

Pair production of heavy quarkonium and $B_c^{(*)}$ mesons at hadron collidersRong Li,¹ Yu-Jie Zhang,¹ and Kuang-Ta Chao^{1,2}¹*Department of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China*²*Center for High Energy Physics, Peking University, Beijing 100871, China*

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We investigate the pair production of S -wave heavy quarkonium at the LHC in the color-singlet mechanism (CSM) and estimate the contribution from the gluon fragmentation process in the color-octet mechanism (COM) for comparison. With the matrix elements extracted previously in the leading-order calculations, the numerical results show that the production rates are quite large for the pair production processes at the LHC. The p_t distribution of double J/ψ production in the CSM is dominant over that in the COM when p_t is smaller than about 10 GeV. For the production of double Y , the contribution of the COM is always larger than that in the CSM. The large differences in the theoretical predictions between the CSM and COM for the p_t distributions in the large p_t region are useful in clarifying the effects of COM on the quarkonium production. We also investigate the pair production of S -wave B_c and B_c^* mesons, and the measurement of these processes is useful to test the CSM and extract the long-distance matrix elements for the B_c and B_c^* mesons. Aside from numerical calculations, analytical expressions for the production differential cross sections of all these processes are also given.

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I. INTRODUCTION

Heavy quarkonium provides an ideal system to investigate both the perturbative and nonperturbative aspects of quantum chromodynamics. Conventionally, the color-singlet mechanism (CSM) is used to describe the decay and production of heavy quarkonium [1]. In the CSM, the processes are factorized into two steps. First the heavy quark pair are created perturbatively at short distances with the same color and angular momentum as the final quarkonium state, and then evolve into the quarkonium nonperturbatively at long distance. There have been, however, some problems in the CSM, e.g., the infrared divergences in the calculation of the decay of P -wave quarkonium [2] and the higher-order correction calculation of S -wave quarkonium [3], and the surplus J/ψ production [4] of which the rate is much higher than that of the color-singlet prediction at the Tevatron. The non-relativistic quantum chromodynamics (NRQCD) factorization formalism [5], which was put forward by Bodwin, Braaten, and Lepage, overcame the infrared divergence difficulties in the color-singlet model [6], and gave the proper prediction for the charmonium production at the Tevatron [7]. In NRQCD, the heavy quark pair at short distance are not necessarily in the color-singlet state but can be in the states with different color and angular-momentum from that of the final state quarkonium. The color-octet pair can evolve to the color-singlet charmonium by emitting soft gluons. This is called the color-octet mechanism (COM).

Lots of work have been done to investigate the validity and limitation of the NRQCD formalism in heavy quark-

onium production. The current experimental results on J/ψ photoproduction at HERA are fairly well described by the NLO color-singlet piece except the J/ψ polarizations [8]. The DELPHI data favor the NRQCD color-octet mechanism for J/ψ production $\gamma\gamma \rightarrow J/\psi X$ [9]. The observed large cross sections of inclusive charmonium production at the Tevatron once gave strong support to the color-octet gluon fragmentation in NRQCD, but recently it is found that the NLO results in the color-singlet piece can bring an order of magnitude enhancement to the J/ψ production rate in the large p_t region [10] with longitudinally polarized J/ψ [11]. The theoretical prediction for p_t distribution of the Y production can properly describe the Tevatron data by including the contributions from the NLO results and the real correction part at the next-to-next-to-leading-order in the CSM [12]. The cross sections of J/ψ exclusive and inclusive production in e^+e^- annihilation at B factories [13] are much larger than the LO NRQCD predictions [14,15], but the discrepancies seem to be resolved by considering the higher-order effects: NLO QCD corrections [16–18] and relativistic corrections [19] without invoking the color-octet contributions [17] (discussions in the light-cone approach or the relativistic quark model can be seen in [20,21]). Recent developments and related topics in quarkonium production can be found in Refs. [22].

The above mentioned developments in heavy quarkonium production indicate that the situation is far from being conclusive, and further tests for the color-singlet and color-octet mechanisms in NRQCD are still needed to clarify various problems involved in heavy quarkonium production.

In order to investigate the effects of the COM on the production of heavy quarkonium, it is useful to study processes which heavily depends on the production mechanism. The pair production of heavy quarkonium can serve as the desired process. In NRQCD, the gluon fragmentation gives the main contribution to the pair production of quarkonium in the large p_t region of the heavy quarkonium. In the pair production processes, there appear two long-distance matrix elements (LDMEs). So the difference of theoretical predictions between the CSM and COM could be more obvious. Moreover, because of charge-parity C conservation the gluon fusion processes $g + g \rightarrow J/\psi + \chi_c$ and $g + g \rightarrow J/\psi + \eta_c$ are forbidden in the CSM. But the gluon fragmentation in the COM can produce these associated final states. So to detect two final heavy quarkonium states with different C -parity may give a good way to test the COM.

The pair production of heavy quarkonium at hadron colliders has been studied by many authors. The color-octet gluon fragmentation into double charmonium at the Tevatron in NRQCD were considered as evidence for the COM [23]. The CSM prediction on the double charmonium production was made and it was found that the contribution with $p_t < 4$ GeV in the CSM is dominant [24]. Only in the large p_t region, the CSM and COM give manifestly different predictions for the double charmonium production. The Large Hadron Collider (LHC) is expected to produce a huge number of heavy quarkonium. Therefore, it is natural to investigate the pair production of heavy quarkonium at the LHC. The double J/ψ production is also studied in $\gamma\gamma$ collision [25] and at the RHIC [26].

At the LHC, it is also interesting to study the production of the double-heavy flavored mesons B_c and B_c^* . The single $B_c^{(*)}$ meson production in hadron collisions has been studied in QCD [27,28]. More literature and recent progress may be found in Ref. [29] and references therein. Some study of $B_c^{(*)}$ pair production was also performed in pp and $\gamma\gamma$ collisions [30], but no discussions on p_T distributions at the LHC were given, and no studies on

the $B_c\bar{B}_c^*$ were presented. Therefore, it is useful to extend the study including the p_T distributions and total cross sections to the pair production of the $B_c^{(*)}$ meson pairs ($B_c\bar{B}_c$, $B_c\bar{B}_c^*$, and $B_c^*\bar{B}_c^*$) at the LHC.

