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Strong Interactions at Large Transverse Momentum

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The models of Blankenbecler, Brodsky, and Gunion for inclusive and exclusive reactions at large transverse momentum are examined in a covariant framework. The discussion of inclusive reactions is much less model-dependent than that of exclusive reactions, but it is argued that for both the rather specific assumptions of Blankenbecler, Brodsky, and Gunion should probably be relaxed. One can argue that, for inclusive reactions, there may be a connection between the power-law falloff in transverse momentum and the behavior of the structure functions for electroproduction and electron-positron annihilation at $\omega=1$ and $\omega=0$. The relation of large transverse-momentum inclusive amplitudes to Regge amplitudes at small transverse momentum is also discussed. For the exclusive process it is shown that a more detailed calculation suggests that the angular dependence is different from that calculated by Blankenbecler, Brodsky, and Gunion.

I. INTRODUCTION

The assumption that hadrons are in some sense composed of quarks appears to account successfully for existing experimental data on electroproduction, deep-inelastic neutrino scattering, and muon pair production.[1] For these processes, suitable well-defined mathematical assumptions enable one to make a precise identification of the dominant contributions at high energy.[2] We have applied similar ideas to elastic hadronic interactions at small momentum transfer and to total hadronic cross sections.[3] There we found it necessary to use physical rather than mathematical arguments to identify the dominant contribution. The main justification for this procedure is its results: We find a natural connection between the quark-counting rule for total cross sections, the Wu-Yang formula for the differential cross section near $t=0$, and s-channel helicity conservation for the coupling of the Pomeranchukon to the nucleon. (It is argued that each of these properties should have no more than approximate validity.)

Blankenbecler, Brodsky, and Gunion[4] (BBG) have recently extended the analysis to elastic interactions at large momentum transfer, and to inclusive reactions at large transverse momentum. These authors have again found it necessary to identify the dominant contributions by physical rather than mathematical arguments. Their calculation and assumptions are in a noncovariant formalism, and in this paper we redevelop the analysis covariantly. In doing so, we shall find it natural to use slightly different basic assumptions, and we shall also suggest that one should probably not confine oneself to so specific a model as they use.

As is by now well known, the dominant contributions to the leptonic processes mentioned above are taken to be[1,2] those having the structure shown in Fig. 1. In each case the internal lines represent off-shell quarks or antiquarks, and T is the amplitude for the hadron to emit a quark or an antiquark. Each diagram has to be cut down the middle and a complete set of intermediate states must be inserted; in the case of Fig. 1(a) this corresponds to taking the imaginary part. [In the case of muon-pair production there is a term additional to Fig. 1(b), where a Pomeranchukon is exchanged between the two bubbles. Apart from a possible logarithmic factor, this term is of the same power in the energy, but the numerical coefficient multiplying this is believed to be smaller.[1,2]] For hadronic elastic scattering at small momentum transfer t, which at $t=0$ yields also the total cross section, we supposed[3] that the main contribution comes from terms having the structure of Fig. 2. Here each hadron emits either a quark or an antiquark, moving slowly in the rest frame of its parent hadron; the relevant emission amplitudes in these circumstances are essentially the same amplitudes T as in Fig. 1. The quarks then scatter in the central amplitude, by Pomeranchukon exchange. Terms like Fig. 2 are expected to dominate only for small t, because unlike Horn and Moshe,[5] we suppose that the central amplitude goes to zero rather rapidly outside the Regge region.

BBG assume[4] that at large t the dominant contribution to elastic scattering corresponds to terms of the structure of Fig. 3, that is, one hadron emits either a quark or an antiquark and this is received directly by the other. The internal amplitudes M in Fig. 3 cannot literally be complete amplitudes T, because this would correspond to overcounting; for example, it would result in double poles in the t channel. BBG take

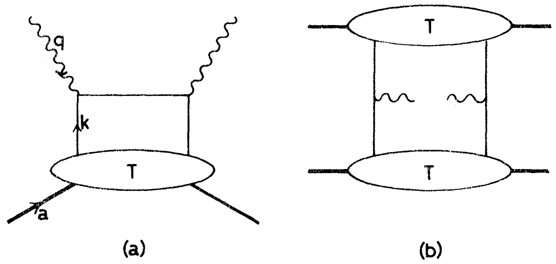

FIG. 1. Dominant contributions to deep-inelastic leptonic processes, (a) to electroproduction or neutrino scattering, and (b) to muon-pair production in pp collisions.

