$\eta(1405)$ in a chiral approach based on mixing of the pseudoscalar glueball with the first radial excitations of η and η'

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The supernumerous $\eta(1405)$ is considered a strong candidate for the pseudoscalar glueball. In a phenomenological chiral approach, we consider a mixing scenario of bare pseudoscalar $n\bar{n}$ and $s\bar{s}$ quarkonia with the glueball. We study the decay properties and point out the peculiarities of this scenario in order to support the possible identification of the $J^{PC} = 0^{-+}$ glueball.

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I. INTRODUCTION

The problem of glueballs is paradigmatic for hadronic physics and has been under intense theoretical and experimental analysis for many years (see e.g. Refs. [1–35]). It is still an important ongoing topic, since the existence and the properties of glueballs are related to very fundamental properties of QCD, such as chiral symmetry breaking and the non-Abelian nature of the gauge group SU(3). The lowest-lying glueball has been predicted by theoretical calculations to have scalar quantum numbers [2,3].

The case for higher glueball excitations, such as the pseudoscalar one, is more difficult: So far, no unquestionable candidate has been observed. The $\eta(2225)$, with appropriate quantum numbers, is excluded by the measured decay pattern and weak coupling to gluons [4,5]. Regarding the $X(1835)$, there is a current debate about its nature, and it has been as well discussed as a glueball candidate [6]. However, the conclusions are not definitive, and further investigation is needed here. The possible nonvanishing gluonium content of the ground state η and η' mesons is discussed in [7–10]. In the mass region of the first radial excitation of the η and η' mesons, a supernumerous candidate, the $\eta(1405)$ has been observed. An excellent review on the experimental status of the $\eta(1405)$ is given in Ref. [11]. This state lies considerably lower than the lattice QCD predictions, which suggest a glueball around 2.5 GeV [12,13]. On the other hand, there are compelling arguments for the pseudoscalar glueball being approximately degenerate in mass with the scalar glueball [14]. Even the scenario that a pseudoscalar glueball is lower in mass than the scalar one is recently discussed in Ref. [15]. Therefore, more phenomenological estimates are needed to resolve this issue, and mixing has to be included as an important ingredient in a model since the three observed isoscalar states, $\eta(1295)$, $\eta(1405)$, and $\eta(1475)$ are close in mass. Besides the "standard scenario," where the first radial excitations of the η states are supposed to reside in the 1300–1500 MeV mass region, other structure interpretations are also discussed. For example, as originally suggested in [16] and recently discussed in [17], an interpretation of the heavy η states as four-quark states including mixing with the conventional quarkonia states is also feasible.

One of the key features to disentangle the properties of pseudoscalar mesons possibly mixed with a glueball is a good understanding of their strong decay patterns. The most convenient language for the treatment of light hadrons at small energies was elaborated in the context of chiral perturbation theory [36–38], the effective low-energy theory of the strong interaction. An extension of this approach above the chiral scale of about 1 GeV has been shown in the past to also reproduce the main dynamical properties of meson resonances. Hence it constitutes a useful phenomenological tool for the estimate of decay widths and ratios. This work is a continuation of a series of papers [27,28,31,39], where we analyzed the decays of scalar (including mixing with the scalar glueball), tensor (including mixing with the tensor glueball), pseudoscalar, and vector mesons using a phenomenological chiral approach.

In the present paper, we consider the possible mixing of the ground state pseudoscalar glueball with the first radial excitations of the η and η' meson. The main contributions to the physical states are assumed to be $\eta(1295) \approx n\bar{n}$, $\eta(1405) \approx G$, $\eta(1475) \approx s\bar{s}$. We give constraints on the nature of the glueball and on the resulting decay patterns. Furthermore, the available, sometimes contradictory, experimental data are confronted with the predictions of the model. A glueball interpretation of the $\eta(1405)$ is consistent with the present, sparse experimental data, but the assignment is still far from unique. Note that an analysis [40] of data on $J/\psi(\psi')$ decays into vector and pseudoscalar mesons also concludes that the $\eta(1405)$ has a dominant pseudoscalar glueball component. Forthcoming data from the planned experiments at BES-III, COMPASS (see, for example, the overview of [41]), and at the upgrade facility FAIR at GSI [42] might allow a full quantitative test of the mixing scenario.

The paper is structured as follows. In Sec. II we discuss our formalism. The results of the calculations are given in

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Sec. III. Conclusions and outlook on future experimental and theoretical directions are presented in Sec. IV.

II. THEORETICAL FOUNDATIONS

A. The Lagrangian

We employ a chiral Lagrangian to describe the coupling of the pseudoscalar excitations to the decay channels: pseudoscalar and vector meson (PV), three pseudoscalar mesons, pseudoscalar and scalar meson. The Lagrangian including the chiral fields (U), the scalar ($\mathcal{S}$), the vector ($\mathcal{V}$), and the excited pseudoscalar ($\mathcal{P}^*$) mesons, defined in the Appendix, reads as

$$\begin{aligned}\mathcal{L} = {} & \frac{F^2}{4}\langle D_\mu U D^\mu U^\dagger + \chi_+\rangle + \mathcal{L}_{\rm mix}^{\mathcal{P}} + \frac{1}{2}\langle \nabla_\mu \mathcal{P}^* \nabla^\mu \mathcal{P}^* - M_{\mathcal{P}^*}^2 \mathcal{P}^{*2}\rangle + \mathcal{L}_{\rm mix}^{\mathcal{P}^*} + \frac{1}{2}\left\langle \nabla_\mu \mathcal{V}^{\mu\nu}\nabla^\rho \mathcal{V}_{\rho\nu} - \frac{1}{2}M_{\mathcal{V}}^2 \mathcal{V}_{\mu\nu}\mathcal{V}^{\mu\nu}\right\rangle \\ & + \mathcal{L}_{\rm mix}^{\mathcal{V}} + \frac{1}{2}\langle \nabla_\mu \mathcal{S}\nabla^\mu \mathcal{S} - M_{\mathcal{S}}^2\mathcal{S}^2\rangle + c_{P^*PV}\langle \mathcal{V}_{\mu\nu}[u^\mu, \nabla^\nu \mathcal{P}^*]\rangle + i c_{P^*PPP}\langle \mathcal{P}^*\chi_-\rangle + c_{P^*PS,1}\langle \mathcal{S}\{\nabla_\mu \mathcal{P}^*, u^\mu\}\rangle \\ & + c_{P^*PS,2}\langle \mathcal{S}\{\mathcal{P}^*, \chi_-\}\rangle. \end{aligned} \tag{1}$$

Here the symbols $\langle\cdots\rangle$, $[\cdots]$, and $\{\cdots\}$ occurring in Eq. (1) denote the trace over flavor matrices, commutator, and anticommutator, respectively. The constants c_{P^*PV}, c_{P^*PPP}, $c_{P^*PS,1}$, $c_{P^*PS,2}$ define the couplings of the excited pseudoscalar fields to the decay channels PV, three pseudoscalar mesons, and pseudoscalar and scalar meson, respectively. The terms $\mathcal{L}_{\rm mix}^{\mathcal{P}}$, $\mathcal{L}_{\rm mix}^{\mathcal{P}^*}$, and $\mathcal{L}_{\rm mix}^{\mathcal{V}}$ describe the mixing between the octet and singlet of the pseudoscalar, excited pseudoscalar, and vector mesons, respectively. Because of the axial anomaly, we also encode an additional contribution to the mass of the η^0 [27].