In this paper, we study the pair productions of heavy quarkonium at the Tevatron and LHC, including J/ψ , η_c , Υ and η_b in the CSM. For comparison, the color-octet contributions to the pair production of J/ψ and Υ are estimated by considering the gluon fragmentation process. We also investigate the pair productions of S -wave B_c and B_c^* mesons where there is no contribution from the gluon fragmentation process and the COM contributions are suppressed by the small v^2 (the relative velocity between quark and antiquark). Therefore, these processes can give a better test of CSM and be used to extract the LDMEs of the B_c and B_c^* mesons.

The outline of our paper is as follows. In Sec. II, some of the definitions and formulas are given for deriving the cross sections of the processes. Then the numerical results are presented in Sec. III. Finally, in Sec. IV we give the summary.

II. THE FORMULATIONS

A. Color-singlet part

At the hadron collider, the pair production of heavy quarkonium at the leading-order (LO) in the CSM have two subprocesses $g + g \rightarrow Q_1 + Q_2$ and $q + \bar{q} \rightarrow Q_1 + Q_2$. But we just consider the gluon fusion process in the calculation since it is the dominant one. There are 31 Feynman diagrams for the processes $g + g \rightarrow J/\psi + J/\psi$ and $g + g \rightarrow B_c(B_c^*) + \bar{B}_c(\bar{B}_c^*)$. The typical Feynman diagrams are presented in Fig. 1. For the process $g + g \rightarrow \eta_c + \eta_c$, there are additional 8 Feynman diagrams which are showed in Fig. 2. The Feynman diagrams for the process of Υ and η_b production are as same as the corresponding process of charmonium production.

Following the color-singlet factorization formalism, the amplitude of the pair production of S -wave heavy quarkonium is written as

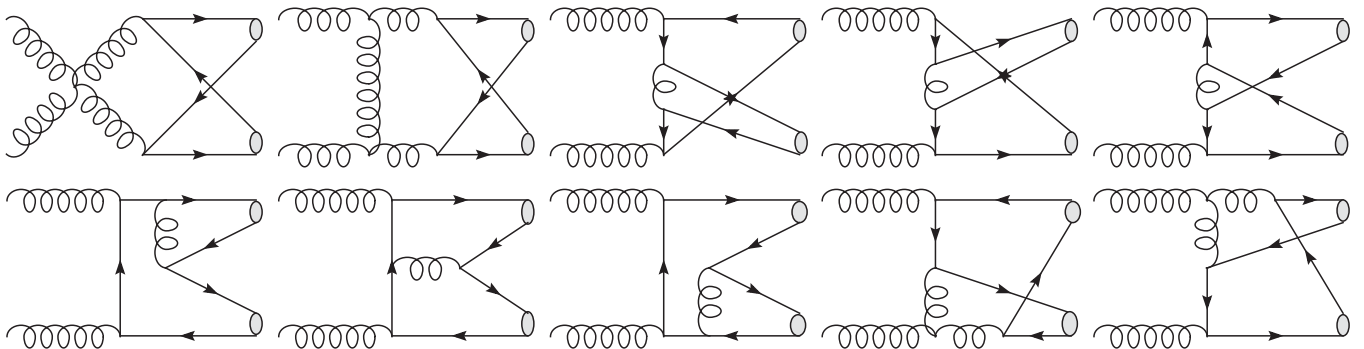


FIG. 1. The typical Feynman diagrams for $g + g \rightarrow J/\psi + J/\psi$. The others can be obtained by reversing the fermion lines or interchanging the initial gluons. As for $g + g \rightarrow B_c(B_c^*) + \bar{B}_c(\bar{B}_c^*)$, there are same diagrams.

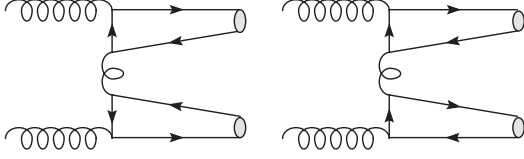


FIG. 2. The additional typical Feynman diagrams of $g + g \rightarrow \eta_c + \eta_c$. The others can be obtained by reversing the fermion lines or interchanging the initial gluons.

$$\begin{aligned} & \mathcal{M}(a + b \rightarrow Q_1 + Q_2) \\ &= \sum_{s_1, s_2, s_3, s_4, i, j, k, l} N_1(\lambda|s_1, s_2) N_2(\lambda|s_3, s_4) \frac{\delta^{ij}}{\sqrt{N_c}} \frac{\delta^{kl}}{\sqrt{N_c}} \frac{R_1(0)}{\sqrt{4\pi}} \\ & \times \frac{R_2(0)}{\sqrt{4\pi}} \mathcal{M}(a + b \rightarrow Q_i \bar{Q}_j(\mathbf{p}_1 = \mathbf{0}; s_1, s_2) \\ & + Q_k \bar{Q}_l(\mathbf{p}_2 = \mathbf{0}; s_3, s_4)) \end{aligned} \quad (1)$$

where the s_i is the spin of the heavy quark in the meson; $R_i(0)$ is the wave function at the origin of the heavy quarkonium; $\frac{\delta^{ij}}{\sqrt{N_c}}$ is the color project operator; $N_i(\lambda|s_1, s_2)$ is the spin project operator as following

$$N(\lambda|s_1, s_2) = \frac{\sqrt{M_{Q\bar{Q}}}\epsilon_\mu^\lambda \bar{v}(\mathbf{P}_{Q\bar{Q}}, s_2) \gamma^\mu u(\mathbf{P}_{Q\bar{Q}}, s_1)}{4m_{Q\bar{Q}}}, \quad (2)$$

where $M_{Q\bar{Q}}$ is the mass of the heavy quarkonium.