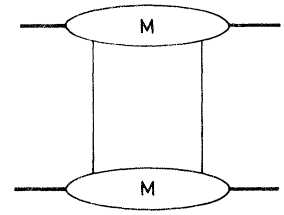

FIG. 3. Model for elastic scattering of hadrons at large momentum transfer.

a specific and simple model for these amplitudes. We discuss the evaluation of the contributions from Fig. 3 in Sec. III. Little can be said about why Fig. 3 should represent the dominant contributions, except that, with not unreasonable assumptions, they do dominate in the large-t region over other particular terms that one might study, such as Fig. 2. The main justification, or otherwise, will come from confrontation with data; BBG show that so far the position is very encouraging. [*Note added in proof.* For a reassessment of the data see P. V. Landshoff and J. C. Polkinghorne, Cambridge Report No. DAMTP 73/4 (unpublished).] However, we note that the analysis of Sec. III raises a question about the angular dependence that BBG calculate.

In this paper we consider also the inclusive reaction

$$a+b\to c+X,$$

where particle c is a pion that emerges at large transverse momentum. We discuss this in the usual way, in terms of the discontinuity of a totally-forward six-point amplitude. Our initial guess is that, in the large-transverse-momentum region, the dominant contribution corresponds to terms having the structure of Fig. 4, though as we discuss at the end of Sec. II, there is reason to suppose that it may be necessary to modify this guess and to consider a more general structure. In Fig. 4 the two central bubbles are coupling functions that couple the pion c to the quark and antiquark, while the upper and lower bubbles are the complete emission amplitudes T; here there are no double-counting problems.

With a suitable choice of pion coupling function, and a particular simple structure for T, Fig. 4 corresponds to the term that BBG assume to be dominant. We show that if one makes assumptions similar to theirs, and carries through a covariant calculation, one obtains a simple connection between this term and the structure functions of deep-inelastic lepton scattering. However, our results differ in detail from those that they obtain from old-fashioned perturbation theory in the infinite-momentum frame[6]; this is in contrast with the elastic process of Fig. 3, where the two calculations give identical results if proper account is taken of logarithmic factors.

As we shall discuss, it is quite likely that the particularly simple structure that BBG assume for T is in fact too simple, and even that a structure more general than Fig. 4 must be considered.[7] Then the connection with deep-inelastic lepton scattering is lost. There remains, however, the qualitative conclusion of BBG that at large trans-

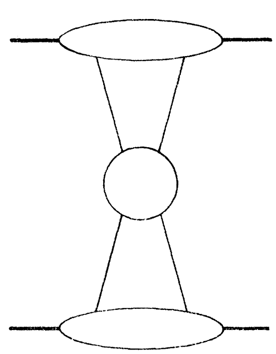

FIG. 2. Model for elastic scattering of hadrons at small momentum transfer.

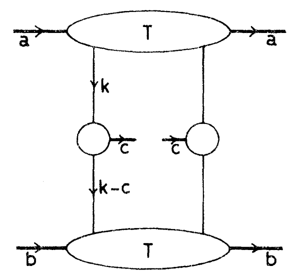

FIG. 4. Model for inclusive process $ab\to cX$ at large transverse momentum. The amplitude must be cut down the middle and intermediate states inserted. The notation for four-momenta is indicated.

verse momentum p_T one may expect to see a power-law falloff $(p_T)^{-n}$ taking over from the exponential decrease found at small p_T. It is possible that this has recently been observed at the CERN Intersecting Storage Rings.[8]

II. INCLUSIVE PROCESSES

We consider first Fig. 4, in the region where all scalar products are large:

$$\begin{aligned} &2a\cdot b = s\,,\\ &2b\cdot c = x_1 s\,,\\ &2a\cdot c = x_2 s\,,\\ &s\to\infty\,. \end{aligned} \tag{2.1}$$

Then

$$\begin{aligned} &{p_T}^2 = x_1 x_2 s\,,\\ &\cot^2(\tfrac{1}{2}\theta) = x_1/x_2\,, \end{aligned} \tag{2.2}$$

where θ is the angle at which c emerges in the center-of-mass frame.

For simplicity, we first suppose that the exchanged particles have zero spin, and pretend that the pion is scalar. We write the loop momentum in the form

$$k = ua + vb + wc + \kappa\,, \tag{2.3}$$

where κ is a one-dimensional spacelike vector orthogonal to a, b, and c. Then in the limit (2.1)

$$\int d^4k \to (x_1x_2)^{1/2}s^{3/2}\int du\,dv\,dw\,d\kappa \tag{2.4}$$

and

$$k^2 \sim s(uv + x_1vw + x_2uw) + M^2(u^2+v^2) + \mu^2w^2 - \kappa^2\,, \tag{2.5}$$

where M is the nucleon mass and μ the pion mass.

