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Dynamics behind the quark mass hierarchy

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We introduce a new class of models describing the quark mass hierarchy. In this class, the dynamics primarily responsible for electroweak symmetry breaking (EWSB) leads to the mass spectrum of quarks with no (or weak) isospin violation. Moreover, the values of these masses are of the order of the observed masses of the down-type quarks. Then, strong (although subcritical) horizontal diagonal interactions for the t quark plus horizontal flavor-changing neutral interactions between different families lead (with no fine-tuning) to a realistic quark mass spectrum. In this scenario, many composite Higgs bosons occur. A concrete model with the dynamical EWSB with the fourth family is described in detail.

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I. INTRODUCTION: SCENARIO

The masses of quarks are [1]:

$$m_t = 171.2 \pm 2.1 \text{ GeV}, \qquad m_b = 4.20^{+0.17}_{-0.07} \text{ GeV}, \quad (1)$$

$$m_c = 1.27^{+0.07}_{-0.11} \text{ GeV}, \qquad m_s = 104^{+26}_{-34} \text{ MeV}, \quad (2)$$

and

$$m_u = 1.5\text{–}3.3 \text{ MeV}, \qquad m_d = 3.5\text{–}6.0 \text{ MeV}. \quad (3)$$

The quark spectrum is characterized by the following striking features: (1) There is a large hierarchy between quark masses from different families,

$$m_u/m_t \sim 10^{-5}, \qquad m_u/m_c \sim 10^{-3}, \qquad m_c/m_t \sim 10^{-2}, \quad (4)$$

$$m_d/m_b \sim 10^{-3}, \qquad m_d/m_s \sim 10^{-2}, \qquad m_s/m_b \sim 10^{-1}. \quad (5)$$

(2) The isospin violation is also hierarchical: It is very strong in the third family, strong (although essentially weaker) in the second family, and mild in the first one:

$$\frac{m_t}{m_b} \simeq 40.8, \qquad \frac{m_c}{m_s} \simeq 11.5, \qquad \frac{m_u}{m_d} = 0.35 - 0.60. \quad (6)$$

The origin of these features is still mysterious: In the standard model (SM), it is required to introduce hierarchical Yukawa couplings by hand, e.g., $y_u/y_t = m_u/m_t \sim 10^{-5}$.

In this paper, we will introduce a new class of models describing the quark mass hierarchy. One of our basic assumptions is the separation of the dynamics triggering the strong isospin violation in the third and second families from that responsible for the generation of the W and Z masses, i.e., electroweak symmetry breaking (EWSB). The latter could be provided by one of the following known mechanisms: (a) An elementary Higgs field (or fields). (b) A modern version of the technicolor (TC) scenario (for recent reviews, see Ref. [2]). (c) At last, it could be a dynamical Higgs mechanism with a Higgs doublet (or doublets) composed of t' and b' quarks of the fourth family [3,4].

We assume that the dynamics primarily responsible for the EWSB leads to the mass spectrum of quarks with no (or weak) isospin violation. *Moreover, we assume that the values of these masses are of the order of the observed masses of the down-type quarks.* In the case of an elementary Higgs field (or fields), they are provided by the conventional Yukawa interactions. In the case of the dynamical Higgs mechanism, in order to generate these masses, one should use flavor-changing-neutral (FCN) interactions: the extended technicolor (ETC) [5] in the case of the TC scenario, and the horizontal interactions between the 4th family and the first three ones in the case of the scenario with the fourth family (see Fig. 1).

Of course, such interactions are restricted by the K^0-$\bar{K}^0$ mixing, for example, and thus for light quarks it is required to introduce heavy exchange vector particles, say, with the masses of order 1000 TeV. Such heavy particles can be a natural source for producing small Yukawa coupling con-

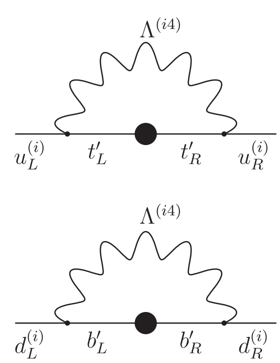

FIG. 1. FCN interactions of the up- and down-quark sectors. Here $u^{(1,2,3)} = u, c, t$ and $d^{(1,2,3)} = d, s, b$, respectively. $\Lambda^{(i4)}$ are masses of exchange vector particles.

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stants for light quarks. For heavier quarks, we introduce lighter vector particles.

The second (central) stage is introducing the horizontal interactions for the quarks in the first three families (this stage is essentially the same for the three EWSB mechanisms mentioned above.) First, following the idea in the model of Mendel *et al.* [6,7], we utilize strong (although *subcritical*) diagonal horizontal interactions for the top quark which lead to the observed ratio $\frac{m_t}{m_b} \simeq 40.8$. The second step is introducing the equal strengths horizontal FCN interactions between the t and c quarks and the b and s ones in order to get the observed ratio $m_c/m_s \simeq 11.5$ in the second family (see Fig. 2). As will be shown in Sec. II B, these interactions can naturally provide such a ratio indeed.

Because of a smallness of the mixing angles for quarks from the different families, neglecting the family mixing in the dynamics responsible for generating the quark masses in the second and third families is a reasonable approximation. Concerning the mild isospin violation in the first family, it should be studied together with the effects of the family mixing, reflected in the Cabibbo-Kobayashi-Maskawa (CKM) matrix. The latter will be considered in Sec. II C.

Thus, in the present scenario, beside the EWSB interactions, the dominant dynamics responsible for the form of the mass spectrum of quarks is connected with the diagonal horizontal interactions for the third family and the horizontal FCN interactions between the second and third ones. The signature of this scenario is the appearance of composite Higgs bosons (resonances) composed of the quarks and antiquarks of the 3rd family (see Sec. II D)

The main source of the isospin violation in this approach is the strong top quark interactions. On the other hand, because these interactions are subcritical, the top quark plays a minor role in electroweak symmetry breaking. This point distinguishes this scenario from the top quark condensate model [8–11].

Two comments are in order. (i) As will be shown below, the characteristic feature in this class of the models is the absence of fine-tuning: What could be called fine-tuning for the near-critical coupling of the t quark (1 part in 10^2) is

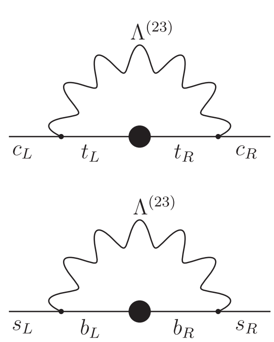

FIG. 2. FCN interactions for the second family.

just a reflection of a ‘‘unnaturally’’ large isospin violation in the third family, $m_b/m_t \simeq 2.5 \times 10^{-2}$. (ii) In this paper, we will concentrate on studying the mass spectrum of quarks. For a discussion concerning the extension of the present approach for the description of lepton masses, see Sec. IV below.