We analytically calculate the amplitude square of these subprocesses and present all the analytical formulas in appendix. The formula for $d\hat{\sigma}/dt$ on the double J/ψ production is consistent with that in Ref. [24]. The final result can be obtained by convoluting the parton level cross section with the parton distribution function $f_{g/p}(x)$ as following

$$\begin{aligned} & d\sigma(p + p(\bar{p}) \rightarrow Q_1 Q_2 + X) \\ &= \int dx_1 dx_2 f_{g_1/p}(x_1, \mu_f) f_{g_2/p(\bar{p})}(x_2, \mu_f) d\hat{\sigma} \\ & \times (g_1 + g_2 \rightarrow Q_1 Q_2, \mu_r), \end{aligned} \quad (3)$$

where the μ_r and μ_f are the renormalization and factorization scale.

B. Color-octet part

As a comparison, we naively estimate the pair production of J/ψ and Y in the COM by using the similar way as in Ref. [23] in which the evolution of the fragmentation function was ignored. In the COM, a gluon can fragment into a $c\bar{c}$ pair with the quantum number ${}^3S_1^{(8)}$ and then hadronize into J/ψ . This process will give large contribution to the double-heavy quarkonium production in the large p_t region and is expressed as

$$\begin{aligned} d\hat{\sigma}_{Q_1+Q_2} &= \int_0^1 dz_1 \int_0^1 dz_2 D_{g \rightarrow Q_1}(z_1, m_{Q_1}) \\ & \times D_{g \rightarrow Q_2}(z_2, m_{Q_2}) d\hat{\sigma}_{gg}(E_1/z_1, E_2/z_2), \end{aligned} \quad (4)$$

where $\hat{\sigma}_{gg}$ is the cross section of two real gluon production; D is the fragmentation function for a gluon to fragment into a quarkonium. In NRQCD, this fragmentation function is written as [5]

$$D_{g \rightarrow Q}(z, \mu^2) = \sum_n d_{g \rightarrow n}(z, \mu^2) \langle O_n^H \rangle. \quad (5)$$

The short distance coefficient can be calculated perturbatively and the result of the LO calculation is

$$d_{g \rightarrow \underline{g}^3 S_1} = \frac{\pi \alpha_s (2m_Q)}{24m_Q^3} \delta(1-z). \quad (6)$$

The contribution from $gg(q\bar{q}) \rightarrow gg$ subprocesses is calculated and the contributions from the feeddown of ψ' , $\chi_{cJ}(1P)$, $Y(2S)$ and $\chi_{bJ}(1P)$ are also included. The final result is expressed as

$$\begin{aligned} d\hat{\sigma}_{Q_1+Q_2} &= d\hat{\sigma}_{gg} \left(\frac{\pi \alpha_s (4m_c^2)}{24m_c^3} \right)^2 [\langle O_8^{J/\psi}({}^3S_1) \rangle + \langle O_8^{\psi'}({}^3S_1) \rangle \\ & \times \text{Br}(\psi' \rightarrow J/\psi) + \sum_{J=0}^2 (2J+1) \\ & \times \langle O_8^{\chi_{c0}(1P)}({}^3S_1) \rangle \text{Br}(\chi_{cJ} \rightarrow J/\psi)]^2. \end{aligned} \quad (7)$$

Here it is noteworthy that the identical particle factor ‘‘2’’ has been put in the calculation of $d\hat{\sigma}_{gg}$.

III. THE NUMERICAL RESULTS AND CONCLUSIONS

In calculating the numerical results, we choose the following parameters $M_c = 1.5$ GeV and $M_b = 4.9$ GeV, and set the renormalization and factorization scale as $\mu_r = \mu_f = \sqrt{M_Q^2 + p_t^2}$, where M_Q is the meson mass. For the gluon fragmentation process, the renormalization and factorization scale are chosen as the transverse momentum $p_t(g)$ of the gluon with $p_t(J/\psi) \simeq p_t(g)$. The center-of-mass energies of the Tevatron and LHC are 1.96 TeV and 14 TeV, respectively. The pseudorapidity cuts on the final quarkonium states are chosen as $-0.6 < \eta < 0.6$ at the Tevatron and $-2.4 < \eta < 2.4$ at the LHC. We use the CTEQ6L1 [31] and CTEQ5L [32] parton distribution functions (PDFs) to calculate the numerical results of $c\bar{c}$ systems for comparison. The corresponding LDMEs are taken from Ref. [33,34] respectively and listed in Table I. Therefore, the running values of α_s are evaluated by the LO formula of CTEQ6 and CTEQ5, respectively. For the $b\bar{b}$ systems, we use the LDMEs in Ref. [35] as

TABLE I. The LDMEs for $c\bar{c}$ systems that are taken from Ref. [33,34]. (unit: GeV^3)

LDMEs	Ref. [33]	Ref. [34]
$\langle O_1^{J/\psi(1S)}(3S_1) \rangle$	1.4	1.4
$\langle O_8^{J/\psi(1S)}(3S_1) \rangle$	0.23×10^{-2}	0.39×10^{-2}
$\langle O_8^{\psi'(2S)}(3S_1) \rangle$	0.20×10^{-2}	0.37×10^{-2}
$\langle O_8^{\chi_{c0}(1P)}(3S_1) \rangle$	0.11×10^{-2}	0.19×10^{-2}

$$\begin{aligned}
 \langle O_1^{Y(1S)}(3S_1) \rangle &= 9.28 \text{ GeV}^3, \\
 \langle O_8^{Y(1S)}(3S_1) \rangle &= 15 \times 10^{-2} \text{ GeV}^3, \\
 \langle O_8^{Y(2S)}(3S_1) \rangle &= 4.5 \times 10^{-2} \text{ GeV}^3, \\
 \langle O_8^{\chi_{b0}(1P)}(3S_1) \rangle &= 4.0 \times 10^{-2} \text{ GeV}^3.
 \end{aligned} \tag{8}$$

and the CTEQ5L PDF is used for consistency. And we use the wave function at the origin of $b\bar{c}$ system as

$$|R(0)|_{b\bar{c}(1S)}^2 = 1.508 \text{ GeV}^3, \tag{9}$$

that is calculated by using the logarithmic potential [36] with quark masses being almost the same as what we use. We also use the CTEQ6L1 PDF to calculate the results of $b\bar{c}$ systems.

