1}]\right\} g^{f2}_{T_2 S_1 \rho}.\end{aligned} \tag{A17}$$

Note that these decay amplitudes may be organized in another way. For example, the tensor structure corresponding to $g^{p0}_{H_1 H_1 \rho}$ was defined as $(e^* \cdot v)(\epsilon^* \cdot \eta_t)$ in Eq. (28) of Ref. [8] rather than $(e^* \cdot q_t)(\epsilon^* \cdot \eta_t)$ in Eq. (A3). Since we have $(e^* \cdot q_t) = -(q \cdot v)(e^* \cdot v)$, the essentially same sum rule as Eq. (24) of Ref. [8] can be obtained if we isolate the tensor structure $(e^* \cdot v)(\epsilon^* \cdot \eta_t)$.

To derive sum rules for these coupling constants, we consider the following correlators:

$$\int d^4x e^{-ik\cdot x}\langle \rho(q)|T\{J_{0,-,(1/2)}(0) J^{\dagger}_{0,-,(1/2)}(x)\}|0\rangle = (e^* \cdot q_t) G^{p0}_{H_0 H_0 \rho}(\omega, \omega'), \tag{A18}$$

$$\int d^4x e^{-ik\cdot x}\langle \rho(q)|T\{J_{0,-,(1/2)}(0) J^{\dagger\alpha}_{1,-,(1/2)}(x)\}|0\rangle = \epsilon^{\alpha e^* q v} G^{p1}_{H_1 H_0 \rho}(\omega, \omega'), \tag{A19}$$

$$\begin{aligned}\int d^4x e^{-ik\cdot x}\langle \rho(q)|T\{J^{\beta}_{1,-,(1/2)}(0) J^{\dagger\alpha}_{1,-,(1/2)}(x)\}|0\rangle &= g_t^{\alpha\beta}(e^* \cdot q_t) G^{p0}_{H_1 H_1 \rho}(\omega, \omega') \\ &+ (e_t^{*\alpha} q_t^{\beta} - q_t^{\alpha} e_t^{*\beta}) G^{p1}_{H_1 H_1 \rho}(\omega, \omega'),\end{aligned} \tag{A20}$$

$$\int d^4x e^{-ik\cdot x}\langle \rho(q)|T\{J_{0,+,(1/2)}(0) J^{\dagger}_{0,+,(1/2)}(x)\}|0\rangle = (e^* \cdot q_t) G^{p0}_{S_0 S_0 \rho}(\omega, \omega'), \tag{A21}$$

$$\int d^4x e^{-ik\cdot x}\langle\rho(q)|T\{J_{0,+,(1/2)}(0)J^{\dagger\alpha}_{1,+,(1/2)}(x)\}|0\rangle = \epsilon^{\alpha e^* q v}G^{p1}_{S_1S_0\rho}(\omega,\omega'), \tag{A22}$$

$$\int d^4x e^{-ik\cdot x}\langle\rho(q)|T\{J^{\beta}_{1,+,(1/2)}(0)J^{\dagger\alpha}_{1,+,(1/2)}(x)\}|0\rangle = g_t^{\alpha\beta}(e^*\cdot q_t)G^{p0}_{S_1S_1\rho}(\omega,\omega') + (e_t^{*\alpha}q_t^\beta - q_t^\alpha e_t^{*\beta})G^{p1}_{S_1S_1\rho}(\omega,\omega'), \tag{A23}$$

$$\int d^4x e^{-ik\cdot x}\langle\rho(q)|T\{J^{\beta}_{1,-,(1/2)}(0)J^{\dagger}_{0,+,(1/2)}(x)\}|0\rangle = e_t^{*\beta}G^{s1}_{S_0H_1\rho}(\omega,\omega') + \left[q_t^\beta(e^*\cdot q_t) - \frac{1}{3}e_t^{*\beta}q_t^2\right]G^{d1}_{S_0H_1\rho}(\omega,\omega'), \tag{A24}$$

$$\int d^4x e^{-ik\cdot x}\langle\rho(q)|T\{J_{0,-,(1/2)}(0)J^{\dagger\alpha}_{1,+,(1/2)}(x)\}|0\rangle = e_t^{*\alpha}G^{s1}_{S_1H_0\rho}(\omega,\omega') + \left[q_t^\alpha(e^*\cdot q_t) - \frac{1}{3}e_t^{*\alpha}q_t^2\right]G^{d1}_{S_1H_0\rho}(\omega,\omega'), \tag{A25}$$