We use the standard notation for the basic blocks of the chiral perturbation theory Lagrangian [36]: $U = u^2 = \exp(i\mathcal{P}\sqrt{2}/F)$ is the chiral field collecting pseudoscalar fields in the exponential parametrization, D_μ and ∇_μ denote the chiral and gauge-invariant derivatives, $u_\mu = iu^\dagger D_\mu U u^\dagger$, $\chi_\pm = u^\dagger \chi u^\dagger \pm u\chi^\dagger u$, $\chi = 2B(s + ip)$, $s = \mathcal{M} + \ldots$ and $F_{\mu\nu}^+ = u^\dagger F_{\mu\nu} Q u + u F_{\mu\nu} Q u^\dagger$, where $F_{\mu\nu}$ is the stress tensor of the electromagnetic field; $Q = {\rm diag}\{2/3, -1/3, -1/3\}$ and $\mathcal{M} = {\rm diag}\{\hat{m}, \hat{m}, m_s\}$ are the charge and the mass matrix of current quarks, respectively, (we restrict to the isospin symmetry limit with $m_u = m_d = \hat{m}$); B is the quark vacuum condensate parameter. Then the masses of the pseudoscalar mesons in the leading order of the chiral expansion are given by $M_\pi^2 = 2\hat{m}B$, $M_K^2 = (\hat{m} + m_s)B$, $M_{\eta^8}^2 = (2/3)(\hat{m} + 2m_s)B$. The glueball configuration is not yet included in this Lagrangian; it will be introduced further on as a flavor singlet which mixes with the corresponding excited η^* states.

We intend to employ the Lagrangian to the tree-level calculation of the strong decays of radially excited pseudoscalar mesons. At the energy scale of interest, $E \sim M_S \sim 1.5$ GeV, a calculation of loops and an application of the power counting rules are not rigorously justified. The aim of the present approach is therefore a phenomenological study of pseudoscalar meson physics, for which a tree-level calculation represents a useful analysis.

B. The mixing scenario

We treat $\eta(1295)$ and $\eta(1475)$ as two members of the nonet of radial pseudoscalar excitations. The glueball G is added as a flavor singlet:

$$\mathcal{L}_G = \tfrac{1}{2}\partial_\mu G \partial^\mu G - \tfrac{1}{4}M_G^2 G^2 + \mathcal{L}_{G,{\rm decay}}, \tag{2}$$

where

$$\begin{aligned}\mathcal{L}_{G,{\rm decay}} = {} & c_{GPV}\langle \mathcal{V}_{\mu\nu}[u^\mu, \partial^\nu G]\rangle + i c_{GPPP}\langle G\chi_-\rangle \\ & + c_{GPS,1}\langle \mathcal{S}\{\partial_\mu G, u^\mu\}\rangle + c_{GPS,2}\langle \mathcal{S}\{G, \chi_-\}\rangle\end{aligned}$$

contains the coupling of the pseudoscalar glueball singlet to its decay channels in analogy to the excited meson octet.

To incorporate mixing, we replace the diagonal term with the bare masses of η_{nn}, η_{ss} and of the unmixed glueball in the Lagrangian with the following nondiagonal mixing term given in general form:

$$\tilde{M}^2 = \begin{pmatrix} M_{n\bar{n}}^2 & \sqrt{2}fr & \epsilon \\ \sqrt{2}fr & M_{gg}^2 & f \\ \epsilon & f & M_{s\bar{s}}^2 \end{pmatrix}, \tag{3}$$

where ϵ denotes the coupling between the $s\bar{s}$ and $n\bar{n} = (u\bar{u} + d\bar{d})/\sqrt{2}$ flavor configurations. This parameter is set to zero in the following discussion. One reason for doing so is traced to an analogous treatment in the scalar sector [20,43], where it was argued that the mixing between excited quarkonia states is suppressed relative to the quarkonia-glueball mixing mechanism. Also, the mass degeneracy between the $\eta(1295)$ and the $\pi(1300)$ suggests a nearly ideal mixing situation for the quarkonia configurations, implying in turn a strongly suppressed mixing between the quarkonia states. The parameter f denotes the mixing of the glueball with the quarkonia states. A possible deviation from the case of flavor symmetric mixing is expressed by the parameter r with $r \neq 1$. In our case, however, we will work in the limit $r = 1$; the same limit is also approximately fulfilled in the scalar case [27].

After diagonalization of $\tilde{M}$, the physical mass matrix reads as

$$M^2 = U\tilde{M}^2U^T = \begin{pmatrix} M^2_{\eta_1} & 0 & 0 \\ 0 & M^2_{\eta_2} & 0 \\ 0 & 0 & M^2_{\eta_3} \end{pmatrix}, \quad (4)$$

where U is the mixing matrix relating the physical states (η_1, η_2, η_3) to the bare states $(\eta_{n\bar{n}}, \eta_{gg}, \eta_{s\bar{s}})$ as

$$U\begin{pmatrix} \eta_{n\bar{n}} \\ \eta_{gg} \\ \eta_{s\bar{s}} \end{pmatrix} = \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{pmatrix}. \quad (5)$$

The mixing strength f is one parameter of interest which will be varied in the following discussion. Furthermore, we will vary the relative decay strength of the bare glueball with respect to the decay strength of the quarkonia states. Thus we set up the relation

$$c_G = gc_{\mathcal{P}^*} \quad (6)$$

applicable to every coupling constant. The limit $g = 0$ indicates no direct decay of the glueball component, interference effects will be observed when changing the sign, e.g. from $g = -1$ to $g = +1$.

III. RESULTS

In the case of decays to the PV channel, we used the results obtained within the $^3P_0$-model by Barnes *et al.* [44,45] to fit the coupling strength to $c_{P^*PV} = 4.95\ \text{GeV}^{-1}$ [39]. This procedure allows the computation of decay widths in physical units. For the three-pseudoscalar channel and the scalar-pseudoscalar channel, no absolute value can be given and we restrict ourselves to the ratios of rates.

In order to obtain the given physical masses, one is restricted to mixing strengths smaller than about $f \approx 0.15\ \text{GeV}^2$. A moderate value for the mixing strength however does not induce considerable mass shifts of the bare values, as can be seen in Fig. 1. Therefore, a three-state mixing scenario cannot explain the discrepancy in the mass values, which persists between the predictions of lattice QCD and the examined candidate.

A glueball-free scenario gives consistent results in decay when $\eta(1295)$ is interpreted as a dominant $n\bar{n}$ quarkonium configuration, and $\eta(1475)$ is seen as the $s\bar{s}$ state [39]. This observation also motivates a mixing scenario with a small mixing strength.

