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[2]See, e.g., Refs. 3–8 where earlier references can be found.

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[4]A. De Rújula, in *Proceedings of the Seventh Rencontre de Moriond,* edited by J. Tran Thanh Van (CNRS, Paris, 1972).

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[6]T. P. Cheng and A. Zee, Phys. Rev. D 6, 3182 (1972).

[7]O. Nachtmann, Phys. Rev. D 6, 1718 (1972).

[8]M. Gourdin, Nucl. Phys. B49, 501 (1972).

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[10]For a review of the data on W_S and W_T see, e.g., H. W. Kendall, in *Proceedings of the International Symposium on Electron and Photon Interactions at High Energies, 1971,* edited by N. B. Mistry (Cornell Univ. Press, Ithaca, N. Y., 1972).

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Phenomenological Model for Diffractive Excitation of Hadron Resonances*

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In order to describe diffractive excitation of hadrons, a vector-vector form for the imaginary amplitude is proposed. The vector transition operator is related to the ordinary vector current. This description gives approximate s-channel helicity conservation in elastic πN scattering and a differential cross section for elastic NN scattering proportional to the fourth power of the magnetic form factor of the nucleon. All inelastic cross sections for excitation of hadron resonances should dip in the very near forward direction. Calculating the matrix elements of the diffractive transition operator in a relativistic quark model with essentially no free parameters, we get quantitative results which are not in obvious disagreement with experiment for most nucleon and pion resonances.

I. INTRODUCTION

High-energy particle reactions mediated by Regge exchanges have cross sections decreasing with increasing energy, while diffractive processes are characterized by approximately constant cross sections. A t-channel description of these latter reactions will involve a $J=1$ singularity in the complex angular momentum plane carrying vacuum quantum numbers. This t-channel object is usually called the Pomeranchukon.

Diffractive reactions can be divided into three subclasses.[1] The most common process is elastic diffraction scattering,

$$A+B\to A+B. \tag{1.1a}$$

Next there can be single diffraction excitation as in

$$A+B\to A^*+B$$

or (1.1b)

$$A+B\to A+B^*,$$

where A^* and B^* have the same internal quantum numbers like charge, hypercharge, and isospin as A and B have, but with different masses and/or spin and parity. Double diffraction excitation should also be possible,

$$A+B\to A^*+B^*. \tag{1.1c}$$

In contrast to the present theory of Regge poles and cuts, we do not have any similar theoretical framework or understanding of diffractive processes.[2]

Some kind of qualitative understanding has been claimed in the form of proposed selection rules. Morrison[3] has suggested that only those reactions of the type (1.1b) will occur where the spin difference ΔJ between B and B^* is related to the parity difference ΔP by

$$\Delta P = (-1)^{\Delta J} .$$

That means for instance that the $D_{13}(1520)$ could be produced from the nucleon, but not the $S_{11}(1535)$. Similarly, the $F_{15}(1688)$ is allowed, but not the $D_{15}(1670)$. In diffraction excitation of the pion, the $A_1(1070)$ should be seen, while the $A_2(1300)$ is forbidden. Carlitz, Frautschi, and Zweig[4] have argued that no change is allowed in the SU(6) representation or the quark spin of the particles in reaction (1.1b). This selection rule would therefore forbid the production of $D_{13}(1520)$, $S_{11}(1535)$, and $D_{15}(1670)$ since they belong to a $\underline{70}$ in SU(6) while the initial nucleon is in a $\underline{56}$. The $F_{15}(1688)$ being in an excited $\underline{56}$, is on the other hand allowed. For mesons, it would imply that both the $A_2(1300)$ and the $A_1(1070)$ are forbidden because they have quark spin $S=1$ and the pion has $S=0$. These two selection rules are seen to be in conflict with each other; both of them cannot be right at the same time.

The experimental situation is not very clear. However, Allaby[5] has reported that the $A_2(1300)$ gives a clear and strong signal at 40 GeV at Serpukhov. If this really is a diffractive excitation of the A_2 and not due to ordinary Regge exchange, it would mean that both the selection rule proposed by Morrison and the one by Carlitz, Frautschi, and Zweig do not apply in this case.

