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[11] This implies, in particular, $\hat{\sigma}(1,b)\equiv 0$.

[12] ... or, at least, the first few terms of it are canonical.

[13] As an example of what we have in mind, consider

$$\sigma(a,b)\propto\theta(a)\,\theta(1-b^2)e^{-a/\mu^2}(1-b^2)^4(1-a/\mu^2).$$

Correspondingly, we have, in the constant-s limit,

$$\begin{aligned}W_2(q^2\to-\infty,s)\sim(q^2)^{-4}\{[(s-M^2)/\mu^2]^3+\cdots&\\ -29+[6(s-M^2)/\mu^2+24]&\\ \times e^{-(s-M^2)/\mu^2}\}&,\end{aligned}$$

$$I_5(q^2\to-\infty,s)\sim(q^2)^{-4}\{[(s-M^2)/\mu^2]^3+\cdots-29\}$$

and, indeed,

$$W_2(q^2,s)\underset{\text{const}-s}{\sim}I_n(q^2,s),\quad s>\mu^2,\ n\geq 5$$

consistently with (6). Notice that the factor $(1-a/\mu^2)$ is put in σ to make $\hat{\sigma}(1,b)=0$, thereby ensuring the right scaling. The threshold behavior of the corresponding scaling function is $F_2(\omega\to 1-)\sim(1-\omega)^3$ and thus saturates the bound (9). The bound (10) is also saturated. Needless to say, since the structure function generated by this choice of σ does not contain any resonances, this is not an example for our discussion of the behavior of W_2 in the region of transition between resonance dominance and light-cone dominance.

[14] Notice that, in Eq. (10), $m=n$ if and only if, in (9), $l=m$.

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PHYSICAL REVIEW D VOLUME 8, NUMBER 3 1 AUGUST 1973

Direct Tests for Violation of *CP* Invariance

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(Received 22 February 1973)

Two tests are proposed for directly establishing violation of *CP* invariance in neutral-kaon decay. Provided that there exist only two neutral kaons, these tests rely only on the principles of quantum theory and relativistic kinematics and, in particular, are independent of the Weisskopf-Wigner approximation made in the theory of Lee, Oehme, and Yang. One of these tests can be made using data that will soon be available and the other can be made with existing techniques.

Despite evidence for violation of *CP* invariance in the neutral-kaon system which has continued to accumulate since the discovery of the long-lived $\pi^+\pi^-$ decays,[1] the validity of this conclusion still rests upon the Weisskopf-Wigner (WW) approximation employed in the Lee-Oehme-Yang (LOY) analysis[2] of the detailed time dependence of these decays (or its equivalent in S-matrix language[3]).[4] This theory has enjoyed remarkable success, an example of which is the agreement between the value of the parameter Δm in the LOY interference term with that obtained for the K_1-K_2 mass difference in experiments having nothing to do with the violation of *CP* invariance.[5] Moreover, the agreement between values of the Wu-Yang parameters[6] as determined from pionic decays[7,8] and the leptonic charge asymmetry[9] constitutes impressive evidence for the validity of these analyses. Nevertheless, *CP* violation ought to be established by direct appeal to experiment in a manner independent of the details of the LOY theory, i.e., free of assumptions such as those entailed in the WW approximation. We seek here to eliminate the remote possibility that the long-lived 2π decays and leptonic charge asymmetry result from some anomalous (non-WW) decay law which does not require the admixture of *CP*-even and *CP*-odd states or other manifestations of *CP* violation.[10,11]

A direct test for *CP* violation has been proposed by Goldhaber and Yang,[12] but it requires observing ~10^7 decays of kaons produced by a stopped antiproton beam. This experiment has not been performed nor is one presently in the planning stage.[13] In the following we propose a direct test applicable to experiments measuring kaon decay into $\pi^+\pi^-$ with statistical accuracy comparable to that obtainable in the current CERN effort.[14] We believe that in this experiment enough events have been observed to provide a clear-cut direct test of *CP* violation. We also propose another test which is independent of the WW assumption made by LOY. This second test involves leptonic charge asymmetry but does not require *CP* self-conjugate production (as does the Goldhaber-Yang test) and appears to be within the range of existing capabilities.