A. Decay to $K\bar{K}^*$

In the chiral approach, the pure glueball configuration does not decay to $K\bar{K}^*$, but only via mixing. The resulting decay width therefore depends very strongly on the mixing strength. The decay widths of the respective η states as a function of the mixing strength are shown in Fig. 2. The decay width $\Gamma(\eta(1475) \to K\bar{K}^*)$ lies in the range of values from 34 to 68 MeV. Concerning data on $\eta(1405)$ the

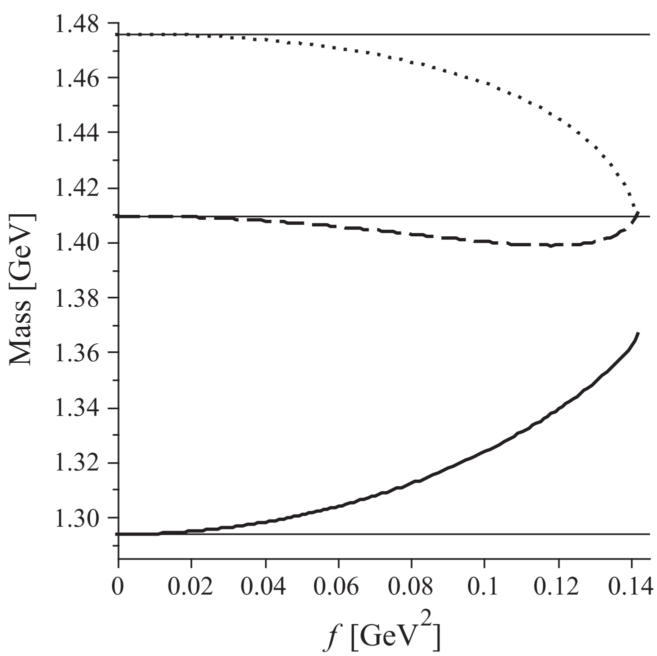

FIG. 1. Bare masses $M_{n\bar{n}}$ (solid line), $M_{s\bar{s}}$ (dotted line), M_{gg} (dashed line). The bare masses shown here generate for different values of f the same physical masses M_{η_1}, M_{η_2}, M_{η_3}. The bare mass of a pseudoscalar glueball is constrained to ≈ 1.39–1.41 GeV.

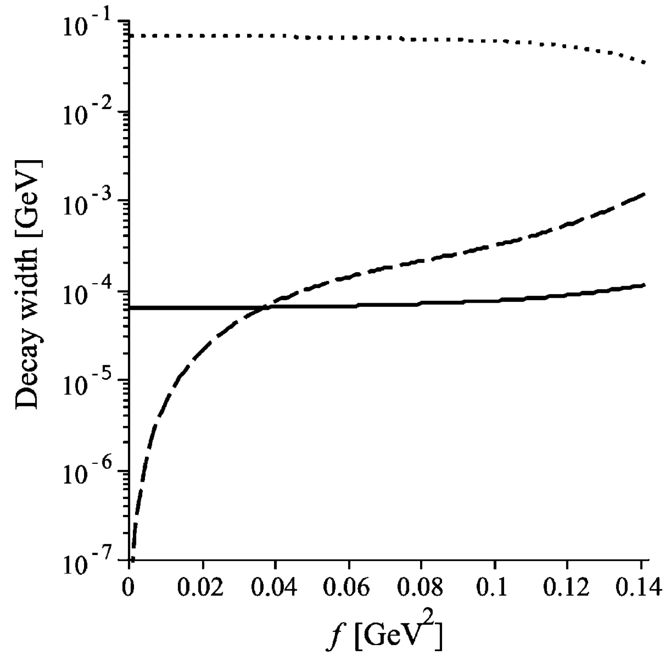

FIG. 2. Decay widths to $K\bar{K}^*$ of $\eta(1295)$ (solid line), $\eta(1475)$ (dotted line), $\eta(1405)$ (dashed line). The decay widths of $\eta(1475)$ and $\eta(1295)$ do not change considerably in the range of f, however the decay width of $\eta(1405)$ changes over several orders of magnitude.

experimental situation is contradictory. Although many experiments do not observe $\eta(1405) \to K\bar{K}^*$ [11], the analysis of E852 [46] indicates a ratio of

$$\mathrm{Br}(\eta(1405) \to K^*\bar{K})/\mathrm{Br}(\eta(1475) \to K^*\bar{K}) = 0.16 \pm 0.04 \tag{7}$$

with statistical error only, but where the systematic error is expected to be large. The last ratio cannot be explained in the present framework, even when additional quarkonia mixing is included. An experimental clarification of the possible $K\bar{K}^*$ decay mode of the $\eta(1405)$ is obviously very useful.

B. Decay to three pseudoscalars

We consider the two kinematically allowed three-body decays to $\pi\pi\eta$ and to $K\bar{K}\pi$. Note that these channels refer to the direct three-body decay modes and not to the final