According to usual ideas,[2] both T and the pion coupling function go fairly rapidly to zero when the mass variables k^2 and $(k-c)^2$ become large. Thus, as $s\to\infty$ one expects the dominant contribution to the integral to arise from the region of integration where these variables remain finite. This is indeed so for the *amplitude* in Fig. 4, but it turns out that this leading term in the asymptotic behavior of the amplitude does not have the necessary discontinuity to describe the inclusive reaction.

Instead it is necessary to consider a region where one of the mass variables, k^2 or $(k-c)^2$, is finite and the other is large. In the first instance we shall suppose that both bubble energies, $(k-a)^2$ and $(k+b-c)^2$, are finite in the important region of the integration.

The region where k^2 is kept finite then corresponds to

$$\begin{aligned} &v = \left(\frac{x_2}{x_1 s}\right)^{1/2}\chi + \bar{y}/s\,,\\ &w = -\chi/(x_1x_2s)^{1/2}\,,\\ &u = \omega^{-1} + \frac{\bar{u}/s}{1-x_2} + \left[\left(\frac{x_1}{x_2 s}\right)^{1/2} + \left(\frac{x_1x_2}{s}\right)^{1/2}\right]\frac{\chi}{1-x_2}\,,\\ &\omega = (1-x_2)/x_1\,, \end{aligned} \tag{2.6}$$

with χ, $\bar{y}$, and $\bar{u}$ finite. In terms of these new variables

$$\begin{aligned} &k^2 \sim \omega^{-1}\bar{y} + \omega^{-2}M^2 - (\chi^2+\kappa^2)\,,\\ &(k-a)^2 \sim (\omega^{-1}-1)\bar{y} + (\omega^{-1}-1)^2M^2 - (\chi^2+\kappa^2)\,,\\ &(k+b-c)^2 \sim \bar{u} + \cdots\,,\\ &(k-c)^2 \sim -x_2\omega^{-1}s\,, \end{aligned} \tag{2.7}$$

with

$$\int d^4k \to \frac{1}{x_1\omega s}\int d\bar{u}\,d\bar{y}\,d\chi\,d\kappa\,.$$

In fact the integral over $\bar{u}$ will conveniently be replaced by an integral over $s' = (k+b-c)^2$, the only variable in which $\bar{u}$ appears. The first two of Eqs. (2.7) are exactly the same as appear in the analysis of Fig. 1(a), with ω playing the role of the usual deep-inelastic scattering variable $-2\nu/q^2$. This is the basis for the comparison made by BBG between deep-inelastic hadron scattering and deep-inelastic lepton scattering, though of course the relationship could be upset by a dependence of the pion coupling function on k^2.

As we outline in the Appendix, in Bethe-Salpeter models for the coupling function there is no such dependence when $k'^2 = (k-c)^2$ is large, so suppose in fact that at large k'^2 the pion coupling function has the asymptotic form

$$C(-k'^2)^{-\gamma_1}\,, \tag{2.8}$$

where C and γ_1 are independent of k^2. Suppose also that for large k'^2 the imaginary part of the lower bubble T has the asymptotic form

$$(-k'^2)^{-\gamma_2}f(s')\,. \tag{2.9}$$

Then altogether we have, up to a constant factor,

$$s^{-2\gamma_1-\gamma_2-1}\omega {x_1}^{-1}(x_2\omega^{-1})^{-2\gamma_1-\gamma_2}F(\omega)\int ds'\,f(s')\,, \tag{2.10}$$

where $F(\omega)$ is the contribution from the quark or antiquark k to the electroproduction structure function $F_2(\omega)$. The result (2.10) can only be valid if the integral $\int ds' f(s')$ is convergent, as it is in the particular model of BBG. Our result then is similar to that of BBG, except that in the third factor they have $x_2(\omega-1)^{-1}$ instead of $x_2\omega^{-1}$. We must add to (2.10) a term obtained by interchanging x_1 and x_2, coming from the region where $(k-c)^2$ is finite instead of k^2.

