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Radiative Corrections to $e^+e^-\to\mu^+\mu^-$ and Neutral Currents in Unified Gauge Theories*

Duane A. Dicus
Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627
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Expressions are given for the part of the radiative corrections to the process $e^+e^-\to\mu^+\mu^-$ which are asymmetric in the cosine of the scattering angle. These corrections are compared with the correction due to the exchange of a neutral vector meson expected in some gauge theories. Given these results it is easy to determine what result should be expected from a measurement of $d\sigma(\theta)-d\sigma(\pi-\theta)$ if the gauge theories are correct (or incorrect). This result will be independent of any unknown parameters of the gauge theory. The effects of the neutral vector meson on the reaction $e^+e^-\to e^+e^-$ are also given.

One method of experimentally testing unified gauge theories[1] is to look for corrections to electromagnetic processes.[2,3] Such tests would require that experiments be done to at least 1% accuracy but have the advantage of depending only on the leptonic part of the theories. In addition, unlike tests involving neutrinos,[4] the results are independent of the values of the masses of the new, so far unobserved, particles.

One proposed experiment of this type is to look for the effects of the neutral vector meson Z in $e^+e^-\to\mu^+\mu^-$ scattering.[2] Among other effects the interference term between vector-meson exchange [Fig. 1(b)] and photon exchange [Fig. 1(a)] will be asymmetric in the cosine of the scattering angle. This can be used to separate it from the lowest-order, purely electromagnetic, contribution by taking the difference of the differential cross sections at θ and $\pi-\theta$.

$$D=\left[\frac{d\sigma}{d\Omega}(\theta)-\frac{d\sigma}{d\Omega}(\pi-\theta)\right]\Big/\left[\frac{d\sigma}{d\Omega}(\theta)+\frac{d\sigma}{d\Omega}(\pi-\theta)\right]. \tag{1}$$

If this experiment is done with colliding beams, at present energies of ~3.5 GeV, the contribution of the neutral vector meson to D will be $\lesssim 1\%$. Therefore one must include in D higher-order, purely electromagnetic, contributions. The interference term between two-photon exchange [Fig. 1(c)] and the usual one-photon exchange will be odd in $\cos\theta$ as will part of the real, soft, photon emission shown in Figs. 1(d) and 1(e).

Our purpose here is to report an evaluation of the part of the radiative corrections to the colliding-beam process $e^+e^-\to\mu^+\mu^-$ which contribute to D.[5] The neutral-vector-meson contribution is enhanced by the transverse polarization of the initial particles. Thus we must include this polarization in our calculation also.[6] Of interest are the asymptotic expressions of the correction for large center-of-mass energy. Hence we will ignore terms of order m/E, where m is a lepton mass, compared to unity or compared to $1\pm z$ ($z\equiv\cos\theta$). Thus our results are not valid if we are close to the forward or backward direction. In the asymptotic limit just defined our results are exact except for a soft-photon approximation to the bremsstrahlung graphs.

The initial electron and positron beams are taken to be along the z axis with polarization in the x direction,

$$p(e^-)=(E,0,0,p),\quad p(e^+)=(E,0,0,-p), \tag{2a}$$

$$s(e^-)=(0,s,0,0)$$
$$=-s(e^+). \tag{2b}$$

The momentum of the μ^- is

$$p(\mu^-)=(E,p'\sin\theta\cos\phi,p'\sin\theta\sin\phi,p'\cos\theta). \tag{3}$$

These relations define E, s^2, and the scattering angles θ and ϕ.

The differential cross section due to one-photon exchange (including polarization) is given by

$$\frac{d\sigma^{(0)}}{d\Omega}=\frac{\alpha^2}{16E^2}W_0, \tag{4}$$

where

$$W_0=1+z^2-s^2(1-z^2)\cos2\phi. \tag{5}$$

The total cross section may then be written as

$$\frac{d\sigma}{d\Omega}=\frac{d\sigma^{(0)}}{d\Omega}(1+\delta), \tag{6}$$

where δ contains all the higher-order effects including those of the neutral vector meson. We will call δ' the part of δ which contributes to (1). To order α^3 D is equal to δ'.

