Chiral logarithms in $\Delta S = 1$ kaon decay amplitudes in general effective flavor theories

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We study the chiral logarithms in $\Delta S = 1$ kaon decay amplitudes from new flavor physics in beyond-standard-model theories. We systematically classify the chiral structures of dimension-5, 6, and 7 effective QCD operators constructed out of light-quark (up, down, and strange) and gluon fields. Using the standard chiral perturbation theory, we calculate the leading chiral logarithms associated with these operators. The result is useful for lattice calculations of the QCD matrix elements in $K \to \pi\pi$ decay necessary, for example, to understand the physical origin of the direct CP violation parameter ϵ'. As a concrete example, we consider the new operators present in minimal left-right symmetric models.

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I. INTRODUCTION

Nonleptonic kaon decay has been a focus for both theoretical and experimental physics for over 40 years since the discovery of CP violation by Christenson, Cronin, Fitch, and Turlay [1] in $K_L \to 2\pi$. Since then, the origin of CP violation has long been a challenge to many theoretical models. The recent data from various experiments have yielded a clear nonvanishing direct CP-violation parameter [2,3]:

$$\mathrm{Re}\,(\epsilon'/\epsilon) = (16.7 \pm 2.6) \times 10^{-4}, \tag{1}$$

which ruled out the so-called superweak theory where no direct CP violation appears in the decay [4]. At present, a full theoretic explanation of the origin of this phenomenon is still lacking. In the framework of the standard model (SM), direct CP violation can be generated by the nonzero phase in the quark flavor-mixing matrix (Cabibbo-Kobayashi-Maskawa matrix), as was suggested by Kobayashi and Maskawa [5]. A precision calculation of the effect, however, is extremely hard due to the nonperturbative nature of the strong interactions at low energy. Results from several groups utilizing different methods differ widely, with error bars much larger than that of the experimental result [6]. The unsatisfying situation of the theoretical calculations has attracted much interest in attributing part of the phenomenon to physics beyond the SM, although it could still be true that the SM contribution can entirely account for the experimental data.

To be able to pin down the contribution to ϵ' from models containing new physics, apart from calculating the relevant Wilson coefficients and their running at two-loop order which is beyond that scope of this paper, one has to make precision calculations of the strong-interaction physics associated with the nonperturbative structure of kaons and pions. Various methods have been used to calculate the hadronic matrix elements, such as lattice [7,8], QCD-inspired models [9,10], chiral expansion together with large-N_c [11], and parametrizations [12]. At present, the lattice field theory is the only approach based on first principles, with controllable systematic errors. However, there are difficulties in lattice calculations which are associated with the fact that the final state contains more than one particle. By the Maiani-Testa theorem [13], it is impossible to extract the physical kaon decay matrix elements by taking the limit $\tau \to \infty$ in the Euclidean space. In practice, there are several ways to avoid it: One can either work with an unphysical choice of momenta [14,15], utilize an unphysical set of meson masses [16,17], or derive the physical matrix elements by unphysical but calculable ones. All of these methods require the chiral perturbation theory (ChPT) to connect what is calculable on the lattice and what is needed in physical amplitudes.

The ChPT assumes that an approximate chiral symmetry exists and describes the low-energy QCD physics below a chiral breaking scale $\Lambda_\chi \sim m_\rho$ by the pseudo-Goldstone bosons, namely, pions, kaons, and eta. Then the low-energy physics can be expanded in powers of the particles' external momenta and masses. It further assumes that the Wilson coefficients of the QCD operators when expressed in terms of meson operators are independent of the external states. Therefore the amplitudes of a large number of reactions can be determined by a relatively small set of coefficients, which gives us the predictive power. In the case of kaon decay, ChPT is used to connect the desired matrix element $\langle \pi\pi|\mathcal{O}|K\rangle$ with some unphysical quantities, such as $\langle \pi|\mathcal{O}|K\rangle$ and $\langle 0|\mathcal{O}|K\rangle$. The results in ChPT are needed before doing relevant lattice calculations. Here we will neglect some subtleties in the ChPT (such as quadratic divergence cancellations, zero pion mass corrections, etc.) and focus on possible operator structures as well as their chiral logarithm corrections for the kaon decay process. It is the goal of this paper to examine the chiral structures of possible QCD operators responsible for $\Delta s = 1$, $\Delta d = -1$ decay in generic beyond-SM theories and to calculate the large chiral logarithms associated with them. Previous calculations have been made for operators present in the SM [18–24]. Our work extends these studies to all possible operators in new physics models.

It is not the goal of this paper to actually compute the contribution to ϵ' from a specific beyond-SM theory and show that it is large. For that, one needs to calculate the relevant Wilson coefficients and hadronic matrix elements to a specific precision altogether. Here we focus on the second part and study the importance of chiral physics in computing those hadronic matrix elements to order $\mathcal{O}(p^2)$ and leading logarithms at $\mathcal{O}(p^4)$. We leave the complete $\mathcal{O}(p^4)$ calculation including counterterms and finite contributions to a future study.

The paper is organized as follows: We start from the operator basis in the SM for the kaon decay, as well as possible new operators coming from physics beyond the SM. A chiral perturbation theory calculation will be presented in the following section, with all corresponding operators and their one-loop corrections of the matrix elements. We end this section by applying our result to a specific example. Concluding remarks and outlook are presented in the last section.

II. EFFECTIVE OPERATORS FROM NEW FLAVOR PHYSICS

In this section, we consider effective QCD operators contributing to CP-violating $K \to \pi\pi$ decay in a generic weak-interaction theory. There is an extensive literature on this topic in the context of the SM [25,26]. Our focus is on new operators arising from novel CP-violating mechanisms beyond the SM. We classify the effective operators in terms of their flavor symmetry properties under chiral group $SU(3)_L \times SU(3)_R$ when up, down, and strange quarks are taken as light.

The direct CP violation parameter ϵ' for $K \to \pi\pi$ decay is defined as [25]

$$\epsilon' = \frac{1}{\sqrt{2}} e^{((\pi/2)+\delta_2-\delta_0)} \frac{\mathrm{Re}A_2}{\mathrm{Re}A_0}\left(\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0} - \frac{\mathrm{Im}A_2}{\mathrm{Re}A_2}\right), \quad (2)$$

where δ_I is the strong-interaction $\pi\pi$ scattering phase shift and A_I is the weak kaon decay amplitude:

$$A_I e^{i\delta_I} = \langle \pi\pi(I=0,2)|(-i\mathcal{H}_W)|K^0\rangle, \quad (3)$$

where $\mathcal{H}_W$ is the effective weak-interaction Hamiltonian which depends on the underlying theory of kaon decay. The small ratio $\omega \equiv \mathrm{Re}A_2/\mathrm{Re}A_0 \approx 1/22$ reflects the well-known $\Delta I = 1/2$ rule. Accurate calculations of ϵ' depend on reliable evaluations of the effective QCD operators present in $\mathcal{H}_W$. Our goal in this paper is to classify these QCD operators and study their chiral behavior.

A. Standard model operators

The standard procedure for calculating ϵ' utilizes an effective field theory approach. The physics at high energy (or short distance) can be calculated perturbatively and is included in Wilson coefficients. The physics at low-energy scales is included in the effective QCD operators composed of light flavor quark fields (u, d, s) and gluon fields. Large QCD radiative corrections or large logarithms are resumed by solving renormalization group equations. The effective operators responsible for the neutral kaon decay have the flavor quantum numbers $\Delta s = 1$ and $\Delta d = -1$. In the SM, it is well known that $\mathcal{H}^{\mathrm{eff}}$ consists of the following 10 operators [26,27]:

$$\begin{aligned}
Q_1 &= (\bar{s}_i u_j)_{V-A}(\bar{u}_j d_i)_{V-A},\\
Q_2 &= (\bar{s}_i u_i)_{V-A}(\bar{u}_j d_j)_{V-A},\\
Q_{3,5} &= (\bar{s}_i d_i)_{V-A}\sum_q (\bar{q}_j q_j)_{V\mp A},\\
Q_{4,6} &= (\bar{s}_i d_j)_{V-A}\sum_q (\bar{q}_j q_i)_{V\mp A},\\
Q_{7,9} &= \frac{3}{2}(\bar{s}_i d_i)_{V-A}\sum_q e_q (\bar{q}_j q_j)_{V\pm A},\\
Q_{8,10} &= \frac{3}{2}(\bar{s}_i d_j)_{V-A}\sum_q e_q (\bar{q}_j q_i)_{V\pm A},
\end{aligned} \quad (4)$$

where $(\bar{q}q')_{V\pm A} = \bar{q}_{L,R}\gamma_\mu q_{L,R}$ with $q_{L,R}$ representing the left- (right-) handed quark fields. The summation in q is over the light-quark flavors u, d, and s; i and j are color indices; and e_q is the algebraic charge factor for flavor q. The $Q_{1,2}$ come from the single W_L-boson exchange tree diagram, and $Q_{3\text{–}6}$ and $Q_{7\text{–}10}$ are derived from one-loop gluon and electroweak penguin diagrams, respectively. In the SM, $(\mathrm{Im}A_0/\mathrm{Re}A_0)$ is dominated by the QCD penguin operators, whereas $(\mathrm{Im}A_2/\mathrm{Re}A_2)$ receives a contribution from the electroweak penguin operators only, because the gluon interaction is flavor-singlet and cannot contribute in the $\Delta I = 3/2$ channel.

A chiral structure analysis of the above ten operators will be useful if we wish to use lattice QCD to calculate the relevant matrix elements in kaon decays. Using (m, n) to denote a representation of group $SU_L(3) \times SU_R(3)$, where m and n are the dimensions of $SU(3)$ representations, it is then easy to see that $Q_{1,2}$ and $Q_{9,10}$ belong to $(8, 1)$ and $(27, 1)$, $Q_{3\text{–}6}$ to $(8, 1)$, and $Q_{7,8}$ to $(8, 8)$ [26]. ChPT calculations have been made to uncover the large logarithms associated with these operators, which in turn help to establish relations of different matrix elements useful for lattice QCD calculations. For the reader's convenience, we have collected the standard chiral results in the appendix.

The question is, what is the general chiral structure of all possible weak operators that might emerge in theories beyond the SM? The rest of this section is devoted to addressing this question.

