*Work supported in part by the U. S. Atomic Energy Commission.

[1]H. Feshbach and E. Lomon, Phys. Rev. **102,** 891 (1956). Such an approach was originally suggested by G. Breit and W. G. Bouricius, Phys. Rev. 75, 1029 (1949).

[2]In applying this analysis to such a system one of course violates the finite-range assumption. However, neglected terms fall off like $e^{-\mu a}$ and should not be important for $a > \mu^{-1}$.

[3]See, for example, W. W. S. Au and E. L. Lomon, Phys. Lett. **4,** 327 (1963); and other references cited therein.

[4]B. Lippmann and J. Schwinger, Phys. Rev. **79,** 469 (1950).

[5]H. Pierre Noyes, Phys. Rev. Lett. **23,** 1201 (1969).

[6]For greater detail as to our conventions, see D. D. Brayshaw, Phys. Rev. Lett. **26,** 659 (1971).

[7]J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (Wiley, New York, 1952), p. 790.

[8]This follows as a simple consequence of the coordinate-space formulation of the Faddeev equations; see, for example, Ref. 5.

[9]Explicit formulas for this purpose are provided in Sec. IV.

[10]Here the sum over l' runs over as many partial waves as one believes are necessary to take into account in each channel; the sum over λ' is of course finite for fixed l' and L. Equation (12) is to be interpreted as a separate equation for each distinct set of the relevant indices $(\alpha l \lambda)$.

[11]D. D. Brayshaw, Phys. Rev. D **7,** 1835 (1973); hereafter we shall refer to this paper as SCI.

[12]Here and in what follows we use the notation of M. E. Rose in *Elementary Theory of Angular Momentum* (Wiley, New York, 1957).

[13]The derivation of this and similar expressions to follow involves standard manipulations with rotation functions similar to those employed by A. Ahmadzadeh and J. A. Tjon, Phys. Rev. **139,** B1085 (1965).

[14]In practice one may easily construct $\overline{Q}$ analytically; a general formula is neither necessary nor illuminating.

[15]The $\overline{N}$ portion of our kernel is essentially identical in structure to kernels which arise in the potential formulation of the three-body problem under the assumption of separable interactions. In particular, the denominator $(q^2 - Q_{\alpha\beta}{}^2)$ in Eq. (37) can vanish for $W > 0$, necessitating contour deformation methods of the type developed by J. H. Hetherington and L. H. Schick, Phys. Rev. **137,** B935 (1965); and extended by R. Aaron and R. D. Amado, Phys. Rev. **150,** 857 (1966), to the type of calculations we have in mind.

[16]The current status of this work has been summarized by I. Šlaus in *Few Particle Problems in the Nuclear Interaction* (North-Holland, Amsterdam, 1972); earlier references are cited therein.

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How to Avoid $\Delta Y = 1$ Neutral Currents*

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The problem is posed of exhibiting a mechanism that avoids $\Delta Y = 1$ neutral currents without invoking experimentally unknown types of particles. The proposed solution rejects the Cabibbo rotation in favor of a mixing, between two types of unit-spin mesons, that is produced by the SU_3-symmetry-breaking interaction. One quantitative prediction that is well satisfied is the identity of the strong-interaction coupling constants appearing in $\pi^{\pm}$ decay and in $\rho^0 \to e^+ + e^-$.

Unified theories of electromagnetic and weak interactions generally face a problem with hadronic neutral currents that change hypercharge. Such currents are strikingly suppressed in nature, but are usually implied by the Cabibbo rotation that introduces the $\Delta Y = 1$ charged currents. This has led to several suggestions, of varying degrees of charm, which are uniformly couched in the language of hypothetical subnuclear constituents.[1] The number of the latter has thereby been increased, from three, to four, five, seven, ... The phenomenological orientation of source theory[2] invites a more conservative attempt. Can one exhibit a mechanism for avoiding unwanted neutral currents that refers only to experimentally recognized types of particles? This note sketches an affirmative answer.