II. MODEL

In this section, the dynamics for generating the quark mass hierarchy will be described in detail. Henceforth we will concentrate on a model of the dynamical EWSB with the fourth family [3]. However, we will also comment on the modifications (if any) for both the scenario with elementary Higgs fields responsible for the EWSB and the TC scenario.

A. Electroweak symmetry breaking dynamics and isospin symmetric quark masses

The first stage is generating the masses with no (or weak) isospin violation and of the order of the observed masses of the down-type quarks. As was pointed in the Introduction, in the present approach, the EWSB dynamics is responsible for that. It is straightforward to produce such masses both in the case of the scenario with elementary Higgs fields (through Yukawa interactions) and in the TC one (through ETC interactions).

Let us now describe this stage in the scenario of the dynamical EWSB with the fourth family [3]. The masses of the 4th family quarks are constrained as [1]

$$m_{b'} > 199 \text{ GeV}, \qquad m_{t'} > 256 \text{ GeV}. \tag{7}$$

Note that if the mixing angles between the 4th family and the rest ones are extremely small, b' and t' quarks behave like long-lived charged massive particles. In this case the constraints are $m_{b'} > 190$ GeV and $m_{t'} > 220$ GeV [12].

At the composite scale $\Lambda^{(4)}$, the 4th family quarks t' and b' condense and thereby they break the electroweak symmetry. By using the Pagels-Stokar (PS) formula [7,13], we can estimate the corresponding decay constants,

$$v_{t'}^2 = \frac{N}{8\pi^2} m_{t'}^2 \ln\left(1 + \frac{(\Lambda^{(4)})^2}{m_{t'}^2}\right), \tag{8a}$$

$$v_{b'}^2 = \frac{N}{8\pi^2} m_{b'}^2 \ln\left(1 + \frac{(\Lambda^{(4)})^2}{m_{b'}^2}\right), \tag{8b}$$

with

$$v_{t'}^2 + v_{b'}^2 = v^2, \tag{9}$$

where $N = 3$ and $v = 246$ GeV. The constraint of the T-parameter suggests that $m_{t'} \simeq m_{b'}$ is favorable and thereby $v_{t'} \simeq v_{b'}$ follows. Note that the masses of t' and b' are essentially determined through the PS formula (8) when the value of $\Lambda^{(4)}$ is fixed.

In order to obtain almost correct masses for the down-type quarks,

$$m_0^{(3)} \sim 1\ \text{GeV}, \qquad m_0^{(2)} \sim 100\ \text{MeV}, \qquad m_0^{(1)} \sim 1\ \text{MeV}, \tag{10}$$

we introduce the following horizontal FCN interactions (see Fig. 1):

$$t' - u^{(i)} - \Lambda^{(i4)}, \qquad b' - d^{(i)} - \Lambda^{(i4)}, \tag{11}$$

where $i = 1, 2, 3$ and $u^{(1,2,3)} = u, c, t$ and $d^{(1,2,3)} = d, s, b$, respectively. These one-loop contributions yield

$$m_0^{(i)} \simeq \frac{C_2 g^2_{t'u^{(i)}}}{4\pi^2} \frac{(\Lambda^{(4)})^2}{(\Lambda^{(i4)})^2} m_{t'} \simeq \frac{C_2 g^2_{b'd^{(i)}}}{4\pi^2} \frac{(\Lambda^{(4)})^2}{(\Lambda^{(i4)})^2} m_{b'}, \tag{12}$$

where C_2 represents the quadratic Casimir invariant and we took into account that the dynamical running $m_{t'}$ and $m_{b'}$ masses rapidly decrease above the scale $\Lambda^{(4)}$ (if these masses are not sharply cutoff at $\Lambda^{(4)}$, there can appear $\log(\Lambda^{(4)})$ factors in Eq. (12), as in QCD [14,15]).

In order to obtain the hierarchical masses $m_0^{(1,2,3)}$, we assume

$$(\Lambda^{(14)})^2 \gg (\Lambda^{(24)})^2 \gg (\Lambda^{(34)})^2 \gg (\Lambda^{(4)})^2. \tag{13}$$

We may expect $C_2 g^2_{t'u^{(i)}} \simeq C_2 g^2_{b'd^{(i)}} \sim \mathcal{O}(1)$. Then, at this stage, the mass spectrum of quarks is isospin symmetric. The running masses are essentially equal to the constants $m_0^{(i)}$ up to the scale of $\Lambda^{(i4)}$ ($i = 1, 2, 3$). Above $\Lambda^{(i4)}$, they rapidly, as $1/q^2$, decrease (q is the momentum of the running masses).

In order to get the appropriate numbers, the scales should be determined by

$$\eta^{(i)}_{t'(b')} \equiv \frac{C_2 g^2_{t'u^{(i)}(b'd^{(i)})}}{4\pi^2} \frac{(\Lambda^{(4)})^2}{(\Lambda^{(i4)})^2} \simeq \frac{m_0^{(i)}}{m_{t'(b')}}. \tag{14}$$

B. Horizontal interactions as the source of the isospin violation in the quark masses

The second (central) stage in the present scenario is introducing the horizontal interactions for the three known fermion families. Note that this stage is essentially identical for the scenarios with the different EWSB dynamics: elementary Higgs fields, TC, and the fourth family.

Let us start from the description of the dynamics generating the large top quark mass. At energy scales less than the mass of a horizontal vector boson $\Lambda^{(3)} \sim \Lambda^{(34)}$, the corresponding horizontal interactions can be presented by the four-fermion Nambu-Jona-Lasinio (NJL) ones. We apply strong (although subcritical) dynamics for the horizontal diagonal interactions for the t quark. The isospin symmetric mass $m_0^{(3)}$, introduced in Sec. II A, plays the role of a bare mass with respect to these interactions. The solution of the Schwinger-Dyson equation for the t quark propagator leads to the following mass m_t [6,7]:

$$m_t \simeq \frac{1}{\Delta g_t} m_0^{(3)}, \tag{15}$$

where Δg_q denotes the difference of the critical coupling and the (normalized) dimensionless NJL one for a q quark, so that

$$\Delta g_t \simeq \frac{m_0^{(3)}}{m_t} \sim 6 \times 10^{-3}, \tag{16}$$

where we used $m_t = 171.2$ GeV and $m_0^{(3)} = 1$ GeV. For the bottom quark, it should be $\Delta g_b \sim \mathcal{O}(1)$. In any case, it is required,

$$\Delta g_b - \Delta g_t \simeq \frac{m_0^{(3)}}{m_b}, \tag{17}$$

where we ignored $m_0^{(3)}/m_t$ because of $m_t \gg m_b$. Concrete models for obtaining such a isospin symmetry breakdown in the third family are described in Appendix A.