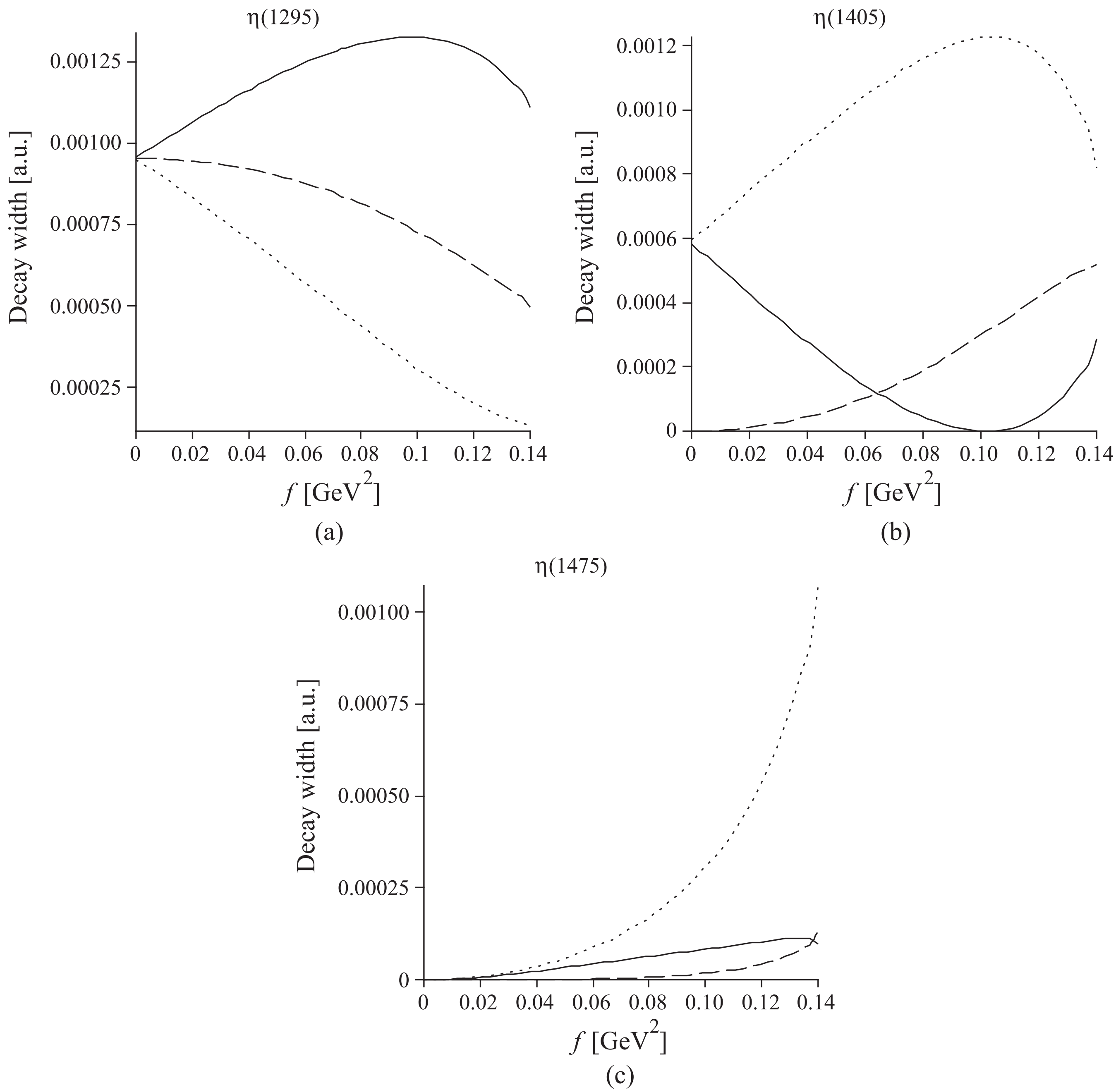

FIG. 3. Decay widths to $\pi\pi\eta$ with arbitrary, absolute normalization. The decay width is shown for three different relative glueball decay strengths: $g = 0$ (no direct glueball decay, dashed line), $g = 1$ (dotted line), $g = -1$ (solid line).

state fed, for example, by intermediate $f_0(980)\eta$ or $K^*\bar{K}$ decay channels. The glueball as a flavor-singlet state can decay into $\eta\pi\pi$ as well as into $\pi K\bar{K}$, therefore an additional dependence on g, indicating a contribution of direct glueball decay, arises. In Fig. 3 we show the decay widths of all three resonances to $\pi\pi\eta$ for different discrete values of the direct glueball decay strength, $g = -1$, 0, 1. The decay widths are given in arbitrary units, but the relative rates are a prediction of the model. It is easy to see that the decay width strongly depends both on the direct glueball decay and on the mixing strength.

The decay of $\eta(1475)$ is strongly suppressed for small values of f, even when the direct glueball decay is very large. In the case of a vanishing direct glueball decay, it can only decay via its $n\bar{n}$ component, which is small. Interference effects are important and lead to very different behavior when considering $g = -1$ instead of $g = 1$. In

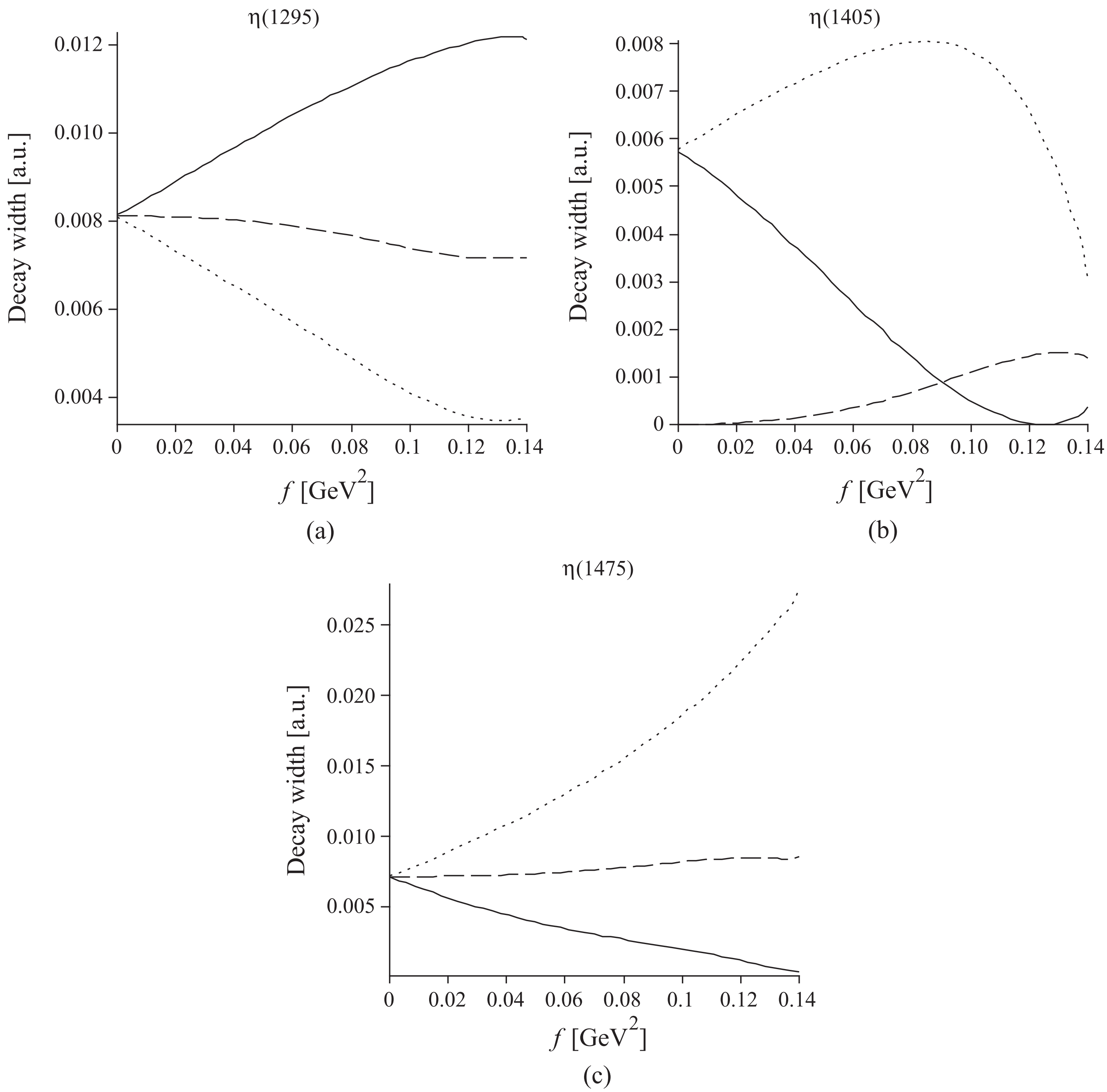

FIG. 4. Decay widths to $\pi K\bar{K}$. The normalization and legend is identical to the one in Fig. 3. A strong admixture of $n\bar{n}$ to the $s\bar{s}$ state leads to a smaller decay width of the $\eta(1475)$ for $g = 0$.

the case of the $\eta(1295)$, for a very large mixing strength, destructive interference leads to a vanishing decay width.

