Transverse-Momentum Distribution in Inclusive Reactions and the Relationship Between the Diffractive Excitation Model and the Thermodynamic Model*

Earl M. Gordon
Institute of Theoretical Science, University of Oregon, Eugene, Oregon 97403
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It is shown that the choice of a Planck distribution for the cluster-decay function in the diffractive excitation model gives good agreement with the inclusive transverse-momentum spectrum of pions in pp collisions. Since this kind of distribution is associated with the thermodynamic model, the relationship between the two models is examined.

Recently, the diffractive excitation model (DEM) has proved very useful in explaining the shapes of the longitudinal-momentum spectrum in inclusive reactions.[1-3] In this model the produced particles are decay products of diffractively excited clusters. Although the DEM specifies the production mechanism of the clusters fairly well, the exact form of the cluster decay is not uniquely specified and the shapes of the longitudinal-momentum spectra are not particularly sensitive to it. The important factor is that the cluster-decay products have a fixed average energy in the cluster frame, and are isotropically distributed about the cluster center with a width determined by this average cluster-frame energy. The transverse-momentum distribution, on the other hand, is sensitive to the nature of the cluster decay, and it is from this distribution that more can be learned about the form of the cluster-decay function.

One of the popular choices for the cluster-decay function has been a Gaussian.[1] Unfortunately, although this leads to good agreement with the shape of the longitudinal-momentum spectra, the transverse spectra fall off much too quickly.[4] Other choices for the cluster-decay function are an exponential[5] and a sum of two Gaussians.[6] The purpose of this note is to propose that a Planck distribution might be a more suitable choice than any of the above. Since such a function is used in the thermodynamic model (TM), the relationship of the latter to the DEM is examined, and the similarities between the two are pointed out.

The motivation for choosing a Planck distribution comes from a measurement[7] of the distribution of pions in the rest frames of systems of particles, which consist of $N\pi$, $N\pi\pi$, and $N\pi\pi\pi$ clusters. The results, shown in Fig. 1, indicate that the pions roughly follow a Planck distribution (with $T=130$ MeV) approximately independently of the number of particles emitted. This distribution can be used to predict, with no free parameters, the shape of the inclusive transverse-momentum spectrum of pions in pp collisions. Of course, the longitudinal spectrum is also reproduced correctly, but since its shape is not so sensitive to the cluster-decay function, it is not discussed further. Only π^+ production is discussed here but equally good results can be obtained for π^- production.

At asymptotic energies, the inclusive π^+ spectrum in the DEM is given by[1-4]

$$E^*\frac{d^3\sigma}{d\vec{p}^{*3}}\xrightarrow[s\to\infty]{} x\,\frac{d^3\sigma}{dx\,d^2p_\perp} = \int_{M_1^0}^{\infty} dM_1\,\frac{d\sigma}{dM_1}\,xg(\vec{p})\frac{dp_\parallel}{dx}\,n_{\pi^+}(M_1), \tag{1}$$

where $x=2p_\parallel^*/\sqrt{s}$ and where starred quantities are in the c.m. frame and unstarred quantities are in the cluster frame. The relationship between the longitudinal momenta in the two frames is given by

$$p_\parallel=\tfrac{1}{2}(xM_1-\mu_\perp^2/xM_1), \tag{2}$$

where $\mu_\perp^2=\mu^2+p_\perp^2$. The number of π^+'s in a cluster of mass M_1 is[4] $n_{\pi^+}(M_1)=\frac{1}{3}(n+1)$, where $n=(M_1-\overline{E}_p)/\overline{E}_\pi$ is the number of pions of any charge in the cluster. $\overline{E}_p$ and $\overline{E}_\pi$ are the average proton and pion cluster-frame energies, taken to be 1.0 GeV and 0.45 GeV, respectively. In the DEM, $d\sigma/dM_1\propto M_1^{-2}$ for large M_1 and this behavior may be expected from duality to be valid in an average sense right down into the resonance region. The value of M_1^0 is taken to be 1.5 GeV corresponding to the lowest-mass resonance that can be diffractively excited and whose decay products can include a π^+. Finally, we take

$$g(\vec{p})\propto\frac{1}{\exp[(\vec{p}^{\,2}+\mu^2)^{1/2}/T]-1} \tag{3}$$

and use $T=130$ MeV as obtained from Fig. 1. The results of the calculation are shown in Fig. 2.