Consider a K^0 created at time $t=0$ and let $|K^0\rangle$ be its state at rest. After a time t, the amplitude for this kaon to decay into a $\pi^+\pi^-$ pair with respective momenta $\vec{k}$ and $-\vec{k}$ is $\langle\pi^+(\vec{k})\pi^-(-\vec{k})|e^{-iHt}|K^0\rangle$, where H is the exact Hamiltonian including all local interactions (strong, electromagnetic, weak, etc.). The resulting invariant intensity is

$$I^0_{+-}(t)=\frac{d}{dt}\sum_{\vec{k}}|\langle\pi^+(\vec{k})\pi^-(-\vec{k})|e^{-iHt}|K^0\rangle|^2 , \tag{1}$$

where t is the proper time. On the other hand, the $\pi^+\pi^-$ intensity resulting from the creation of a $\bar{K}^0$ at $t=0$ is

$$\bar{I}^0_{+-}(t)=\frac{d}{dt}\sum_{\vec{k}}|\langle\pi^+(\vec{k})\pi^-(-\vec{k})|e^{-iHt}|\bar{K}^0\rangle|^2 , \tag{2}$$

where $|\bar{K}^0\rangle$ is the state of a $\bar{K}^0$ at rest. Since $|\pi^+\pi^-\rangle$ is a *CP* eigenstate with eigenvalue +1 and since $CP|K^0\rangle=e^{i\alpha}|\bar{K}^0\rangle$ (where $\alpha=$ real constant, conventionally zero), the assumption of *CP* invariance (i.e., $[CP,H]=0$) implies

$$\bar{I}^0_{+-}(t)=I^0_{+-}(t) . \tag{3}$$

This states that the $\pi^+\pi^-$ intensity is the same for an initial $\bar{K}^0$ as for an initial K^0.

This exact consequence of *CP* invariance differs from the prediction of the LOY theory and *CP* violation, especially in the proper-time interval from about 8×10^{-10} to 15×10^{-10} sec where the LOY interference term contributes. Hence, we would have an ideal test for *CP* violation, except for the difficulty in obtaining a sufficiently intense initially pure K^0 beam—and, especially, an initially pure $\bar{K}^0$ beam. Usually the beam is a mixture produced incoherently (at proper time $t=0$) as K^0's and $\bar{K}^0$'s with momentum-dependent probabilities $S(p)$ and $\bar{S}(p)$, respectively. Thus the laboratory intensity is

$$I_{+-}(t,p)=S(p)I^0_{+-}(t)+\bar{S}(p)\bar{I}^0_{+-}(t) . \tag{4}$$

This well-known result follows independently of *CP* violation. As indicated, t and p (the kaon lab momentum) are independent variables in spite of the kinematical relation

$$t=zm/p \tag{5}$$

since the kaon flight distance z is not fixed (m is the kaon mass).

By combining Eq. (3) with Eq. (4) we obtain

$$I_{+-}(t,p)=[S(p)+\bar{S}(p)]I^0_{+-}(t) , \tag{6}$$

which provides us with our basic test: Assuming only two neutral kaons, *CP* invariance implies that the observed $\pi^+\pi^-$ intensity (corrected for spectrometer acceptance) is independent of momentum p except for a scale factor, $[S(p)+\bar{S}(p)]$. In contrast, the LOY theory and *CP* violation imply a nontrival momentum dependence because of the "dilution" factor[7] $[S(p)-\bar{S}(p)]/[S(p)+\bar{S}(p)]$ multiplying only the interference term.[15] The predictions differ in the proper-time interval 8×10^{-10} to 15×10^{-10} sec. We believe that the current CERN experiment[14] has detected enough events (~3×10^6) to enable one to apply this test with statistically significant results—assuming, of course, that the apparatus response function can be determined with sufficient accuracy to rule it out as a source of spurious momentum dependence. It is anticipated that $I_{+-}(t,p)$ will be determined within a few percent,[14] which is sufficient for our purpose. To avoid possible confusion, we stress that the value of $S(p)+\bar{S}(p)$ *per se* need *not* be determined to comparable accuracy. It is only the *factorization property* $I_{+-}(t,p)=f(p)g(t)$, implied by Eq. (6), that is the subject of our test.[16]