$$\delta'=\delta_0'+\delta_s'+\delta_B'+\delta_Z'. \tag{7}$$

The pieces δ_0' and δ_s' come from the interference between the graphs that exchange two virtual photons and the lowest-order (one-virtual-photon exchange) graph. δ_0' is the part of that contribution which is independent of the polarization s^2. What has happened is that the spin dependence appears in the form W_0 [Eq. (5)] and is factored out when we form (6). The second piece, δ_s', does not factor in this manner.[7] We find

$$\delta_0' = -\frac{2\alpha}{\pi}\left(\ln\frac{4E^2}{\lambda^2}\right)\ln\frac{1-z}{1+z} - \frac{\alpha}{\pi}\left[\frac{1}{1-z^2}\ln\frac{1+z}{1-z} + \frac{z}{1-z^2}\ln\frac{1-z^2}{4} + \frac{z}{(1+z)^2}\ln^2\left(\frac{1-z}{2}\right) + \frac{z}{(1-z)^2}\ln^2\left(\frac{1+z}{2}\right)\right], \tag{8}$$

where λ is the photon mass required by the infrared divergence. The spin-dependent correction is

$$\delta_s' = \frac{2\alpha}{\pi}\frac{1}{W_0}\left[\frac{z^2}{(1-z)^2}\ln^2\left(\frac{1+z}{2}\right) - \frac{z^2}{(1+z)^2}\ln^2\left(\frac{1-z}{2}\right) + \frac{z^2}{1-z^2}\ln\frac{1+z}{1-z} + \frac{z}{1-z^2}\ln\frac{1-z^2}{4}\right]. \tag{9}$$

Notice that even though this is spin-dependent it is not zero when s^2 is zero.

The contribution to δ' from the emission of real photons, δ_B', comes only from the interference between the graphs where the photon is emitted by one of the initial particles [Fig. 1(d)] and the graphs where the photon is emitted by one of the final particles [Fig. 1(e)]. For this contribution we make the soft-photon approximation of neglecting the momentum of the photon in the numerators of our expressions and assuming the emission of the photon does not alter the momentum of the muons, i.e., that p_3+p_4 still equals p_1+p_2. Within this soft-photon approximation we calculate exactly, in the asymptotic limit explained above. The spin dependence again factors out, as for δ_0',

$$\delta_B' = -\frac{2\alpha}{\pi}\left[\left(\ln\frac{4\epsilon^2}{\lambda^2}\right)\ln\frac{1+z}{1-z} + \frac{1}{2}\left(\ln\frac{1-z^2}{4}\right)\ln\frac{1+z}{1-z} + I'(p_1,p_4) - I'(p_1,p_3)\right]. \tag{10}$$

The quantity ϵ is the soft-photon energy cutoff.

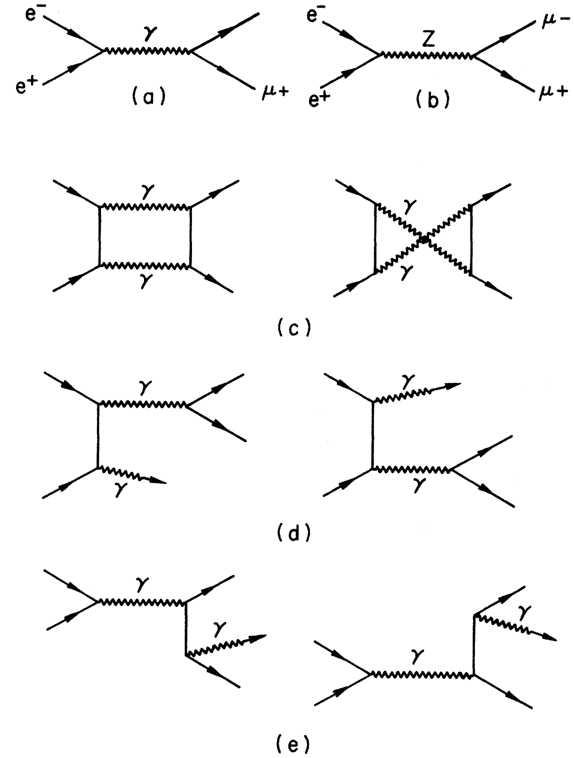

FIG. 1. The graphs which contribute to the process $e^+e^-\to\mu^+\mu^-$; (a) is single-photon exchange, (b) is the exchange of a neutral vector meson, (c) is two-photon exchange. The emission of a real photon from an initial particle is shown in (d), emission from a final particle is shown in (e).