A third selection rule of a different kind has been proposed by Chou and Yang.[6] If the product of all the parities of the particles involved in reaction (1.1c) is -1, then the cross section should be zero in the forward direction. This would apply to single diffraction excitation of the $S_{11}(1535)$, $D_{13}(1520)$, $A_2(1300)$, and $A_1(1070)$.

What we need is a quantitative description of diffractive processes. The first steps towards this have already been made in the framework of the quark model by Hendry and Trefil[7] and Le Yaouanc, Oliver, Pene, and Raynal.[8] If one allows oneself to talk about diffractive scattering of quarks off particles or other quarks, one can relate all the different diffractive reactions. At the first stages of these calculations one did not allow any change of the quark spin in the fundamental quark diffractive amplitudes. This led therefore to the selection rule of Carlitz, Frautschi, and Zweig. To improve the agreement with experiment it was pointed out by Clegg[9] that one needed diffractive spin flip of quarks. In that way one was left with no selection rules. A quantitative calculation including spin flip and taking all the quark diffractive amplitudes from elastic scattering so as to give a result with no free parameters, has recently been made by Le Yaouanc *et al.*[10] in good agreement with existing data for excitation of nucleon resonances.

In this paper we want to propose a phenomenological model for diffractive excitation of hadron resonances which in principle is somewhat more general than previous models, but in effect gives results similar to the ones obtained by Le Yaouanc *et al.* We will postulate the existence of an operator θ which in reaction (1.1c) takes the state A into A^* and B into B^*. It is possible to argue for this to be a local vector operator with even charge conjugation. In this way, we are able to relate the transition matrix elements of this operator to matrix elements of the electromagnetic current. Our model is therefore a generalization of the Wu-Yang[11] conjecture for elastic scattering and we easily recover their result that the cross section for elastic NN scattering is proportional to the fourth power of the magnetic form factor. Another immediate consequence is that the amplitudes for diffractive excitation of nucleon resonances are given directly in terms of the amplitudes for electromagnetic excitation of the proton and neutron.

It turns out to be very difficult to test the proposed model without any specific model for strongly interacting particles. Hence in the following section we will use the partially successful relativistic quark model of Feynman, Kislinger, and Ravndal[12] to calculate the matrix elements of the diffractive transition operator. There are essentially no free parameters in these calculations and the results obtained are not in obvious disagreement with most of the existing data for diffraction production of nucleon and pion resonances.

II. DIFFRACTIVE TRANSITION OPERATOR

In a previous, short paper[13] the existence of a diffractive transition operator θ was first proposed for the imaginary amplitude of reaction (1.1c):

$$\mathrm{Im}T(A+B\to A^*+B^*) \sim \langle A^*|\theta|A\rangle\langle B^*|\theta|B\rangle . \quad (2.1)$$

The real amplitude is assumed to be negligibly small or zero. It was guessed that this operator with even charge conjugation should be a local vector operator $\overline{V}_\mu$ so that the amplitude could be written as

$$\mathrm{Im}T(A+B\to A^*+B^*)=P(t)\langle A^*|\overline{V}_\mu|A\rangle\langle B^*|\overline{V}_\mu|B\rangle\,, \tag{2.2}$$

where $P(t)$ is some unknown function of the squared four-momentum transfer t. Making the assumption that the Pomeranchukon is a SU(3) singlet, it was argued that the matrix elements of this operator were given by the matrix elements of the ordinary conserved SU(3)-singlet vector current operator V_μ^0. The only difference is that V_μ^0 couples to charges which have opposite signs for particle and antiparticle while $\overline{V}_\mu$ couples to particles and antiparticles with the same sign. How this can come about has been explained by Feynman.[14] Consider the process

$$Q+B\to Q+B, \tag{2.3}$$

where Q is a particle, for instance a quark. Let this process be described by an effective vector exchange so to give a real and imaginary part to the amplitude which will be proportional to the energy-squared variable s:

$$T(s)=as+ibs. \tag{2.4}$$

The amplitude for the crossed reaction

$$\overline{Q}+B\to\overline{Q}+B, \tag{2.5}$$

where $\overline{Q}$ is the antiparticle of Q, will now be given by

$$\overline{T}(s)=T^*(-s)=-as+ibs\,. \tag{2.6}$$

In other words, when the amplitude is real as in ordinary photon exchange, we would say that the particle and antiparticle couple by equal but opposite charges. However, for a purely imaginary amplitude which we are interested in, we have to say that both particle and antiparticle couples to equal charges of the same sign.