First we must review the archetypal treatment of the leptons.[3] These particles are grouped into leptonic charge triplets,[4] $L = +1$: μ^+, ν, e^-, and the chiral charge-bearing currents represented by

$$j^{\mu}_{ab} = \tfrac{1}{2}\psi\gamma^0\gamma^{\mu} T_{ab}\psi\,, \quad ab = 12, 21\,. \tag{1}$$

Here we have introduced the antisymmetrical matrices

$$T_{ab} = \tfrac{1}{2}\left(t_{ab} + i\gamma_5\{t_3, t_{ab}\}\right), \quad ab = 12, 21 \tag{2}$$

where[5]

$$\sqrt{2}\, t_{12} = t_1 + it_2\,, \quad \sqrt{2}\, t_{21} = t_1 - it_2\,, \tag{3}$$

and the t_a, $a = 1, 2, 3$ are the 3×3 imaginary, antisymmetrical matrices of unit isotopic spin. The T matrices obey the commutation relations of the group U_2, as illustrated by

$$[T_{12}, T_{21}] = T_{11} - T_{22}, \tag{4}$$

in which

$$T_{11} = t_3 \tag{5}$$

is the electric charge matrix and

$$T_{11} + T_{22} = i\gamma_5 + \tfrac{3}{2} t_3 (1 - i\gamma_5 t_3) \tag{6}$$

commutes with all the T matrices. The simplest dynamical hypothesis introduces a U_2-invariant coupling with four-vector fields A^μ_{ab}, a, $b=1,2$:

$$e \sum_{a,b=1,2} A^\mu_{ab} j_{\mu ba}, \tag{7}$$

where the association between particles and fields is $A_{11} \leftrightarrow \gamma$, $A_{12,21} \leftrightarrow W^\pm$, $A_{22} \leftrightarrow Z$. The U_2 invariance is a partial symmetry that is broken by the large masses assigned to the particles W and Z (as discussed in Ref. 3).

The coupling between the vector fields A^μ_{ab} and the hadrons is pictured as proceeding through the intermediary of fields associated with the nonuplets of $1^+(0^-)$ and 1^- particles. The fields of such particles are conveniently represented by spinors of the second rank,

$$\psi_{\zeta\zeta'} \sim \psi_\zeta \psi_{\zeta'}, \quad \zeta, \zeta' = 1, \ldots, 4. \tag{8}$$

We emphasize that the factorization into individual spinors indicated here is purely symbolic, designed to facilitate contact with the leptonic structures; it carries *no* implication concerning the compositeness of the particles. Nonuplets of vector and axial-vector fields are thus symbolized by

$$\begin{aligned} v^\mu_{\alpha\beta} &\sim \tfrac{1}{2}\psi\gamma^0\gamma^\mu t_{\alpha\beta}\psi, \\ a^\mu_{\alpha\beta} &\sim \tfrac{1}{2}\psi\gamma^0\gamma^\mu i\gamma_5 t_{\alpha\beta}\nu\psi, \quad \alpha, \beta = 1, 2, 3 \end{aligned} \tag{9}$$

in which the $t_{\alpha\beta}$ are antisymmetrical matrices obeying U_3 group commutation relations,

$$[t_{\alpha\beta}, t_{\gamma\delta}] = \delta_{\beta\gamma} t_{\alpha\delta} - \delta_{\alpha\delta} t_{\gamma\beta}, \tag{10}$$

together with

$$\sum_{\alpha=1}^{3} t_{\alpha\alpha} = \nu, \tag{11}$$

and ν is a 2×2 antisymmetrical imaginary matrix representing a unit nucleonic charge. The explicit structure of the $t_{\alpha\beta}$ is given by

$$t_{\alpha\beta} = \tfrac{1}{2}(1+\nu)\tau_{\alpha\beta} - \tfrac{1}{2}(1-\nu)\tau_{\beta\alpha}, \tag{12}$$

where the $\tau_{\alpha\beta}$ are the elementary 3×3 matrices with a single unit entry in the α row, β column. They have the multiplication property

$$\tau_{\alpha\beta}\tau_{\gamma\delta} = \delta_{\beta\gamma}\tau_{\alpha\delta}. \tag{13}$$