Let us now turn to the generation of the realistic masses for the second family. We assume that there exist FCN interactions between the t and c quarks and similarly between the b and s ones (see Fig. 2),

$$t - c - \Lambda^{(23)}, \qquad b - s - \Lambda^{(23)}. \tag{18}$$

These one-loop diagrams yield the following masses for charm and strange quarks:

$$m_c = m_0^{(2)} + \eta_t^{(23)} m_t, \qquad m_s = m_0^{(2)} + \eta_b^{(23)} m_b, \tag{19}$$

where $m_0^{(2)} \sim 100$ MeV is the isospin symmetric mass for the second family (see Sec. II A), and $\eta^{(23)}_{t,b}$ are

$$\eta^{(23)}_{t(b)} \equiv \frac{C_2 g^2_{tc(bs)}}{4\pi^2} \frac{(\Lambda^{(34)})^2}{(\Lambda^{(23)})^2} \tag{20}$$

for $\Lambda^{(23)} \gg \Lambda^{(34)}$.

As described above, the ratio $m_b/m_t \simeq 1/40$ is obtained via the near-critical dynamics in this model. Now, taking $m_0^{(2)} = 100$ MeV and $\eta_t^{(23)} = \eta_b^{(23)} = 1/100$, we get

$$m_c = 100\ \text{MeV} + m_t/100 \sim 1\ \text{GeV}, \tag{21}$$

$$m_s = 100\ \text{MeV} + m_b/100 \sim 140\ \text{MeV}. \tag{22}$$

In this way, we can obtain the correct mass enhancement for the charm quark via the large m_t. *Let us emphasize that the presence of the isospin symmetric mass* $m_0^{(2)} \sim 100$ MeV $\sim m_s$ *is crucial here: with* $m_0^{(2)} \ll 100$ MeV, *the ratio* m_s/m_c *would be close to* m_b/m_t.

As to the horizontal FCN gauge bosons which couple to the quarks of the 1st and 2nd families, we assume that they are very heavy,

$$c - u - \Lambda^{(12)}, \qquad s - d - \Lambda^{(12)}, \tag{23}$$

with $\Lambda^{(12)} \gtrsim \mathcal{O}(1000\ \mathrm{TeV})$. As a result, their contributions to the masses of the u and d quarks are very small.

C. The CKM mass matrix

So far we have neglected the family mixing effects. Because the mixing angles between quarks from the different families are small, such an approach can be considered as a reasonable approximation for the description of generating quark masses in the second and third quark families. Here we will turn to the structure of the CKM mass matrix.

Recall that the number of the CP phases is three in the 4th family quark model, whereas the three generation model has only one CP phase [16]. This can offer richer phenomenology, for example, in the B physics. In this paper, however, we ignore the CP violation and concentrate on the family mixing effects.

There are several approaches to this problem: (1) Mass texture ansätze (for example, the Fritzsch-type mass matrix [17], the democratic family mixing, etc.). (2) The Froggatt-Nielsen mechanism [18]. (3) Dynamical approaches, e.g., ETC models [19], the top loops mechanism [20], etc. We will employ a modification of the dynamical approach in Ref. [19] that is appropriate for the model with the 4th family.

Let us start from the down-type quark masses. We assume that

(1) There exist horizontal FCN interactions with a mixing of $V_i^{(1)}$ and $V_j^{(1)}$ gauge bosons related to two different families i and j, one of which is the first one (see Fig. 3). We further assume that the values of all the relevant parameters (the masses of $V_i^{(1)}$ and $V_j^{(1)}$, and the gauge boson mixing parameters) are around the scale $\Lambda^{(14)}$. In this case, we obtain naturally a universal mass $m_{\mathrm{off}}^{(1)}$ with $m_{\mathrm{off}}^{(1)} \sim m_d$.

(2) Similarly, when neither i nor j are 1, there exist horizontal FCN interactions with a mixing for another set of $V_i^{(2)}$ and $V_j^{(2)}$ gauge bosons. In this case, the values of all the relevant parameters are assumed to be around $\Lambda^{(24)}$. This leads to a universal mass $m_{\mathrm{off}}^{(2)} \sim m_s$.

We can then explicitly write the mass matrix M_D for the down-type quark as

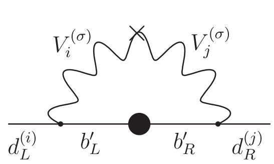

FIG. 3. FCN interactions with a gauge boson mixing. The parameter $\sigma = 1, 2$ is described in text.

$$M_D = \begin{pmatrix} m_d & \xi_1 m_d & \xi_1 m_d & \xi_1 m_d \\ \xi_1 m_d & m_s & \xi_2 m_s & \xi_2 m_s \\ \xi_1 m_d & \xi_2 m_s & m_b & \xi_2 m_s \\ \xi_1 m_d & \xi_2 m_s & \xi_2 m_s & m_{b'} \end{pmatrix}. \tag{24}$$

The parameters $\xi_{1,2}$ will be determined by $|V_{us}|$ and $|V_{cb}|$ later. As to the diagonal mass terms, the values of m_s, m_b, and $m_{b'}$ are almost the same as the mass eigenvalues, whereas it is required to adjust numerically the value of m_d in order to obtain the correct mass eigenvalue for the down quark.

For the up-type quarks, the mass matrix has a similar structure with the replacement of m_d, m_s, m_b, $m_{b'}$ by m_u, m_c, m_t, $m_{t'}$, respectively.

Since the mass matrix M_D is symmetric, it can be diagonalized by a single orthogonal matrix D_L. Similarly, the up-type quark mass matrix can be diagonalized by an orthogonal matrix U_L. The 4×4 CKM matrix $V_{\mathrm{CKM}}^{4\times 4}$ is given by

$$V_{\mathrm{CKM}}^{4\times 4} = U_L^\dagger D_L. \tag{25}$$

Noting that $m_d \ll m_s \ll m_b \ll m_{b'}$, we approximately obtain the matrix D_L as

$$D_L \simeq \begin{pmatrix} 1 - \frac{\xi_1^2}{2}(\frac{m_d}{m_s})^2 & \xi_1 \frac{m_d}{m_s} & \xi_1 \frac{m_d}{m_b} & \xi_1 \frac{m_d}{m_{b'}} \\ -\xi_1 \frac{m_d}{m_s} & 1 - \frac{\xi_1^2}{2}(\frac{m_d}{m_s})^2 & \xi_2 \frac{m_s}{m_b} & \xi_2 \frac{m_s}{m_{b'}} \\ -\xi_1 \frac{m_d}{m_b} & -\xi_2 \frac{m_s}{m_b} & 1 & \xi_2 \frac{m_s}{m_{b'}} \\ -\xi_1 \frac{m_d}{m_{b'}} & -\xi_2 \frac{m_s}{m_{b'}} & -\xi_2 \frac{m_s}{m_{b'}} & 1 \end{pmatrix}, \tag{26}$$

where we took into account that the quadratic term $m_d^2/m_s^2 \sim O(0.01)$.