B. Dimension-5 and 6 operators

Let us systematically consider the possible operators and their chiral structures in a general low-energy description of kaon decay, independent of the underlying short-distance flavor physics that can be taken into account by Wilson coefficients. The lowest dimensional operator is a dimension-5 chromomagnetic operator

$$Q_M = \bar{s}(\sigma^{\mu\nu})t^a d G^a_{\mu\nu}. \tag{5}$$

This operator does appear in the standard model through a penguin diagram as shown in the left panel in Fig. 1, although it is proportional to the strange or down quark masses, which are chirally suppressed. It also appears naturally in the left-right symmetric model (LRSM) with left-right-handed gauge-boson mixing, proportional to charm or top quark masses [28], as shown in the right panel in Fig. 1. Under chiral symmetry, this operator transforms as $(\bar{3}, 3) + (3, \bar{3})$. The chiral logarithms appearing with the matrix elements of this operator have been studied before in the literature and are collected in the appendix.

Next, consider dimension-6 four-quark operators. We define the following flavor tensor:

$$\Theta^{jl}_{ik} = (\bar{q}^i \Gamma q_j)(\bar{q}^k \Gamma' q_l), \tag{6}$$

where the flavor indices i, j, k, and l go through 1, 2, and 3, or up, down, and strange quarks. Γ and Γ' are possible Dirac matrix structures. In addition, there are two independent color structures $(\mathbf{1})(\mathbf{1})$ and $(t^a)(t^a)$ which are not essential for the following discussion. Assuming that all fields are projected to their helicity states, the possible helicities are as follows:

(i) All four-quark fields have the same chiral projection.
(ii) Both $\bar{q}^i$ and $\bar{q}^k$ (also j and l) have the opposite chiral projection.
(iii) Both $\bar{q}^i$ and $\bar{q}^k$ (also j and l) have the same chiral projection.

These are the only possibilities because the operators must be Lorentz scalars, and the numbers of left- and right-handed fields apart from the first case must be exactly 2, respectively. In the first and second cases, the operators are the ones appearing in the SM weak interactions, as shown in Eq. (4), and their parity partners. They correspond to chiral structures (8, 1), (1, 8), (27, 1), and (1, 27) from the first case and (8, 1), (1, 8), and (8, 8) from the second case.

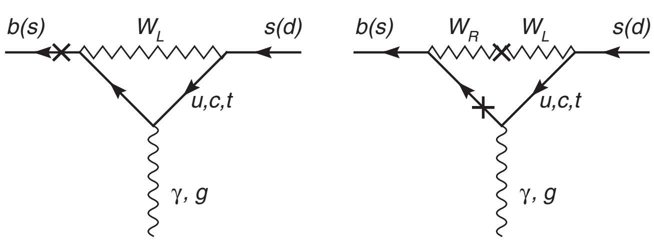

FIG. 1. Dimension-5 effective operators generated from the weak-interaction vertex corrections in the SM (left) and in the LRSM (right). The crosses on fermion lines represent mass insertion, needed to flip the chirality of the quarks.

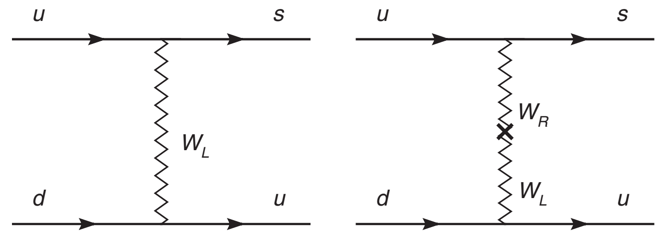

FIG. 2. Feynman diagrams generating dimension-6 quark operators in the SM and LRSM.

The new (1, 8) and (1, 27) structures will appear in, for example, LR symmetric models where the right-handed gauge boson plays the same role as the left-handed one in the SM. Since the strong interactions conserve parity, the new operators in the LRSM have the same matrix elements as Q_i's in the SM up to a parity sign. The corresponding Feynman diagrams in both the SM and LRSM for the first and second cases are shown in Fig. 2.

In the last case, there are new chiral structures arising from operators of type

$$(\bar{s}_L \Gamma d_R)(\bar{q}_L \Gamma q_R), \qquad (\bar{s}_R \Gamma d_L)(\bar{q}_R \Gamma q_L), \tag{7}$$

where two Γ's must be the same. The new chiral structures are $(\bar{6}, 6)$, $(\bar{6}, \bar{3})$, $(3, 6)$, and their parity conjugates. However, $(\bar{6}, \bar{3})$ and $(3, 6)$ involve symmetrization of two flavor indices and, at the same time, antisymmetrization of the other two. It is easy to check that the result vanishes, and we are left with just $(\bar{6}, 6)$ and $(6, \bar{6})$.

Let us consider the following tensor with up (and hence lower) indices symmetrized:

$$\Theta^{kl}_{(ij)} = \tfrac{1}{2}(\bar{q}^i_L \Gamma q_{Rk})(\bar{q}^j_L \Gamma q_{Rl}) + \tfrac{1}{2}(\bar{q}^j_L \Gamma q_{Rk})(\bar{q}^i_L \Gamma q_{Rl}), \tag{8}$$

where the upper indices represent the left-handed fields and the lower indices the right-handed. Without loss of generality, we take $i = 3$. If $j = 3$, and k or l is 3 and the other indices must be 2, one gets an isospin-1/2 operator

$$\Theta^{(\bar{6},6)}_{1/2,A} \equiv \Theta^{23}_{33} = \bar{s}_L \Gamma d_R \bar{s}_L \Gamma s_R. \tag{9}$$

If we define a tensor T^{ij}_{kl} which multiplies the quark operator Θ^{kl}_{ij} to generate the above operator $T^{ij}_{kl}\Theta^{kl}_{ij}$, we have

$$T^{33}_{23} = T^{33}_{32} = 1/2, \tag{10}$$

and other components are zero.

On the other hand, if j, k, and l take 1's and 2's, one can subtract the trace with respect to j and k, and j and l, and one obtains an isospin-3/2 operator

$$\Theta^{(\bar{6},6)}_{3/2} \equiv \Theta^{12}_{(31)} + \Theta^{21}_{(31)} - \Theta^{22}_{(32)} = \bar{s}_L \Gamma u_R \bar{u}_L \Gamma d_R + \bar{s}_L \Gamma d_R \bar{u}_L \Gamma u_R - \bar{s}_L \Gamma d_R \bar{d}_L \Gamma d_R, \tag{11}$$

with corresponding nonzero tensor components

$$T^{31}_{12} = T^{31}_{21} = T^{13}_{12} = T^{13}_{21} = -T^{32}_{22} = -T^{23}_{22} = 1/2. \quad (12)$$

Another isospin-$1/2$ operator can be obtained by its trace part

$$\begin{aligned}\Theta^{(\bar{6},6)}_{1/2,S} &\equiv \Theta^{12}_{(31)} + \Theta^{21}_{(31)} + 2\Theta^{22}_{(32)} \\ &= \bar{s}_L\Gamma u_R \bar{u}_L \Gamma d_R + \bar{s}_L \Gamma d_R \bar{u}_L \Gamma u_R + 2\bar{s}_L \Gamma d_R \bar{d}_L \Gamma d_R,\end{aligned} \quad (13)$$

and the corresponding tensor components are

$$T^{31}_{12} = T^{31}_{21} = T^{13}_{12} = T^{13}_{21} = 1/2; \qquad T^{32}_{22} = T^{23}_{22} = 1. \quad (14)$$

An example of these new operators in the LRSM through flavor-changing neutral and charged currents is shown in Fig. 3. The relevant QCD four-quark operator will be

$$\begin{aligned}\mathcal{O}^{\Delta s=1} &= (\bar{s}_L d_R)\sum_q (\bar{q}_L q_R) \\ &= \frac{1}{2}[\Theta^{(\bar{6},6)}_{1/2,S} + 2\Theta^{(\bar{6},6)}_{1/2,A} - \Theta^{(3,\bar{3})}],\end{aligned} \quad (15)$$

where $\Theta^{(3,\bar{3})} \equiv (\bar{s}_L u_R)(\bar{u}_L d_R) - (\bar{s}_L d_R)(\bar{u}_L u_R)$.

C. Dimension-7 operators

Dimension-7 operators come in two types. The first is the chromomagnetic operators with an insertion of two additional derivatives, which does not change the original chiral structure. The second type is an insertion of one derivative into four-quark operators discussed above. Since the covariant derivative has one Lorentz index, it must be contracted with another Lorentz index appearing on a Dirac matrix. An example of this type of operators is

$$\begin{aligned}\mathcal{O}^{LR}_{P,g} &= \bar{s}_{i,L}(i\sigma^{\mu\nu})d_{j,R}\sum_q \bar{q}_{j,L}(\gamma_\mu D_\nu)q_{i,L}, \\ \mathcal{O}^{LR}_{P,EW} &= \bar{s}_{i,L}(i\sigma^{\mu\nu})d_{i,R}\sum_q e_q \bar{q}_{j,L}(\gamma_\mu D_\nu)q_{j,L},\end{aligned} \quad (16)$$

which comes from the gluon and electromagnetic penguin diagram in the LRSM as shown in Fig. 1. These operators contain either 3 left-handed fields and 1 right-handed one or 3 right-handed fields and 1 left-handed one. They have novel chiral structures $(15,\bar{3})$, $(\overline{15},3)$, $(6,3)$, $(\bar{6},\bar{3})$, and parity partners. Let us classify them all in detail.

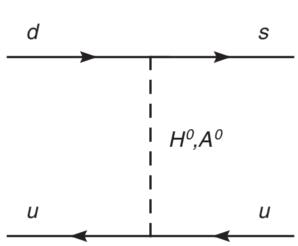

FIG. 3. Feynman diagrams generating scalar quark interactions through neutral-current Higgs exchanges.

1. $(\overline{15},3)$

Let us use Θ^{kl}_{ij} to represent an operator $\bar{q}^i_L \Gamma q_{Lk} \bar{q}^j_L \Gamma' q_{Rl}$, where l is the flavor index of the right-handed field and Γ and Γ' are not just Dirac matrices. We first construct $\hat{\Theta}^{kl}_{ij}$ which forms $(\overline{15},3)$ after symmetrizing the up two indices and subtracting the traces:

$$2\hat{\Theta}^{kl}_{ij} = \Theta^{kl}_{ij} + \Theta^{kl}_{ji} - \tfrac{1}{4}\delta^k_i[\Theta^{\alpha l}_{\alpha j} + \Theta^{\alpha l}_{j\alpha}] - \tfrac{1}{4}\delta^k_j[\Theta^{\alpha l}_{\alpha i} + \Theta^{\alpha l}_{i\alpha}], \quad (17)$$

where α sums over 1, 2, and 3. Clearly, i, or equivalently j, has to be an $\bar{s}$. One isospin-$1/2$ operator that one can immediately identify is when j is 3, k is 2, and l is 3, namely,

$$\Theta^{(\overline{15},3)}_{1/2} \equiv \hat{\Theta}^{23}_{33} = \Theta^{23}_{33} = \bar{s}_L \Gamma d_L \bar{s}_L \Gamma' s_R, \quad (18)$$

where the only nonzero tensor component is

$$T^{33}_{23} = 1. \quad (19)$$

Other independent operators can be obtained by considering j, k, and l as up and down quarks. Others, such as $\hat{\Theta}^{32}_{33}$, can be related to these through traceless conditions.