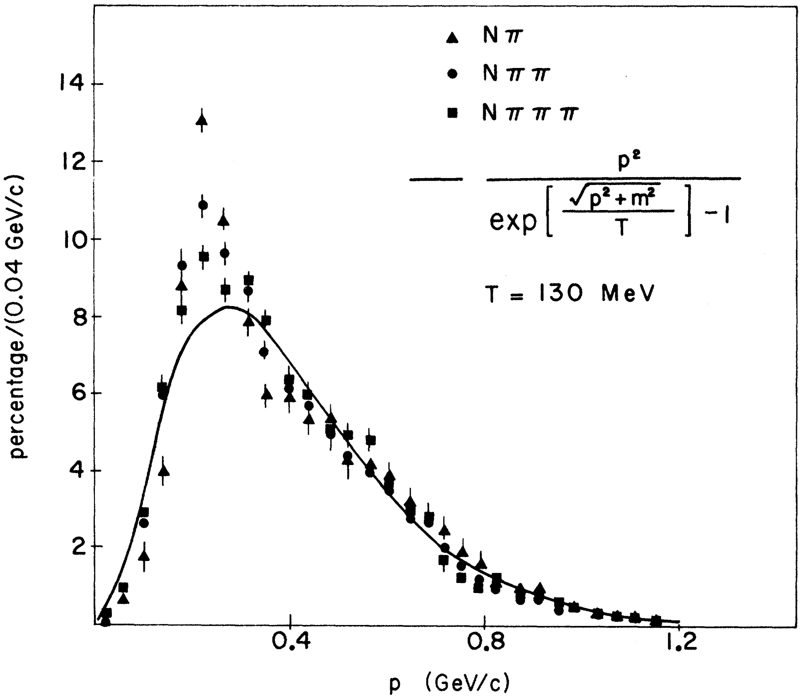

FIG. 1. Pion momentum spectrum in the rest frame of systems of particles consisting of $N\pi$, $N\pi\pi$, and $N\pi\pi\pi$. Solid line is Planck distribution.

The over-all normalization has been chosen to fit the data.[8] The decrease with $p_\perp$ is well reproduced over the entire range of the data. The additional peaking at small x is a natural feature of the model, as a consequence of the $p_\perp$ dependence in Eq. (2). This gives rise to the well-known seagull effect since the value of $\langle p_\perp\rangle$ is smaller at small x than at medium x. This seagull effect is over and above that caused by the usual E^{-1} phase-space factor.[9]

Since the cluster-decay function used here [Eq. (3)] is usually associated with the TM, we feel it is worthwhile to clearly point out the correspondence between the two models. The comparison is made at asymptotic energies where the kinematics is simplest. We consider the most recent version of the TM, namely the strong thermodynamic bootstrap model.[10,11] In this model the inclusive spectrum is given by an integral over fireballs moving with different velocity parameters, λ. At infinite energy, the spectrum for $x > 0$ receives contributions only from forward moving fireballs ($\lambda > 0$):

$$E^*\frac{d^3\sigma}{d\vec{p}^{*3}} = \int_0^1 d\lambda\, F(\lambda) q(\lambda) E f(E), \qquad (4)$$

