Science Press, Jerusalem, 1971).
[6]A. Mueller, Phys. Rev. D 2, 2963 (1970).
[7]S. D. Drell and T.-M. Yan, Phys. Rev. Letters 24, 855 (1970).
[8]H. D. I. Abarbanel and D. Gross, Phys. Rev. Letters 26, 732 (1971).
[9]R. N. Cahn, J. W. Cleymans, and E. W. Colglazier, Phys. Letters 43B, 323 (1973).

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Model Calculation of Correlations at High Energy*

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The inclusive two-body correlation function for particles produced in high-energy hadronic collisions is calculated in a multiperipheral-like model, and compared with experiment.

It has been recognized lately that different models of particle production processes at high energy that may agree with each other and with the data on single-particle inclusive spectra may greatly differ in their predictions for two-particle spectra, and that studies of two-particle spectra could therefore be useful in distinguishing between these models.[1] One class of model which is often considered is the class of multiperipheral-like or short-range-correlation models. It is the purpose of this note to report a calculation of the two-particle spectrum at high energy in a simple yet hopefully representative model of this class which had been proposed earlier by Peccei and myself[2] and by Krzywicki and Petersson.[3]

In this model, particle production is described by distinguishing one particle (called the "leading particle") from the rest (the "fireball"); the distribution of particles in the fireball is assumed to be, in the proper Lorentz frame, the same as the distribution of all the particles in a typical event. Let $f_1(x)$ be the scaling limit of the normalized inclusive single-particle distribution:

$$f_1(x) \equiv \lim_{s\to\infty} \frac{x}{\sigma_{\text{tot}}(s)} \frac{d\sigma(x,s)}{dx} \quad . \tag{1}$$

In the model described in Refs. 2 and 3, $f_1(x)$ is related to $g(x)$, which is the distribution function for leading particles, by the integral equation

$$f_1(x) = g(x) + \int \frac{dy}{y} g(y) f_1\left(\frac{x}{1-y}\right) \quad . \tag{2}$$

Once the function $g(x)$ is known, then the two- (and in fact the many-) particle distribution can be calculated. In particular, the two-particle distribution with both particles in the central ($x \approx 0$) region can be expressed rather simply in terms of g and f_1. Let $f_2(x_1, x_2)$ be the scaling limit of the two-particle inclusive distribution, defined analogously with Eq. (1), and let

$$f_2(R) \equiv \lim_{x\to 0} f_2(x, Rx). \tag{3}$$

Then, according to the model,

$$f_2(R) = f_1(0) \int \frac{dx}{x} \left[g(x) f\left(\frac{Rx}{1-x}\right) + g(Rx) f\left(\frac{x}{1-Rx}\right) \right] . \tag{4}$$

The reader is referred to Ref. 2 for a derivation and discussion of this equation. The variable R is closely related to the relative rapidity of the two particles:

$$\ln R \approx y_2 - y_1 \qquad (|y_2 - y_1| \text{ large}). \tag{5}$$

In the central region, the two-particle distribution is independent of the sum of the two rapidities.

We adopt the following strategy to calculate $f_2(R)$: First, we use data on the single-particle distribution to guess at the form of $f_1(x)$; second, we make use of this $f_1(x)$ to obtain $g(x)$ through Eq. (2); finally, we calculate $f_2(R)$ through Eq. (4). The model as originally formulated makes no reference to any internal quantum numbers; thus the distributions f_1 and f_2 should be understood to be the distribution of particles of all types, neutral as well as charged. The model can be generalized to recognize the existence of several types of particles;[4] however, for the present purpose of illustrating the kind of distribution one obtains in multiperipheral-like models, this extra complication is probably not warranted, and so will not be considered here.

A more serious limitation of this type of model stems from the fact that, as pointed out by Le Bellac,[5] no model with only short-range correlations can be entirely correct if total cross sections are asymptotically constant, since such models can never describe diffraction. We thus have to be able to ignore diffraction; for example, if individual events can be classified as diffractive or nondiffractive, then the quantities f_1 and f_2 must be understood to represent the distributions of particles from nondiffractive events, and the quantity σ_{tot} in Eq. (1) should be understood to be the total nondiffractive cross section, which is presumably somewhat less than the inelastic cross section.