One can get an isospin-$3/2$ operator by symmetrizing k and l while taking away $SU(2)$ traces between j and k, and j and l. Thus we have the following combination:

$$\hat{\Theta}^{kl}_{3j} + \hat{\Theta}^{lk}_{3j} - \tfrac{1}{3}\delta^k_j(\hat{\Theta}^{al}_{3a} + \hat{\Theta}^{la}_{3a}) - \tfrac{1}{3}\delta^l_j(\hat{\Theta}^{ak}_{3a} + \hat{\Theta}^{ka}_{3a}), \quad (20)$$

where a sums over 1 and 2 only, and j, k, and l can take value in 1 or 2. There is only one independent operator

$$\begin{aligned}\Theta^{(\overline{15},3)}_{3/2} &\equiv 2(\hat{\Theta}^{21}_{31} + \hat{\Theta}^{12}_{31} - \hat{\Theta}^{22}_{32}) \\ &= \bar{s}_L\Gamma u_L \bar{u}_L \Gamma' d_R + \bar{s}_L \Gamma d_L \bar{u}_L \Gamma' u_R + \bar{u}_L \Gamma u_L \bar{s}_L \Gamma' d_R \\ &\quad + \bar{u}_L \Gamma d_L \bar{s}_L \Gamma' u_R - \bar{s}_L \Gamma d_L \bar{d}_L \Gamma' d_R \\ &\quad - \bar{d}_L \Gamma d_L \bar{s}_L \Gamma' d_R.\end{aligned} \quad (21)$$

The corresponding tensor components are

$$T^{31}_{12} = T^{13}_{12} = T^{31}_{21} = T^{13}_{21} = -T^{32}_{22} = -T^{23}_{22} = 1. \quad (22)$$

The trace part of the above operator produces an $I = 1/2$ operator $\hat{\Theta}^{a2}_{3a} + \hat{\Theta}^{2a}_{3a}$. We can subtract from the result with another isospin-$1/2$ operator $\hat{\Theta}^{33}_{32}$ to cancel the unwanted trace part $-\frac{1}{4}\delta^k_i[\Theta^{\alpha l}_{\alpha j}] - \frac{1}{4}\delta^k_j[\Theta^{\alpha l}_{\alpha i}]$ in Eq. (17). The resulting $I = 1/2$ operator is

$$\begin{aligned}\Theta^{(\overline{15},3)}_{1/2,S} &\equiv 2(\hat{\Theta}^{12}_{31} + \hat{\Theta}^{21}_{31} + 2\hat{\Theta}^{22}_{32}) - 3\hat{\Theta}^{32}_{33} \\ &= \bar{s}_L\Gamma u_L \bar{u}_L \Gamma' d_R + \bar{u}_L \Gamma u_L \bar{s}_L \Gamma' d_R + \bar{s}_L \Gamma d_L \bar{u}_L \Gamma' u_R \\ &\quad + \bar{u}_L \Gamma d_L \bar{s}_L \Gamma' u_R + 2\bar{s}_L \Gamma d_L \bar{d}_L \Gamma' d_R \\ &\quad + 2\bar{d}_L \Gamma d_L \bar{s}_L \Gamma' d_R - 3\bar{s}_L \Gamma s_L \bar{s}_L \Gamma' d_R,\end{aligned} \quad (23)$$

with the following tensor components:

$$T^{31}_{21} = T^{13}_{21} = T^{31}_{12} = T^{13}_{12} = \tfrac{1}{2}T^{32}_{22} = \tfrac{1}{2}T^{23}_{22} = \tfrac{1}{3}T^{33}_{32} = 1. \tag{24}$$

Finally, one can antisymmetrize k and l to generate another isospin-$1/2$ operator. Again we add $\hat{\Theta}^{32}_{33}$ to cancel the unwanted trace part:

$$\begin{aligned}\Theta^{(\overline{15},3)}_{1/2,A} &\equiv 2(\hat{\Theta}^{21}_{31} - \hat{\Theta}^{12}_{31}) + \hat{\Theta}^{32}_{33} \\ &= \bar{s}_L\Gamma d_L \bar{u}_L\Gamma' u_R + \bar{u}_L\Gamma d_L \bar{s}_L\Gamma' u_R - \bar{s}_L\Gamma u_L \bar{u}_L\Gamma' d_R \\ &\quad - \bar{u}_L\Gamma u_L \bar{s}_L\Gamma' d_R + \bar{s}_L\Gamma s_L \bar{s}_L\Gamma' d_R,\end{aligned} \tag{25}$$

with the following tensor components:

$$T^{31}_{21} = T^{13}_{21} = -T^{31}_{12} = -T^{13}_{12} = T^{33}_{32} = 1. \tag{26}$$

Note that the operators in $(3, \overline{15})$ can be obtained from the above through parity transformation.

2. $(\bar{3}, 15)$

Define $\Theta^{kl}_{ij} = \bar{q}^i_L\Gamma q_{Rk}\bar{q}^j_R\Gamma' q_{Rl}$, and construct the general $(\bar{3}, 15)$ operators

$$2\hat{\Theta}^{kl}_{ij} = \Theta^{kl}_{ij} + \Theta^{lk}_{ij} - \tfrac{1}{4}\delta^k_j[\Theta^{\alpha l}_{i\alpha} + \Theta^{l\alpha}_{i\alpha}] - \tfrac{1}{4}\delta^l_j[\Theta^{\alpha k}_{i\alpha} + \Theta^{k\alpha}_{i\alpha}], \tag{27}$$

where the α trace is over 1, 2, and 3. Either index i or j can be identified as the strange quark field. In either case, one can construct isospin-$3/2$ operators by subtracting the $SU(2)$ trace. With the left-handed strange quark, we have

$$\begin{aligned}\Theta^{(\bar{3},15)}_{3/2,L} &\equiv \hat{\Theta}^{12}_{31} + \hat{\Theta}^{21}_{31} - \hat{\Theta}^{22}_{32} = \Theta^{12}_{31} + \Theta^{21}_{31} - \Theta^{22}_{32} \\ &= \bar{s}_L\Gamma u_R \bar{u}_R\Gamma' d_R + \bar{s}_L\Gamma d_R \bar{u}_R\Gamma' u_R \\ &\quad - \bar{s}_L\Gamma d_R \bar{d}_R\Gamma' d_R,\end{aligned} \tag{28}$$

with tensor components

$$T^{31}_{12} = T^{31}_{21} = -T^{32}_{22} = 1. \tag{29}$$

With the right-handed strange quark,

$$\begin{aligned}\Theta^{(\bar{3},15)}_{3/2,R} &\equiv \hat{\Theta}^{12}_{13} + \hat{\Theta}^{21}_{13} - \hat{\Theta}^{22}_{23} = \Theta^{12}_{13} + \Theta^{21}_{13} - \Theta^{22}_{23} \\ &= \bar{u}_L\Gamma u_R \bar{s}_R\Gamma' d_R + \bar{u}_L\Gamma d_R \bar{s}_R\Gamma' u_R \\ &\quad - \bar{d}_L\Gamma d_R \bar{s}_R\Gamma' d_R,\end{aligned} \tag{30}$$

with tensor components,

$$T^{13}_{12} = T^{13}_{21} = -T^{23}_{22} = 1. \tag{31}$$

There are also two isospin-$1/2$ operators. The first one is with the left-handed strange quark

$$\begin{aligned}\Theta^{(\bar{3},15)}_{1/2,L} &\equiv 4(\hat{\Theta}^{12}_{31} + \hat{\Theta}^{21}_{31} + 2\hat{\Theta}^{22}_{32}) \\ &= \bar{s}_L\Gamma u_R \bar{u}_R\Gamma' d_R + \bar{s}_L\Gamma d_R \bar{u}_R\Gamma' u_R \\ &\quad + 2\bar{s}_L\Gamma d_R \bar{d}_R\Gamma' d_R - 3\bar{s}_L\Gamma s_R \bar{s}_R\Gamma' d_R \\ &\quad - 3\bar{s}_L\Gamma d_R \bar{s}_R\Gamma' s_R,\end{aligned} \tag{32}$$

with tensor components

$$T^{31}_{12} = T^{31}_{21} = \tfrac{1}{2}T^{32}_{22} = -\tfrac{1}{3}T^{33}_{32} = -\tfrac{1}{3}T^{33}_{23} = 1, \tag{33}$$

and the second one has the right-handed strange quark

$$\begin{aligned}\Theta^{(\bar{3},15)}_{1/2,R} &\equiv \hat{\Theta}^{12}_{13} + \hat{\Theta}^{21}_{13} + 2\hat{\Theta}^{22}_{23} = \Theta^{12}_{13} + \Theta^{21}_{13} + 2\Theta^{22}_{23} \\ &= \bar{u}_L\Gamma u_R \bar{s}_R\Gamma' d_R + \bar{u}_L\Gamma d_R \bar{s}_R\Gamma' u_R \\ &\quad + 2\bar{d}_L\Gamma d_R \bar{s}_R\Gamma' d_R,\end{aligned} \tag{34}$$

with tensor components

$$T^{13}_{12} = T^{12}_{21} = \tfrac{1}{2}T^{23}_{22} = 1. \tag{35}$$

The $\hat{\Theta}^{32}_{33}$ is not independent by the same reason as for $(\overline{15}, 3)$.

3. $(6, 3)$

Define $\Theta^{kl}_{ij} = \bar{q}^i_L\Gamma q_{Lk}\bar{q}^j_L\Gamma' q_{Rl}$, and construct the $(6, 3)$ operator

$$\hat{\Theta}^{\;kl}_{ij} = \epsilon_{ijm}\hat{\Theta}^{(mk)l} = \tfrac{1}{2}\epsilon_{ijm}(\epsilon^{\alpha\beta m}\Theta^{kl}_{\alpha\beta} + \epsilon^{\alpha\beta k}\Theta^{ml}_{\alpha\beta}), \tag{36}$$

where k and m are symmetric and α and β run over 1 to 3. To get the $\Delta s = -\Delta d = 1$ operators, none of the m, k, and l can be a 3: When 3 is on the ϵ, it prevents both i and j from being a strange quark, and when 3 is a lower index, both i and j must be 3 which is impossible because of the antisymmetry. In fact, the only possible combination for m, k, and l is 2, 2, 1.