However, in a realistic model it is likely that in fact the integral $\int ds' f(s')$ diverges. Then (2.10) is not correct, the reason evidently being that the initial assumption, that only finite values of s' matter, is not correct. Thus, we must abandon the last constraint in (2.6), and instead of (2.7) we have

$$\begin{aligned} &k^2 \sim u\bar{y}+u^2M^2-(\chi^2+\kappa^2),\\ &(k-a)^2 \sim (u-1)\bar{y}+(u-1)^2M^2-(\chi^2+\kappa^2),\\ &(k+b-c)^2 \sim (1-x_2)(u-\omega^{-1})s,\\ &(k-c)^2 \sim -x_2us. \end{aligned} \tag{2.11}$$

When k'^2 and s' are both large, simple models suggest that a reasonable assumption for the behavior of $\mathrm{Im}\,T$ corresponds to a generalized scaling relation

$$\mathrm{Im}\,T(s',k'^2) \sim (-k'^2)^{-\gamma_2} s'^{\delta-1}\phi(-s'/k'^2), \tag{2.12}$$

with $\delta>0$ and ϕ a finite function such that $\phi(0)\neq 0$. [The case $\delta<0$ then corresponds to the previous calculation leading to (2.10).] In this case (2.10) is replaced by

$$s^{-2\gamma_1-\gamma_2+\delta-1}x_1^{\delta-1}x_2^{-2\gamma_1-\gamma_2} \times \int_{\omega^{-1}}^{1} du\, u^{-2\gamma_1-\gamma_2-2}(uw-1)^{\delta-1}F(u^{-1})\phi\left(\frac{x_1(u\omega-1)}{x_2u}\right). \tag{2.13}$$

We must again add to this a term obtained by interchanging x_1 and x_2.

We have shown in a recent paper[9] that the constants γ_2 and δ can be measured in high-energy electron-positron annihilation. Let $\bar{F}_2(\omega)$ be the structure function for

$$e^+e^- \to pX.$$

Then, near $\omega=0$

$$\begin{aligned} \bar{F}_2(\omega) &\sim \omega^{\gamma_2-3}, \quad \delta<0\\ &\sim \omega^{\gamma_2-\delta-3}, \quad \delta>0. \end{aligned} \tag{2.14}$$

Further, γ_2 can be measured[2,9] from the behavior of either $F_2(\omega)$ or $\bar{F}_2(\omega)$ near $\omega=1$,

$$F_2(\omega) \text{ or } \bar{F}_2(\omega) \sim (\omega-1)^{\gamma_2-1} \tag{2.15}$$

so that present electroproduction data suggest[10] that $\gamma_2=4$. There are no data yet from which we can determine the value of δ, though we have shown[9] that, in order that $F_2(\omega)$ and $\bar{F}_2(\omega)$ exist,

$$\gamma_2-\delta>1. \tag{2.16}$$

Until there are data from annihilation we can say little else about the value of δ, and in particular we cannot predict whether $\delta<0$, with the result (2.10), or $\delta>0$, with the result (2.13).

Notice that, whatever the value of δ, we may expect that both the terms (2.10) and (2.13) are present. The value of δ merely determines which of the two is the leading term. If $\delta>0$ the coefficient of the term (2.10) is no longer represented by the integral $\int ds' f(s')$, but rather by the analytic continuation in δ of this integral from the region $\delta<0$ where it is defined. Similarly, the representation given in (2.13) for the coefficient of the other term is well defined only for $\delta>0$, and for $\delta<0$ it must be obtained by analytic continuation.

It is of interest to consider the behavior of (2.10) and (2.13) for small values of x_1 or x_2. The limits $x_1=O(1/s)$ or $x_2=O(1/s)$ correspond to finite p_T fragmentation limits with the pion c a fragment of a or of b, respectively, while the limit $x_1=O(s^{-\lambda})$, $x_2=O(s^{\lambda-1})$, $0<\lambda<1$, corresponds to the finite p_T pionization limit. Hence, for these limits one might expect to obtain the powers of s that correspond to Regge limits. However, this does not have to be so, since if one considers any of the finite p_T limits directly, instead of first going to the large p_T limit, terms additional to Fig. 4 also contribute. The situation is analogous to the relation between the large ω behavior of inelastic structure functions and Regge theory.[2]

The limit $x_1=O(1/s)$ automatically gives the power $s^{\alpha_P(0)}$ in both (2.10) and (2.13), since then $\omega=O(s)$ and we know[2] that, for large ω, $F(\omega) \sim \omega^{\alpha_P(0)-1}$. In the case of (2.13) a small calculation is involved since the dominant contribution to the integral comes from the lower end point, $u\sim\omega^{-1}$, and this must be exhibited by the change of variable

$$u=\omega^{-1}+\bar{u}/s.$$

For (2.10) the other fragmentation limit, $x_2=O(1/s)$, gives the power s^{-1}, and the pionization limit does not correspond to Regge behavior in this case either. In the case of (2.13) the behavior in these limits depends on the behavior of $\phi(-s'/k'^2)$ as $-s'/k'^2\to\infty$. This is the Regge limit for $\mathrm{Im}\,T$, so that it is reasonable to assume the behavior

$$\phi(-s'/k'^2) \sim (-s'/k'^2)^{\alpha_P(0)+1-\delta} \tag{2.17}$$

to give Regge behavior in (2.12). This then leads to the behavior $s^{\alpha_P(0)}$ for (2.13) in both fragmentation limits and in the pionization limit also.