The infrared divergence, $\lambda^2\to 0$, cancels with (8), as it must. $I'(p_i,p_j)$ is an integral given by[8]

$$I'(p_i,p_j) = -p_i\cdot p_j\int_0^1\frac{dx}{p_x^{\,2}}\left(2\ln\frac{E_x+|\vec{p}_x|}{2E_x} + \frac{E_x-|\vec{p}_x|}{|\vec{p}_x|}\ln\frac{E_x+|\vec{p}_x|}{E_x-|\vec{p}_x|}\right), \tag{11}$$

where

$$p_x = p_i x + p_j(1-x). \tag{12}$$

This integral was done numerically. The results of the numerical integration can be accurately fit (±2%) to the following polynomial in z:

$$I'(p_1,p_4) = -(1+z)[0.825 - 0.272z + 0.176z^2], \tag{13}$$

with $I'(p_1,p_3)$ given by (13) with z replaced by $-z$. The contribution to D of the I' terms in (10) is less than 0.6% but nevertheless significant.

The final piece of δ' is the contribution of the neutral vector meson in a gauge theory.[2] If the coupling of the Z to the leptons is of the form

$$\bar{l}\gamma^\alpha(g_V + g_A\gamma_5)l\, Z_\alpha, \tag{14}$$

then δ_Z' is given by

$$\delta_Z' = \frac{B}{A}\frac{z}{W_0}, \tag{15}$$

where

$$A = \frac{e^4}{32E^4} + \frac{e^2 g_V{}^2}{4E^2(4E^2 - m_Z{}^2)}, \tag{16}$$

$$B = \frac{e^2 g_A{}^2}{2E^2(4E^2 - m_Z{}^2)}. \tag{17}$$

Specializing to Weinberg's theory[9] we have[2]

$$\frac{B}{A} = -\frac{4\sqrt{2}\,G}{e^2}E^2, \tag{18}$$

where G is the weak Fermi coupling constant. From here on when we refer to δ_Z' we will mean (15) with the value (18) for B/A. The important feature of δ_Z' is that it is independent of any unknown masses or parameters.

An example of our results is shown in Fig. 2.

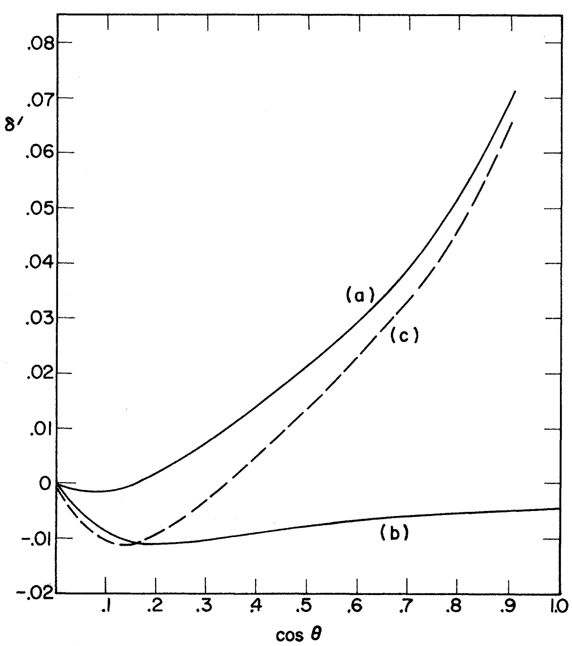

FIG. 2. δ' is that part of the correction to the reaction $e^+e^- \to \mu^+\mu^-$ which is asymmetric in the cosine of the scattering angle. (a) shows the radiative correction when the soft-photon cutoff is $\frac{1}{10}$ the center-of-mass energy of one particle, E. (b) the correction due to the neutral vector meson when E is 3.5 GeV. In both (a) and (b) the polarization is taken as $s^2 = 0.924$. The curve (c) is the total correction, (a) plus (b). An experiment, performed under these conditions would see the curve (c) if the gauge theory is correct or (a) if there is no neutral vector meson.

The curve (a) is the radiative correction $\delta_0' + \delta_s' + \delta_B'$, where we have taken ϵ equal to $\frac{1}{10}$ of E. The neutral-vector-meson effect, δ_Z', is shown in curve (b). δ_Z' has a maximum at

$$z = \left(\frac{1-s^2}{1+s^2}\right)^{1/2},$$

where it is enhanced by a factor of[2]

$$\frac{1}{(1-s^4)^{1/2}}.$$

Thus, in the curves (a) and (b) we have chosen s^2 to be large, $s^2 = 0.924$ and $\cos 2\phi = 1$. Also δ_Z' is quadratic in the center-of-mass energy, E. The curve (b) corresponds to $E = 3.5$ GeV. The curve (c) is the sum of (a) and (b). If these conditions on the energies and polarization are met and the experiment performed the results for D will be (a) if there is no neutral vector meson or (c) if Weinberg's theory[9] is correct. The result is independent of any masses or parameters of the gauge theory. Thus a measurement of D for z between 0.1 and 0.2 which yielded -0.01 ± 0.005 would be definite evidence of the neutral vector meson.