In order to determine the shape of $f_1(x)$, we use data on particles produced in proton-proton collisions. At small values of x, most of the particles produced are pions. The π^+ and π^- distributions are reasonably well determined[6] at the CERN Intersecting Storage Rings (ISR), where they are seen to scale. Because the Pomeron has isospin zero, the limiting π^0 distribution should be the average of π^+ and π^-; thus the total pion distribution is three times the average of π^+ and π^-. At large values of x, protons dominate. The proton distribution is seen[7] to have a peak near $x=1$; since this peak is presumably a diffractive effect, it should not be included in our nondiffractive distribution function $f_1(x)$. A detailed (and t-dependent) triple-Regge analysis near $x=1$ would presumably enable us to subtract the diffractive from the measured distribution; otherwise, we can use data on $pp\to pX$ for x outside the peak ($x<0.9$) and then extrapolate to a constant value at $x=1$. In fact, since scaling is seen to be reasonably well obeyed in this region, we can use data at lower energy.[8] Finally, although not quite all of the produced particles are protons or pions, we assume in what follows that the distribution of all particles has the same shape as the sum of the proton and the pion distributions.

Some further constraints on $f_1(x)$, which are valid for the type of model we consider, are:

(i) The energy-conservation sum rule, applied to nondiffractive events, reads

$$\int_0^1 dx\, f_1(x)=1\ . \tag{6}$$

Thus the normalization of $f_1(x)$ is determined once its shape is known.

(ii) If $\langle n\rangle$ is the average number of produced particles of all types, then, as $s\to\infty$,

$$\langle n\rangle \sim f(0)\ln s\ . \tag{7}$$

An estimate of $f(0)$ which is consistent with the observed energy dependence of multiplicities is $f(0)\approx 3$.

(iii) For small x,

$$f(x)\approx f(0)+Ax^{1/2}\ , \tag{8}$$

where the parameter A also appears in the expression for the approach to scaling in the central region[9]

$$\frac{x}{\sigma_{tot}}\frac{d\sigma}{dx}\bigg|_{x=0}\sim f(0)+2A\left(\frac{s}{m^2+P_\perp^2}\right)^{-1/4},\quad s\to\infty\ . \tag{9}$$

If the form given in (9) is valid down to conventional accelerator energies, then the value of A can be seen[10] to be about -3.3.

A parametrization of $f_1(x)$, which respects these three constraints and which has the shape suggested by the sum of the proton and pion spectra, is

$$f_1(x)=3.0-3.3x^{1/2}-x+1.75x^{3/2}. \tag{10}$$

This function is displayed in Fig. 1, together with data points from Refs. 6 and 8 representing $\frac{3}{2}(f_1^{\pi^+}+f_1^{\pi^-})+f_1^p$; the data points have been normalized to make $f_1(x=0)=3$.

Using $f_1(x)$ as given in (10), it is straightforward to compute numerically $g(x)$ from (2), and then to compute $f_2(R)$ from (4). The two-body correlation function $C_2(R)$ is defined by

$$C_2(R)\equiv f_2(R)-[f_1(0)]^2\ . \tag{11}$$

The calculated values of $C_2(R)$ are displayed in Fig. 2; I have checked that these values are reasonably insensitive to the precise form of $f_1(x)$ assumed. The predicted values for the fractional correlation, which is defined as $(f_2-f_1^2)/f_1^2$, can be obtained by dividing $C_2(R)$ by nine.

In models which exhibit short-range correla-

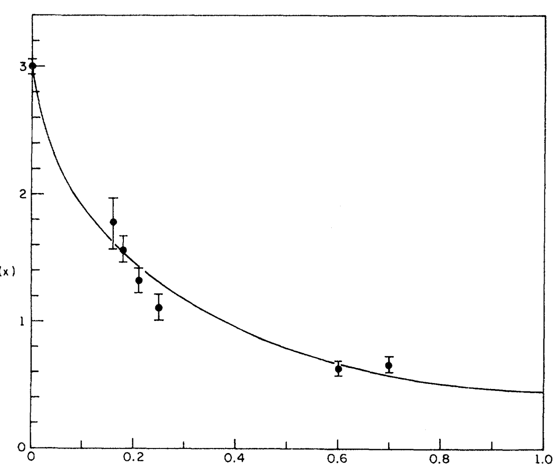

FIG. 1. The single-particle distribution from p-p scattering. The data points (Refs. 6 and 8) represent the proton distribution plus three times the average of the π^+ and π^- distribution, normalized to the value 3 at $x=0$. The solid line is the parametrization used in the calculation of the two-particle distribution.

tions, the integrated two-particle correlation function $\langle C_2 \rangle$ grows logarithmically with s:

$$\langle C_2 \rangle \to C_2 \ln s. \qquad (12)$$

The constant C_2 is given in terms of $C_2(R)$ by

$$C_2 = 2 \int_1^\infty \frac{dR}{R} C_2(R) , \qquad (13)$$

and, in the calculation reported here, has a numerical value of about 7.6; thus we predict that

$$\langle C_2 \rangle \approx 2.5 \langle n \rangle . \qquad (14)$$

At large values of R, $C_2(R)$ behaves as

$$C_2(R) \to (\text{const}) \times R^{-1/2}. \qquad (15)$$

The form given in (15) corresponds to a correlation length of $\frac{1}{2}$; the calculated value of the constant in (15) is about 2.2. In the approximation suggested by Frazer *et al.*,[11] which is to use the asymptotic form given in (15) for all R in order to calculate C_2 through Eq. (13), we would have obtained $C_2 = 8.8$, which is about 16% higher than the actual value of the integral in (13).