On the other hand, since numerically $m_u/m_c \ll m_d/m_s$, $m_u/m_t \ll m_d/m_b$, and $m_c/m_t \ll m_s/m_b$, we can neglect the off-diagonal entries of U_L in the 3×3 part of the CKM matrix. Then we get:

$$|V_{ud}| \simeq |V_{cs}| \simeq 1 - \frac{\xi_1^2}{2}\left(\frac{m_d}{m_s}\right)^2, \tag{27}$$

$$|V_{tb}| \simeq 1, \tag{28}$$

$$|V_{us}| \simeq |V_{cd}| \simeq \xi_1 \frac{m_d}{m_s}, \tag{29}$$

$$|V_{ub}| \simeq |V_{td}| \simeq \xi_1 \frac{m_d}{m_b}, \tag{30}$$

$$|V_{cb}| \simeq |V_{ts}| \simeq \xi_2 \frac{m_s}{m_b}. \tag{31}$$

The relation $|V_{ub}/V_{us}| = m_s/m_b = 0.02$ is noticeable. Note that the PDG value is $|V_{ub}/V_{us}| = 3.93 \times 10^{-3}/0.2255 = 0.0174$ [1].

By using $|V_{us}| = 0.23$ and $|V_{cb}| = 0.04$ [1], we fix the values of $\xi_{1,2}$,

$$\xi_1 = \frac{23}{m_d\ (\mathrm{MeV})}, \qquad \xi_2 = 0.04 \times \frac{m_b}{m_s} = 2. \qquad (32)$$

With these values of ξ_1 and ξ_2, and the masses of quarks for D_L and U_L, we thereby obtain the 4×4 CKM matrix:

$$V_{\mathrm{CKM}}^{4\times4} = \begin{pmatrix} 0.97 & 0.23 & -0.006 & 0.00009 \\ -0.23 & 0.97 & -0.04 & -0.008 \\ -0.003 & 0.04 & 1.0 & 0.02 \\ -0.002 & 0.007 & -0.02 & 1.0 \end{pmatrix}, \qquad (33)$$

where we used $m_{t'} = m_{b'} = 300$ GeV, which is responsible only for the 4th column and row. Actually, the values of $|V_{ud}|$, $|V_{cs}|$, $|V_{tb}|$, $|V_{cd}|$, and $|V_{ts}|$ are "correct" [1]. Although our $|V_{ub}| = 0.006$ and $|V_{td}| = 0.003$ are a bit different from the PDG values [1], the orders, $|V_{ub}| \sim |V_{td}| \sim O(0.001)$, are correct.

The 4th generation mixing terms are approximately given by

$$|V_{t'd}| \simeq \xi_1 \frac{m_d}{m_s} \cdot \xi_2 \frac{m_c}{m_{t'}} \sim 0.23 \times \xi_2 \frac{m_c}{m_{t'}} \sim \mathcal{O}(10^{-3}), \qquad (34)$$

and

$$|V_{t's}| \simeq |V_{t'b}| \simeq \xi_2 \frac{m_c}{m_{t'}} \sim \mathcal{O}(10^{-2}). \qquad (35)$$

Thus the contributions of t' to the $B^0 - \bar{B}^0$ mixing are roughly proportional to $m_{t'}^2 |V_{t'd}^* V_{t'b}|^2 \sim m_c^4/m_{t'}^2 \times 10^{-2}$ for B_d and $m_{t'}^2 |V_{t's}^* V_{t'b}|^2 \sim m_c^4/m_{t'}^2$ for B_s. On the other hand, the corresponding SM contributions are proportional to $m_t^2 |V_{td}^* V_{tb}|^2 \simeq 6.4 10^{-5} m_t^2$ and $m_t^2 |V_{ts}^* V_{tb}|^2 \simeq 1.6 10^{-3} m_t^2$, respectively. Therefore the 4th generation contributions are negligible. Similarly, the processes $b \to s\gamma$ and $Z \to b\bar{b}$ are also suppressed.

Although the dynamics underlying the CKM matrix is still far from being completely understood, it is noticeable that by using a simple extension of the mechanism for producing the quark masses used in Secs. II A and II B, the essential features of the CKM matrix can be extracted.

D. Composite Higgs bosons

In this scenario, there potentially appear many composite Higgs bosons (compare with Refs. [6,7,21]). In the scenario with the 4th family quarks, the masses of the bound states of the t' and b' quarks should be of the order of the EWSB scale. Since we consider the condensation both of the t' and b', there appear at least two composite Higgs doublets. For the 3rd family, we may estimate the mass of the top-Higgs doublet (resonance) ϕ_t via the NJL relation [6,7,22]:

$$M_{\phi_t} \sim \Lambda^{(3)} \left(\frac{2\Delta g_t}{\ln \frac{1}{2\Delta g_t}} \right)^{1/2} \sim 0.05 \Lambda^{(3)}, \qquad (36)$$

where we used $\Delta g_t \sim 6 \times 10^{-3}$. For the bottom-Higgs resonance ϕ_b, it should be $M_{\phi_b} \sim \Lambda^{(3)}$, i.e., it is very heavy and unstable. Note that the quark structures of the composites ϕ_t and ϕ_b are $\phi_t \sim (\Lambda^{(3)})^{-2} \bar{t}_R (t, b)_L$ and $\phi_b \sim (\Lambda^{(3)})^{-2} \bar{b}_R (b, -t)_L$, respectively.

Note that in the case of the scenario with elementary Higgs fields responsible for the EWSB, there should appear (beside the elementary Higgs fields) at least one composite Higgs resonance ϕ_t. In the TC scenario, such a Higgs resonance exists in addition to technihadrons.

Since we assume that the scales $\Lambda^{(1)}$ and $\Lambda^{(2)}$ (related to the 1st and 2nd families) are very large, the corresponding Higgs composites should be very heavy and unstable, and therefore they are irrelevant for the electroweak dynamics.

III. PHENOMENOLOGICAL ANALYSIS

In this section, we describe a phenomenology in the simplest model with the 4th family of the class described in Sec. II. In this model, the scale $\Lambda^{(3)}$ is assumed to be sufficiently large, such that the mass M_{ϕ_t} (36) is much heavier than the masses of the Higgs doublets composed of the t' and b'. Otherwise, the mass of the top-Higgs field ϕ_t would be also of the order of the EWSB scale. In that case, there appear three relevant Higgs doublets. This interesting possibility will be considered elsewhere.