The two symbolic fields appearing in the construction (9) are associated with opposite nucleonic charge, which is the counterpart of the more usual procedure where the two unitary indices α, β are assigned to inequivalent, complex conjugate representations. In connection with nucleonic charge, we note the relation between the generators $T_{\alpha\beta}$ of the group U_3, where

$$N = \tfrac{1}{3}\sum_{\alpha=1}^{3} T_{\alpha\alpha} \tag{14}$$

represents nucleonic charge, and those of the reduced group SU_3 ($T'_{\alpha\beta}$) from which the nucleonic charge concept has been deleted. It is

$$T'_{\alpha\beta} = T_{\alpha\beta} - \delta_{\alpha\beta} N. \tag{15}$$

Also, the U_2 strong-interaction group having generators that combine isotopic spin and hypercharge, electric charge in particular, is identified with the corresponding subgroup of SU_3, so that

$$Q = T'_{11} = T_{11} - N. \tag{16}$$

As an aspect of this relation we expect that 1^- fields coupled to the photon are of the form

$$v^\mu_{11} - \tfrac{1}{3}\sum_{\alpha=1}^{3} v^\mu_{\alpha\alpha} \sim \tfrac{1}{2}\psi\gamma^0\gamma^\mu(t_{11} - \tfrac{1}{3}\nu)\psi. \tag{17}$$

The problem in constructing hadronic couplings on the leptonic model begins with the choice of charge-bearing currents analogous to (1, 2), since the electric charge axis of the unitary space could be combined with either of the other two axes. Leaving that decision open for the moment, we introduce analogs of the matrices (2) as ($\beta=2$ or 3)

$$\begin{aligned} T_{12} &= \tfrac{1}{2}(t_{1\beta} + i\gamma_5\{t_{11}, t_{1\beta}\}) \\ &= \tfrac{1}{2}(1 + i\gamma_5\nu)t_{1\beta}, \\ T_{21} &= \tfrac{1}{2}(t_{\beta 1} + i\gamma_5\{t_{11}, t_{\beta 1}\}) \\ &= \tfrac{1}{2}(1 + i\gamma_5\nu)t_{\beta 1}, \end{aligned} \tag{18}$$

which uses the equivalence illustrated by

$$\begin{aligned} \{t_{11}, t_{1\beta}\} &= \tfrac{1}{2}(1+\nu)\tau_{1\beta} + (1-\nu)\tau_{\beta 1} \\ &= \nu t_{1\beta}. \end{aligned} \tag{19}$$

Now we evaluate the commutator

$$\begin{aligned} [T_{12}, T_{21}] &= \tfrac{1}{2}(1 + i\gamma_5\nu)(t_{11} - t_{\beta\beta}) \\ &= T_{11} - T_{22}, \end{aligned} \tag{20}$$

where

$$\begin{aligned} T_{11} &= t_{11} - \tfrac{1}{3}\nu, \\ T_{22} &= \tfrac{1}{2}(1 - i\gamma_5\nu)t_{11} + \tfrac{1}{2}(1 + i\gamma_5\nu)t_{\beta\beta} - \tfrac{1}{3}\nu, \end{aligned} \tag{21}$$

and

$$T_{11} + T_{22} = (1 - i\gamma_5\nu)t_{11} + \tfrac{1}{2}(1 + i\gamma_5\nu)(t_{11} + t_{\beta\beta}) - \tfrac{2}{3}\nu. \tag{22}$$

The latter matrix commutes with all the T matrices, specifically, because

$$[t_{1\beta}, t_{11}+t_{\beta\beta}]=[t_{\beta 1}, t_{11}+t_{\beta\beta}] = 0 \tag{23}$$

and

$$(1-i\gamma_5\nu)(1+i\gamma_5\nu)=0\,. \tag{24}$$

Note that all four of the currents obtained in this way can be represented as linear combinations of the v and a fields.

The conventional response given to the problem of choosing between the unitary axes 2 and 3 is that the weak interactions select a particular direction in the 23 plane, one that is inclined at a fairly small angle $\theta_C \sim 0.2$ relative to the second axis (Cabibbo rotation). But there is another possibility. Perhaps nature utilizes both axes, with the respective currents constructed as different combinations of $1^\pm$ fields, which combinations also reflect the mixing action of the strong SU_3-symmetry-breaking interactions. To explore this idea, we place an appropriate superscript on the hadronic fields to distinguish the choice of $\beta=2$ or 3, and write out the U_2-invariant coupling, apart from a common factor, as