To get an isospin-$3/2$ operator, one must symmetrize m, k, and l, yielding

$$\begin{aligned}\Theta^{(6,3)}_{3/2} &\equiv \hat{\Theta}^{221} + \hat{\Theta}^{212} + \hat{\Theta}^{122} = \hat{\Theta}^{21}_{31} + \hat{\Theta}^{12}_{31} + \hat{\Theta}^{22}_{23} \\ &= \bar{s}_L\Gamma d_L \bar{u}_L\Gamma' u_R + \bar{s}_L\Gamma u_L \bar{u}_L\Gamma' d_R - \bar{u}_L\Gamma d_L \bar{s}_L\Gamma' u_R \\ &\quad - \bar{u}_L\Gamma u_L \bar{s}_L\Gamma' d_R - \bar{s}_L\Gamma d_L \bar{d}_L\Gamma' d_R \\ &\quad + \bar{d}_L\Gamma d_L \bar{s}_L\Gamma' d_R.\end{aligned} \tag{37}$$

The corresponding tensor components are

$$T^{31}_{12} = -T^{13}_{12} = T^{31}_{21} = -T^{13}_{21} = -T^{32}_{22} = T^{23}_{22} = 1. \tag{38}$$

There is also an isospin-$1/2$ operator by antisymmetrizing k and l:

$$\begin{aligned}\Theta^{(6,3)}_{1/2} &\equiv 2\hat{\Theta}^{221} - \hat{\Theta}^{212} - \hat{\Theta}^{122} = 2(\hat{\Theta}^{21}_{31} - \hat{\Theta}^{12}_{31}) \\ &= 2\bar{s}_L\Gamma d_L\bar{u}_L\Gamma' u_R - 2\bar{u}_L\Gamma d_L\bar{s}_L\Gamma' u_R \\ &\quad - \bar{s}_L\Gamma u_L\bar{u}_L\Gamma' d_R + \bar{u}_L\Gamma u_L\bar{s}_L\Gamma' d_R \\ &\quad - \bar{d}_L\Gamma d_L\bar{s}_L\Gamma' d_R + \bar{s}_L\Gamma d_L\bar{d}_L\Gamma' d_R. \end{aligned} \tag{39}$$

The tensor components are

$$\tfrac{1}{2}T^{31}_{21} = -\tfrac{1}{2}T^{13}_{21} = -T^{31}_{12} = T^{13}_{12} = T^{32}_{22} = -T^{23}_{22} = 1. \tag{40}$$

4. $(\bar{3}, \bar{6})$

Define $\Theta^{kl}_{ij} = \bar{q}^i_L\Gamma q_{Rk}\bar{q}^j_R\Gamma' q_{Rl}$, and construct the $(\bar{3}, \bar{6})$ operator

$$\hat{\Theta}{}^{kl}_{ij} = \epsilon^{klm}\hat{\Theta}_{i(jm)} = \tfrac{1}{2}\epsilon^{klm}(\epsilon_{\alpha\beta m}\Theta^{\alpha\beta}_{ij} + \epsilon_{\alpha\beta j}\Theta^{\alpha\beta}_{im}), \tag{41}$$

where j and m are symmetric and α and β run from 1 to 3. The only choice which will generate the $\Delta s = -\Delta d = 1$ operators is i, j, and m takes 3, 3, 1. If i is 3, and j and m take 3 and 1, we have the isospin-$1/2$ operator

$$\begin{aligned}\Theta^{(\bar{3},\bar{6})}_{1/2,L} &\equiv \hat{\Theta}_{313} + \hat{\Theta}_{331} = 2\hat{\Theta}^{12}_{31} \\ &= \bar{s}_L\Gamma u_R\bar{u}_R\Gamma' d_R - \bar{s}_L\Gamma d_R\bar{u}_R\Gamma' u_R + \bar{s}_L\Gamma d_R\bar{s}_R\Gamma' s_R \\ &\quad - \bar{s}_L\Gamma s_R\bar{s}_R\Gamma' d_R, \end{aligned} \tag{42}$$

with the following tensor components:

$$T^{31}_{12} = -T^{31}_{21} = T^{33}_{23} = -T^{33}_{32} = 1. \tag{43}$$

On the other hand, if i takes 1, one gets another isospin-$1/2$ operator

$$\Theta^{(\bar{3},\bar{6})}_{1/2,R} \equiv \hat{\Theta}_{133} = \hat{\Theta}^{12}_{13} = \bar{u}_L\Gamma u_R\bar{s}_R\Gamma' d_R - \bar{u}_L\Gamma d_R\bar{s}_R\Gamma' u_R, \tag{44}$$

with the following tensor components:

$$T^{13}_{12} = -T^{13}_{21} = 1. \tag{45}$$

One could consider operators with dimension 8 and higher. However, generally they are suppressed by $1/\Lambda^2$ relative to those we have considered, where Λ is some weak-interaction scale. We summarize the above result in Table I.

We emphasize that these operators are completely general, independent of the underlying mechanisms (supersymmetry, large extra dimension, or little Higgs, etc.) for flavor and *CP* violations in beyond-SM theories.

III. CHIRAL EXPANSION AT LEADING ORDER

Chiral perturbation theory (ChPT) for kaon decay is useful for two reasons: First, it allows one to connect the physical matrix elements $\langle K|\mathcal{O}|\pi\pi\rangle$ to some unphysical, but easier-to-calculate matrix elements on the lattice. Second, it yields dependence of the matrix elements on meson mass parameters. Since lattice calculations are usually done at larger and unphysical meson masses because of limited computational resources, this dependence can be used to extrapolate the calculated matrix elements to physical ones. In this section, we will build a set of effective operators in ChPT up to the lowest order and use an example to illustrate how to connect the unphysical processes to the physical process $K \to \pi\pi$ in which we are interested.

In lattice calculations the quenched approximation to QCD has usually been applied in the past, where valance quark fields are "quenched" by corresponding ghost quark fields with the same masses and quantum numbers but opposite statistics. The ChPT can be adapted with the quenched QCD by introducing the "super-η'" field into the effective Lagrangian [29]. In this paper we will work with the full dynamical QCD only.

A. ChPT and SM operators

The standard ChPT starts with the nonlinear Goldstone meson field Σ by:

$$\Sigma \equiv \exp\left(\frac{2i\phi}{f}\right), \tag{46}$$

where ϕ is the Goldstone meson matrix

$$\phi = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix}, \tag{47}$$

and $f \approx 135$ MeV is the bare pion decay constant. We separate the effective ChPT Lagrangian into two parts:

$$\mathcal{L}_{\mathrm{ChPT}} = \mathcal{L}_s + \mathcal{L}_w, \tag{48}$$

TABLE I. Chiral representations appearing in dimension-5, 6, and 7 operators.

	Dimension-5	Dimension-6	Dimension-7
(L, R)	$(3, \bar{3})$, $(\bar{3}, 3)$	$(8, 1)$, $(1, 8)$ $(27, 1)$, $(1, 27)$ $(8, 8)$ $(6, \bar{6})$, $(\bar{6}, 6)$ $(3, \bar{3})$, $(\bar{3}, 3)$	$(15, \bar{3})$, $(\bar{3}, 15)$, $(\overline{15}, 3)$, $(3, \overline{15})$ $(\bar{6}, \bar{3})$, $(\bar{3}, \bar{6})$, $(6, 3)$, $(3, 6)$ $(3, \bar{3})$, $(\bar{3}, 3)$

where $\mathcal{L}_s$ corresponds the QCD strong interaction which preserves the flavor symmetry; the $\mathcal{L}_w$ is an effective Lagrangian for nonleptonic weak interaction and is responsible for the $\Delta s = 1$ processes. The lowest-order terms for the strong-interaction part is

$$\mathcal{L}_s^{(2)} = \frac{f^2}{8}\,\mathrm{Tr}(\partial_\mu \Sigma \partial^\mu \Sigma) + v\,\mathrm{Tr}[M\Sigma + (M\Sigma)^\dagger], \quad (49)$$

where $M \equiv \mathrm{diag}(m_u, m_d, m_s)$ is the quark mass matrix and $v \sim -\frac{1}{2}\langle \bar{u}u\rangle$ is proportional to the quark chiral condensate at the chiral limit. We demand the fields transform under $SU(3)_L \times SU(3)_R$ as

$$\Sigma \to L\Sigma R^\dagger, \qquad M \to R\Sigma L^\dagger, \quad (50)$$

to keep the Lagrangian invariant under an $SU(3)_L \times SU(3)_R$ transformation. Higher-order terms in the effective Lagrangian contain higher derivatives and can be written in systematic derivative and mass expansion. For our purpose here, however, only the leading large logarithms are calculated, and the higher-order terms are irrelevant.

At one loop, the physical masses and wave-function renormalizations are given by [30]

$$m_\pi^2 = m_{\pi,0}^2[1 + L(m_\pi) - \tfrac{1}{3}L(m_\eta) + \cdots], \quad (51)$$

$$m_K^2 = m_{K,0}^2[1 + \tfrac{2}{3}L(m_\eta) + \cdots], \quad (52)$$

$$Z_\pi = 1 + \tfrac{4}{3}L(m_\pi) + \tfrac{2}{3}L(m_K) + \cdots, \quad (53)$$

$$Z_K = 1 + \tfrac{1}{4}L(m_\pi) + \tfrac{1}{2}L(m_K) + \tfrac{1}{4}L(m_\eta) + \cdots, \quad (54)$$

$$f_\pi = f[1 - 2L(m_\pi) - L(m_K) + \cdots], \quad (55)$$

$$f_K = f[1 - \tfrac{3}{4}L(m_\pi) - \tfrac{3}{2}L(m_K) - \tfrac{3}{4}L(m_\eta) + \cdots], \quad (56)$$

for the pion and kaon fields, respectively. $L(m)$ is the chiral logarithm defined as

$$L(m) \equiv \frac{m^2}{(4\pi f)^2}\ln\frac{m^2}{\mu_\chi^2}, \quad (57)$$

with μ_χ the cutoff scale. The dots represent nonlogarithm contributions from $\mathcal{O}(p^4)$ and higher-order Lagrangian terms. In this paper we focus only on the large chiral logarithmic corrections and will not, for simplicity, include the dots explicitly in the results.