For $v_{t'} = v_{b'}$, the PS formula (8) yields $m_{t'} = m_{b'} \sim 0.3$ TeV with $\Lambda^{(4)} = 10$ TeV. More precisely, by using the RGE's [23] with the compositeness conditions [11,24], we obtain

$$m_{t'} = 0.292\ \mathrm{TeV}, \qquad m_{b'} = 0.291\ \mathrm{TeV}, \qquad (37)$$

which gives the T-parameter contribution $T_f = 10^{-5}$. Smaller $\Lambda^{(4)}$ provides larger $m_{t'}$ and $m_{b'}$ with relaxing the cost of the fine-tuning, due to a combination of the gap equation and the PS formula,

$$\frac{v^2}{(\Lambda^{(4)})^2} = \frac{N}{8\pi^2}\left(2 - \frac{1}{g_{t'}^{\mathrm{eff}}} - \frac{1}{g_{b'}^{\mathrm{eff}}}\right) \simeq \frac{N}{4\pi^2}\left(1 - \frac{1}{g_{t'}^{\mathrm{eff}}}\right), \qquad (38)$$

where we used near-equality for the effective dimensionless NJL couplings: $g_{b'}^{\mathrm{eff}} \simeq g_{t'}^{\mathrm{eff}}$, because $m_{t'} \simeq m_{b'}$.

As to the masses of the Higgs bosons composed of t' and b', we take the mass M_A of the CP odd Higgs as a free parameter. The rest masses are determined through the RGE's [23,24]. For example, we may take

$$M_A = 0.30, 0.40, 0.50, 0.60\ \mathrm{TeV}, \qquad (39)$$

and in this case, we obtain the charged and CP even Higgs masses,

$$M_{H^\pm} = 0.43, 0.50, 0.59, 0.67\ \mathrm{TeV}, \qquad (40)$$

$$M_h = 0.42, 0.44, 0.46, 0.47\ \mathrm{TeV}, \qquad (41)$$

$$M_H = 0.43, 0.50, 0.59, 0.67\ \mathrm{TeV}, \qquad (42)$$

respectively. Note that $\tan\beta \equiv v_{t'}/v_{b'} = 1$ in our model. The HZZ-, $h\bar{t}'t'$- and $H\bar{t}'t'$- couplings are proportional to [25]

$$\cos(\beta - \alpha) = -0.02, -0.002, -0.0009, -0.0005, \tag{43}$$

$$\cos\alpha/\sin\beta = 0.98, 1.0, 1.0, 1.0, \tag{44}$$

$$\sin\alpha/\sin\beta = -1.0, -1.0, -1.0, -1.0, \tag{45}$$

respectively, where α denotes the mixing angle of the two CP even Higgs bosons. We can immediately read the relative $h\bar{b}'b'$- and $H\bar{b}'b'$-couplings, $-\sin\alpha/\cos\beta$ and $\cos\alpha/\cos\beta$, from above, because of $\tan\beta = 1$ in our model. Because of $M_{H^\pm} \simeq M_H$ in this parameter regime, the contributions of the Higgs bosons to the S- and T-parameters are small, at most $S_H = 0.03$ and $T_H = -0.05$ for the reference value of the SM Higgs boson $M_h^{\rm ref} = 300$ GeV.

Let us fix $m_0^{(3)} = 1.0$ GeV and thereby obtain

$$\Delta g_t = \frac{m_0^{(3)}}{m_t} = (5.8 \pm 0.1) \times 10^{-3}, \tag{46}$$

$$\Delta g_b = \frac{m_0^{(3)}}{m_b} = 0.24^{+0.00}_{-0.01}, \tag{47}$$

with the error bars. The Higgs masses are estimated as

$$M_{\phi_t} \simeq \Lambda^{(3)}\left(\frac{2\Delta g_t}{\ln\frac{1}{2\Delta g_t}}\right)^{1/2} = 0.051\Lambda^{(3)}, \tag{48}$$

$$M_{\phi_b} \simeq \Lambda^{(3)}\left(\frac{2\Delta g_b}{\ln\frac{1}{2\Delta g_b}}\right)^{1/2} = 0.80\Lambda^{(3)}, \tag{49}$$

where we used only the central value. Recall that it is assumed in the present model that the top-Higgs ϕ_t is decoupled. It requires, say, $M_{\phi_t} \gtrsim 1$ TeV, i.e., $\Lambda^{(3)} \gtrsim 20$ TeV. We also find

$$\Lambda^{(34)} \simeq \Lambda^{(4)}\sqrt{\frac{C_2 g_{t't}^2}{4\pi^2}\frac{m_{t'}}{m_0^{(3)}}} = 2.7\sqrt{C_2}g_{t't}\Lambda^{(4)}, \tag{50}$$

$$\simeq \Lambda^{(4)}\sqrt{\frac{C_2 g_{b'b}^2}{4\pi^2}\frac{m_{b'}}{m_0^{(3)}}} = 2.7\sqrt{C_2}g_{b'b}\Lambda^{(4)}. \tag{51}$$

For the masses of the 2nd family, assuming $\eta_t^{(23)} = \eta_b^{(23)} \equiv \eta^{(23)}$, the following relation is crucial;

$$\eta^{(23)} = \frac{m_c - m_s}{m_t - m_b} = (7.0^{+0.7}_{-0.9}) \times 10^{-3}, \tag{52}$$

so that we obtain

$$m_0^{(2)} = m_s - \eta^{(23)} m_b = 75^{+30}_{-39}\ \text{MeV}, \tag{53}$$

$$C_2 g_{tc(bs)}^2 \frac{(\Lambda^{(34)})^2}{(\Lambda^{(23)})^2} \simeq 0.28^{+0.03}_{-0.04}, \tag{54}$$

i.e.,

$$\Lambda^{(23)} = (1.9 \pm 0.1)\sqrt{C_2}g_{tc}\Lambda^{(34),} \tag{55}$$

$$= (1.9 \pm 0.1)\sqrt{C_2}g_{bs}\Lambda^{(34)}. \tag{56}$$

The mass $m_0^{(2)}$ yields

$$\Lambda^{(24)} \simeq \Lambda^{(4)}\sqrt{\frac{C_2 g_{t'c}^2}{4\pi^2}\frac{m_{t'}}{m_0^{(2)}}} = (10^{+4}_{-2})\sqrt{C_2}g_{t'c}\Lambda^{(4)}, \tag{57}$$

$$\simeq \Lambda^{(4)}\sqrt{\frac{C_2 g_{b's}^2}{4\pi^2}\frac{m_{b'}}{m_0^{(2)}}} = (10^{+4}_{-2})\sqrt{C_2}g_{b's}\Lambda^{(4)}. \tag{58}$$

Finally, for the 1st family, we directly get

$$\Lambda^{(14)} \simeq \Lambda^{(4)}\sqrt{\frac{C_2 g_{t'u}^2}{4\pi^2}\frac{m_{t'}}{m_0^{(1)}}} = 61\sqrt{C_2}g_{t'u}\Lambda^{(4)}, \tag{59}$$

$$\simeq \Lambda^{(4)}\sqrt{\frac{C_2 g_{b'd}^2}{4\pi^2}\frac{m_{b'}}{m_0^{(1)}}} = 61\sqrt{C_2}g_{b'd}\Lambda^{(4)}, \tag{60}$$

where we used

$$m_0^{(1)} = 2.0\ \text{MeV}. \tag{61}$$

Note that in order to get a more realistic ratio,

$$\frac{m_u}{m_d} = 0.35\text{–}0.60, \tag{62}$$

one may tune $g_{t'u}^2/g_{b'd}^2$ to the latter. One should however remember that the eigenvalues m_u and m_d are determined after diagonalizing the quark mass matrices discussed in Sec. II C. In general, the numerical calculations of m_u and m_d beyond the order-estimates are highly sensitive to the fine-structure of the mass matrices.