$$\lambda = \frac{\gamma - 1}{\gamma_0 - 1},$$

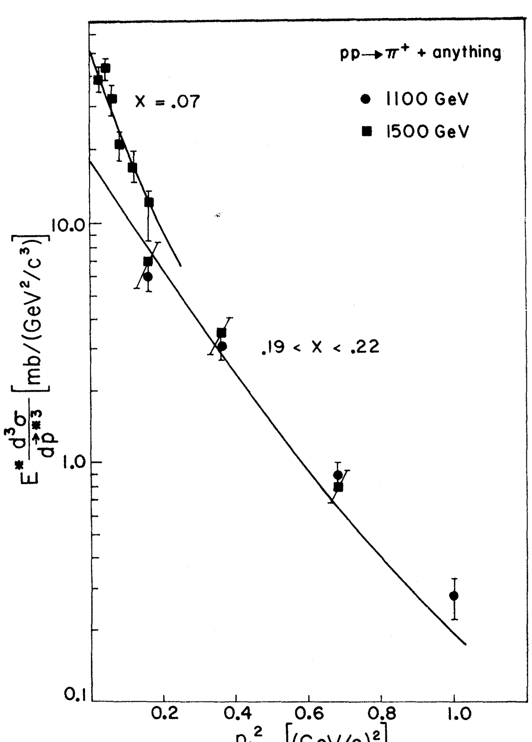

FIG. 2. Predicted transverse-momentum spectrum in the reaction $pp \to \pi^+ +$anything. For the lower curve, data are for $0.19 \le x \le 0.22$ and solid line is for $x = 0.205$.

where γ and γ_0 are the Lorentz parameters of the fireball and incoming particle in the c.m. system. $F(\lambda)$ is an empirical fireball velocity distribution, $f(E)$ is the Planck distribution, and $q(\lambda)$ is called the decay-chain multiplicity. The latter is a linear function of M_F, the maximum mass kinematically allowed for a fireball moving with velocity λ. Clearly a fireball moving with velocity λ will have mass M_F, if it is the only forward moving fireball. Most applications of the TM have assumed only single fireball production. If E_F^* is the c.m. fireball energy and $E^*_{\text{c.m.}}$ is the c.m. energy of the incoming proton then

$$E_F^* = \gamma M_F ,$$

$$E^*_{\text{c.m.}} = \gamma_0 M_p ,$$

and

$$\lambda \xrightarrow[M_F^2 \ll s]{s\to\infty} \frac{M_p}{M_F} . \tag{5}$$

Equation (4) can then be written as follows:

$$E^* \frac{d^3\sigma}{d\vec{p}^{*3}} = \int_{M_p}^{\infty} M_p \frac{dM_F}{M_F^2} q(M_F) E f(E) F\left(\frac{M_p}{M_F}\right). \tag{6}$$

Since from Eq. (2) $x\,dp_{\parallel}/dx = E$, the correspondence between the DEM and TM is obvious: $n(M_1)$ corresponds to $q(M_F)$, and most importantly

$$\frac{d\sigma}{dM_1} \leftrightarrow \frac{1}{M_F^2} F\left(\frac{M_p}{M_F}\right).$$

In the TM F is arbitrary, except that as $M_F \to \infty$, $F(M_p/M_F) \to$ const. Different applications have involved widely different functions.[10-12] In the DEM, $d\sigma/dM_1$ is closely specified. It behaves as M_1^{-2} from large values of M_1 right down into the resonance region, where this behavior is true in an average sense. It is sharply damped for $M_1 < M_1^0$, where M_1^0 corresponds to the mass of the lightest resonance that can be diffractively excited. The shape of inclusive spectra depends crucially on M_1^0 and it is the different values for this quantity that successfully account for the different shapes of pion spectra in pp, Kp, πp, and γp collisions.[1,2] The small fireball mass region is equally important in the TM but it is precisely in this region where least is known about $F(M_p/M_F)$.

We conclude, therefore, that recent applications of the TM are very close in spirit to the DEM. Whereas the TM may suggest an improved specification for the cluster decay (i.e., the use of a Planck distribution), we feel that the DEM specifies the cluster production mechanism reliably, while the TM specification is largely arbitrary. Combining the best features of both gives good results (with no free parameters) for $pp \to \pi^+ X$ and we may hope that this can be extended to the more difficult problem of predicting (with no free parameters) the magnitudes of particle production ratios.

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