We have indicated that δ_Z' is very sensitive to E and s^2. The radiative correction, (a) in Fig. 2, also is sensitive to the parameters. In Fig. 2 it is very small in the region $0 \leq z \leq 0.3$ because of cancellations between δ_s' and $\delta_0' + \delta_B'$. For $s^2 = 0$, however, the radiative corrections are $+0.5\%$ at $z = 0.1$ and rise to $+1.5\%$ at $z = 0.3$. Figure 2 has $\epsilon = 0.1E$. Taking $\epsilon = 0.01E$ would add $+0.4\%$ to the radiative correction at $z = 0.1$ and $+1.3\%$ at $z = 0.3$. Precisely what value is used for the ratio of ϵ to E is not too important; we are simply trying to indicate that Fig. 2 might look quite different. What is important is that δ_Z' is around 1% over a large range of z and thus a measurement of D to $\pm 0.5\%$ can establish the validity of some of the most popular gauge theories or rule them out completely.

Other than simple miscalculation there are only two possible sources of error in our results. The first is our asymptotic approximation, $1 \pm z \gg m^2/E^2$. For center-of-mass energies greater than one GeV our calculations should be good for $-0.8 < z < +0.8$. A more serious source of error is our soft-photon approximation to real-photon emission. Hard-photon corrections to the process $e^+e^- \to (\mu^+\mu^- + \mu^-\mu^+)$ have recently been calculated[10] and it was found that the soft-photon approximation could be in error by as much as 40%.

We have calculated the radiative corrections of order α^3. The next higher-order purely electromagnetic contribution to D will be of order α^4 and can be neglected compared to neutral vector-meson effects provided the order α^4 contribution has no $\ln(E/m)$ terms.

Since observation of the effects of a Z look promising in the electron-muon reaction it seems reasonable to ask if the Z can also be seen in the process $e^+e^- \to e^+e^-$. Here the cross section due to single-photon exchange in both the s and t channels is

$$\frac{d\sigma^{(0)}}{d\Omega} = \frac{\alpha^2}{16E^2}\frac{1}{(1-z)^2}W_1, \tag{19}$$

where

$$W_1 = (3+z^2)^2 + (1-z^2)^2 s^2 \cos 2\phi. \tag{20}$$

The quantities E, s^2, z, and ϕ are defined as in (2) and (3).

The cross section, including the neutral vector meson, can be written as

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma^{(0)}}{d\Omega}[1 + \delta_Z + O(\alpha)]. \tag{21}$$

If we sum over the helicities of both final-state particles and take the limit $m_Z \gg E$ then δ_Z is given by

$$\begin{aligned}\delta_Z = \frac{a^2}{4}\frac{(1-z)}{W_1}&[3(1+z)(3+z^2) \\ &+ (1-3z)(1-z^2)s^2\cos 2\phi] \\ + \frac{b^2}{4}\frac{(1-z)}{W_1}&[(1+z)(-5+8z+z^2) \\ &+ (3-z)(1-z^2)s^2\cos 2\phi],\end{aligned} \tag{22}$$

where a is related to g_V of (14) and, in Weinberg's theory, is given by

$$a^2 = \frac{\sqrt{2}\,G}{\pi\alpha}E^2\left(1 - 4\frac{m_0^2}{m_W^2}\right)^2. \tag{23}$$

The quantity m_W is the (unknown) mass of the charged vector meson and m_0 is the minimum allowed value of m_W, 37.3 GeV. The coupling constant b is related to g_A of (4). Again, in Weinberg's theory,

$$b^2 = \frac{\sqrt{2}\,G}{\pi\alpha}E^2. \tag{24}$$

If $m_W > 60$ GeV (Ref. 4) the effect of the neutral vector meson, δ_Z, is almost an order of magnitude smaller for this process than δ_Z' is for $e^+e^- \to \mu^+\mu^-$. Here δ_Z is never greater than 0.25%. This is due to a combination of factors. The purely electromagnetic graphs are enhanced by the proximity to the photon pole in the t channel. The nonzero polarization of the initial particles does not substantially increase δ_Z for this process. There is some cancellation between the a^2 and the b^2 terms in (22). Finally there is no combination of cross sections, similar to (1), which will emphasize δ_Z.