In summary, the suppression factors for getting the masses $m_0^{(3)} = 1.0$ GeV, $m_0^{(2)} = 75$ MeV and $m_0^{(1)} = 2.0$ MeV should be equal to

$$\eta_{t'}^{(3)} = \frac{m_0^{(3)}}{m_{t'}} = 3.4 \times 10^{-3}, \quad \eta_{b'}^{(3)} = \frac{m_0^{(3)}}{m_{b'}} = 3.4 \times 10^{-3}, \tag{63}$$

$$\eta_{t'}^{(2)} = \frac{m_0^{(2)}}{m_{t'}} = 2.6 \times 10^{-4}, \quad \eta_{b'}^{(2)} = \frac{m_0^{(2)}}{m_{b'}} = 2.6 \times 10^{-4}, \tag{64}$$

TABLE I. Numerical estimates for $\Lambda^{(4)} = 20$, 30 TeV.

$\Lambda^{(4)}$ (TeV)	30	30	20	20
$m_{t'}$ (TeV)	0.26	0.26	0.27	0.27
$m_{b'}$ (TeV)	0.26	0.26	0.27	0.27
$m_0^{(3)}$ (GeV)	1.0	2.0	1.0	2.0
Δg_t	5.8×10^{-3}	0.012	5.8×10^{-3}	0.012
Δg_b	0.24	0.48	0.24	0.48
$M_{\phi_t}/\Lambda^{(3)}$	0.051	0.079	0.051	0.079
$M_{\phi_b}/\Lambda^{(3)}$	0.80	4.4	0.80	4.4
$(\Lambda^{(34)})^2/[C_2 g_{q'q}^2 (\Lambda^{(4)})^2]$	6.8	3.2	6.8	3.6
$m_0^{(2)}$ (MeV)	75^{+30}_{-39}	75^{+30}_{-39}	75^{+30}_{-39}	75^{+30}_{-39}
$\eta^{(23)}$	$(7.0^{+0.7}_{-0.9}) \times 10^{-3}$	$(7.0^{+0.7}_{-0.9}) \times 10^{-3}$	$(7.0^{+0.7}_{-0.9}) \times 10^{-3}$	$(7.0^{+0.7}_{-0.9}) \times 10^{-3}$
$(\Lambda^{(24)})^2/[C_2 g_{q'q}^2 (\Lambda^{(4)})^2]$	88^{+96}_{-25}	88^{+96}_{-25}	92^{+99}_{-27}	92^{+99}_{-27}
$(\Lambda^{(23)})^2/[C_2 g_{qq}^2 (\Lambda^{(34)})^2]$	$3.6^{+0.5}_{-0.3}$	$3.6^{+0.5}_{-0.3}$	$3.6^{+0.5}_{-0.3}$	$3.6^{+0.5}_{-0.3}$
$m_0^{(1)}$ (MeV)	1	2	1	2
$(\Lambda^{(14)})^2/[C_2 g_{q'q}^2 (\Lambda^{(4)})^2]$	6.6×10^3	3.2×10^3	6.9×10^3	3.5×10^3

$$\eta_{t'}^{(1)} = \frac{m_0^{(1)}}{m_{t'}} = 6.8 \times 10^{-6},$$
$$\eta_{b'}^{(1)} = \frac{m_0^{(1)}}{m_{b'}} = 6.9 \times 10^{-6}. \tag{65}$$

They are obtained by taking appropriate values for the ratios $\Lambda^{(i4)}/\Lambda^{(4)}$ as above.

Roughly speaking, in our scenario, the masses of t' and b' are

$$m_{t'} \simeq 300 \text{ GeV}, \qquad m_{b'} \simeq 300 \text{ GeV}, \tag{66}$$

the cutoff scale is

$$\Lambda^{(4)} \sim 10 \text{ TeV}, \tag{67}$$

and the other FCN scales are estimated as

$$\Lambda^{(14)} \sim 100\Lambda^{(4)}, \qquad \Lambda^{(24)} \sim 10\Lambda^{(4)}, \tag{68}$$

$$\Lambda^{(34)} \sim 3\Lambda^{(4)}, \qquad \Lambda^{(23)} \sim 5\Lambda^{(4)}. \tag{69}$$

Although the exchange of $\Lambda^{(34)}$ contributes to R_b, it is tiny, $\delta R_b/R_b \sim 10^{-6}$ for $\Lambda^{(34)} = 30$ TeV with $C_2 g_{b'b}^2 \sim 1$. The constraint from the B_s^0-$\bar{B}_s^0$ mixing suggests $\Lambda^{(23)} \gtrsim 100$ TeV, so that the above estimate $\Lambda^{(23)} \sim 5\Lambda^{(4)}$ is a bit dangerous. (If we take a smaller $m_0^{(3)}$ or a bigger $\Lambda^{(4)}$, we can evade this problem.)

As we discussed in Sec. II C, the constraints from the B^0-$\bar{B}^0$ mixing, $b \to s\gamma$ and R_b via the t' loop are suppressed, because the relevant mixing angles are tiny, $|V_{t'd}| \sim |V_{cd}| m_c/m_{t'} \sim 10^{-3}$, and $|V_{t's}| \simeq |V_{t'b}| \sim m_c/m_{t'} \sim 10^{-2}$. The contributions of the charged Higgs are also suppressed.

The numerical estimates of all the relevant parameters of the model for the values $\Lambda^{(4)} = 30$ TeV, $\Lambda^{(4)} = 20$ TeV, and $\Lambda^{(4)} = 10$ TeV, $\Lambda^{(4)} = 5$ TeV are presented in Tables I and II, respectively.

TABLE II. Numerical estimates for $\Lambda^{(4)} = 5{,}10$ TeV.