The same analysis can be repeated for the decay into $K\bar{K}\pi$. The results are shown in Fig. 4. The decay pattern is comparable to the previous case. Since the component $\eta_{s\bar{s}}$ can now also feed the decay channel $K\bar{K}\pi$, the decay width of $\eta(1475)$ does not vanish in case of $f = 0$ and $g = 0$.

Since the decay strength—although unknown—is the same for both three-body decays analyzed here, we can consider the decay ratio

$$\frac{\Gamma(\eta_i \to \pi K\bar{K})}{\Gamma(\eta_i \to \pi\pi\eta)}. \tag{8}$$

Its dependence on the various choices of parameters is indicated in Fig. 5. Similarly, the predictions for the ratios

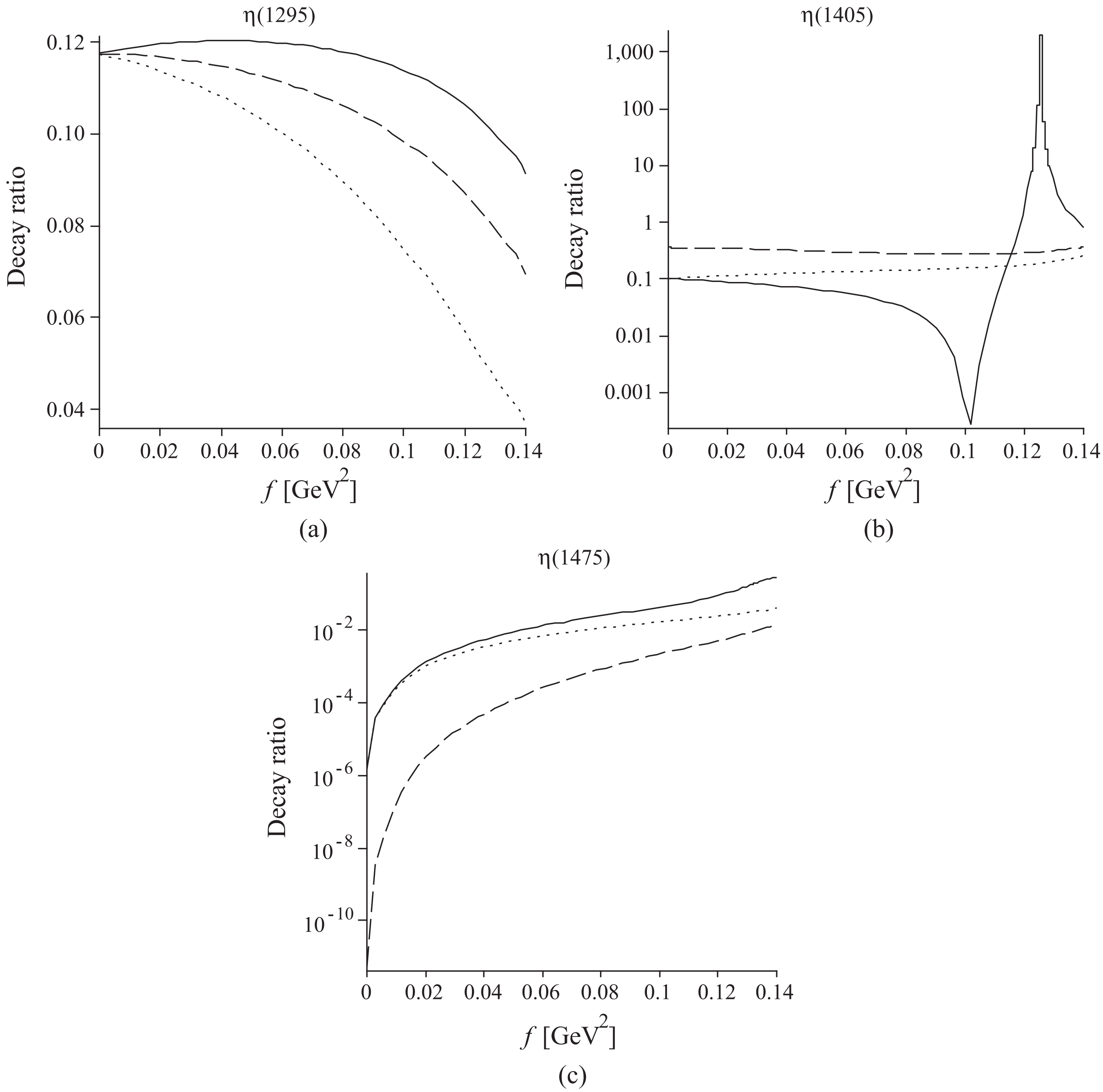

FIG. 5. Decay ratios $\frac{\Gamma(\eta_i \to \pi K\bar{K})}{\Gamma(\eta_i \to \pi\pi\eta)}$. We consider the cases $g = 0$, $g = 1$, $g = -1$ (legend analogous to previous two figures). Because of the wide range of values, the plots for $\eta(1405)$ and $\eta(1475)$ are shown in logarithmic scale. Compared to the cases of $\eta(1475)$ and $\eta(1405)$, the f dependence of the decay ratio for $\eta(1295)$ is less pronounced.

$\Gamma(\eta_i \to \pi K\bar{K})/\Gamma(\eta_j \to \pi K\bar{K})$ and $\Gamma(\eta_i \to \pi\pi\eta)/\Gamma(\eta_j \to \pi\pi\eta)$ are independent of the coupling strength. We note that the decay ratios of Fig. 5 for $\eta(1295)$ and $\eta(1405)$ remain rather unaffected when taking a reasonable glueball admixture, independent of the direct decay strength. For $\eta(1475)$, the situation is different: Since $\eta\pi\pi$ is totally suppressed for a pure $s\bar{s}$ configuration, even small $n\bar{n}$ admixtures lead to a nonvanishing decay width and a strong change in the ratio.