In the standard electroweak theory, there are 7 independent four-quarks operators which can be classified into $(8, 1)$, $(27, 1)$, and $(8, 8)$. Defining $\Theta = T^{ik}_{jl}\bar{q}^i_L\gamma_\mu q_{Lj}\bar{q}^k_L\gamma^\mu q_{Ll}$, we can obtain four-independent quark operators with the following tensor components:

$$(27,1)_{3/2}\colon\ T^{31}_{21} = T^{31}_{12} = T^{13}_{21} = T^{13}_{12} = -T^{32}_{22} = -T^{23}_{22} = \tfrac{1}{2}, \quad (58)$$

$$\begin{aligned}(27,1)_{1/2}\colon\ & T^{31}_{21} = T^{31}_{12} = T^{13}_{21} = T^{13}_{12} = \tfrac{1}{2},\\ & T^{32}_{22} = T^{23}_{22} = 1, \qquad T^{33}_{23} = T^{33}_{32} = -\tfrac{3}{2},\end{aligned} \quad (59)$$

$$\begin{aligned}(8,1)_{1/2,S}\colon\ & T^{31}_{21} = T^{13}_{12} = T^{31}_{12} = T^{13}_{21} = \tfrac{1}{2},\\ & T^{32}_{22} = T^{23}_{22} = T^{33}_{23} = T^{33}_{32} = 1,\\ (8,1)_{1/2,A}\colon\ & T^{31}_{21} = T^{13}_{12} = -T^{31}_{12} = -T^{13}_{21} = \tfrac{1}{2}.\end{aligned} \quad (60)$$

On the other hand, defining a $(8, 8)$ operator $\Theta = T^{ik}_{jl}\bar{q}^i_L\gamma_\mu q_{Lj}\bar{q}^k_R\gamma^\mu q_{Rl}$, we have three quark operators with following tensor components:

$$(8,8)_{3/2}\colon\ T^{31}_{21} = T^{31}_{12} = -T^{32}_{22} = 1, \quad (61)$$

$$(8,8)_{1/2,S}\colon\ T^{31}_{21} = T^{31}_{12} = T^{32}_{22}/2 = -T^{33}_{23}/3 = 1, \quad (62)$$

$$(8,8)_{1/2,A}\colon\ T^{31}_{21} = -T^{31}_{12} = -T^{33}_{23} = 1. \quad (63)$$

One can similarly define other $(8, 8)$ operators with different color indices contractions. For the sake of convenience, we have broken the operators into representations of definite isospins. This has the advantage of easily building up reducible operators from linear combinations of these simple ones. For example, the SM electromagnetic penguin operators $Q_{7,8}$:

$$Q_7 = \tfrac{1}{2}[\Theta^{(8,8)}_{3/2} + \Theta^{(8,8)}_{1/2,A}], \quad (64)$$

and Q_8 is similar to Q_7 but with different color indices contraction.

In the ChPT, one can match the above QCD operators to the hadronic operators made of Goldstone boson fields [23,31,32]:

$$\begin{aligned}\tilde{\Theta}^{(8,1)}_1 &\equiv \mathrm{Tr}[\Lambda\partial_\mu\Sigma\partial^\mu\Sigma^\dagger],\\ \tilde{\Theta}^{(8,1)}_2 &\equiv \frac{8v}{f^2}\,\mathrm{Tr}[\Lambda\Sigma M + \Lambda(\Sigma M)^\dagger],\\ \tilde{\Theta}^{(27,1)}_{\Delta I} &\equiv [T^{(27,1)}_{\Delta I}]^{ij}_{kl}(\Sigma\partial_\mu\Sigma^\dagger)^k{}_i(\Sigma\partial^\mu\Sigma^\dagger)^l{}_j,\\ \tilde{\Theta}^{(8,8)}_{\Delta I} &\equiv [T^{(8,8)}_{\Delta I}]^{ij}_{kl}(\Sigma)^k{}_j(\Sigma^\dagger)^l{}_i,\\ \tilde{\Theta}^{(\bar{3},3)} &\equiv \mathrm{Tr}[\Lambda\Sigma^\dagger],\end{aligned} \quad (65)$$

where $\Lambda = \delta_{i,3}\delta_{j,2}$ and T's are tensor structures defined above. The expansions go like

$$\begin{aligned}\Theta^{(8,1)}_i &= \alpha^{(8,1)}_{1i}\tilde{\Theta}^{(8,1)}_1 + \alpha^{(8,1)}_{2i}\tilde{\Theta}^{(8,1)}_2 + \cdots,\\ \Theta^{(27,1)}_{\Delta I} &= \alpha^{(27,1)}\tilde{\Theta}^{(27,1)}_{\Delta I} + \cdots,\\ \Theta^{(8,8)}_{\Delta I} &= \alpha^{(8,8)}\tilde{\Theta}^{(8,8)}_{\Delta I} + \cdots,\end{aligned} \quad (66)$$

where $\alpha^{(L,R)}$'s are "Wilson coefficients" which are universal in different processes and dots represent higher dimensional operators. The subscript i on the $(8, 1)$ operator indicates different quark operators in the same chiral representation, including ones with two right-handed fields coupled to the singlet.

The one-loop results of these operators in various processes can be found in [18–24]. Because of the different definitions of the operators and the nonlinear meson fields, there might be sign differences among these results. Overall speaking, the $(8, 1)$ and $(27, 1)$ operators dominate in the CP-conserve process. The $(8, 8)$ operators, corresponding to the $Q_{7,8}$ operators, play a significant role in CP-violation processes [23,33]. The lowest-order mass-dependent term $\Theta_2^{(8,1)}$ will vanish in physical process $K \to \pi\pi$ to all orders. This property was pointed out by [32] first and has been well-studied by [20,34]. We will come back to this issue later.

B. Chiral matching of new operators

Now we can proceed in constructing new hadronic operators for new interactions arising from physics beyond the SM. We label operators by the irreducible representatives and their isospin quantum numbers. Similar to the case in the SM, we define the effective operators at their lowest order as

$$\text{dimension-6:}\ \ \tilde{\Theta}^{(\bar{6},6)}_{\Delta I} = [T^{(\bar{6},6)}_{\Delta I}]^{ij}_{kl}(\Sigma^\dagger)^k{}_i(\Sigma^\dagger)^l{}_j, \tag{67}$$

$$\text{dimension-7:}\ \ \tilde{\Theta}^{(\overline{15},3)}_{\Delta I} = [T^{(\overline{15},3)}_{\Delta I}]^{ij}_{kl}(\Sigma\partial^\mu\Sigma^\dagger)^k{}_i(\partial_\mu\Sigma^\dagger)^l{}_j, \tag{68}$$

$$\tilde{\Theta}^{(6,3)}_{\Delta I} = [T^{(6,3)}_{\Delta I}]^{ij}_{kl}(\Sigma\partial^\mu\Sigma^\dagger)^k{}_i(\partial_\mu\Sigma^\dagger)^l{}_j, \tag{69}$$

$$\tilde{\Theta}^{(\bar{3},15)}_{\Delta I} = [T^{(\bar{3},15)}_{\Delta I}]^{ij}_{kl}(\partial^\mu\Sigma^\dagger)^k{}_i(\Sigma^\dagger\partial_\mu\Sigma)^l{}_j, \tag{70}$$

$$\tilde{\Theta}^{(\bar{3},\bar{6})}_{\Delta I} = [T^{(\bar{3},\bar{6})}_{\Delta I}]^{ij}_{kl}(\partial^\mu\Sigma^\dagger)^k{}_i(\Sigma^\dagger\partial_\mu\Sigma)^l{}_j. \tag{71}$$

We can also construct operators with one insertion of quark masses:

$$X^L_\pm \equiv (\Sigma M) \pm (\Sigma M)^\dagger, \qquad X^R_\pm \equiv (M\Sigma) \pm (M\Sigma)^\dagger. \tag{72}$$

They transform under $SU(3)_L \times SU(3)_R$ as

$$X^L_\pm \to LX^L_\pm L^\dagger; \qquad X^R_\pm \to RX^R_\pm R^\dagger. \tag{73}$$

With the insertion of $X_\pm$ we can build two additional sets of the dimension-7 operators at the lowest order:

$$\tilde{\Theta}'^{(L,R)}_{\Delta I,X_\pm} = [T^{(L,R)}_{\Delta I}]^{ij}_{kl}(X^L_\pm)^k{}_i(\Sigma^\dagger)^l{}_j, \tag{74}$$

for (L, R) belonging to $(\overline{15}, 3)$ or $(6, 3)$, and

$$\tilde{\Theta}'^{(L,R)}_{\Delta I,X_\pm} = [T^{(L,R)}_{\Delta I}]^{ij}_{kl}(\Sigma^\dagger)^k{}_i(X^R_\pm)^l{}_j, \tag{75}$$

for (L, R) belonging to $(\bar{3}, 15)$ or $(\bar{3}, \bar{6})$. Therefore, the dimension-6 and 7 QCD operators should be matched to hadron operators as follows:

$$\Theta^{(L,R)}_{D6} \to \alpha^{(L,R)}\tilde{\Theta}^{(L,R)}, \tag{76}$$

$$\Theta^{(L,R)}_{D7} \to \alpha^{(L,R)}\tilde{\Theta}^{(L,R)} + \alpha^{(L,R)}_{X_+}\tilde{\Theta}'^{(L,R)}_{X_+} + \alpha^{(L,R)}_{X_-}\tilde{\Theta}'^{(L,R)}_{X_-}, \tag{77}$$

where higher-order terms have been omitted.

In the SM we need operators with X_+ only since all QCD operators obey the CPS symmetry, the CP transformation followed by an exchange of s and d quarks [32]. However, the four-quark operators derived from new physics do not necessarily have this symmetry, and hence we can have an additional set of operators in the effective theory.

Just as the $(8, 8)$ operators in the SM, the $(\bar{6}, 6)$ dimension-6 operators will contribute at $\mathcal{O}(p^0)$ order in the ChPT. This set of operators can be derived from the Higgs (or some new heavy bosons) exchange and will contribute to the CP violation phase in the same manner as Q_7 and Q_8 in the SM. We will consider an example of applying our result later.