In order to estimate the results of the gluon fragmentation processes for comparison, the values of α_s should be given at scales of $M_{J/\psi}$ and M_Y . From the LO formula of α_s running of CTEQ6, the corresponding α_s in the fragmentation function for the J/ψ and Y are chosen as $\alpha_s(M_{J/\psi}) = 0.286$, $\alpha_s(M_Y) = 0.201$ respectively. The running values of CTEQ5L give $\alpha_s(M_{J/\psi}) = 0.274$ and $\alpha_s(M_Y) = 0.195$. The branching ratios in Eq. (7) are taken from the PDG08 [37].

In the Table II, we give the cross sections of pair production of J/ψ (Y) and B_c (B_c^*) at the Tevatron and LHC with $p_t > 3$ GeV in the CSM. From the table, the cross section of each process is enhanced by about 2 orders or more in magnitude at the LHC than that at the Tevatron. Therefore, the LHC will be a good place to study the pair

production processes. It also can be seen that the total cross sections of $c\bar{c}$ systems with CTEQ6L1 are smaller than that with CTEQ5L about 10%. This comes from the difference of PDFs because their color-singlet matrix elements are the same.

Figure 3 shows the p_t distribution of the pair production for J/ψ and η_c at the Tevatron and LHC with CTEQ6L1 PDF. The result of the gluon fragmentation process in the COM is also plotted in the figure. We can see that whether at the Tevatron or at the LHC the p_t distributions of J/ψ and η_c are similar and the numerical results at the LHC are enhanced by about 2 orders or more in magnitude at large p_t region than that at the Tevatron. Therefore, the LHC will provide a chance to measure the p_t distribution of J/ψ pair production. Comparing the J/ψ pair production in the CSM with that in the COM, the formal is dominant as the p_t is smaller than about 10 GeV. And the result in the COM become dominant and even larger than that in the CSM for three orders in the large p_t region. For the color-singlet production of double $c\bar{c}$ systems, the differences between the p_t distributions of CTEQ6L1 and CTEQ5L are less than 20% in the region $3 \text{ GeV} < p_t < 50 \text{ GeV}$, which is consistent with the situation of total cross section. And the p_t distribution from the gluon fragmentation estimated by CTEQ5L PDFs is about 2 times of that by CTEQ6L1 as shown in Fig. 3.

The p_t distributions of Y and η_b pair production are shown in Fig. 4. The p_t distribution is enhanced by about 2 orders or more in magnitude at the LHC than that at the Tevatron. But unlike the case of J/ψ pair production, the pair production of Y in the COM dominate over that in the CSM in the whole p_t region, and even is 2 orders or more in magnitude larger than that in the CSM at large p_t .

Because the B_c and B_c^* are consist of quarks with different flavors, there is no contribution from gluon fragmentation processes in the COM. So in Fig. 5, we only give the p_t distribution of the pair production for $B_c\bar{B}_c$, $B_c\bar{B}_c^*$ and $B_c^*\bar{B}_c^*$ in the CSM. The pair production of $B_c^*\bar{B}_c^*$ is dominant in the whole p_t region. There are little difference between the production of the three final states with double-heavy flavor mesons in the CSM.

 TABLE II. The cross sections of pair production of J/ψ , Y , B_c and B_c^* at the Tevatron and LHC with $p_t > 3$ GeV. The results of $c\bar{c}$ systems with different PDFs are listed as comparison.

Final States	CTEQ6L1		CTEQ5L	
	$\sigma_{\text{Tevatron}}[nb]$	$\sigma_{\text{LHC}}[nb]$	$\sigma_{\text{Tevatron}}[nb]$	$\sigma_{\text{LHC}}[nb]$
$\eta_c\eta_c$	4.99×10^{-3}	4.10	4.35×10^{-3}	4.2
$J/\psi J/\psi$	8.46×10^{-2}	4.25	7.49×10^{-2}	4.6
$\eta_b\eta_b$			2.66×10^{-5}	1.16×10^{-2}
YY			1.74×10^{-4}	2.46×10^{-2}
$B_c\bar{B}_c$	3.86×10^{-3}	2.72×10^{-1}		
$B_c\bar{B}_c^*$	1.00×10^{-3}	8.37×10^{-2}		
$B_c^*\bar{B}_c^*$	8.23×10^{-3}	7.08×10^{-1}		