C. Decay to scalar and pseudoscalar mesons

Several decay channels to scalar and pseudoscalar mesons open in the final state: The most important ones are $\kappa(K_0^*(800))K$, $\sigma(f_0(600))\eta$, $f_0(980)\eta$, $a_0(980)\pi$. We assume that flavor symmetry remains unbroken in this scenario. It can however be shown that a flavor-symmetry–breaking term does not induce dramatic changes in the decay pattern. Since we are rather interested in the depen-

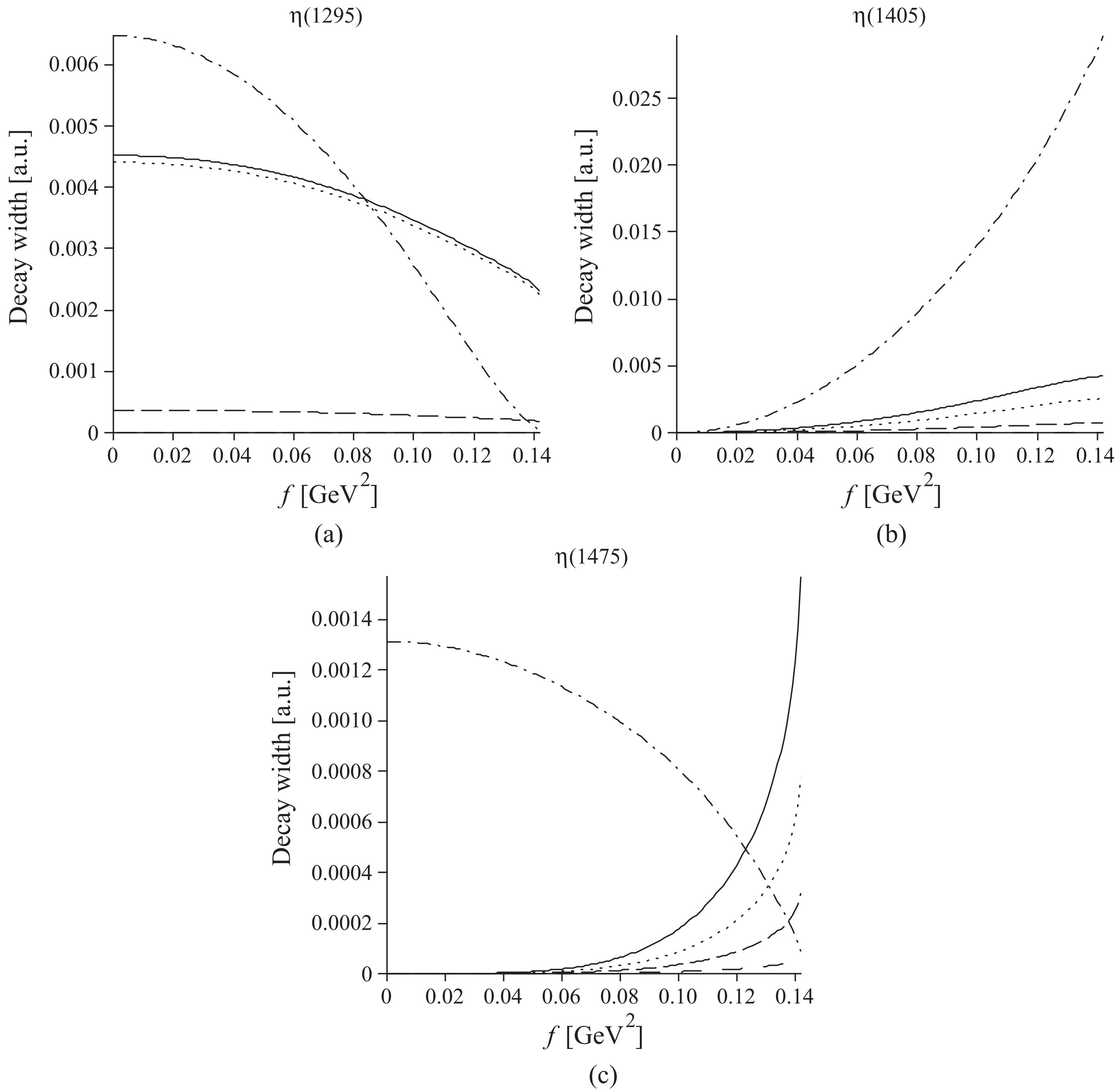

FIG. 6. Partial decay widths of η_i to scalar and pseudoscalar mesons in arbitrary normalization. Flavor symmetry breaking and direct glueball decay are suppressed. The solid line denotes decays to $a_0\pi$, the dashed line $\sigma\eta'$, the dots denote $\sigma\eta$, the dash-dotted line κK, and the space-dashed line [only visible for $\eta(1475)$] denotes the decay to $f_0\eta$. Please note that the dominant decay width $\Gamma(\eta(1475) \to \kappa K)$ has been rescaled by a factor of 0.05 to facilitate comparison for convenience.

dence on the mixing strength and the direct glueball decay strength, we omit an analysis of this effect here.

1. Omission of direct glueball decay

The strength of the direct glueball decay is not known *a priori*, and it is interesting to study the effect of vanishing strength on the decay pattern. The resonance $\eta(1405)$ can now only decay via mixing; its decay is strongly suppressed for small mixing strengths. The partial decay widths, in arbitrary normalization are shown in Fig. 6. The total decay width to scalar and pseudoscalar mesons (normalized to the sum of all decay widths) is shown in Fig. 7.

Three-body decays fed by intermediate resonances dominate in all three channels and contribute the main part to the total decay width of the resonances. We note that the full width of the $\eta(1405)$ of 51.1 MeV implies that either the glueball decays directly to scalar and pseudoscalar mesons, or the mixing with the quarkonia is strong, larger than $\approx 0.08\ \mathrm{GeV}^2$. On the other hand, if we assume that the mass of the bare $\eta_{n\bar{n}}$ is significantly smaller than the mass of the bare $\eta_{s\bar{s}}$, the mixing strength has to be small (see Fig. 1). Furthermore, no mixing mechanism that would be strong enough is known for this sector. While in the scalar sector, the absence of a direct glueball decay is a realistic option based on theoretical arguments and might be phenomenologically successful in some cases; the inclusion of the direct decay is therefore needed for the scenario under consideration.

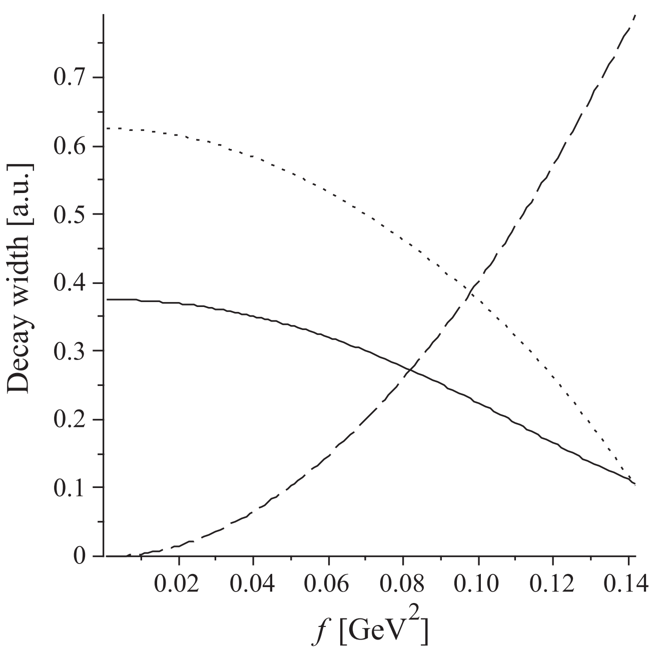

FIG. 7. Sum of partial decay widths of η_i to scalar and pseudoscalar mesons in arbitrary normalization. Flavor symmetry breaking and direct glueball decay are suppressed. The solid line denotes $\eta(1295)$, the dashed line $\eta(1405)$, and the dotted line $\eta(1475)$.