We must now consider the realistic case of spin-$\frac{1}{2}$ constituents and also take into account the pseudoscalar character of the pion. The amplitudes T now become Dirac matrices. For example, the upper amplitude can be expanded in the form

$$T = T_0 + T_1\gamma\cdot a + T_2\gamma\cdot k + T_3\sigma_{\mu\nu}k^\mu a^\nu\,. \qquad (2.18)$$

The simplest way of taking account of the pseudoscalar nature of the pion is to include a γ_5 factor with its coupling function. A trace around the quark loop must also be taken. This produces an extra factor of s from the terms T_1, T_2, and T_3 in (2.18). The general direct connection with electroproduction is then lost since $F(\omega)$ involves only T_1 and T_2. [This is to be contrasted with the apparently similar case of Fig. 1(b), where in fact neither T_0 nor T_3 terms contribute.] The Regge properties of T_1, T_2, and T_3 are such that the small-x limits retain their Regge form.

Further complications are provided by the fact that $\gamma_5\gamma\cdot k$ and $\gamma_5\gamma\cdot k'$ terms should also be included in the general covariant expansion of the pion coupling function. Further factors proportional to s then arise in the trace, though again the Regge behavior is obtained in the appropriate limits of small x_1 or small x_2.

Even in the spinless-constituent case the connection with electroproduction is not a necessary general property, for it proceeds only from our choice of the amplitude of Fig. 4. Consideration of the kinematic arguments given above shows that one should in fact consider the more general contribution Fig. 5, where the masses and subenergies indicated by arrows are large. Figure 4 corresponds to a particular disconnected contribution to the eight-point amplitude of Fig. 5. It requires a dynamical theory if one is to say that this disconnected contribution is the dominant part of the amplitude.

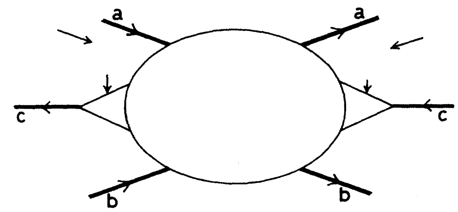

FIG. 5. A more general contribution to the inclusive process, which is also likely to be important. The arrows denote the variables that are large.

III. ELASTIC SCATTERING

The case of elastic scattering at large transverse momentum is even more model-dependent. This is no surprise, since a similar situation exists in electroproduction, where the discussion of exclusive processes[11] involves many more assumptions than that of inclusive processes.

As we have said in Sec. I, we suppose that the central bubble in Fig. 2 goes to zero rapidly outside the Regge region, so that it may be natural to assume that the diagrams of Fig. 3, where the quarks are exchanged directly, are the dominant terms in the large-momentum-transfer limit. We have explained that the two internal amplitudes M in Fig. 3 must be reduced amplitudes, to avoid double counting. It is natural to suppose that they are the same reduced amplitudes as the outer bubbles of Fig. 2, but this identification is not very helpful because in Fig. 3 the amplitudes M are evaluated in a completely different kinematic region. Here we content ourselves with assuming that in the kinematic region where they are needed they have a structure closely similar to that assumed by BBG; our object is to show that in a covariant calculation one can obtain the same result as they derive from old-fashioned perturbation theory, though the result must be modified to take account of logarithmic factors.

BBG suppose that, in the region of their variables that matters, the amplitudes M are well represented by contributions that resemble either s-channel or u-channel Born terms. The internal "particle," represented by broken lines in Fig. 6, does not have to have an associated pole, but its "propagator" is supposed to behave like s^{-1} at large s. It is called the "core" of its parent hadron. The coupling functions that describe the breakup of the hadron into core plus quark (or antiquark) are supposed to have the asymptotic form (discussed in the Appendix)

$$\int_0^1 d\mu\,\eta(\mu)[\mu k^2 + (1-\mu)k'^2]^{-\Gamma}, \qquad (3.1)$$

when both the quark momentum variable k^2 and the core momentum variable k'^2 are large. When just one of these is large, the behavior is $(k^2)^{-\Gamma}$ or $(k'^2)^{-\Gamma}$. The form of (3.1) seems to be the covariant counterpart of the assumption made by BBG about the dependence of infinite-momentum-frame wave functions on a single variable.