Another test involves the leptonic charge asymmetry. The intensity for, say, an initial $\bar{K}^0$ decaying into $\pi^-l^+\nu_l$ (where $l^\pm=\mu^\pm$ or $e^\pm$) is

$$\bar{I}^0_{l^+}(t)=\frac{d}{dt}\sum_{\substack{\text{final}\\ \text{momenta}}}|\langle\pi^-l^+\nu_l|e^{-iHt}|\bar{K}^0\rangle|^2 , \tag{7}$$

with similar expressions for the three other possible intensities. For a mixed kaon beam the laboratory intensities are

$$I_{l^\pm}(t,p)=S(p)I^0_{l^\pm}(t)+\bar{S}(p)\bar{I}^0_{l^\pm}(t) . \tag{8}$$

With *CP* invariance, a derivation similar to that of Eq. (3) yields

$$\bar{I}^{0}_{l\pm}(t)=I^{0}_{l\mp}(t)\ , \tag{9}$$

and the charge asymmetry parameter $\delta \equiv (I_{l^+}-I_{l^-})/(I_{l^+}+I_{l^-})$ becomes

$$\delta(t,p)=\left(\frac{S(p)-\bar{S}(p)}{S(p)+\bar{S}(p)}\right)\left(\frac{I^{0}_{l^+}(t)-\bar{I}^{0}_{l^+}(t)}{I^{0}_{l^+}(t)+\bar{I}^{0}_{l^+}(t)}\right)\ . \tag{10}$$

This furnishes us with a second test.[17] The cleanest example occurs when $\bar{S}(p)=S(p)$ as in $K^0\bar{K}^0$ production via $p\bar{p}$ annihilation when Eq. (10) implies $\delta=0$, the Goldhaber-Yang[12] result. We remark that Eq. (10) can provide a test even when $\bar{S}(p) \neq S(p)$, provided that $\bar{S}(p)/S(p)$ is not negligible compared with unity and that the flight path z can be varied sufficiently so as to render t and p effectively independent [cf. Eq. (5)].[18] For example, at fixed proper decay time t, *CP* invariance implies a momentum dependence for δ of the form exhibited in Eq. (10), whereas for times long compared with 10^{-9} sec the LOY theory and *CP* violation yield δ = constant, independent of t and p.[18] For short times, as well, the momentum dependence in Eq. (10) differs from that of the LOY theory and *CP* violation.[19]

Added note. Since submitting this paper we have learned of the experiment of Banner *et al.*[20] which intercompares the $\pi^+\pi^-$ decays resulting from initially pure K^0 ($\bar{K}^0$) states produced via charge exchange of K^+ (K^-) in a carbon target to provide direct evidence of *CP* violation. That is, they show that Eq. (3) is disobeyed. The factorization implied by our Eq. (6) affords a test even for the case of initially mixed K^0-$\bar{K}^0$ beams such as those which are obtained in the current CERN experiment.[14]

We wish to thank J. Steinberger for providing us with some preliminary data from the CERN experiment. We also thank D. G. Hill, D. A. Jensen, G. J. Lasher, I. G. Schröder, M. Schwartz, P. H. Steinberg, V. L. Telegdi, L. Wolfenstein, and G. T. Zorn for helpful conversations.

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[10] Such an explanation would require that somehow the long-lived time dependence of the *CP*-even state closely approximate that of the *CP*-odd state (an unlikely accident).

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[14] J. Steinberger (private communication). The experiment is of the vacuum regeneration type reported in Ref. 7.

[15] For a 24-GeV/c primary proton beam $\bar{S}/S$ varies from 0.3 at p = 6 GeV/c to 0.08 at p = 12 GeV/c. [J. Steinberger, in *Evolution of Particle Physics, a Volume Dedicated to E. Amaldi*, edited by M. Conversi (Academic, New York, 1970), p. 268].

[16] Differences between the dilution factors in the two experiments of Ref. 7 are suggestive of a violation of the factorization property implied by our Eq. (6), but it would appear safer to test the momentum dependence of $I_{+-}(t,p)$ quantitatively in a single high-statistics experiment.

[17] Of course, within the LOY framework *CP* invariance implies $\delta = 0$, contrary to experiment, but this need not be the case generally.

[18] $\bar{S}/S$ is typically $\sim$ 1/3 in the recent SLAC experiment, where it is found that δ is a constant for variations in flight distance $\Delta z/z \sim$ 7 m/77 m and for considerably wider variations in the proper decay time. [M. Schwartz (private communication). See also R. Piccioni *et al.*, Ref. 9.].

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