$$\begin{aligned}A_{11}&[(v_{11}-\tfrac{1}{3}\textstyle\sum v_{\alpha\alpha})^{(2)}+(v_{11}-\tfrac{1}{3}\sum v_{\alpha\alpha})^{(3)}]+A_{21}[\tfrac{1}{2}(v_{12}+a_{12})^{(2)}+\tfrac{1}{2}(v_{13}+a_{13})^{(3)}]\\ &+A_{12}[\tfrac{1}{2}(v_{21}+a_{21})^{(2)}+\tfrac{1}{2}(v_{31}+a_{31})^{(3)}]+A_{22}[\tfrac{1}{2}(v_{11}-a_{11})^{(2)}+\tfrac{1}{2}(v_{22}+a_{22})^{(2)}-\tfrac{1}{3}\textstyle\sum v_{\alpha\alpha}^{(2)}\\ &+\tfrac{1}{2}(v_{11}-a_{11})^{(3)}+\tfrac{1}{2}(v_{33}+a_{33})^{(3)}-\tfrac{1}{3}\textstyle\sum v_{\alpha\alpha}^{(3)}]\,,\end{aligned} \tag{25}$$

where vector indices are suppressed.

As the simplest realization of the fields $v^{(2)}, a^{(2)}$ and $v^{(3)}, a^{(3)}$, we consider just two sets of particle fields v, a and v', a', which are mixed together by the strong symmetry-breaking interaction. A crude picture of the latter will be based on the situation encountered in the well-established 1^- nonuplet, where ρ and ω are approximately degenerate in mass, while K^* and ϕ are displaced upward. Thus, field components with one or two 3-indices are perturbed, and mixed:

$$\begin{aligned}&v_{13}^{(2)}=v_{13}\cos\theta_3-v_{13}'\sin\theta_3\,,\quad v_{13}^{(3)}=v_{13}\sin\theta_3+v_{13}'\cos\theta_3\,,\\ &v_{33}^{(2)}=v_{33}\cos\theta_{33}-v_{33}'\sin\theta_{33}\,,\quad v_{33}^{(3)}=v_{33}\sin\theta_{33}+v_{33}'\cos\theta_{33}\,,\end{aligned} \tag{26}$$

while the other components are identified directly with distinct particle fields, as illustrated by

$$v_{11}^{(2)}=v_{11}\,,\quad v_{11}^{(3)}=v_{11}'\,,\quad v_{12}^{(2)}=v_{12}\,. \tag{27}$$

Our rough view of strong interaction effects also assumes the same mixing angles for 1^+ and 1^- fields. To avoid unessential complications, we do not include the mixing described by θ_{33} in stating the resulting form of the coupling (and write $\theta_3=\theta$),

$$\begin{aligned}A_{11}&[v_{11}-\tfrac{1}{3}\textstyle\sum v_{\alpha\alpha}+v_{11}'-\tfrac{1}{3}\sum v_{\alpha\alpha}']+A_{21}[\tfrac{1}{2}(v_{12}+a_{12})+\tfrac{1}{2}(v_{13}+a_{13})\sin\theta+\tfrac{1}{2}(v_{13}'+a_{13}')\cos\theta]\\ &+A_{12}[\tfrac{1}{2}(v_{21}+a_{21})+\tfrac{1}{2}(v_{31}+a_{31})\sin\theta+\tfrac{1}{2}(v_{31}'+a_{31}')\cos\theta]\\ &+A_{22}[\tfrac{1}{2}(v_{11}-a_{11})+\tfrac{1}{2}(v_{22}+a_{22})-\tfrac{1}{3}\textstyle\sum v_{\alpha\alpha}+\tfrac{1}{2}(v_{11}'-a_{11}')+\tfrac{1}{2}(v_{33}'+a_{33}')-\tfrac{1}{3}\sum v_{\alpha\alpha}']\,.\end{aligned} \tag{28}$$

The two kinds of hadronic fields, v, a and v', a', have been distinguished through their (somewhat mixed) roles in mediating the weak interactions. Now we add a final assumption concerning their disparate roles in strong interactions. It is that the fields v', a' are only slightly coupled to the quasistable hadrons that are of interest in weak-interaction measurements. If we ignore that coupling completely, we can effectively strike out the primed fields in (28). The outcome is a coupling with the charged bosons $W^\pm$ that differs from the result of a Cabibbo rotation ($\theta \simeq \theta_C$) only in the absence of the factor $\cos\theta_C \cong 0.98$, and a coupling with the neutral boson Z that contains no hypercharge transitions (in contrast, the Cabibbo rotation would introduce the field combination $v_{23}+a_{23}+v_{32}+a_{32}$, for which $\Delta Y=1$). This is the principal consequence of our investigation.