$\Lambda^{(4)}$ (TeV)	10	10	5	5
$m_{t'}$ (TeV)	0.29	0.29	0.32	0.32
$m_{b'}$ (TeV)	0.29	0.29	0.32	0.32
$m_0^{(3)}$ (GeV)	1.0	2.0	1.0	2.0
Δg_t	5.8×10^{-3}	0.012	5.8×10^{-3}	0.012
Δg_b	0.24	0.48	0.24	0.48
$M_{\phi_t}/\Lambda^{(3)}$	0.051	0.079	0.051	0.079
$M_{\phi_b}/\Lambda^{(3)}$	0.80	4.4	0.80	4.4
$(\Lambda^{(34)})^2/[C_2 g_{q'q}^2 (\Lambda^{(4)})^2]$	7.3	3.6	8.4	4.0
$m_0^{(2)}$ (MeV)	75^{+30}_{-39}	75^{+30}_{-39}	75^{+30}_{-39}	75^{+30}_{-39}
$\eta^{(23)}$	$(7.0^{+0.7}_{-0.9}) \times 10^{-3}$	$(7.0^{+0.7}_{-0.9}) \times 10^{-3}$	$(7.0^{+0.7}_{-0.9}) \times 10^{-3}$	$(7.0^{+0.7}_{-0.9}) \times 10^{-3}$
$(\Lambda^{(24)})^2/[C_2 g_{q'q}^2 (\Lambda^{(4)})^2]$	99^{+106}_{-29}	99^{+106}_{-29}	109^{+119}_{-31}	109^{+119}_{-31}
$(\Lambda^{(23)})^2/[C_2 g_{qq}^2 (\Lambda^{(34)})^2]$	$3.6^{+0.5}_{-0.3}$	$3.6^{+0.5}_{-0.3}$	$3.6^{+0.5}_{-0.3}$	$3.6^{+0.5}_{-0.3}$
$m_0^{(1)}$ (MeV)	1	2	1	2
$(\Lambda^{(14)})^2/[C_2 g_{q'q}^2 (\Lambda^{(4)})^2]$	7.4×10^3	3.7×10^3	8.3×10^3	4.1×10^3

The following comments are in order. (i) While the contribution of the particles of the 4th family to the T-parameter is almost vanishing in the case of degenerate masses of both the quarks and the leptons, their contribution to the S-parameter is a bit large, $S_f \sim 0.2$, if no Majorana neutrinos are present. One can avoid this difficulty by introducing a Majorana neutrino with a mass smaller than that of the charged lepton [26,27]. At the same time, the T-parameter can be kept small even in this case [26,28,29]. (ii) In the present model, the maximum value for the mass of t' and b' is realized for $\Lambda^{(4)} = m_{t'(b')}$. The PS formula (8) yields $m_{t'(b')}^{(\max)} \simeq 1$ TeV for it. The fact that $m_{t'} \simeq m_{b'} < 1$ TeV in this model is noticeable: the 4th family quarks with masses of 1 TeV or lighter can be discovered at LHC [30].

IV. DISCUSSION

The two crucial ingredients in the class of models described in this paper are (i) the assumption that the EWSB dynamics leads to the isospin symmetric quark mass spectrum, with the masses of the order of the down-type quarks, and (ii) the existence of strong (although subcritical) horizontal diagonal interactions for the t quark plus horizontal flavor-changing neutral interactions between different families. The signature of such dynamics is the presence of composite Higgs bosons. It is noticeable that this dynamics can be build into the scenarios with different EWSB mechanisms.

The concrete model with the 4th family considered above shows that these two ingredients quite naturally lead to the realistic masses for quarks. Moreover, as was pointed out in Sec. II, in the present approach it is necessary to choose the mass $m_0^{(2)}$ (generated by the EWSB dynamics) to be of the order of the mass of the s quark: only in this case one can obtain the correct m_c/m_s ratio. We also demonstrated that by using a simple extension of the present mechanism for producing the quark masses, the essential features of the CKM matrix can be extracted. Another noticeable feature in the model is the absence of fine-tuning: the near criticality (1 part in 10^2) of the coupling of the t quark is determined by the small ratio $m_b/m_t \simeq 2.5 \times 10^{-2}$.

As the next steps, it would be important to include leptons and to study the dynamics underlying the CKM matrix in more detail. As to the leptons, the fact that the masses of the charged leptons are of the order of the masses of the corresponding down-type quarks suggests that it is not unreasonable to assume that the origin of the former is similar to that of the latter. The main specific issues for leptons are of course connected with neutrinos, in particular, with a large mixing between the muon and tau neutrinos and a possible existence of Majorana neutrinos. Note that the latter occur quite naturally in the 4th family models [3]. Last but not least, it would be interesting to embed the present scenario into an extra dimensional one [4,31].

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APPENDIX A: MORE ABOUT ISOSPIN SYMMETRY BREAKING IN THE THIRD FAMILY

In this section, we will briefly describe several models which could provide strong isospin symmetry breaking in the third family.

For example, we here employ the first version of the topcolor model [32,33]. In this case, the QCD sector in the SM is extended to a $SU(3)_1 \times SU(3)_2$ one, with a stronger coupling for the $SU(3)_1$. The $SU(3)_1$ and $SU(3)_2$ charges are assigned as

$$(u, d)_L \to (\mathbf{1}, \mathbf{3}), \qquad u_R \to (\mathbf{1}, \mathbf{3}), \qquad d_R \to (\mathbf{1}, \mathbf{3}), \tag{A1}$$

$$(c, s)_L \to (\mathbf{1}, \mathbf{3}), \qquad c_R \to (\mathbf{1}, \mathbf{3}), \qquad s_R \to (\mathbf{1}, \mathbf{3}), \tag{A2}$$

$$(t, b)_L \to (\mathbf{3}, \mathbf{1}), \qquad t_R \to (\mathbf{3}, \mathbf{1}), \qquad b_R \to (\mathbf{1}, \mathbf{3}), \tag{A3}$$

$$(t', b')_L \to (\mathbf{3}, \mathbf{1}), \qquad t'_R \to (\mathbf{3}, \mathbf{1}), \qquad b'_R \to (\mathbf{3}, \mathbf{1}), \tag{A4}$$

while their $SU(2)_L \times U(1)_Y$ charges are conventional. Recall also that for the anomaly cancellation, $SU(2)_L$ singlet fermions are required,

$$Q_L \to (\mathbf{1}, \mathbf{3}), \qquad Q_R \to (\mathbf{3}, \mathbf{1}), \tag{A5}$$

with the same hypercharge as b_R [32]. Besides this topcolor scheme, we also introduce an additional $U(1)_{4F}$ gauge boson which couples (with the same strength) only to the fourth family. We may assign the $U(1)_{4F}$ charge as the $U(1)_{B-L}$ one, for example.