When calculating CP conserving matrix elements, the new operators are usually negligible compared to the SM weak-interaction operators defined in Eqs. (58)–(65). The new operators are mainly responsible for the CP-violating phase and are worth investigating as the case in [23,33]. We observe that some of these new operators, notably $(\bar{6}, 6)$, $(\overline{15}, 3)$, $(6, 3)$, and $(\bar{3}, 15)$, have contributions in the $\Delta I = 3/2$ channel, in addition to $(27, 1)$ and $(8, 8)$ operators in the SM. Furthermore, as we shall see, they all receive large chiral logarithmic corrections in the one-loop ChPT. Therefore, the operators from new flavor theories beyond the SM can help explain the $\Delta I = 1/2$ selection rule and the direct CP violation parameter ϵ'.

C. Results at tree level

In this subsection, we consider tree-level relations among the matrix elements of the QCD operators in different states. These relations reflect chiral symmetry and can also be derived using old-fashioned current algebra.

There are three processes that we are mainly interested in: $K^0 \to$ vacuum, $K^+ \to \pi^+$, and $K^0 \to \pi^0\pi^0$. For $K^0 \to \pi^+\pi^-$, one can obtained the matrix elements through angular momentum relation, as shown in the appendix. At tree level, the dimension-7 momentum operators will not contribute to the $K^0 \to$ vacuum process. For the mass-dependent operators, the result will be proportional to either $(m_s - m_d) \sim m^2_{K,0} - m^2_{\pi,0}$ or $(m_s + m_d) \sim m^2_{K,0}$, by the lowest-order expansion of $X_\pm$. However, beyond tree level, the result will no longer be proportional to $(m_s - m_d)$ or $(m_s + m_d)$.

For dimension-6 operators in $(\bar{6}, 6)$, we have the tree-level results:

$$\langle 0|\Theta_{D6}|K^0\rangle_{\text{Tree}} = \frac{ib_0}{f}\alpha_{D6}, \tag{78}$$

TABLE II. Tree-level contributions from dimension-6 operators.

	$K^0 \to$ vacuum	$K^+ \to \pi^+$	$K^0 \to \pi^0\pi^0$
$(L, R)_{\Delta I}$	b_0	c_0	d_0
$(\bar{6}, 6)_{3/2}$	0	-4	-8
$(\bar{6}, 6)_{1/2,S}$	-6	-10	16
$(\bar{6}, 6)_{1/2,A}$	-2	-2	0

$$\langle \pi^+ | \Theta_{D6} | K^+ \rangle_{\text{Tree}} = \frac{c_0}{f^2} \alpha_{D6}, \tag{79}$$

$$\langle \pi^0 \pi^0 | \Theta_{D6} | K^0 \rangle_{\text{Tree}} = \frac{i d_0}{f^3} \alpha_{D6}, \tag{80}$$

with b_0,c_0, and d_0 coefficients listed in Table II, which are different from different isospin projections. The corresponding results for $\pi^+\pi^-$ final state can be obtained from relations in the appendix. The nonperturbative coefficient α_{D6} is the same for different operators and final states.

Similarly, the tree-level matrix elements for dimension-7 operators are

$$\begin{aligned}\langle 0 | \Theta_{D7} | K^0 \rangle_{\text{Tree}} &= \frac{4iv}{f^3} [b'_{0,+}(m_s - m_d)\alpha_{D7,+} \\ &\quad + b'_{0,-}(m_s + m_d)\alpha_{D7,-}] \\ &= \frac{i}{f}[b'_{0,+}(m^2_{K,0} - m^2_{\pi,0})\alpha_{D7,+} \\ &\quad + b'_{0,-} m^2_{K,0} \alpha_{D7,-}],\end{aligned} \tag{81}$$

$$\begin{aligned}\langle \pi^+ | \Theta_{D7} | K^+ \rangle_{\text{Tree}} &= \frac{m^2_{M,0}}{f^2} [c'_0 \alpha_{D7} + c'_{0,+} \alpha_{D7,+} \\ &\quad + c'_{0,-} \alpha_{D7,-}],\end{aligned} \tag{82}$$

TABLE III. Tree-level contributions from dimension-7 operators.

	$K^0 \to$ vacuum		$K^+ \to \pi^+$			$K^0 \to \pi^0\pi^0$		
$(L, R)_{\Delta I}$	$b'_{0,+}$	$b'_{0,-}$	c'_0	$c'_{0,+}$	$c'_{0,-}$	d'_0	$d'_{0,+}$	$d'_{0,-}$
$(\overline{15}, 3)_{3/2}$	0	0	-8	0	2	8	-2	-2
$(\overline{15}, 3)_{1/2,S}$	9/2	3/2	-8	$-3/2$	2	8	-2	-2
$(\overline{15}, 3)_{1/2,A}$	$-1/2$	1/2	0	$-1/2$	0	8	-2	-2
$(\overline{15}, 3)_{1/2}$	1/2	1/2	0	$-1/2$	0	0	0	0
$(6, 3)_{3/2}$	0	0	0	0	0	8	-2	-2
$(6, 3)_{1/2}$	3/2	3/2	12	$-3/2$	-3	16	-4	-4
$(\bar{3}, 15)_{3/2,L}$	0	0	4	0	-1	0	0	0
$(\bar{3}, 15)_{3/2,R}$	0	0	4	0	-1	8	2	-2
$(\bar{3}, 15)_{1/2,L}$	$-3/2$	$-3/2$	4	3/2	-1	0	0	0
$(\bar{3}, 15)_{1/2,R}$	$-3/2$	3/2	4	$-3/2$	-1	-16	2	-2
$(\bar{3}, \bar{6})_{1/2,L}$	3/2	$-1/2$	4	1/2	-1	0	0	0
$(\bar{3}, \bar{6})_{1/2,R}$	$-1/2$	1/2	-4	$-1/2$	1	0	2	-2

$$\begin{aligned}\langle \pi^0 \pi^0 | \Theta_{D7} | K^0 \rangle_{\text{Tree}} &= \frac{m^2_K}{f^3} [d'_0 \alpha_{D7} + d'_{0,+} \alpha_{D7,+} \\ &\quad + d'_{0,-} \alpha_{D7,-}],\end{aligned} \tag{83}$$

with coefficients listed in Table III. Here the nonperturbative coefficients $\alpha' s$ are different for different chiral representations. From the above equations, it is clear that one can obtained the two-pion matrix elements from the vacuum and one-pion ones, if only one of the X_+ and X_- types of operators is present, such as in the SM case.

IV. CHIRAL LOGARITHMS AT ONE LOOP

ChPT calculations of the kaon decay matrix elements up to higher chiral orders are needed for understanding the size of chiral corrections and for extrapolating matrix elements from unphysical quark masses to physical ones. In lattice calculations, unphysically large quark masses are usually used to make calculations feasible. Then one needs to extrapolate the matrix elements to the physical region. In this section, we calculate the large chiral logarithms of the dimension-6 and 7 operators for the process $K \to 0$, $K \to \pi$, and $K \to \pi\pi$ hoping to get the leading corrections as the function of quark masses.

In our calculations, we have made the simplifying assumption $m_u = m_d$. For the $\langle 0 | \mathcal{O} | K^0 \rangle$ matrix element, we have kept all of the Goldstone boson masses independent. For the matrix element $\langle \pi^+ | \mathcal{O} | K^+ \rangle$, we utilize a common mass m_M for all of the mesons to conserve momentum. In the calculation of $\langle \pi^0 \pi^0 | \mathcal{O} | K^0 \rangle$ matrix elements, the pion masses are neglected. Since $m^2_\pi / m^2_K \approx 10^{-1}$ with physical pion and kaon masses, this is a reasonable approximation for the physical processes.

A. $K^0 \to$ vacuum

The diagram we need to consider is shown in Fig. 4 below, where and henceforth the square dot represents an effective weak-interaction operator while the round dots represent strong-interaction insertions. We have not shown the wave-function renormalization diagrams, but they have to be included in the final result.

For dimension-6 operators, the results for $\langle 0 | \mathcal{O} | K^0 \rangle$ up to one loop can be written as

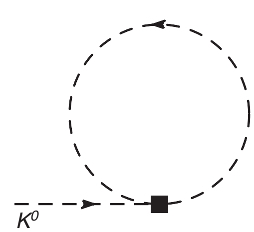

FIG. 4. Feynman diagram for $K^0 \to$ vacuum at one loop.

TABLE IV. One-loop contributions from dimension-6 operators.

	$K^0 \to$ vacuum			$K^+ \to \pi^+$	$K^0 \to \pi^0\pi^0$
$(L, R)_{\Delta I}$	b_η	b_K	b_π	c_M	d_K
$(\bar{6}, 6)_{3/2}$	0	0	0	112/3	80/9
$(\bar{6}, 6)_{1/2,S}$	1/2	33	57/2	196/3	−1024/9
$(\bar{6}, 6)_{1/2,A}$	25/6	15	3/2	28/3	−4

$$\langle 0|\Theta_{D6}|K^0\rangle = \frac{i\alpha_{D6}}{f}[b_0 + b_\eta L(m_\eta^2) + b_K L(m_K^2) + b_\pi L(m_\pi^2)], \tag{84}$$

with coefficients listed in Table IV. Note that we have three different chiral logarithms corresponding to eta, kaon, and pion, respectively. The isospin-3/2 operator does contribute for the obvious reason.

For dimension-7 operators, the results are more complicated:

$$\begin{aligned}\langle 0|\Theta_{D7}|K^0\rangle = \langle 0|\Theta_{D7}|K^0\rangle_{\rm Tree} + \frac{i}{f}\Big\{&[b'_\eta m_\eta^2 L(m_\eta^2) \\ &+ b'_K m_K^2 L(m_K^2) + b'_\pi m_\pi^2 L(m_\pi^2)]\cdot\alpha_{D7} \\ &+ \sum_\pm [(b'_{\eta,K} m_K^2 + b'_{\eta,\pi} m_\pi^2) L(m_\eta^2) \\ &+ (b'_{K,K} m_K^2 + b'_{K,\pi} m_\pi^2) L(m_K^2) \\ &+ (b_{\pi,K} m_K^2 + b'_{\pi,\pi} m_\pi^2) L(m_\pi^2)]\cdot\alpha_{D7,\pm}\Big\},\end{aligned} \tag{85}$$

where each chiral logarithm now has different meson mass factors. The coefficients are listed in Tables V and VI.

B. $K^+ \to \pi^+$

For the $K^+ \to \pi^+$ matrix elements, we utilize a common mass for mesons $m_M^2 = m_\pi^2 = m_K^2$ in the calculation. The Feynman diagrams are shown in Fig. 5. The matrix elements up to the leading chiral logarithms are

TABLE V. One-loop contributions from dimension-7 operators (I).