Notice that the assumption that the reduced amplitude M has a simple core structure does not necessarily require a similar assumption concerning the complete amplitude T, so that the results of the calculations below need bear no relation at all to those of Sec. II.

The two diagrams in Fig. 6 are related by crossing symmetry, so we concentrate on Fig. 6(a). We are concerned with the asymptotic limit

$$\begin{aligned} &2p_1\cdot p_2 \sim \nu a\,,\\ &2p_1\cdot p_3 \sim \nu b\,,\\ &2p_1\cdot p_4 \sim \nu c\,,\\ &\nu\to\infty\,,\quad a+b+c=0\,. \end{aligned} \qquad (3.2)$$

For simplicity we suppose that the quarks and the cores have zero spin. The calculation goes through in much the same way as that of Sec. II, beginning with an expansion of k as a linear combination of p_1, p_2, p_3 and a transverse momentum, as in (2.3). It turns out that the dominant contribution arises from regions of integration where one of the quark masses k^2 or $(k+p_2+p_3)^2$ is finite and the other is large, and where also one of the core masses $(k-p_1)^2$ or $(k+p_2)^2$ is finite and the other is large. For the region where k^2 and $(k-p_1)^2$ are finite, the appropriate expansion of k is

$$k = up_1 + \left[\left(\frac{b}{ac\nu}\right)^{1/2}\chi + \frac{\bar{y}}{a\nu}\right]p_2 - \left(\frac{a}{bc\nu}\right)^{1/2}\chi p_3 + \kappa , \tag{3.3}$$

and one obtains

$$\nu^{-3\Gamma-2}(-ac)^{-\Gamma-1}F(a,c), \tag{3.4}$$

with

$$F(a,c) = \int_0^1 d\mu \int du\, d\bar{y}\, d\chi\, d\kappa\, g(k^2, k'^2)Q(k^2)C(k'^2)\eta(\mu)[u(1-u)]^{-\Gamma-1}[\mu a u + (1-\mu)c(1-u)]^{-\Gamma},$$
$$k^2 = u\bar{y} + u^2M^2 - (\chi^2+\kappa^2), \quad k'^2 = (u-1)\bar{y} + (u-1)^2M^2 - (\chi^2+\kappa^2). \tag{3.5}$$

Here Q and C are the propagators for quark and core, respectively; for large k^2 it is supposed that $Q(k^2) \sim 1/k^2$, and for large k'^2, $C(k'^2) \sim 1/k'^2$. If the constants Γ associated with the vertices are not all the same, the necessary change in (3.4) and (3.5) is straightforward to make.

According to usual ideas, the functions g, Q, C are such that their singularities are all just below the real axis in the k^2 and k'^2 planes, so that the $\bar{y}$ integration vanishes unless $0<u<1$. Thus, we may write

$$F(a,c) = \int_0^1 d\mu\, \eta(\mu) \int_0^1 du [u(1-u)]^{-\Gamma-1}[\mu a u + (1-\mu)c(1-u)]^{-\Gamma} f(u), \tag{3.6}$$

which corresponds to the result of BBG. When the quarks and the core have spin, there are extra factors in the integral, but the results of the covariant calculation again agree with those obtained by BBG using their time-ordered formalism.

Actually, the form of $f(u)$ is such that the integral (3.6) diverges logarithmically at its upper and lower end points. This is because the other significant regions of integration, corresponding to other possible choices of invariants kept large or small, overlap with the region corresponding to (3.4). This means that the contributions of these different regions are not additively related, as BBG seem to suggest, but combine to produce a $\ln\nu$ factor modifying the straight power of ν in (3.4). Since the divergencies occur at $u=0$ and $u=1$ in (3.6), one would expect that the resulting angular dependence corresponds to a sum of two terms with dependences

$$a^{-\Gamma-1}c^{-2\Gamma-1}, \quad a^{-2\Gamma-1}c^{-\Gamma-1}, \tag{3.7}$$

respectively, arising from the sum of the behaviors of the integral at its two end points. These conclusions can all be verified by straightforward calculation using the Fourier-transform type of argument we have used elsewhere.[12] It is however the behavior (3.4), with $F(a,c)$ taken as a slowly varying function of a and c, that BBG use in their successful comparison with experiment.