Data on the two-particle correlation in the central region are now becoming available from the ISR; we consider in particular the preliminary data of the Pisa-Stony Brook collaboration.[12] These data show a fractional correlation[13] which is consistent with the parametrization $0.6 \times \exp(-\frac{1}{2}|y_2 - y_1|)$. This is somewhat larger than the correlation predicted by our calculation; from Eqs. (5) and (15), we would say that the coefficient of $\exp(-\frac{1}{2}|y_2 - y_1|)$ should be about $2.2 \div 9 = 0.24$. On the other hand, the fractional correlation as reported in Ref. 12 is defined with σ_{tot} [e.g., in Eq. (1)] replaced by $\sigma_{\text{inelastic}}$, while for the purpose of the present discussion we want it replaced by $\sigma_{\text{nondiffractive}}$; thus the reported correlation is expected to be larger than the one which we predict. In other words, the presence of diffractive inelastic events produces a positive long-range correlation which adds to the short-range correlation we have been discussing.

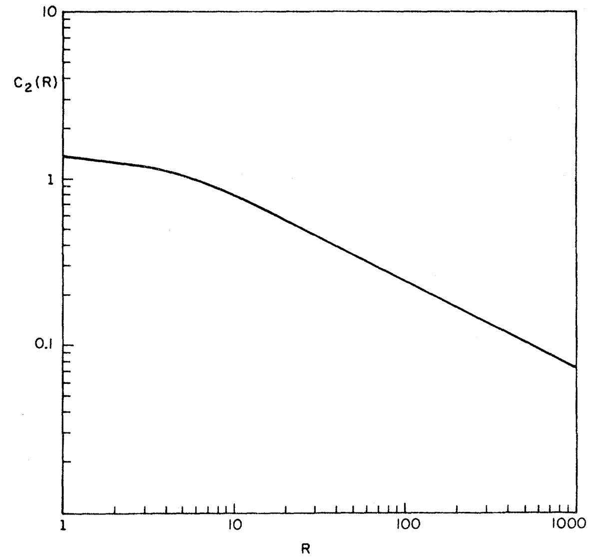

FIG. 2. The calculated two-particle correlation in the central region, as a function of $R \approx \exp(|y_2 - y_1|)$.

Although our calculation was done within a specific model, we suspect that the general features of the calculated correlation would be common to almost any multiperipheral-like model; in particular, we think it would be difficult for a model of this type which fits the single-particle spectrum to produce a fractional correlation as large as 60%. If the preliminary data[12] are correct in their suggestion that correlations are in fact this large, we should consider this to indicate a failure of this type of model as applied to *all* inelastic events. The question of whether or not these models can be reprieved by being applied to a subset of inelastic events is left for future investigation.

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[1]For a review of models for high-energy production processes, see M. Jacob, rapporteur's talk in *Proceedings of the Sixteenth International Conference on High Energy Physics, National Accelerator Laboratory, Batavia, Ill., 1972*, edited by J. D. Jackson and A. Roberts (NAL, Batavia, Ill., 1973), Vol. 3, p. 373.

[2]J. Finkelstein and R. D. Peccei, Phys. Rev. D **6**, 2606 (1972).

[3]A. Krzywicki and B. Petersson, Phys. Rev. D **6**, 924 (1972); B. Petersson, in *Proceedings of the Seventh Rencontre de Moriond*, edited by J. Tran Thanh Van (CNRS, Paris, 1972).

[4]R. Kronenfeld and R. D. Peccei, Phys. Rev. D **7**, 1556 (1973).

[5]M. Le Bellac, Phys. Lett. **37B**, 413 (1971).

[6]M. Banner *et al.*, Phys. Lett. **41B**, 547 (1972); M. G. Albrow *et al.*, Phys. Lett. **42B**, 279 (1972); A. Bertin *et al.*, Phys. Lett. **42B**, 493 (1972).

[7]M. G. Albrow *et al.*, contribution to the Sixteenth International Conference on High Energy Physics, National Accelerator Laboratory, Batavia, Ill., 1972, and discussed in Ref. 1.

[8]J. V. Allaby *et al.*, CERN Report No. CERN 70-12, 1970 (unpublished).