$$\int d^4x e^{-ik\cdot x}\langle\rho(q)|T\{J^{\beta}_{1,-,(1/2)}(0)J^{\dagger\alpha}_{1,+,(1/2)}(x)\}|0\rangle = \epsilon^{\alpha\beta e^* v}G^{s1}_{S_1H_1\rho}(\omega,\omega') + \left[\epsilon^{\alpha\beta q v}(e^*\cdot q_t) - \frac{1}{3}\epsilon^{\alpha\beta e^* v}q_t^2\right]G^{d1}_{S_1H_1\rho}(\omega,\omega'), \tag{A26}$$

$$\int d^4x e^{-ik\cdot x}\langle\rho(q)|T\{J_{0,-,(1/2)}(0)J^{\dagger\alpha}_{1,+,(3/2)}(x)\}|0\rangle = e_t^{*\alpha}G^{s1}_{T_1H_0\rho}(\omega,\omega') + \left[q_t^\alpha(e^*\cdot q_t) - \frac{1}{3}e_t^{*\alpha}q_t^2\right]G^{d1}_{T_1H_0\rho}(\omega,\omega'), \tag{A27}$$

$$\int d^4x e^{-ik\cdot x}\langle\rho(q)|T\{J_{0,+,(1/2)}(0)J^{\dagger\alpha}_{1,+,(3/2)}(x)\}|0\rangle = \epsilon^{\alpha e^* q v}G^{p1}_{T_1S_0\rho}(\omega,\omega'), \tag{A28}$$

$$\int d^4x e^{-ik\cdot x}\langle\rho(q)|T\{J^{\beta}_{1,+,(1/2)}(0)J^{\dagger\alpha}_{1,+,(3/2)}(x)\}|0\rangle = g_t^{\alpha\beta}(e^*\cdot q_t)G^{p0}_{T_1S_1\rho}(\omega,\omega') + (e_t^{*\alpha}q_t^\beta - q_t^\alpha e_t^{*\beta})G^{p1}_{T_1S_1\rho}(\omega,\omega'), \tag{A29}$$

$$\int d^4x e^{-ik\cdot x}\langle\rho(q)|T\{J_{0,-,(1/2)}(0)J^{\dagger\alpha_1\alpha_2}_{2,+,(3/2)}(x)\}|0\rangle = (\epsilon^{\alpha_1 e^* q v}q_t^{\alpha_2} + \epsilon^{\alpha_2 e^* q v}q_t^{\alpha_1})G^{d2}_{T_2H_0\rho}(\omega,\omega'), \tag{A30}$$

$$\begin{aligned}\int d^4x e^{-ik\cdot x}&\langle\rho(q)|T\{J^{\beta}_{1,-,(1/2)}(0)J^{\dagger\alpha_1\alpha_2}_{2,+,(3/2)}(x)\}|0\rangle \\ =&\left[g_t^{\alpha_1\beta}e_t^{*\alpha_2} + g_t^{\alpha_2\beta}e_t^{*\alpha_1} - \frac{2}{3}g_t^{\alpha_1\alpha_2}e_t^{*\beta}\right]G^{s1}_{T_2H_1\rho}(\omega,\omega') + \left\{\left[g_t^{\alpha_1\beta}q_t^{\alpha_2} + g_t^{\alpha_2\beta}q_t^{\alpha_1} - \frac{2}{3}g_t^{\alpha_1\alpha_2}q_t^\beta\right](e^*\cdot q_t)\right. \\ &\left. - \frac{1}{3}\left[g_t^{\alpha_1\beta}e_t^{*\alpha_2} + g_t^{\alpha_2\beta}e_t^{*\alpha_1} - \frac{2}{3}g_t^{\alpha_1\alpha_2}e_t^{*\beta}\right]q_t^2\right\}G^{d1}_{T_2H_1\rho}(\omega,\omega') + [2(e_t^{*\alpha_1}q_t^{\alpha_2}q_t^\beta + q_t^{\alpha_1}e_t^{*\alpha_2}q_t^\beta - 2q_t^{\alpha_1}q_t^{\alpha_2}e_t^{*\beta}) \\ &+ (g_t^{\alpha_1\beta}q_t^{\alpha_2} + g_t^{\alpha_2\beta}q_t^{\alpha_1} - 2g_t^{\alpha_1\alpha_2}q_t^\beta)(e^*\cdot q_t) - (g_t^{\alpha_1\beta}e_t^{*\alpha_2} + g_t^{\alpha_2\beta}e_t^{*\alpha_1} - 2g_t^{\alpha_1\alpha_2}e_t^{*\beta})q_t^2]G^{d2}_{T_2H_1\rho}(\omega,\omega'),\end{aligned} \tag{A31}$$