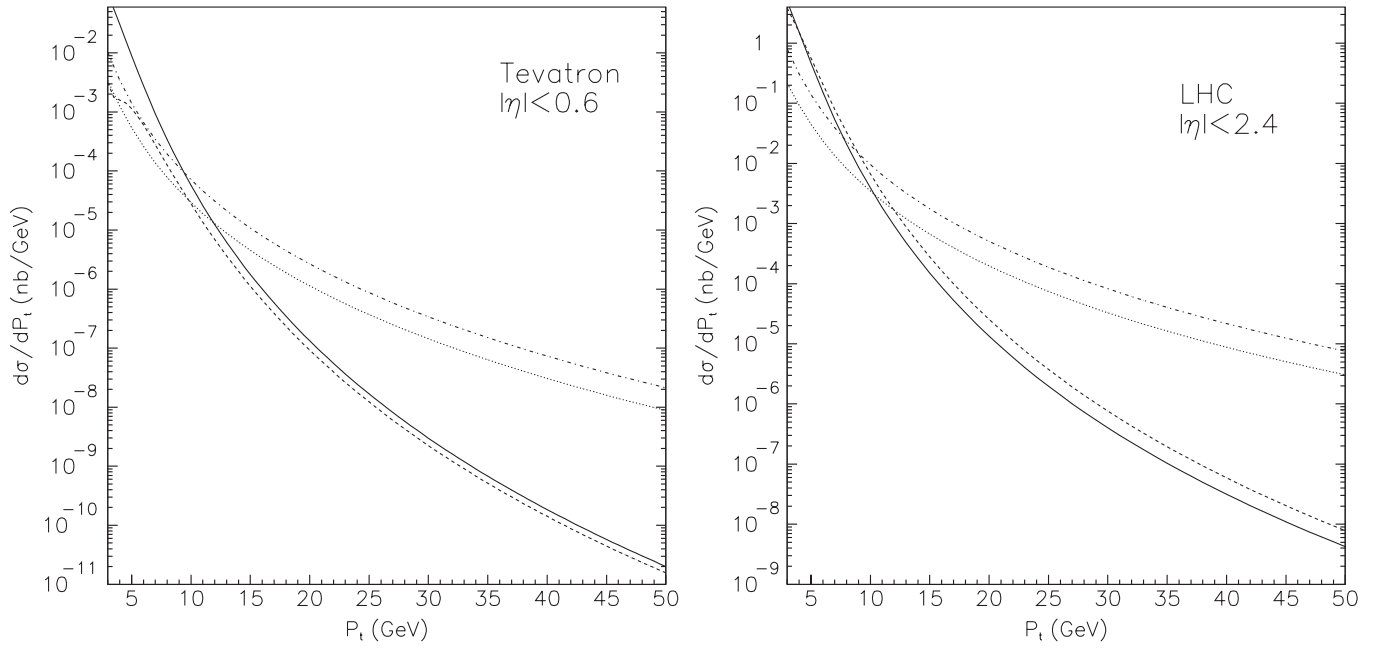


FIG. 3. The p_t distributions on the pair production of J/ψ (solid line) and η_c (dashed line) in the CSM at the hadron colliders with CTEQ6L1 PDF. The dotted line and dot-dashed line correspond to the pair production of J/ψ that come from the gluon fragmentation process in the COM with CTEQ6L1 and CTEQ5L PDFs, respectively.

From the Table II and the figures, it can be seen that all of the cross sections are enhanced when the center-of-mass energy is increased. This is because that with the fixed p_t , the larger the \sqrt{s} is, the smaller the momentum fraction x of the parton is. In small x region, the parton distribution function of gluon increases rapidly.

IV. SUMMARY

In this paper, we have investigated the leading-order pair production of S -wave heavy quarkonium at hadron colliders in the color-singlet mechanism (CSM) and estimated the contributions from the gluon fragmentation process in the color-octet mechanism (COM) for comparison. With

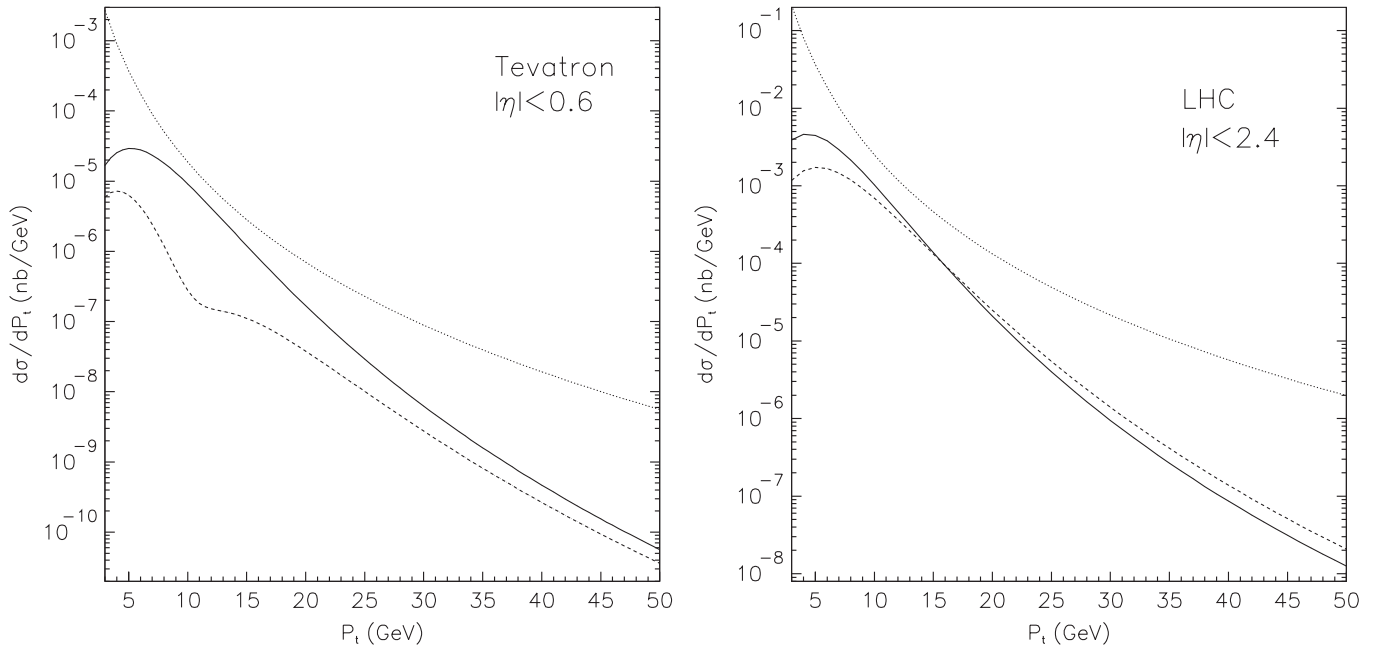


FIG. 4. The p_t distributions on the pair production of Y (solid line) and η_b (dashed line) in the CSM at the hadron colliders. The dotted line corresponds to the pair production of Y that come from the gluon fragmentation process in the COM.