As pointed out in Ref. 2 the interference term between the vector and axial-vector coupling of the Z can lead to nonzero helicity for the final-state particles.[11] If we define the momentum of the final electron as (3) then the helicity of final electron is defined by the spin vector

$$s' = \frac{h}{m_e}(p', E\sin\theta\cos\phi, E\sin\theta\sin\phi, E\cos\theta). \tag{25}$$

If we sum over the spin of the final positron, the polarization is defined in terms of the square of the matrix element as

$$P = \frac{|M|_{h=+1}^2 - |M|_{h=-1}^2}{|M|_{h=+1}^2 + |M|_{h=-1}^2}. \tag{26}$$

For the electron-positron final state under consideration we find

$$\begin{aligned}P = ab\,\frac{1-z}{W_1}[&z(1+z)(2+z) \\ &+ (1-z)(1-z^2)s^2\cos 2\phi].\end{aligned} \tag{27}$$

In Weinberg's theory

$$ab = -\frac{\sqrt{2}\,G}{\pi\alpha}E^2\left(1 - 4\frac{m_0^2}{m_W^2}\right). \tag{28}$$

The factor of $1 - 4m_0^2/m_W^2$ acts to suppress the amount of polarization expected if m_W is within the probable range of $m_W > 60$ GeV (Ref. 4) and $m_W < 150$ GeV (Ref. 12). (This same factor occurs for $e^+e^- \to \mu^+\mu^-$ and may make the polarization there so small as to be unobservable.)

Again, for the same reasons that δ_Z is considerably smaller for this process than for $e^+e^- \to \mu^+\mu^-$, P is also smaller by more than an order of magnitude. For $m_W > 60$ GeV, P is always less than 0.2%. Even if δ_Z and P were not smaller than for the reaction with muons in the final state, they would have the disadvantage of depending upon an unknown parameter through (28) or (23) and (24). Thus an unambiguous prediction, like Fig. 2 for example, is impossible for this process.

Note added. After this paper was submitted for publication we were made aware of two other reports which give calculations of the radiative corrections to $e^+e^- \to \mu^+\mu^-$. One paper is by Khriplovich[13] and is very similar to ours, the other is a slightly more general calculation by Brown, Cung, Mikaelian, and Paschos.[14] Our correction $\delta_0' + \delta_S' + \delta_B'$ agrees exactly with the relevant part of the calculation by Khriplovich while our δ_Z' has the opposite sign. Brown *et al.* used a better technique for calculating the bremstrahlung contribution than the soft-photon approximation used by Khriplovich and us. Our answer for the virtual-photon part, $\delta_0' + \delta_s'$, and for the weak part, δ_Z', agrees exactly with the corresponding parts of their work.

I would like to thank Professor H. L. Lynch for sending me a copy of the paper by Khriplovich and Dr. E. A. Paschos for sending me the paper by Brown *et al.*

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Bremsstrahlung Model Calculation of High-Energy Nucleon-Nucleon Scattering*

Héctor Moreno†
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305
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A model calculation of nucleon-nucleon scattering is presented which results in an amplitude that in the elastic high-energy limit realizes the Wu-Yang idea regarding the relation between the elastic form factor of the nucleon and the differential cross section, and which is consistent with the behavior of the differential cross section for the production of nucleon resonances. The calculation makes use of results obtained with the same model in the study of electroproduction.

This paper presents a calculation of nucleon-nucleon scattering at very high energies using a model which has proved useful[1] to describe deep-inelastic electroproduction, scaling, and the nucleon's elastic and inelastic form factors.

The basic idea of the model is that at high energies the nucleon trajectories can be parametrized by the four-momentum, which remains constant except for sudden changes due to hard collisions.

The nucleon is coupled to a neutral, massive- and soft[2]-vector-meson field. The probability that this field adjusts itself in such a way that no real mesons are radiated upon the nucleon's collision has been previously calculated[1] and shown to be of the right form to account for the nucleon's form factors. If additionally the nucleon possesses a spectrum of excited states of an appropriate density,[3] the scaling behavior of the νW_2 structure function of the electroproduction experiments can also be accounted for.

These ideas will be applied here to study the very-high-energy scattering of two nucleons, under the following considerations and assumptions:

(a) An inelastic collision between two nucleons