[9]R. C. Brower and J. Ellis, Phys. Rev. D **5**, 2253 (1972). We take the intercept of secondary Regge trajectories to be 1/2 .

[10]Compare the analysis of T. Ferbel, Phys. Rev. Lett. **29**, 448 (1972).
[11]W. R. Frazer, R. D. Peccei, S. S. Pinsky, and C.-I Tan, Phys. Rev. D **7**, 2647 (1973).
[12]Pisa-Stony Brook collaboration, contribution to the Sixteenth International Conference on High Energy Physics, National Accelerator Laboratory, Batavia, Ill., 1972, and discussed in Ref. 1.
[13]The data of Ref. 12 are for charged correlations; however, as long as most of the produced particles are pions, we expect the fractional correlation in the central region for all particles to be the same as that for charged particles, except possibly for very small rapidity separation. This is because the middle of the three Reggeons in the Mueller diagram for the two-particle distribution has isospin zero or one; the isospin-one part cancels when one sums π^+ and π^-, while the isospin zero couples equally to π^+, π^-, and π^0.

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Magnetic-Transition Form Factor of the Δ(1236) in a Veneziano-Type Representation

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A Veneziano-type expression is proposed for the magnetic-transition form factor of the Δ(1236). The agreement with present experimental data up to $-t = 2.34$ (GeV/c)^2 is found to be quite satisfactory.

In the past few years many efforts have been devoted to the study of Veneziano three-point functions. As a result of this it is now established that for a photon-hadron-hadron vertex a typical form factor can be represented by

$$F(t)=C\,\frac{\Gamma(1-\alpha_\rho(t))}{\Gamma(n-\alpha_\rho(t))}\,, \tag{1}$$

where $\alpha_\rho(t)\sim 0.5+t$ is the ρ-meson Regge trajectory, C a normalization factor, and n a half-integer number that determines the correct asymptotic behavior of $F(t)$. Very satisfactory agreement with the experimental data has been obtained in the case of the pion,[1] nucleon,[2] K_{l3} decay,[3] and kaon[4] form factors. Moreover, it has been shown recently[5] that a Veneziano-type $NN\pi$ vertex function can account, in order of magnitude, for the corrections to the Goldberger-Treiman relation. Finally, we mention that an expression like Eq. (1) has been obtained for the pion electromagnetic form factor as a possible solution of the Omnés equation when the Veneziano formula for $\pi\pi$ scattering is used as an input.[6]

The physical picture behind Eq. (1) is indeed quite simple and appealing, i.e., the form factor is built up from the contribution of an infinite number of equally spaced poles. In other words the photon couples not only to the ρ meson but to all its (infinite) daughters. In a way this is like a generalization of the vector-meson-dominance model (ρ dominance). The recent discovery of the ρ' meson[7] lends further support to the above-mentioned picture.

The purpose of the present paper is to show that an expression like Eq. (1) can also represent quite well the Δ(1236) magnetic-transition form factor $G_M^*(t)$.

The available data[8-11] for $G_M^*(t)$ go up to $t=-2.34$ (GeV/c)^2 and there are more than 30 points. The most popular expression that fits the data well is the Gutbrod-Simon formula[12]

$$\frac{G_M^*(t)}{G_M^*(0)}=\frac{G_M^N(t)}{G_M^N(0)}\gamma_N\left(\frac{1}{1-t/\Omega_1^2}\right)+F_\pi(t)\gamma_\pi\left(\frac{1}{1-t/\Omega_2^2}\right)\,, \tag{2}$$

where $G_M^N(t)$ is the magnetic form factor of the nucleon, $F_\pi(t)$ is the pion form factor, and γ_N, γ_π, Ω_1^2 and Ω_2^2 are adjustable parameters. Using the dipole fit for $G_M(t)$, and for $F_\pi(t)$ the expression

$$F_\pi(t)=\frac{1}{1-t/m_\rho^2}\,,$$

and $\gamma_N=0.85$, $\gamma_\pi=0.15$, $\Omega_1^2=2.72$ (GeV/c)^2, and $\Omega_2^2=0.97$ (GeV/c)^2, a quite good agreement with experiment is obtained. As can be seen from Eq. (2), $G_M^*(t)$ falls faster than $G_M^N(t)$ and $F_\pi(t)$ by one power of t.

The starting point of our model is the Veneziano-type expression for[2] $G_M(t)$

$$G_M(t)=C\,\frac{\Gamma(1-\alpha_\rho(t))}{\Gamma(\frac{7}{2}-\alpha_\rho(t))}\,, \tag{3}$$

which fits the present data very well up to $t=-25$ (GeV/c)^2.