2. Inclusion of direct glueball decay

The inclusion of direct glueball decay changes the picture in the case of the $\eta(1405)$ at low values for the mixing strength f dramatically. The relative sign between the glueball and the quarkonium decay constants plays an important role. Negative, as well as positive, interference effects appear in the decay pattern of all three states. The decay rates for the most important scalar and pseudoscalar decays are shown in Fig. 8 for various choices of the direct glueball decay strength. Interference effects are especially dramatic in the case of the $\eta(1405)$ and $\eta(1295)$: While mixing leads to an increase of the decay widths of $\eta(1295)$ to $\sigma\eta$ and $a_0\pi$ for $g = 1$, they are lowered for $g = -1$. The decay mode $\eta(1475) \to \kappa K$ is totally dominant for the case of suppressed direct glueball decay (please compare to Fig. 4), for intermediate values of the mixing angles other decay modes begin to be important.

Current data on the $a_0\pi$ and $\sigma\eta$ [or $(\pi\pi)_{S\text{-wave}}\eta$] decay channels are available for the $\eta(1295)$ and $\eta(1405)$. The Crystal Barrel Collaboration [47] reports a value for the ratio

$$\mathrm{Br}(\eta(1405) \to \eta\sigma)/\mathrm{Br}(\eta(1405) \to a_0\pi, a_0 \to \eta\pi) = 0.78 \pm 0.12 \pm 0.10, \tag{9}$$

consistent with the inverse ratio

$$\mathrm{Br}\,(\eta(1405) \to a_0\pi)/\mathrm{Br}(\eta(1405) \to \eta\sigma) = 0.91 \pm 0.12 \tag{10}$$

of Ref. [48]. These values should be compared to the BES result [49] of

$$\mathrm{Br}(\eta(1405) \to a_0\pi)/\mathrm{Br}(\eta(1405) \to \eta\sigma) = 0.70 \pm 0.12 \pm 0.20. \tag{11}$$

The E852 Collaboration published a value of

$$\mathrm{Br}\,(\eta(1405) \to a_0\pi)/\mathrm{Br}(\eta(1405) \to \eta\sigma) = 0.15 \pm 0.04 \tag{12}$$

in [50], statistical error indicated only, in conflict with above mentioned values. Whereas first analyses indicate that the $a_0\pi$ and $\sigma\eta$ decay modes are roughly of equal strength, the E852 result implies a dominant $\eta\sigma$ mode. Last scenario cannot be reproduced in the present model, even when the value of direct glueball decay strength g is increased dramatically. From our predictions we always find that $a_0\pi$ dominates with varying strength over $\sigma\eta$. This effect is traced to the fact that we place σ and κ in the chiral formalism in the same flavor octet.

For the $\eta(1295)$ the GAMS Collaboration [51] reports the ratio

$$\mathrm{Br}(\eta(1295) \to \eta\sigma)/\mathrm{Br}(\eta(1295) \to a_0\pi, a_0 \to \eta\pi) = 0.54 \pm 0.22. \tag{13}$$

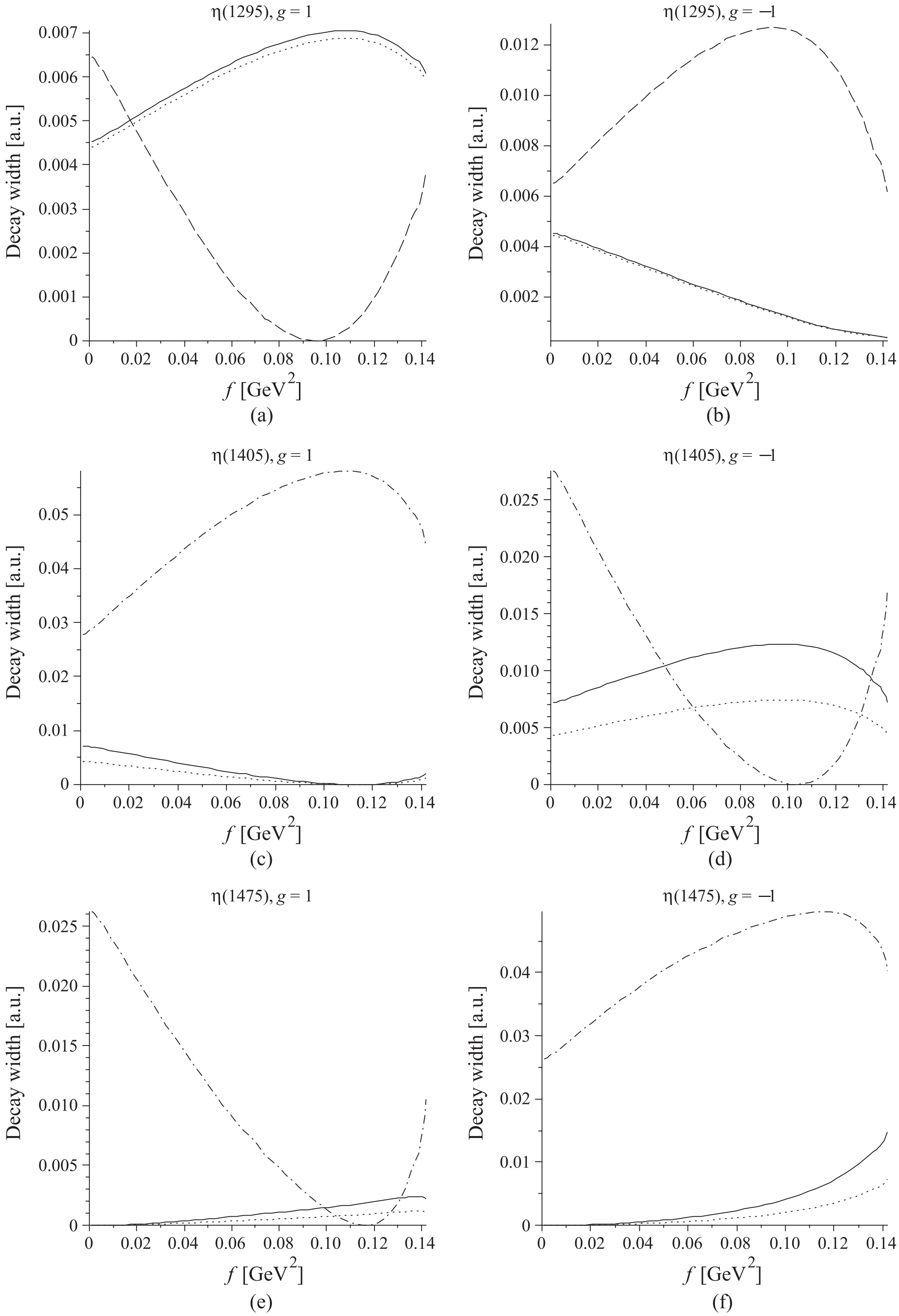

FIG. 8. Decay widths of η_i to scalar and pseudoscalar mesons in arbitrary normalization including direct glueball decay. The solid line denotes decay to $a_0\pi$, the dotted line denotes $\sigma\eta$, and the dash-dotted line denotes κK. Note the difference in the behavior of the curves for positive $g = 1$ and negative $g = -1$ glueball decay strength.