Then, after the spontaneous breakdown of $SU(3)_1 \times SU(3)_2$ down to $SU(3)_c$ at the scale $\Lambda^{(3)}$ ($= \Lambda^{(4)}$ in this case), the NJL couplings for the top and bottom are $g_c^2 \cot^2\theta/(\Lambda^{(3)})^2$ and $g_c^2/(\Lambda^{(3)})^2$, respectively, where g_c represents the QCD coupling constant and θ is the mixing angle of the $SU(3)_{1,2}$ gauge bosons. Since, unlike the topcolor model, we utilize the subcritical dynamics, the following relation holds,

$$\frac{3}{2\pi}\cot^2\theta\,\alpha_c(\Lambda^{(3)}) \lesssim 1. \tag{A6}$$

Therefore Eq. (17) now reads

$$\frac{3}{2\pi}(\cot^2\theta - 1)\alpha_c(\Lambda^{(3)}) \simeq \frac{m_0^{(3)}}{m_b}, \tag{A7}$$

where $\alpha_c(\Lambda^{(3)}) = g_c^2/(4\pi)$ is the QCD coupling at the scale $\Lambda^{(3)}$.

As to t' and b', in order to make their NJL couplings supercritical, the contribution from $U(1)_{4F}$ is crucial,

$$\frac{3}{2\pi}\cot^2\theta\alpha_c(\Lambda^{(4)}) + \frac{3}{2\pi}\alpha_{4F}(\Lambda^{(4)}) \gtrsim 1. \tag{A8}$$

where $\alpha_{4F}(\Lambda^{(4)}) = g_{4F}^2/(4\pi)$ is the gauge coupling of $U(1)_{4F}$ at the scale $\Lambda^{(4)}(= \Lambda^{(3)})$.

In this case, the scenario with three Higgs doublets as the composite fields of t', b' and t is likely.

For a model with $\Lambda^{(3)} \gtrsim \Lambda^{(4)}$, we may further extend the QCD sector,

$$SU(3)_1 \times SU(3)_{2+b} \times SU(3)_t \times SU(3)_4, \tag{A9}$$

with the following quark representations:

$$(u,d)_L \to (\mathbf{3},\mathbf{1},\mathbf{1},\mathbf{1}), \quad u_R \to (\mathbf{3},\mathbf{1},\mathbf{1},\mathbf{1}), \quad d_R \to (\mathbf{3},\mathbf{1},\mathbf{1},\mathbf{1}), \tag{A10}$$

$$(c,s)_L \to (\mathbf{1},\mathbf{3},\mathbf{1},\mathbf{1}), \quad c_R \to (\mathbf{1},\mathbf{3},\mathbf{1},\mathbf{1}), \quad s_R \to (\mathbf{1},\mathbf{3},\mathbf{1},\mathbf{1}), \tag{A11}$$

$$(t,b)_L \to (\mathbf{1},\mathbf{1},\mathbf{3},\mathbf{1}), \quad t_R \to (\mathbf{1},\mathbf{1},\mathbf{3},\mathbf{1}), \quad b_R \to (\mathbf{1},\mathbf{3},\mathbf{1},\mathbf{1}), \tag{A12}$$

$$(t',b')_L \to (\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{3}), \quad t'_R \to (\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{3}), \quad b'_R \to (\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{3}). \tag{A13}$$

The charges of the SM gauge group $SU(2)_L \times U(1)_Y$ are conventional. For anomaly cancellation, we also introduce $SU(2)_L$-singlet quarks,

$$Q_L \to (\mathbf{1},\mathbf{3},\mathbf{1},\mathbf{1}), \qquad Q_R \to (\mathbf{1},\mathbf{1},\mathbf{3},\mathbf{1}), \tag{A14}$$

with the same hypercharge as b_R.

At the scale $\Lambda^{(3)}$, a part of the gauge symmetry is spontaneously broken down to a diagonal subgroup,

$$SU(3)_{2+b} \times SU(3)_t \to SU(3)', \tag{A15}$$

and also, at the scale $\Lambda^{(4)}$, the rest part is broken down to

$$SU(3)_1 \times SU(3)_4 \to SU(3)''. \tag{A16}$$

The two gauge groups are broken down to the conventional QCD at some scale Λ_c ($\sim \Lambda^{(4)}$),

$$SU(3)' \times SU(3)'' \to SU(3)_c. \tag{A17}$$

The gauge coupling constants then satisfy the following relations,

$$\frac{1}{g_{2c}^2} + \frac{1}{g_{tc}^2} = \frac{1}{g_c'^2}, \qquad \frac{1}{g_{1c}^2} + \frac{1}{g_{4c}^2} = \frac{1}{g_c''^2}, \tag{A18}$$

and

$$\frac{1}{g_c'^2} + \frac{1}{g_c''^2} = \frac{1}{g_c^2}, \tag{A19}$$

where g_{ic} ($i = 1, 2, t, 4$), g'_c and g''_c denote the gauge couplings for $SU(3)_{1,(2+b),t,4}$, $SU(3)'$ and $SU(3)''$, respectively. Let us introduce the mixing angles θ'_c, θ''_c and θ_c between $SU(3)_{2+b}$ and $SU(3)_t$, between $SU(3)_1$ and $SU(3)_4$, and between $SU(3)'$ and $SU(3)''$, respectively. At the scale $\Lambda^{(3)}$, the four-top interaction is generated with the strength

$$G_t \equiv g_c'^2 \cot^2\theta'_c/(\Lambda^{(3)})^2, \tag{A20}$$

whereas the strengths of the NJL interactions for t' and b' are

$$G_4 \equiv g_c''^2 \cot^2\theta''_c/(\Lambda^{(4)})^2, \tag{A21}$$

provided at the scale $\Lambda^{(4)}$. When $g'_c \sim g''_c \sim g_c$, i.e., $\tan\theta_c \sim 1$, the four-fermion interactions generated at the scale Λ_c are irrelevant. In our scenario, we require that G_t and G_4 are subcritical and supercritical, respectively, so that

$$\frac{3}{2\pi}\frac{\cot^2\theta'_c}{\sin^2\theta_c}\alpha_c(\Lambda^{(3)}) \lesssim 1, \tag{A22}$$

at $\Lambda^{(3)}$, and

$$\frac{3}{2\pi}\frac{\cot^2\theta''_c}{\cos^2\theta_c}\alpha_c(\Lambda^{(4)}) \gtrsim 1, \tag{A23}$$

at $\Lambda^{(4)}$, where we expressed the gauge couplings g'_c and g''_c through g_c and the mixing angle θ_c, i.e., $g'_c = g_c/\sin\theta_c$ and $g''_c = g_c/\cos\theta_c$. Note that the NJL couplings for the 3rd family are restricted by the current mass enhancement relations. Equation (17) then reads

$$\frac{3}{2\pi}\frac{\cot^2\theta'_c - 1}{\sin^2\theta_c}\alpha_c(\Lambda^{(3)}) \simeq \frac{m_0^{(3)}}{m_b}. \tag{A24}$$

Another possibility for the isospin symmetry breaking is to use the $U(1)$-tilting mechanism, which can be realized in the model with extended QCD and hypercharge sectors, $SU(3)_1 \times SU(3)_2 \times U(1)_1 \times U(1)_2$ [21,34].

In this paper, we did not discuss the origin of the FCN interactions. For such a purpose, concrete ETC models could provide a useful hint [19,27,35].

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