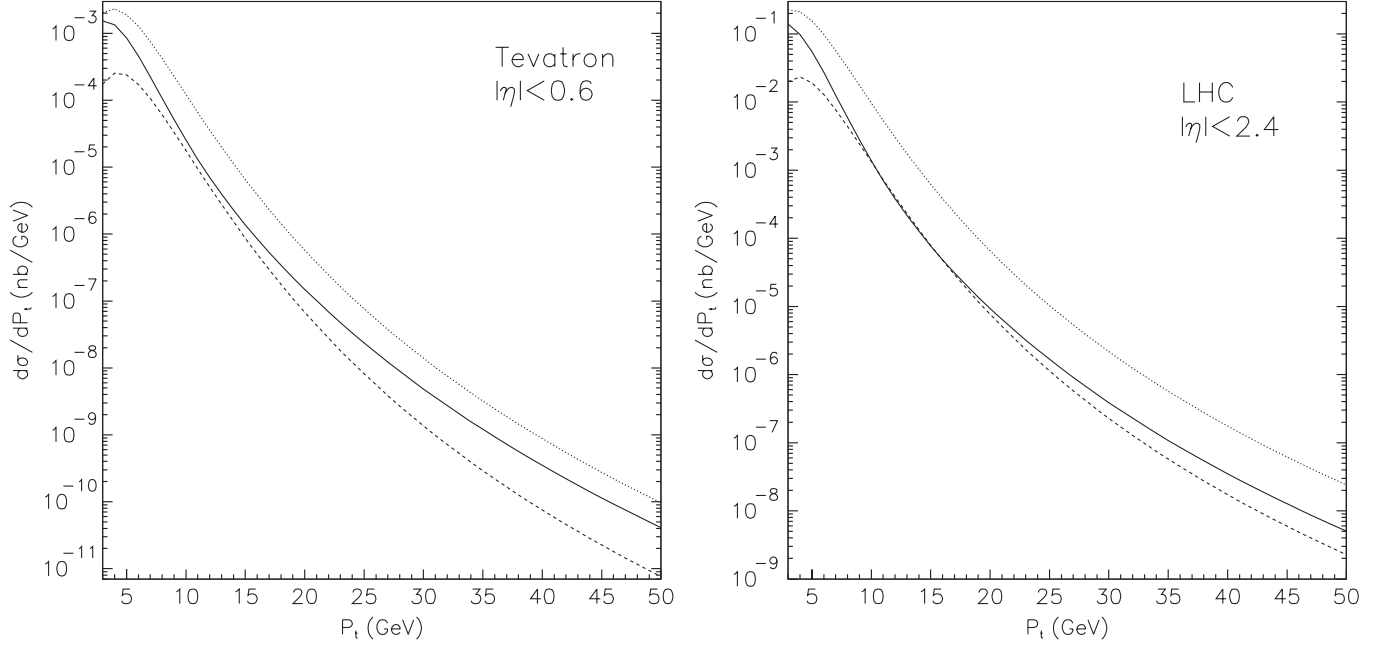


FIG. 5. The p_t distribution on the production of $B_c \bar{B}_c$ (solid line), $B_c \bar{B}_c^*$ (dashed line) and $B_c^* \bar{B}_c$ (dotted line) in the CSM at the Tevatron and LHC.

the matrix elements extracted previously in leading-order calculations, the numerical results show that the production rates are quite large for the pair production processes at the LHC. The p_t distribution of double J/ψ production in the CSM is dominant over that in the COM when p_t is smaller than about 10 GeV. For the production of double Υ , the contribution of the COM is always larger than that in the CSM. There are large differences in the theoretical predictions between the CSM and COM for the p_t distributions in the large p_t region, and this is useful in clarifying the effects of COM on the quarkonium production. Furthermore, since to produce a pair of quarkonium states with different C -parity is forbidden in the CSM at the leading-order, the observation of these processes could be a positive support for the COM.

We also investigate the pair productions of S -wave B_c and B_c^* mesons ($B_c \bar{B}_c$, $B_c \bar{B}_c^*$, and $B_c^* \bar{B}_c$), and the measurement of these processes may be useful to test the CSM and extract the LDMEs for the B_c and B_c^* mesons. The study of $B_c \bar{B}_c^*$ and the p_t distribution for $B_c \bar{B}_c$ and $B_c^* \bar{B}_c$ at the LHC were given for the first time.

Aside from the numerical results, we present the analytical results on the subprocesses for all these pair productions, which are new except the one of double J/ψ production process.

After our work was completed [38], a paper appeared [39], in which Qiao, Sun, and Sun calculated the double J/ψ production at the LHC. They focused on the polarizations of the double J/ψ . We focused on the cross sections of double-heavy quarkonia J/ψ , η_c , Υ , η_b , as well as the double-heavy flavored B_c , and B_c^* mesons. Both the two papers discuss the test of the COM. Our color-octet

and color-singlet double J/ψ cross sections are consistent with their result [39].

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APPENDIX

In this appendix we give the parton level cross section for the pair production of the processes mentioned above, respectively. The Mandelstam variables are defined as

$$s = (k_1 + k_2)^2, \quad t = (k_1 - P)^2, \quad u = (k_2 - P)^2, \quad (\text{A1})$$

where k_1 , k_2 and P are the momenta of the initial gluons and one of the final state heavy mesons.

For the $c\bar{c}$ systems, we also define the following variables

$$\bar{s} = s/m_c^2, \quad \theta = \arccos \left[\frac{-8m_c^2 + s + 2t}{4\sqrt{s^2/16 - sm_c^2}} \right], \quad (\text{A2})$$

where θ is the angle between the out going particle and the beam axis in the center-of-mass frame.

For the $b\bar{c}$ systems, we define the following variables

$$\bar{s} = s/m_b^2, \quad r = m_c/m_b, \quad \theta = \arccos \left[\frac{-2(m_b + m_c)^2 + s + 2t}{\sqrt{s^2 - 4s(m_b + m_c)^2}} \right], \quad (\text{A3})$$

where θ is the angle between the out going particle and the beam axis in the center-of-mass frame.