The factor $\cos\theta_C$ is ranked as one of the minor successes of the Cabibbo theory. One would be more concerned about its absence were there not larger discrepancies outstanding in the usual theory. We cite in particular the ~10% difference between the (Goldberger-Treiman) prediction of 84 MeV for the pion decay constant and the observed value of 94 MeV. It is worth recognizing, then, that the general viewpoint advocated here can accommodate such deviations from the special model just discussed. The modifications to be introduced are twofold: The fields v, a and v', a' are replaced by linear combinations of different particle fields of the respective types; the assumption that the primed fields are completely uncoupled from the low-lying hadrons is removed.

If the primed fields give an effective baryon contribution in the γ coupling that is $\sim +2\%$ of the unprimed value, the result is essentially equivalent to introducing $\cos\theta_C$ in the $\Delta Y = 0$ part of the W coupling with baryons.[6] There are two possibilities here: The various primed particles are coupled with normal strength to the familiar hadrons but there is almost complete destructive interference among the contributions of all the particles of this type for small momentum transfer; or, every primed particle is subnormally coupled to the usual baryons (is there a connection with a recent cosmic ray observation[7] of a particle that decays into hadrons with an anomalously long lifetime?). Next, let us recall that the charge-bearing components of the a field associated with the lightest nonuplets have the following meaning in terms of fields attached to the particles A_1 and π:

$$a = A_1 + \frac{1}{m_A}\,\partial\pi\,, \quad m_A = \sqrt{2}\,m_\rho\,. \tag{29}$$

Now suppose that, both in the photon and W couplings, the effective baryon contribution is reduced by ~10% through the mediation of more massive (unprimed) families of unit spin particles. The consequence is an increase of ~10% in the π-W coupling, in comparison with the usual theory. So far, this is simply data fitting. But there is an important implication here for the value of the coupling constant g associated with the unit-spin particles; it is best inferred through the behavior of π, which is an associate member of the lightest particle family, rather than by reference to baryons, the properties of which are also influenced by other families of such particles.

To express the last point more quantitatively, let us supply (28) with the factor

$$\sqrt{2}\,\frac{e}{g}\,m_\rho^{\,2}\cos\theta_C\,. \tag{30}$$

We have introduced $\cos\theta_C$ to compensate for the additional contribution of the primed particles to the electromagnetic interactions of the baryons. The factor of $\sqrt{2}$ refers to the field identification, expressed for the usual 1^- particles, by

$$v_{12} = \rho^+, \quad v_{21} = \rho^-, \quad v_{11} - v_{22} = \sqrt{2}\,\rho^0, \tag{31}$$

having in mind that

$$v_{11} = \tfrac{1}{2}(v_{11} - v_{22}) + \tfrac{1}{2}(v_{11} + v_{22})\,. \tag{32}$$

The constant g is not to be equated to the constant g_ρ that characterizes the lightest mesons, since the heavier mesons are supposed to produce a reduction by the factor $\frac{84}{94}$:

$$g = (\tfrac{84}{94})g_\rho\,. \tag{33}$$

Now let us exhibit the electromagnetic coupling specifically associated with ρ^0:

$$(e/g)\,m_\rho^{\,2}\cos\theta_C A\rho^0\,, \tag{34}$$

and the W coupling of the pion:

$$2^{-1/2}e\cos\theta_C\left[W^-\left(\frac{m_\rho}{\sqrt{2}\,g}\right)\partial\pi^+ + W^+\left(\frac{m_\rho}{\sqrt{2}\,g}\right)\partial\pi^-\right]. \tag{35}$$