		$K^0 \to$ vacuum	
$(L, R)_{\Delta I}$	b'_η	b'_K	b'_π
$(\overline{15}, 3)_{1/2,S}$	0	−12	12
$(\overline{15}, 3)_{1/2,A}$	4	−4	0
$(\overline{15}, 3)_{1/2}$	−4	4	0
$(6, 3)_{1/2}$	6	12	−18
$(\bar{3}, 15)_{1/2,L}$	18	−12	−6
$(\bar{3}, 15)_{1/2,R}$	6	−12	6
$(\bar{3}, \bar{6})_{1/2,L}$	2	4	−6
$(\bar{3}, \bar{6})_{1/2,R}$	2	4	−6

TABLE VI. One-loop contributions from dimension-7 operators (II).

			$K^0 \to$ vacuum			
$(L, R)_{\Delta I, X\pm}$	$b'_{\eta,K}$	$b'_{\eta,\pi}$	$b'_{K,K}$	$b'_{K,\pi}$	$b'_{\pi,K}$	$b'_{\pi,\pi}$
$(\overline{15}, 3)_{1/2,S,X_+}$	−43/8	27/8	−123/4	111/4	−51/8	99/8
$(\overline{15}, 3)_{1/2,S,X_-}$	7/8	0	−33/4	0	−33/8	−3
$(\overline{15}, 3)_{1/2,A,X_+}$	25/24	−17/24	27/4	−15/4	−21/8	−3/8
$(\overline{15}, 3)_{1/2,A,X_-}$	−25/24	1/3	−3/4	0	−27/8	0
$(\overline{15}, 3)_{1/2,X_+}$	−3/8	17/24	−15/4	15/4	−3/8	3/8
$(\overline{15}, 3)_{1/2,X_-}$	−3/8	−1/3	−15/4	0	−3/8	0
$(6, 3)_{1/2,X_+}$	3/2	5/8	−21/4	9/4	−57/8	45/8
$(6, 3)_{1/2,X_-}$	3/2	1/2	−21/4	0	−57/8	9/2
$(\bar{3}, 15)_{1/2,L,X_+}$	25/8	−5/8	45/4	−33/4	9/8	−45/8
$(\bar{3}, 15)_{1/2,L,X_-}$	25/8	1/2	45/4	0	9/8	−3/2
$(\bar{3}, 15)_{1/2,R,X_+}$	9/8	−5/8	33/4	−33/4	33/8	−45/8
$(\bar{3}, 15)_{1/2,R,X_-}$	−9/8	1/2	−33/4	0	−33/8	−3/2
$(\bar{3}, \bar{6})_{1/2,L,X_+}$	−43/24	13/8	−45/4	33/4	−9/8	21/8
$(\bar{3}, \bar{6})_{1/2,L,X_-}$	−7/24	1/2	15/4	0	3/8	−3/2
$(\bar{3}, \bar{6})_{1/2,R,X_+}$	3/8	−5/24	3/4	−3/4	27/8	−15/8
$(\bar{3}, \bar{6})_{1/2,R,X_-}$	−3/8	1/6	−3/4	0	−27/8	3/2

$$\langle \pi^+|\Theta_{D6}|K^+\rangle = \frac{\alpha_{D6}}{f^2}[c_0 + c_M L(m_M^2)], \tag{86}$$

$$\begin{aligned}\langle \pi^+|\Theta_{D7}|K^+\rangle = \langle \pi^+|\Theta_{D7}|K^+\rangle_{\rm Tree} + \frac{m_{M,0}^2}{f^2} L(m_M^2) \\ \times [c'_M \alpha_{D7} + c'_{M,+}\alpha_{D7,+} + c'_{M,-}\alpha_{D7,-}],\end{aligned} \tag{87}$$

for dimension-6 and 7 operators, respectively. The coefficients are listed in Tables IV and VII.

C. $K^0 \to \pi^0\pi^0$

For $K^0 \to \pi^0\pi^0$, we neglect the pion mass in the calculation. The diagrams we need to consider for are shown in Fig. 6. The results are

$$\langle \pi^0\pi^0|\Theta_{D6}|K^0\rangle = \frac{i\alpha_{D6}}{f^3}[d_0 + d_K L(m_K^2)], \tag{88}$$

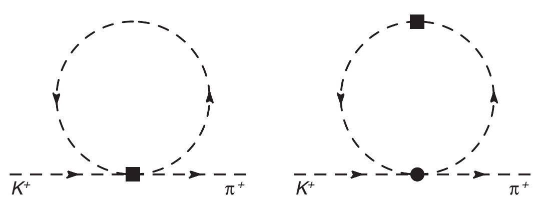

FIG. 5. Feynman diagrams for $K^+ \to \pi^+$ at one loop.

TABLE VII. One-loop contributions from dimension-7 operators (III).

$(L, R)_{\Delta I}$	$K^+ \to \pi^+$ (c'_M)	$(c'_M)_{X_+}$	$(c'_M)_{X_-}$	$K^0 \to \pi^0\pi^0$ (d'_K)	$(d'_K)_{X_+}$	$(d'_K)_{X_-}$
$(\overline{15}, 3)_{3/2}$	208/3	3/4	−52/3	8	−2/9	−2/9
$(\overline{15}, 3)_{1/2,S}$	196/3	28/3	−49/3	−640/9	214/9	160/9
$(\overline{15}, 3)_{1/2,A}$	−4/3	8/3	1/3	−352/9	70/9	88/9
$(\overline{15}, 3)_{1/2}$	−4/3	8/3	1/3	−8	2	2
$(6, 3)_{3/2}$	0	0	0		8/9 −2/9	−2/9
$(6, 3)_{1/2}$	−80	12	9	−848/9	212/9	212/9
$(\bar{3}, 15)_{3/2,L}$	−16	2/3	26/3	0	0	0
$(\bar{3}, 15)_{3/2,R}$	−16	2/3	26/3	−16	2/9	−2/9
$(\bar{3}, 15)_{1/2,L}$	0	−22/3	29/3	0	−3	−3
$(\bar{3}, 15)_{1/2,R}$	−32	26/3	23/3	128	−133/9	133/9
$(\bar{3}, \bar{6})_{1/2,L}$	−80/3	−4	3	−8	5	−3
$(\bar{3}, \bar{6})_{1/2,R}$	80/3	4	−3	0	−97/9	97/9

$$\langle \pi^0\pi^0|\Theta_{D7}|K^0\rangle = \langle \pi^0\pi^0|\Theta_{D7}|K^0\rangle_{\text{Tree}} + \frac{m_K^2}{f^2}L(m_K^2)[d'_M\alpha_{D7} + d'_{M,+}\alpha_{D7,+} + d'_{M,-}\alpha_{D7,-}], \quad (89)$$

with coefficients listed in Tables IV and VII.

There are several comments we would like to make. First, the quark-mass-dependent operators (operators constructed with $X_\pm$) contribute in the physical process $K \to \pi\pi$, contrary to $\Theta_2^{(8,1)}$ in the SM case. The reason is similar to that for the higher-order operators of (27, 1) [22]: They cannot be expressed as a total divergence by the equations of motion. Similarly, these new operators will not act as a generator for rotation in $s - d$ plane like the $\Theta_2^{(8,1)}$ does [20,32,34]. Therefore the one-loop matrix elements of $K \to 0$ will no longer be proportional to $(m_s \pm m_d)$ as they were at tree level.

Second, the masses appearing in our result are either bare masses or the renormalized one depending on the processes. For the unphysical processes $K \to 0$ and $K \to \pi$, we use the bare masses, whereas for the physical $K \to \pi\pi$, the one-loop renormalized mass is implied. It makes the comparison to the experimental result feasible.

Third, in [35] the author claimed that infrared-sensitive terms like $m_K^2 \log m_\pi^2$, which diverges in the $m_\pi \to 0$ limit, will emerge in the $K \to \pi\pi$ matrix element. We have checked the result by keeping pion masses explicit in our calculations and found that all such terms canceled when summing all of the diagrams. Therefore it is safe to take the limit $m_\pi \to 0$.

Finally, there are a large number of unknown nonperturbative coefficients in the new operators. For dimension-

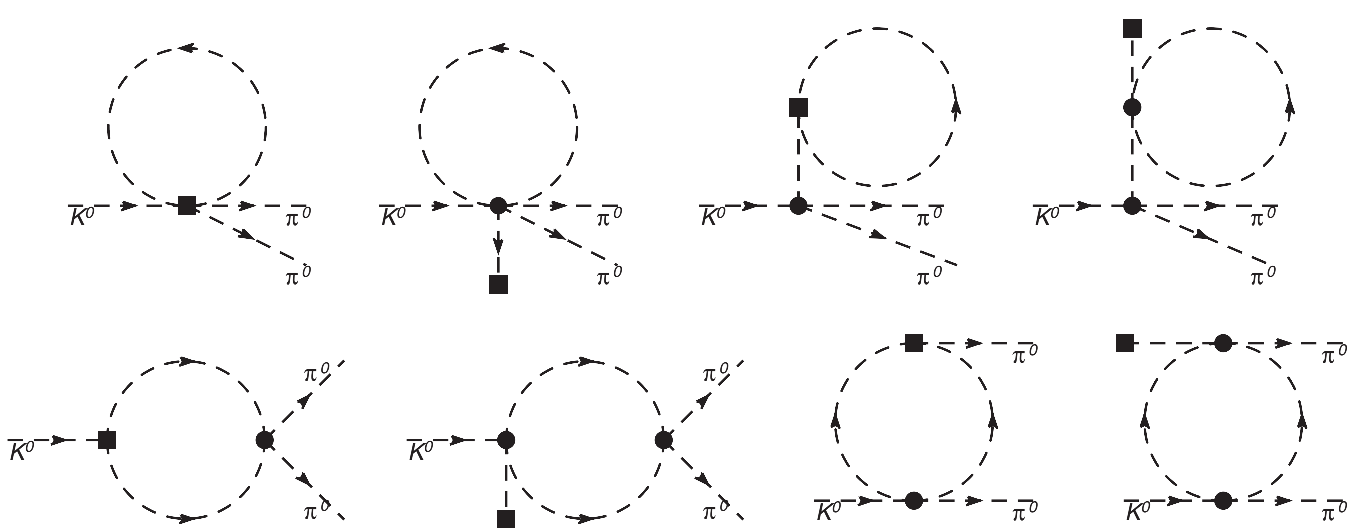

FIG. 6. Feynman diagrams for $K^0 \to \pi^0\pi^0$ at the one-loop ChPT.