(1) $g + g \rightarrow \eta_c + \eta_c$

$$\begin{aligned} \frac{d\hat{\sigma}}{dt} = & \frac{\alpha_s^4 \pi |R(0)|_{c\bar{c}(1S)}^4}{20736m_c^6 s^2 \bar{s}^4 (16\cos^2(\theta) + \bar{s}\sin^2(\theta))^4 (\bar{s}^2 \sin^2(\theta) - 16\bar{s}\sin^2(\theta) + 64)^2} \\ & \times ((64120 \cos(2\theta) + 8479 \cos(4\theta) - 5396 \cos(6\theta) + 3442 \cos(8\theta) - 612 \cos(10\theta) + 81 \cos(12\theta) + 77342) \\ & \times \sin^4(\theta) \bar{s}^{10} - 32(90386 \cos(2\theta) + 15380 \cos(4\theta) - 8543 \cos(6\theta) + 9662 \cos(8\theta) - 1971 \cos(10\theta) \\ & + 324 \cos(12\theta) + 152810) \sin^4(\theta) \bar{s}^9 - 64(666414 \cos(2\theta) - 79140 \cos(4\theta) + 150533 \cos(6\theta) \\ & - 112834 \cos(8\theta) + 68593 \cos(10\theta) - 15156 \cos(12\theta) + 2268 \cos(14\theta) - 828134) \sin^2(\theta) \bar{s}^8 \\ & + 2048(350790 \cos(2\theta) - 185850 \cos(4\theta) + 90677 \cos(6\theta) - 58186 \cos(8\theta) + 45865 \cos(10\theta) \\ & - 12222 \cos(12\theta) + 2268 \cos(14\theta) - 380798) \sin^2(\theta) \bar{s}^7 + 8192(-892244 \cos(2\theta) + 646030 \cos(4\theta) \\ & - 337651 \cos(6\theta) + 175822 \cos(8\theta) - 122451 \cos(10\theta) + 62938 \cos(12\theta) - 17766 \cos(14\theta) \\ & + 2835 \cos(16\theta) + 519351) \bar{s}^6 - 131072(-226484 \cos(2\theta) + 280537 \cos(4\theta) - 136300 \cos(6\theta) \\ & + 60522 \cos(8\theta) - 57764 \cos(10\theta) + 33991 \cos(12\theta) - 12348 \cos(14\theta) + 2268 \cos(16\theta) + 92442) \bar{s}^5 \\ & + 1048576(-51336 \cos(2\theta) + 88498 \cos(4\theta) - 77070 \cos(6\theta) + 6517 \cos(8\theta) - 26726 \cos(10\theta) \\ & + 22570 \cos(12\theta) - 11844 \cos(14\theta) + 2268 \cos(16\theta) - 8173) \bar{s}^4 - 33554432(19822 \cos(2\theta) \\ & + 8249 \cos(4\theta) - 3158 \cos(6\theta) - 352 \cos(8\theta) + 1762 \cos(10\theta) + 2039 \cos(12\theta) - 2106 \cos(14\theta) \\ & + 324 \cos(16\theta) + 13356) \bar{s}^3 + 268435456(41624 \cos(2\theta) + 7444 \cos(4\theta) - 6210 \cos(6\theta) - 554 \cos(8\theta) \\ & + 3974 \cos(10\theta) + 244 \cos(12\theta) - 1116 \cos(14\theta) + 81 \cos(16\theta) + 31313) \bar{s}^2 + 34359738368 \cos^2(\theta) \\ & \times (2428 \cos(2\theta) + 2679 \cos(4\theta) + 1477 \cos(6\theta) - 260 \cos(8\theta) - 225 \cos(10\theta) + 81 \cos(12\theta) + 1116) \bar{s} \\ & + 1099511627776 \cos^4(\theta) (1418 \cos(2\theta) - 35 \cos(4\theta) + 18 \cos(6\theta) + 81 \cos(8\theta) + 1406) \end{aligned} \quad (A4)$$

(2) $g + g \rightarrow J/\psi + J/\psi$

$$\begin{aligned} \frac{d\hat{\sigma}}{dt} = & \frac{\alpha_s^4 \pi |R(0)|_{c\bar{c}(1S)}^4}{81m_c^6 s^2 \bar{s}^4 (16\cos^2(\theta) + \bar{s}\sin^2(\theta))^4} ((1332 \cos(2\theta) + 243 \cos(4\theta) + 1217) \bar{s}^6 \sin^8(\theta) - 2(1365 \cos(2\theta) \\ & + 2854 \cos(4\theta) + 1899 \cos(6\theta) + 729 \cos(8\theta) - 2239) \bar{s}^5 \sin^4(\theta) + (18860 \cos(2\theta) - 9913 \cos(4\theta) \\ & - 14990 \cos(6\theta) + 12805 \cos(8\theta) - 990 \cos(10\theta) + 3645 \cos(12\theta) + 6967) \bar{s}^4 - 64(10208 \cos(2\theta) \\ & + 8809 \cos(4\theta) + 8629 \cos(6\theta) + 7555 \cos(8\theta) + 1035 \cos(10\theta) + 1215 \cos(12\theta) + 3509) \bar{s}^3 \\ & + 256(175240 \cos(2\theta) + 127259 \cos(4\theta) + 78808 \cos(6\theta) + 27740 \cos(8\theta) + 7200 \cos(10\theta) \\ & + 3645 \cos(12\theta) + 103372) \bar{s}^2 + 3145728 \cos^4(\theta) (1418 \cos(2\theta) - 35 \cos(4\theta) + 18 \cos(6\theta) \\ & + 81 \cos(8\theta) + 1406) - 32768 \cos^2(\theta) (21756 \cos(2\theta) + 14212 \cos(4\theta) + 3275 \cos(6\theta) + 801 \cos(8\theta) \\ & + 729 \cos(10\theta) + 10939) \bar{s} \end{aligned} \quad (A5)$$

(3) $g + g \rightarrow B_c + \bar{B}_c$

$$\begin{aligned} \frac{d\hat{\sigma}}{dt} = & \frac{\alpha_s^4 (m_b + m_c)^2 \pi (r+1)^4 |R(0)|_{b\bar{c}(1S)}^4}{331776m_b^4 m_c^4 r^2 s^2 \bar{s}^4 (4(r+1)^2 \cos^2(\theta) + \bar{s}\sin^2(\theta))^4} (16(-80r^6 + 768r^5 - 72\bar{s}r^4 + 3792r^4 + 320\bar{s}r^3 \\ & + 5888r^3 + 23\bar{s}^2 r^2 + 784\bar{s}r^2 + 3792r^2 + 64\bar{s}^2 r + 320\bar{s}r + 768r + 23\bar{s}^2 + 9(r^2 + 1) \\ & \times \cos(4\theta) (\bar{s} - 4(r+1)^2)^2 - 72\bar{s} + 16(r+1)^2 \cos(2\theta) (4(r^2 + 17r + 1)(r+1)^2 - 2\bar{s}^2 + (7r^2 - r + 7)\bar{s} \\ & - 80)^2 (r+1)^4 + (\bar{s} - 4(r+1)^2)^2 (-80r^6 + 1632r^5 - 200\bar{s}r^4 + 7248r^4 - 80\bar{s}r^3 + 11072r^3 - 9\bar{s}^2 r^2 \\ & + 240\bar{s}r^2 + 7248r^2 + 46\bar{s}^2 r - 80\bar{s}r + 1632r - 9\bar{s}^2 + 9(r+1)^2 \cos(4\theta) (\bar{s} - 4(r+1)^2)^2 - 200\bar{s} \\ & + 16 \cos(2\theta) (4(r^2 + 35r + 1)(r+1)^4 - (r^2 + 19r + 1)\bar{s}(r+1)^2 - 4r\bar{s}^2) - 80)^2 \sin^4(\theta)) \end{aligned} \quad (A6)$$