IV. DISCUSSION

We have seen that the covariant analysis, when applied to the BBG model, confirms the character of their results, though with some differences of detailed consequences. However, the covariant analysis also suggests that, if one considers kinematic arguments alone, one should consider a less specific model for large transverse-momentum processes. This is made clear by the analysis of the inclusive process in Sec. II, where not only are certain problems encountered in the general analysis of Fig. 4 in the case of constituents with spin, but also there is no general reason for excluding the connected contribution of Fig. 5.

Our discussion has also shown that it is possible in a natural way to make a smooth transition from the large-transverse-momentum region into the fragmentation and pionization regions

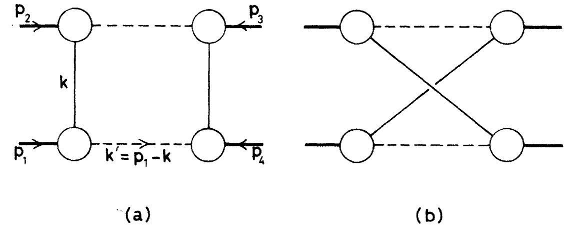

FIG. 6. The result of replacing the amplitudes M in Fig. 3 by simple core structures.

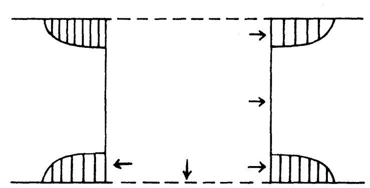

FIG. 7. Feynman-graph model for Fig. 6(a).

which are Regge dominated. Of course not all the contributions important in these latter regions come from terms which dominate at large p_T, as we have already remarked. Indeed, it is possible that large and small p_T are not closely related. This would be the case if constituent hadron amplitudes consisted of two types of terms: (a) Regge exchanges, or more generally dual amplitudes, whose off-shell behavior was strongly (perhaps exponentially) damped; (b) core terms which were not important at large energies, but which had less rapidly decreasing off-shell power-law behavior. In the fragmentation and pionization regions (a) would dominate, while at very large transverse momentum (b) would play the leading role. Such a picture might well provide the dynamical model which would lead to the BBG structure, even though this appears oversimplified if one considers kinematic arguments alone.

It is instructive to compare the results of the BBG model for elastic processes with other calculations of the behavior at high energy and fixed angle, in particular, with perturbation-theory models.[13] The simplest such contributions correspond to "end-point" terms,[14] in which a set of Feynman parameters are set equal to zero, the set being chosen so that contracting out the corresponding lines gives a diagram independent of s, t, and u. The difficulty of assessing the significance of such contributions has always been that they are highly model-dependent. The BBG terms are contributions of just this end-point type. In Feynman integral terms Fig. 6 can be rewritten in the form of Fig. 7. The term we calculated in Sec. III is just an end-point contribution associated with contracting out the lines marked with arrows in Fig. 7. The novelty of the BBG term is that it is framed in terms of a model having connections with other phenomena (form factors and deep-inelastic scattering), though as we have remarked, this need not be so if the simple core structure applies only to the reduced amplitude M and not to the complete quark-hadron amplitude T in Fig. 1.

In addition to these end-point terms there may be "pinch" terms[13] associated with more complicated nonplanar diagrams. These are little understood at present, and it is not possible to say if they should be expected to modify the simple BBG picture in some significant way. This matter is under investigation.

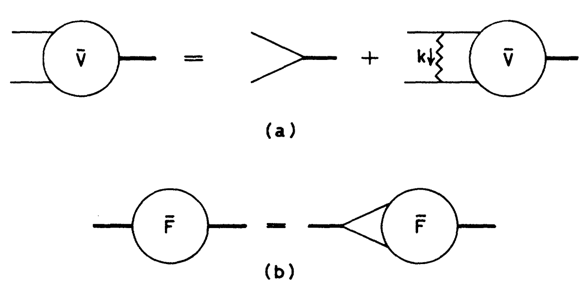

FIG. 8. Integral equations for the functions $\overline{V}$ and $\overline{F}$ in (A1).

APPENDIX

We here discuss the assumption (2.8) that the asymptotic form of the pion coupling function at large k'^2 is independent of the finite variable k^2 and also the assumption (3.1) on the asymptotic form of the function that couples a hadron to quark and core. Both of these functions arise from a picture of the hadron as a composite system, so it is natural to consider a Bethe-Salpeter model, and consider the hadron as being a particle on the Regge trajectory provided by the model. A very convenient formalism for this purpose is provided by the work of Swift and Tucker,[15] who showed that expressions for the residues and trajectories of Regge poles in the Bethe-Salpeter model are equivalent to those derived originally from an analysis of Feynman integrals.[16,3] The formalism is somewhat complicated at first sight, but the general picture is in fact rather simple.