6 operators, the traditional way of determining these coefficients by calculating simple processes like $K^+ \to \pi^+$ is sufficient. In dimension-7 cases, however, the two simple processes $K^0 \to 0$ and $K^+ \to \pi^+$ are not enough to determine all of the coefficients, unless there is the so-called CPS symmetry. Adding other simple processes like $K^0 \to \pi^0$ and $K^0 \to \eta$ will not improve the situation since they are not independent in the $SU(3)$ limit on which we are working. We can, in principle, get more relationships when away from $SU(3)$ chiral and isospin symmetries, but many more new coefficients will enter as well, and then we need even more relationships to determine all of the coefficients. Therefore we could either rely on some model-dependent assumptions or calculate more complicate processes on the lattice directly. In any case, the ChPT calculations can serve as a check for relations among coefficients from lattice or other nonperturbative model calculations.

V. CONCLUSION

The standard model calculations for the direct CP violation in nonleptonic kaon decay have not been entirely settled due to the difficulty in nonperturbative QCD physics. This leaves the possibility of understanding the phenomenon from beyond-standard-model physics, although a precision SM calculation may prove otherwise. We do not know yet what form the new physics will take, either supersymmetry, left-right symmetry, large extra dimensions, or little Higgs, or something else. Presumably, the Large Hadron Collider will help us to identify it in the next few years.

In this paper, we aim to study a general effective theory for nonleptonic kaon decay which has its origin from beyond-SM physics. Our goal is to understand an important step in a complete calculation, namely, the chiral effects in the hadronic matrix elements. We systematically classify the dimension-5, 6, and 7 quark and gluon operators according to their chiral structures. Using chiral symmetry, we derive tree-level relations between the matrix elements involving zero, one, and two pions. This is useful because lattice calculations of multiparticle matrix elements are much harder than these for a few particles. We have also calculated the leading chiral logarithmic behavior of these operators in the ChPT. The result again will be useful for calculating matrix elements of these operators on the lattice. We have not considered them in quenched QCD formulations, as the rapid progress in lattice QCD calculations makes quenched studies less useful than in the past.

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APPENDIX: LEADING CHIRAL LOGARITHMS IN SM OPERATORS

The leading chiral logarithms in SM operators have been calculated by many authors [18,19,22,23,33], and for completeness we list the result here. Notice that the results quoted in Eqs. (80) and (81) in [36] contain sign errors. The result for $K^0 \to \pi^0\pi^0$ here is different from that in [19], as pointed out in [35]. Results for $K^0 \to$ vacuum and $K^+ \to \pi^+$ are presented in terms of bare masses and couplings, while for $K^0 \to \pi^0\pi^0$ we use the physical mass.

The operators we use are defined in the main body of the paper [Eqs. (58)–(65)]. We first consider the matrix elements between K^0 and the vacuum:

$$\begin{aligned}\langle 0|\Theta^{(8,1)}|K^0\rangle &= \frac{2i\alpha_1^{(8,1)}}{f}[m_\eta^2 L(m_\eta) + 2m_K^2 L(m_K) \\ &\quad - 3m_\pi^2 L(m_\pi)] + \frac{4i\alpha_2^{(8,1)}}{f}(m_{K,0}^2 - m_{\pi,0}^2) \\ &\quad \times \left[1 - \frac{1}{12}L(m_\eta) - \frac{3}{2}L(m_K) - \frac{3}{4}L(m_\pi)\right],\end{aligned} \tag{A1}$$

$$\begin{aligned}\langle 0|\Theta_{1/2}^{(27,1)}|K^0\rangle &= \frac{6i\alpha^{(27,1)}}{f}[3m_\eta^2 L(m_\eta) - 4m_K^2 L(m_K) \\ &\quad + m_\pi^2 L(m_\pi)],\end{aligned} \tag{A2}$$

$$\langle 0|\Theta_{1/2,A}^{(8,8)}|K^0\rangle = \frac{12i\alpha^{(8,8)}}{f}[L(m_K) - L(m_\pi)], \tag{A3}$$

$$\begin{aligned}\langle 0|\Theta_{1/2,S}^{(8,8)}|K^0\rangle &= -\frac{12i\alpha^{(8,8)}}{f}\left[1 - \frac{3}{4}L(m_\eta) - \frac{13}{2}L(m_K) \right.\\ &\quad \left. - \frac{7}{4}L(m_\pi)\right],\end{aligned} \tag{A4}$$

$$\begin{aligned}\langle 0|\Theta^{(\bar{3},3)}|K^0\rangle &= -\frac{2i\alpha^{(\bar{3},3)}}{f}\left[1 - \frac{1}{12}L(m_\eta) - \frac{3}{2}L(m_K) \right.\\ &\quad \left. - \frac{3}{4}L(m_\pi)\right],\end{aligned} \tag{A5}$$

where f is the bare meson decay constant and $m_{\pi,0}$ and $m_{K,0}$ are bare masses of mesons. Because of the isospin conservation, only the $I = 1/2$ part of the operator can contribute.

For $K^+ \to \pi^+$ matrix elements, we apply a common mass m_M for all of the mesons. Therefore the momentum is conserved in the process.

$$\begin{aligned}\langle \pi^+|\Theta^{(8,1)}|K^+\rangle &= \frac{4m_{M,0}^2}{f^2}\left\{\alpha_1^{(8,1)}\left[1 + \frac{1}{3}L(m_M)\right] \right.\\ &\quad \left. - \alpha_2^{(8,1)}[1 + 2L(m_M)]\right\},\end{aligned} \tag{A6}$$

$$\langle\pi^+|\Theta^{(27,1)}_{3/2}|K^+\rangle = \langle\pi^+|\Theta^{(27,1)}_{1/2}|K^+\rangle = -\frac{4m^2_{M,0}\alpha^{(27,1)}}{f^2}\left[1-\frac{34}{3}L(m_M)\right], \tag{A7}$$

$$\langle\pi^+|\Theta^{(8,8)}_{3/2}|K^+\rangle = \frac{4\alpha^{(8,8)}}{f^2}[1-8L(m_M)], \tag{A8}$$

$$\langle\pi^+|\Theta^{(8,8)}_{1/2,A}|K^+\rangle = \frac{8\alpha^{(8,8)}}{f^2}[1-5L(m_M)], \tag{A9}$$

$$\langle\pi^+|\Theta^{(8,8)}_{1/2,S}|K^+\rangle = \frac{4\alpha^{(8,8)}}{f^2}[1-8L(m_M)], \tag{A10}$$

$$\langle\pi^+|\Theta^{(\bar{3},3)}|K^+\rangle = \frac{2\alpha^{(\bar{3},3)}}{f^2}[1+2L(m_M)]. \tag{A11}$$

This result is useful in lattice calculations where the pion mass can be adjusted through quark mass parameters. The $K^0 \to \pi^0$ matrix elements can be obtained from the above by using

$$\begin{aligned}\langle\pi^0|\mathcal{O}_{\Delta I=1/2}|K^0\rangle &= -\sqrt{\frac{1}{2}}\langle\pi^+|\mathcal{O}_{\Delta I=1/2}|K^+\rangle,\\ \langle\pi^0|\mathcal{O}_{\Delta I=3/2}|K^0\rangle &= \sqrt{2}\langle\pi^+|\mathcal{O}_{\Delta I=3/2}|K^+\rangle.\end{aligned} \tag{A12}$$

Finally, for $K\to\pi\pi$, we take the limit $m_\pi \to 0$ and keep the kaon mass dependency only:

$$\langle\pi^0\pi^0|\Theta^{(8,1)}|K^0\rangle = \frac{4i\alpha_1^{(8,1)}m_K^2}{f^3}\left[1-\frac{5}{4}L(m_K)\right], \tag{A13}$$

$$\langle\pi^0\pi^0|\Theta^{(27,1)}_{3/2}|K^0\rangle = \frac{8i\alpha^{(27,1)}m_K^2}{f^3}\left[1-\frac{3}{2}L(m_K)\right], \tag{A14}$$

$$\langle\pi^0\pi^0|\Theta^{(27,1)}_{1/2}|K^0\rangle = -\frac{4i\alpha^{(27,1)}m_K^2}{f^3}[1-15L(m_K)], \tag{A15}$$

$$\langle\pi^0\pi^0|\Theta^{(8,8)}_{3/2}|K^0\rangle = \frac{8i\alpha^{(8,8)}}{f^3}[1+L(m_K)], \tag{A16}$$

$$\langle\pi^0\pi^0|\Theta^{(8,8)}_{1/2,A}|K^0\rangle = -\frac{8i\alpha^{(8,8)}}{f^3}\left[1-\frac{7}{2}L(m_K)\right], \tag{A17}$$

$$\langle\pi^0\pi^0|\Theta^{(8,8)}_{1/2,S}|K^0\rangle = \frac{8i\alpha^{(8,8)}}{f^3}\left[1-\frac{19}{2}L(m_K)\right]. \tag{A18}$$

Here the physical mass of the kaon is used. Note that the weak mass operator $\Theta_2^{(8,1)}$ will not contribute to the $K \to \pi\pi$ matrix element as pointed out in [20,31,32,34].

The matrix elements for the final state $|\pi^+\pi^-\rangle$ are related to the above ones simply by

$$A_{+-} = \frac{1}{\sqrt{3}}(A_2+\sqrt{2}A_0), \qquad A_{00} = \sqrt{\frac{2}{3}}(-\sqrt{2}A_2+A_0). \tag{A19}$$

Compared with the angular momentum relation, the A_0 amplitude has a factor of $-\sqrt{2}$. The minus sign arises from the definition of $\pi^+ = (\pi^1 + i\pi^2)/\sqrt{2}$ which has a different sign from the usual spherical tensor definition. The $\sqrt{2}$ accounts for the identical particle nature of two π^0's, which is usually accounted for by a factor of $1/2$ in the final state phase space. From the above relation, we derive

$$\begin{aligned}\langle\pi^+\pi^-|\mathcal{O}_{\Delta I=1/2}|K^0\rangle &= \langle\pi^0\pi^0|\mathcal{O}_{\Delta I=1/2}|K^0\rangle,\\ \langle\pi^+\pi^-|\mathcal{O}_{\Delta I=3/2}|K^0\rangle &= -\tfrac{1}{2}\langle\pi^0\pi^0|\mathcal{O}_{\Delta I=3/2}|K^0\rangle.\end{aligned} \tag{A20}$$

By using the relation (A12) and (A20), it is easy to check that the result for (8, 8) operators is consistent with that in Ref. [23].

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