More recently Kislinger[15] has shown how the amplitude in Eq. (2.2) could arise in a world where there was f and f' dominance of the Pomeranchukon couplings, exchange-degenerate vector and tensor trajectories and universality of the vector-meson trajectory residues.

Consider the reaction

$$M^i(k_1)+B\overset{V^k}{\to}M^j(k_2)+B', \tag{2.7}$$

mediated by the exchange of a vector-meson trajectory with SU(3) label k. M^i and M^j are pseudoscalar mesons belonging to the same multiplet and B is some baryon. Universality then tells you that the Regge residue for this process is given by

$$\beta_{V^k}(M^i+B\to M^j+B') =b(t)\langle M^j|V_\mu^k|M^i\rangle\langle B'|V_\mu^k|B\rangle s^{-1}, \tag{2.8}$$

where $b(t)$ is some function of t multiplying the matrix elements of the conserved SU(3) vector current:

$$\langle M^j|V_\mu^k|M^i\rangle=f_{ijk}(k_{1\mu}+k_{2\mu})f(t), \tag{2.9}$$

$$\langle B'|V_\mu^k|B\rangle=\overline{u}'[f_1(t)\gamma_\mu+f_2(t)\sigma_{\mu\nu}q_\nu]u\,. \tag{2.10}$$

Here $f(t)$, $f_1(t)$, and $f_2(t)$ are form factors. This very strong hypothesis for the residues of the vector-meson trajectories will be tested in detail in the near future. Some consequences have already been shown by Lipkin[16] to be in very good agreement with experiment.

Exchange degeneracy is known to work well for the vector and tensor trajectories. It relates the residues for reaction (2.7) to the residues for the same reaction mediated by the tensor trajectory with the same SU(3) quantum numbers:

$$\beta_{T^k}(M^i+B\overset{T^k}{\to}M^j+B')\sim\beta_{V^k}(M^i+B\overset{V^k}{\to}M^j+B')\,. \tag{2.11}$$

Combining this relation with universality and remembering that the tensor trajectory has even charge conjugation so that it couples to mesons through D coupling and not F couplings as for the vector trajectory, we get the following tensor trajectory residue:

$$\beta_{T^k}(M^i+B\to M^j+B')=+d_{ijk}b(t)(k_{1\mu}+k_{2\mu}) \times f(t)\langle B'|V_\mu^k|B\rangle s^{-1}\,. \tag{2.12}$$

For the crossed reaction

$$M^i(k_1)+\overline{B}\to M^j(k_2)+\overline{B}'\,, \tag{2.13}$$

we would similarly get for the tensor residue:

$$\beta_{T^k}(M^i+\overline{B}\to M^j+\overline{B}')=-d_{ijk}b(t)(k_{1\mu}+k_{2\mu}) \times f(t)\langle\overline{B}'|V_\mu^k|\overline{B}\rangle s^{-1}\,. \tag{2.14}$$

It is easy to show (as done in the next section) that this kind of tensor trajectory residue functions will imply that the f exchange (like ω exchange) approximately conserves s-channel helicity in elastic meson-nucleon scattering. Present experimental data is consistent with this result as shown by Zarmi[17] and Barger and Halzen.[18]

The last assumption made by Kislinger was f and f' dominance of the Pomeranchukon. This idea has recently been extensively discussed by Carlitz, Green, and Zee[19] and is known to hold in dual theories as first pointed out by Freund and Rivers[20] and calculated by Lovelace.[21] Its implications for diffraction scattering have been discussed by Freund, Jones, and Rivers.[22] Also in the work of Chew and Snider[23] the Pomeranchukon is related to the f and f'.