$$\begin{aligned}\int d^4x e^{-ik\cdot x}&\langle\rho(q)|T\{J_{0,+,(1/2)}(0)J^{\dagger\alpha_1\alpha_2}_{2,+,(3/3)}(x)\}|0\rangle \\ =&\left[e_t^{*\alpha_1}q_t^{\alpha_2} + q_t^{\alpha_1}e_t^{*\alpha_2} - \frac{2}{3}g_t^{\alpha_1\alpha_2}(e^*\cdot q_t)\right]G^{p2}_{T_2S_0\rho}(\omega,\omega') + \left\{q_t^{\alpha_1}q_t^{\alpha_2}(e^*\cdot q_t)\right. \\ &\left. - \frac{q_t^2}{5}\left[g_t^{\alpha_1\alpha_2}(e^*\cdot q_t) + e_t^{*\alpha_1}q_t^{\alpha_2} + q_t^{\alpha_1}e_t^{*\alpha_2}\right]\right\}G^{f2}_{T_2S_0\rho}(\omega,\omega'),\end{aligned} \tag{A32}$$

$$\int d^4x e^{-ik\cdot x}\langle\rho(q)|T\{J^{\beta}_{1,+,(1/2)}(0)J^{\dagger\alpha_1\alpha_2}_{2,+,(3/2)}(x)\}|0\rangle$$
$$= \Big[-\epsilon^{\alpha_1 e^* qv}g_t^{\alpha_2\beta} - \epsilon^{\alpha_2 e^* qv}g_t^{\alpha_1\beta} + \frac{2}{3}g_t^{\alpha_1\alpha_2}\epsilon^{\beta e^* qv}\Big]G^{p1}_{T_2S_1\rho}(\omega,\omega')$$
$$+ [\epsilon^{\alpha_1\beta e^* v}q_t^{\alpha_2} + \epsilon^{\alpha_2\beta e^* v}q_t^{\alpha_1} + \epsilon^{\alpha_1\beta qv}e_t^{*\alpha_2} + \epsilon^{\alpha_2\beta qv}e_t^{*\alpha_1}]G^{p2}_{T_2S_1\rho}(\omega,\omega')$$
$$+ \Big\{\epsilon^{\alpha_1\beta qv}q_t^{\alpha_2}(e^*\cdot q_t) + \epsilon^{\alpha_2\beta qv}q_t^{\alpha_1}(e^*\cdot q_t) - \frac{q_t^2}{5}\Big[\epsilon^{\alpha_1\beta qv}e_t^{*\alpha_2} + \epsilon^{\alpha_2\beta qv}e_t^{*\alpha_1} + \epsilon^{\alpha_1\beta e^* v}q_t^{\alpha_2} + \epsilon^{\alpha_2\beta e^* v}q_t^{\alpha_1}\Big]\Big\}$$
$$\times G^{f2}_{T_2S_1\rho}(\omega,\omega'). \tag{A33}$$

APPENDIX B: THE DEFINITIONS OF THE ρ MESON LIGHT-CONE DISTRIBUTION AMPLITUDES

The definitions of the distribution amplitudes used in the text read as [15]

$$\langle 0|\bar u(z)\gamma_\mu d(-z)|\rho^-(P,\lambda)\rangle = f_\rho m_\rho\Big[p_\mu\frac{e^{(\lambda)}\cdot z}{p\cdot z}\int_0^1 du e^{i\xi p\cdot z}\varphi_\parallel(u,\mu^2) + e^{(\lambda)}_{\perp\mu}\int_0^1 du e^{i\xi p\cdot z}g_\perp^{(v)}(u,\mu^2) - \frac{1}{2}z_\mu\frac{e^{(\lambda)}\cdot z}{(p\cdot z)^2}m_\rho^2\int_0^1 du e^{i\xi p\cdot z}g_3(u,\mu^2)\Big], \tag{B1}$$