The E852 Collaboration extracts from their analysis the result [50]

$$\mathrm{Br}\,(\eta(1295) \to a_0\pi)/\mathrm{Br}(\eta(1295) \to \eta\sigma) = 0.48 \pm 0.22. \tag{14}$$

Similar to the behavior of the $\eta(1405)$ for the $\eta(1295)$, we deduce that the $a_0\pi$ mode slightly dominates over $\sigma\eta$, nearly independent of values for the mixing and glueball decay strength.

D. Discussion

We can safely assume that the bare η_{nn} mass is considerably lighter than the mass of η_{ss}. To be able to explain the mass values of the η states, we furthermore have to restrict to a small glueball-quarkonia mixing strength. For these small values of the mixing strength and, in addition, a vanishing direct glueball decay, we expect the width of the $\eta(1405)$ to be much smaller than the values for $\eta(1295)$ and $\eta(1475)$. This is however not the case, since $\Gamma(\eta(1405)) \approx 51$ MeV which is comparable to the total width of $\eta(1295)$. While in the scalar sector the direct glueball decay process may be suppressed, it has to play a dominant role in the pseudoscalar channel in order to explain the width of the $\eta(1405)$.

A Dalitz plot analysis of the $\pi K\bar{K}$ and $\pi\pi\eta$ mode in $\eta(1405)$ would be interesting for various reasons: For a very small mixing strength the decays to $K\bar{K}^*$ are suppressed, since these channels are not fed by the direct glueball decay. Therefore, the decay to $KK\pi$ cannot proceed by an intermediate vector resonance; an enhancement would be a signal for considerable $q\bar{q}$ admixture if it cannot be attributed to κK. Similarly, we may extract information on the glueball or $n\bar{n}$ content of the $\eta(1475)$ by the search for the decay modes $a_0\pi$, $\sigma\eta$ in three-body decays. The measurement of $K\bar{K}^*$ in $\eta(1405)$ would give a good estimate of the possible $q\bar{q}$ components and the mixing strength.

The situation is more difficult when trying to estimate the glueball or $s\bar{s}$ admixture in the $\eta(1295)$, since the decay modes are all open and are only slightly modified when varying the mixing strength. The actual known decay pattern and its consequence for the interpretation of the pseudoscalar mesons is the following:

(i) The dominant decay modes for $\eta(1475)$ are $K\bar{K}^*$ and κK, which are revealed by the $K\bar{K}\pi$ final state in the experiment. A Dalitz plot analysis of the $KK\pi$ mode is important to extract the scalar (κ) and vector meson (K^*) resonance contributions. Nonstrange decay modes are strongly suppressed, and their observation would point to a glueball component or $n\bar{n}$ configuration. The actual observation of $a_0\pi$, although relatively weak, points to some glueball admixture. The strong dominance of $KK\pi$ is compatible with our model, in which κK dominates the scalar-pseudoscalar channel and $K\bar{K}^*$ the vector-pseudoscalar channel.

(ii) We expect the decays to scalar and pseusoscalar mesons for the $\eta(1295)$ to be dominant, the $K\bar{K}^*$ being suppressed kinematically. The decays to $K\bar{K}\pi$ and $\pi\pi\eta$ do not depend strongly on the mixing strength.

(iii) For reasonable mixing strengths, $K\bar{K}^*$ is suppressed for $\eta(1405)$, and we expect the $K\bar{K}\pi$ to mainly arise from scalar resonances. The decays to $\rho\gamma$ and $\phi\gamma$, which have been observed according to PDG [52], are compatible with a glueball interpretation as well as the decay to $a_0\pi$. On the other hand, $K\bar{K}^*$ and $\gamma\gamma$ are weak in comparison to the decays into scalar and pseudoscalars. Our model shows that their bare observation points to a nonvanishing $s\bar{s}$ or $n\bar{n}$ component.

IV. CONCLUSIONS AND OUTLOOK

Qualitatively, the scenario we have considered shows reasonable agreement with the decay processes observed so far. Further experimental input would be appreciated, especially for the $\eta(1405)$ and $\eta(1475)$. The glueball contribution to $\eta(1295)$ should be approximately as large as the contribution to $\eta(1475)$, but it is reflected more strongly in the decay pattern of the latter. Similarly, an $s\bar{s}$ admixture to $\eta(1405)$ would be as large as the $n\bar{n}$ admixture, but would lead to greater changes in the decay modes. The dominant $n\bar{n}$ structure of $\eta(1295)$ is rather well established, especially due to the mass degeneracy with the $\pi(1300)$. Quantitative determination of glueball and $s\bar{s}$ admixtures are difficult, since no new channels open up when mixing is included; only subtle changes in the decay pattern would be observable. On the other hand, the observation of nonstrange decay modes in $\eta(1475)$ or further strange decays, such as κK in $\eta(1405)$-decays, would point to considerable mixing in this region and help to quantify it.

On the experimental side, exciting results can be expected in the next years: planned experiments at BES-III, COMPASS, and at the upgrade facility FAIR at GSI might give essential contributions to map out the decay modes of the η states [41].

If the glueball interpretation was confirmed by experiment, the first question which naturally arises would be why the theoretical predictions for the $J^{\mathrm{PC}} = 0^{-+}$ glueball are dominantly in a very different mass region. We have seen that mixing cannot change the physical mass of the glueball dramatically and hence cannot offer an explanation for the strong deviation from the mainstream theoretical predictions.

The question whether the $\eta(1405)$ may be the lowest pseudoscalar glueball cannot be answered conclusively at this stage. We have shown however that the scenario under consideration is in qualitative agreement with the available experimental data. We have also pointed out how the problem of the pseudoscalar glueball may be solved in the future at a more quantitative level.

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APPENDIX: MATRICES $\mathcal{S}$, $\mathcal{V}_{\mu\nu}$, AND $\mathcal{P}^*$

$$\mathcal{S} = \begin{pmatrix} \frac{a_0}{\sqrt{2}} + \frac{\sigma}{\sqrt{2}} & a_0^+ & \kappa^+ \\ a_0^- & -\frac{a_0}{\sqrt{2}} + \frac{\sigma}{\sqrt{2}} & \kappa^0 \\ \kappa^- & \bar{\kappa}^0 & f_0 \end{pmatrix}, \tag{A1}$$

$$\mathcal{V}_{\mu\nu} = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho_0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}, \tag{A2}$$

$$\mathcal{P}^* = \begin{pmatrix} \frac{\pi^0(1300)}{\sqrt{2}} + \frac{\eta(1295)}{\sqrt{2}} & \pi^+(1300) & K^+(1460) \\ \pi^-(1300) & -\frac{\pi^0(1300)}{\sqrt{2}} + \frac{\eta(1295)}{\sqrt{2}} & K^0(1460) \\ K^-(1460) & \bar{K}^0(1460) & \eta(1475) \end{pmatrix}. \tag{A3}$$

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