The latter displays the conventional π coupling constant for weak interactions,

$$\frac{m_\rho}{\sqrt{2}\,g} = 94\ \text{MeV}\,, \tag{36}$$

from which it follows that

$$\frac{g^2}{4\pi} = 2.7\,. \tag{37}$$

Apart from the factor $\cos\theta_C$, the same coupling constant appears in the electromagnetic interaction (34), which predicts the rate for the decay $\rho^0 \to e^+ + e^-$. A recent colliding-beam measurement[8] gives

$$\frac{1}{4\pi}\left(\frac{g}{\cos\theta_C}\right)^2 = 2.8 \pm 0.16\,, \tag{38}$$

or

$$\frac{g^2}{4\pi} = 2.7 \pm 0.15\,, \tag{39}$$

which is quite satisfactory agreement.

The value deduced for g_ρ is measured by

$$\frac{g_\rho^{\,2}}{4\pi} = 3.4\,. \tag{40}$$

It is interesting that a very similar number is obtained from the identification of low-energy s-wave πN scattering with the consequence of ρ exchange.[9] Of course, we must now reconcile this agreement with our picture in which π and N interact, not only through the exchange of ρ, but also by means of more massive particles of the same type. Since we are, at the moment, quite free to adjust unknown coupling constants and masses, there is no immediate difficulty here. As an illustration, let us suppose that the same constant g appears in coupling the various mesons to the photon and to W. In addition, imagine that the ~10% contribution associated with the heavier mesons is dominated by one particle family with a squared mass ~3 and a hadronic coupling constant $\sim\frac{1}{3}$ relative to the lightest mesons. Then the additional contribution associated with the heavier mesons in πN coupling, where squared coupling constants appear, will be only several percent.

Another application where the distinction between g and g_ρ can be significant is in the calculation of magnetic moments. We follow the discussion of Ref. 9, Sec. 3.8, where it is remarked that the predicted magnitudes of magnetic moments are some 15% too low, as illustrated by

$$\mu_p - \mu_n = \frac{5}{3}\frac{e}{m_\rho} = 4.1\,\mu_N . \tag{41}$$

The factor

$$\frac{g_\rho}{g}\cos\theta_C = 1.10 \tag{42}$$

describes a 10% increase associated with the lightest 1^- mesons. Presumably, the primed particles will produce a small additional increase, while the heavier particles of the same type as ρ^0, ω, ϕ will make a negative contribution. To the extent that the magnetic moments associated with heavier mesons have a corresponding inverse mass factor, a substantial fraction of the 10% increase could remain.

The last remarks are characteristic of the situation produced by this approach to electromagnetic and weak interactions. The quantitative predictive power of the theory is limited, but the qualitative situation seems to be improved. The application of these ideas to electromagnetic mass splittings and nonleptonic decays will be deferred to another publication.

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[1]A survey of such models is given by B. Zumino, Lectures at Cargèse Summer Institute, CERN report, 1972 (unpublished).

[2]A systematic development of source theory will be found in J. Schwinger, *Particles, Sources and Fields* (Addison-Wesley, Reading, Mass.), Vol. I (1970) and Vol. II (1973).

[3]It is also reviewed briefly in J. Schwinger, Phys. Rev. D **7**, 908 (1973).

[4]The first use of such a classification is in E. Konopinski and H. Mahmoud, Phys. Rev. **92**, 1045 (1953). However, the independent introduction of this concept of leptonic charge [J. Schwinger, Ann. Phys. (N.Y.) **2**, 407 (1957)] owed nothing to the earlier work but was a reaction to the then new situation of parity nonconservation in which helicity had become the only discrete quantum number generally attributed to the neutrino. The denial of the latter view was the later vindicated prediction of two neutrinos. See also K. Nishijima, *Fundamental Particles* (Benjamin, New York, 1963), Sec. 7-8.

[5]Note the change in definition from that used in Ref. 3.

[6]One should also note that, at the accuracy level of 1%, electromagnetic differences between ρ^0 and $\rho^\pm$ may be significant.

[7]K. Niu, E. Mikumo, and Y. Maeda, Prog. Theor. Phys. **46**, 1644 (1971).

[8]As reported by the Particle Data Group, Phys. Lett. **39B**, 1 (1972).

[9]See, for example, J. Schwinger, *Particles and Sources* (Gordon and Breach, New York, 1969), Sec. 3.3.