$$\langle 0|\bar u(z)\gamma_\mu\gamma_5 d(-z)|\rho^-(P,\lambda)\rangle = \frac{1}{2}f_\rho m_\rho\epsilon_\mu^{\nu\alpha\beta}e^{(\lambda)}_{\perp\nu}p_\alpha z_\beta\int_0^1 du e^{i\xi p\cdot z}g_\perp^{(a)}(u,\mu^2), \tag{B2}$$

$$\langle 0|\bar u(z)\sigma_{\mu\nu}d(-z)|\rho^-(P,\lambda)\rangle = if_\rho^T\Big[(e^{(\lambda)}_{\perp\mu}p_\nu - e^{(\lambda)}_{\perp\nu}p_\mu)\int_0^1 du e^{i\xi p\cdot z}\varphi_\perp(u,\mu^2) + (p_\mu z_\nu - p_\nu z_\mu)\times\frac{e^{(\lambda)}\cdot z}{(p\cdot z)^2}m_\rho^2\int_0^1 du e^{i\xi p\cdot z}h_\parallel^{(t)}(u,\mu^2) + \frac{1}{2}(e^{(\lambda)}_{\perp\mu}z_\nu - e^{(\lambda)}_{\perp\nu}z_\mu)\frac{m_\rho^2}{p\cdot z}\int_0^1 du e^{i\xi p\cdot z}h_3(u,\mu^2)\Big], \tag{B3}$$

$$\langle 0|\bar u(z)d(-z)|\rho^-(P,\lambda)\rangle = -if_\rho^T(e^{(\lambda)}z)m_\rho^2\int_0^1 du e^{i\xi p\cdot z}h_\parallel^{(s)}(u,\mu^2). \tag{B4}$$

The vector and tensor decay constants f_ρ and f_ρ^T are defined as

$$\langle 0|\bar u(0)\gamma_\mu d(0)|\rho^-(P,\lambda)\rangle = f_\rho m_\rho e_\mu^{(\lambda)}, \tag{B5}$$

$$\langle 0|\bar u(0)\sigma_{\mu\nu}d(0)|\rho^-(P,\lambda)\rangle = if_\rho^T(e_\mu^{(\lambda)}P_\nu - e_\nu^{(\lambda)}P_\mu). \tag{B6}$$

The distribution amplitude $\varphi_\parallel$ and $\varphi_\perp$ are of twist-2, $g_\perp^{(v)}$, $g_\perp^{(a)}$, $h_\parallel^{(s)}$, and $h_\parallel^{(t)}$ are twist-3 and g_3, h_3 are twist-4. All functions $\phi = \{\varphi_\parallel, \varphi_\perp, g_\perp^{(v)}, g_\perp^{(a)}, h_\parallel^{(s)}, h_\parallel^{(t)}, g_3, h_3\}$ are normalized to satisfy $\int_0^1 du\phi(u) = 1$.

The 3-particle distribution amplitudes are defined as [15]

$$\langle 0|\bar u(z)g\tilde G_{\mu\nu}\gamma_\alpha\gamma_5 d(-z)|\rho^-(P,\lambda)\rangle = f_\rho m_\rho p_\alpha[p_\nu e^{(\lambda)}_{\perp\mu} - p_\mu e^{(\lambda)}_{\perp\nu}]\mathcal{A}(v,pz) + f_\rho m_\rho^3\frac{e^{(\lambda)}\cdot z}{pz}[p_\mu g^\perp_{\alpha\nu} - p_\nu g^\perp_{\alpha\mu}]\tilde\Phi(v,pz) + f_\rho m_\rho^3\frac{e^{(\lambda)}\cdot z}{(pz)^2}p_\alpha[p_\mu z_\nu - p_\nu z_\mu]\tilde\Psi(v,pz) \tag{B7}$$

$$\langle 0|\bar u(z)gG_{\mu\nu}i\gamma_\alpha d(-z)|\rho^-(P)\rangle = f_\rho m_\rho p_\alpha[p_\nu e^{(\lambda)}_{\perp\mu} - p_\mu e^{(\lambda)}_{\perp\nu}\mathcal{V}(v,pz) + f_\rho m_\rho^3\frac{e^{(\lambda)}\cdot z}{pz}[p_\mu g^\perp_{\alpha\nu} - p_\nu g^\perp_{\alpha\mu}]\Phi(v,pz) + f_\rho m_\rho^3\frac{e^{(\lambda)}\cdot z}{(pz)^2}p_\alpha[p_\mu z_\nu - p_\nu z_\mu]\Psi(v,pz), \tag{B8}$$