The function that couples the Reggeon to the two constituent particles is given by

$$\frac{\overline{V}(\alpha, s)}{1+\overline{F}(\alpha, s)/(\alpha+1)}, \tag{A1}$$

where the functions $\overline{V}$ and $\overline{F}$ are defined by integral equations schematically represented in Fig. 8. There the loop integrations are in a Euclidean space of dimension $2\alpha+4$. The exchanged wavy line is a function of the momentum k, derived from the interaction potential; for large k^2 it is supposed to have the behavior $(k^2)^{-\gamma}$, where γ depends on α and on the short-distance behavior of

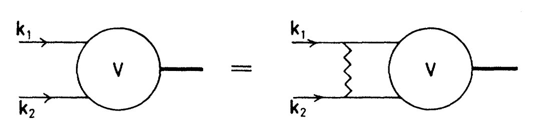

FIG. 9. Integral equation for the residue V of the pole in $\overline{V}$.

the potential.

The Regge trajectory $\alpha(s)$ satisfies the equation

$$\alpha+1=\frac{\bar{F}(\alpha, s)}{1+\bar{F}(\alpha, s)/(\alpha+1)}. \tag{A2}$$

Evidently this equation requires $\bar{F}(\alpha, s)$ to have a pole at $\alpha=\alpha(s)$. In order that (A1) shall not vanish, $\bar{V}(\alpha, s)$ must also have this pole; this may be confirmed by analysis of the integral equations in Fig. 8. The pole is to be factored out of the numerator and the denominator in (A1), with the result

$$\frac{(\alpha+1)V(\alpha, s)}{F(\alpha, s)}, \tag{A3}$$

with F the residue of the pole in $\bar{F}$, and where the residue V of the pole in $\bar{V}$ satisfies the homogeneous integral equation in Fig. 9. The function F depends only on the squared mass s of the Reggeon, but V depends also on the squared masses k_1^2 and k_2^2 of the constituents. Analysis of the diagram on the right-hand side of Fig. 9 shows that when both k_1^2 and k_2^2 are large,

$$V\sim\int_0^1 d\mu\, \eta(\mu)[\mu k_1^2+(1-\mu)k_2^2]^{-\gamma}, \tag{A4}$$

where μ is a certain Sudakov parameter associated with the loop integration, and $\eta(\mu)$ is a dynamically determined weight function. This result corresponds to (3.1). Consideration of the case with k_1^2 only large leads to (2.8). These results have been established for the case of spinless constituents. There are considerable technical difficulties in extending the discussion to include spin, though this is not expected to change the picture in any radical way.

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[1]See, for instance, the review by P. V. Landshoff [Cambridge Report No. DAMTP 72/35 (unpublished)] which contains references to original work (to appear in the proceedings of the 1972 Cargèse Summer School).

[2]P. V. Landshoff and J. C. Polkinghorne, Phys. Reports 5C, 1 (1972).

[3]P. V. Landshoff and J. C. Polkinghorne, Nucl. Phys. B32, 541 (1972). Similar ideas were subsequently developed by other authors, for example, F. Ravndal, this issue, Phys. Rev. D 8, 847 (1973).

[4]R. Blankenbecler, S. Brodsky, and F. Gunion, Phys. Letters 39B, 649 (1972); 42B, 461 (1972).

[5]D. Horn and M. Moshe, Tel-Aviv report (unpublished).

[6]However, we understand that an independent reconsideration of the noncovariant calculation is likely to bring the two methods into agreement (private communication from BBG).

[7]We shall not consider here the possible existence of an extra term resulting from vector gluon exchange; see S. M. Berman, J. D. Bjorken, and J. B. Kogut, Phys. Rev. D 4, 3388 (1971).

[8]M. Banner *et al.*, CERN report (unpublished). See also the recent results of Davis *et al.* [Phys. Rev. Letters 29, 1356 (1972)] who find an unexpectedly large cross section for the production of photons which are near their mass shell and which have transverse momentum of about 1 GeV/c.

[9]P. V. Landshoff and J. C. Polkinghorne, Nucl. Phys. B53, 473 (1973).

[10]See, for example, H. W. Kendall, in *Proceedings of the International Symposium on Electron and Photon Interactions at High Energies, 1971,* edited by N. B. Mistry (Cornell Univ. Press, Ithaca, New York, 1972).

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[15]A. R. Swift and R. W. Tucker, Phys. Rev. D 1, 2894 (1970); 2, 397 (1970).

[16]J. C. Polkinghorne, J. Math. Phys. 5, 431 (1964).