(4) $g + g \rightarrow B_c + \bar{B}_c^*$

$$\frac{d\hat{\sigma}}{dt} = -\frac{\alpha_s^4(m_b + m_c)^2 \pi(r-1)^2(r+1)^6(4(r+1)^2 - \bar{s})|R(0)|_{b\bar{c}(1S)}^4}{5184m_b^4m_c^4r^2s^2\bar{s}^3(4(r+1)^2\cos^2(\theta) + \bar{s}\sin^2(\theta))^2} (2\cos^2(\theta)(76\cos(2\theta) + 81\cos(4\theta) - 149)(r+1)^4 + (200\cos(2\theta) + 81\cos(4\theta) + 375)\bar{s}\sin^2(\theta)(r+1)^2 + 81\cos^2(\theta)\bar{s}^2\sin^4(\theta)) \quad (A7)$$

(5) $g + g \rightarrow B_c^* + \bar{B}_c^*$

$$\begin{aligned} \frac{d\hat{\sigma}}{dt} = & -\frac{\alpha_s^4(m_b + m_c)^2 \pi(r+1)^4|R(0)|_{b\bar{c}(1S)}^4}{663552m_b^4m_c^4r^2s^2\bar{s}^4(4(r+1)^2\cos^2(\theta) + \bar{s}\sin^2(\theta))^4} \left(12288\cos^4(\theta)(-5027r^4 + 27172r^3 - 66786r^2 \right. \\ & + 27172r + 8(r(r(944r - 4615) + 4506) - 4615) + 944)\cos(2\theta) - 4(r(r(787r - 1448) + 1182) \\ & - 1448) + 787)\cos(4\theta) + 72(r+1)^2(r(8r - 17) + 8)\cos(6\theta) - 81(r+1)^4\cos(8\theta) - 5027)(r+1)^{12} \\ & - 512\cos^2(\theta)(-729\cos(10\theta)(r+1)^4 + 18(r(195r - 568) + 195)\cos(8\theta)(r+1)^2 \\ & + 2r(r(24241r - 104742) + 73490) - 104742) - 6(r(r(13059r - 55604) + 143106) - 55604 \\ & + 13059)\cos(2\theta) + 8(r(r(5661r - 27206) + 14666) - 27206) + 5661)\cos(4\theta) \\ & + (r(r((25388 - 20245)r - 62686) + 25388) - 20245)\cos(6\theta) + 48482\bar{s}(r+1)^{10} \\ & + 16(-3645\cos(12\theta)(r+1)^4 + 180(r(51r - 262) + 51)\cos(10\theta)(r+1)^2 + 2r(r(3(70796 - 16625)r \\ & - 1152002) + 212388) + 8(r(r(33335r - 121408) - 174334) - 121408) + 33335)\cos(2\theta) \\ & + (r(r((810676 - 226611)r - 3204274) + 810676) - 226611)\cos(4\theta) + 4(r(r(r(28059r - 174944) \\ & - 21462) - 174944) + 28059)\cos(6\theta) - 2(r(r(r(34165r - 14204) + 181998) - 14204) + 34165) \\ & \times \cos(8\theta) - 99750\bar{s}^2(r+1)^8 - 256(180(r+1)^2(r(15r - 53) + 15)\cos^5(2\theta) - (1215\cos(4\theta)(r+1)^4 \\ & + r(r(3r(3247r + 400) + 46334) + 1200) + 9741)\cos^4(2\theta) + 8(r(r(894r^2 - 3675r + 3835) - 3675) \\ & + 894)\cos^3(2\theta) + 2(729\cos(4\theta)(r+1)^4 + r(r((22896 - 473r)r - 18818) + 22896) - 473)\cos^2(2\theta) \\ & + 4(r(r(r(609r - 2431) + 6270) - 2431) + 609)\cos(2\theta) - r(r(r(1601r + 2352) - 2978) + 2352) \\ & - 243(r+1)^4\cos(4\theta) - 1601\bar{s}^3(r+1)^6 + (-3645\cos(12\theta)(r+1)^4 + 180(r(39r - 56) + 39) \\ & \times \cos(10\theta)(r+1)^2 - 2(r(r(r(54979r - 412780) + 771338) - 412780) + 54979) \\ & + 8(r(r(r(21403r - 170226) + 259926) - 170226) + 21403)\cos(2\theta) + (r(r(3(234748 - 39057r)r \\ & - 1015538) + 704244) - 117171)\cos(4\theta) + 4(r(r(r(20975r - 41466) + 100942) - 41466) + 20975) \\ & \times \cos(6\theta) - 2(r(r(r(15685r - 4244) + 79558) - 4244) + 15685)\cos(8\theta))\bar{s}^4(r+1)^4 \\ & + 32\left(\frac{729}{4}\cos(8\theta)(r+1)^4 + 9(r(48r + 115) + 48)\cos(6\theta)(r+1)^2 + \frac{1}{4}(-r(r(r(6533r + 6684) + 9390) \right. \\ & + 6684) - 6533) + (r(r(r(592r + 2453) - 630) + 2453) + 592)\cos(2\theta) + (r(r(r(427r + 1198) \\ & + 8166) + 1198) + 427)\cos(4\theta)\Big)\bar{s}^5\sin^4(\theta)(r+1)^2 - 16(243\cos(4\theta)(r+1)^4 + 36(r(45r + 58) + 45) \\ & \left. \times \cos(2\theta)(r+1)^2 + r(r(9r(153r + 484) + 8006) + 4356) + 1377\right)\bar{s}^6\sin^8(\theta) \quad (A8) \end{aligned}$$

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