$$\begin{aligned}
&\langle 0|\bar{u}(z)\sigma_{\alpha\beta}gG_{\mu\nu}(vz)d(-z)|\rho^-(P,\lambda)\rangle \\
&= f_\rho^T m_\rho^3 \frac{e^{(\lambda)}\cdot z}{2(p\cdot z)}[p_\alpha p_\mu g^\perp_{\beta\nu} - p_\beta p_\mu g^\perp_{\alpha\nu} - p_\alpha p_\nu g^\perp_{\beta\mu} + p_\beta p_\nu g^\perp_{\alpha\mu}]\mathcal{T}(v,pz) \\
&\quad + f_\rho^T m_\rho^2[p_\alpha e^{(\lambda)}_{\perp\mu} g^\perp_{\beta\nu} - p_\beta e^{(\lambda)}_{\perp\mu} g^\perp_{\alpha\nu} - p_\alpha e^{(\lambda)}_{\perp\nu} g^\perp_{\beta\mu} + p_\beta e^{(\lambda)}_{\perp\nu} g^\perp_{\alpha\mu}]T_1^{(4)}(v,pz) \\
&\quad + f_\rho^T m_\rho^2[p_\mu e^{(\lambda)}_{\perp\alpha} g^\perp_{\beta\nu} - p_\mu e^{(\lambda)}_{\perp\beta} g^\perp_{\alpha\nu} - p_\nu e^{(\lambda)}_{\perp\alpha} g^\perp_{\beta\mu} + p_\nu e^{(\lambda)}_{\perp\beta} g^\perp_{\alpha\mu}]T_2^{(4)}(v,pz) \\
&\quad + \frac{f_\rho^T m_\rho^2}{pz}[p_\alpha p_\mu e^{(\lambda)}_{\perp\beta} z_\nu - p_\beta p_\mu e^{(\lambda)}_{\perp\alpha} z_\nu - p_\alpha p_\nu e^{(\lambda)}_{\perp\beta} z_\mu + p_\beta p_\nu e^{(\lambda)}_{\perp\alpha} z_\mu]T_3^{(4)}(v,pz) \\
&\quad + \frac{f_\rho^T m_\rho^2}{pz}[p_\alpha p_\mu e^{(\lambda)}_{\perp\nu} z_\beta - p_\beta p_\mu e^{(\lambda)}_{\perp\nu} z_\alpha - p_\alpha p_\nu e^{(\lambda)}_{\perp\mu} z_\beta + p_\beta p_\nu e^{(\lambda)}_{\perp\mu} z_\alpha]T_4^{(4)}(v,pz) + \ldots
\end{aligned} \tag{B9}$$

$$\begin{aligned}
\langle 0|\bar{u}(z)gG_{\mu\nu}(vz)d(-z)|\rho^-(P,\lambda)\rangle &= if_\rho^T m_\rho^2[e^{(\lambda)}_{\perp\mu}p_\nu - e^{(\lambda)}_{\perp\nu}p_\mu]S(v,pz), \langle 0|\bar{u}(z)ig\tilde{G}_{\mu\nu}(vz)\gamma_5 d(-z)|\rho^-(P,\lambda)\rangle \\
&= if_\rho^T m_\rho^2[e^{(\lambda)}_{\perp\mu}p_\nu - e^{(\lambda)}_{\perp\nu}p_\mu]\tilde{S}(v,pz),
\end{aligned} \tag{B10}$$

where

$$\mathcal{A}(v,pz) = \int \mathcal{D}\underline{\alpha} e^{-ipz(\alpha_u - \alpha_d + v\alpha_g)}\mathcal{A}(\underline{\alpha}), \tag{B11}$$

etc. The integration measure is

$$\int \mathcal{D}\underline{\alpha} \equiv \int_0^1 d\alpha_d \int_0^1 d\alpha_u \int_0^1 d\alpha_g \delta\Big(1 - \sum \alpha_i\Big). \tag{B12}$$

The distribution amplitudes $\mathcal{A}$, $\mathcal{V}$, and $\mathcal{T}$ are twist-3 and the others are twist-4.

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