Let us assume that the Pomeranchukon is a

SU(3) singlet. Then f and f' dominance of its coupling gives for the reaction

$$M(k_1)+B\to M'(k_2)+B', \quad (2.15)$$

$$\mathrm{Im}T(M+B\to M'+B')\sim\beta_{T^0}(M+B\to M'+B'), \quad (2.16)$$

where T^0 is the SU(3)-singlet tensor meson with the quark structure

$$T^0=(\tfrac{1}{3})^{1/2}(\bar{u}u+\bar{d}d+\bar{s}s) = (\tfrac{2}{3})^{1/2}f+(\tfrac{1}{3})^{1/2}f'. \quad (2.17)$$

How relation (2.16) looks like in terms of quark graphs is shown in Fig. 1.

The blob in the middle of Fig. 1(b) can only be a scalar function of t. Then combining Eq. (2.16) with Eq. (2.12) where $d_{0ij}=(\frac{2}{3})^{1/2}\delta_{ij}$, we get finally for the imaginary part of the diffractive amplitude for process (2.15)

$$\mathrm{Im}T(M+B\to M'+B')=P(t)\langle M'|\bar{V}_\mu|M\rangle\langle B'|\bar{V}_\mu|B\rangle, \quad (2.18)$$

where

$$\langle M'|\bar{V}_\mu|M\rangle=\mathrm{const}(k_{1\mu}+k_{2\mu})f(t) \quad (2.19)$$

and $\langle B'|\bar{V}_\mu|B\rangle$ is given in Eq. (2.10).

During this long series of arguments we made the implicit assumption that simple Regge theory could be used. Since this is the case only at small t, we expect that our amplitude Eq. (2.18) will not be applicable to reactions with large momentum transfers.

This long and detailed discussion will hopefully make it clear what is meant in Eq. (2.2). In short we can sum up the content of this equation by saying that the matrix elements are given in terms of the SU(3)-singlet current operator where the charges are taken to be equal and of the same sign for particles and antiparticles. To fix the normalization, we will set the corresponding quark charges equal to +1 for all quarks and antiquarks. In addition we will have the condition that

$$q_\mu\langle|\bar{V}_\mu|\rangle=0, \quad (2.20)$$

since this holds for the corresponding matrix elements of the ordinary vector currents.

This description of diffractive processes is not really new. It was first suggested by Feynman[24]

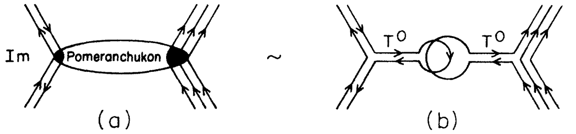

FIG. 1. Quark graphs illustrating f and f' dominance of the Pomeranchukon.

to use "a point current-current interaction which would make for the ultimate diffraction." This idea was subsequently taken up by Abarbanel, Drell, and Gilman.[25] However, they used it for the real amplitude and would therefore get zero total cross sections by the optical theorem.

In the next section we will extract some consequences from our imaginary amplitude in Eq. (2.2). We will at this stage try not to depend too much on any specific quark model for the hadrons so as to make the results more plausible.

III. SOME ALMOST GENERAL RESULTS

First of all we do not recover the Morrison nor the Carlitz-Frautschi-Zweig selection rules. This is a consequence of f and f' dominance of the Pomeranchukon as shown by Carlitz, Green, and Zee[19] and Freund, Jones, and Rivers.[22] It also follows automatically from the diffractive quark spin-flip model of Le Yaouanc *et al.*[10]

We are not very surprised by this lack of selection rules. The Morrison rule has no theoretical or experimental justification while the Carlitz-Frautschi-Zweig rule is a generalization of the belief that no change should take place in the internal quantum numbers of the particles in a diffractive process. For instance, since the meson η belongs to a SU(3) octet and the η' is a SU(3) singlet, assuming no mixing, the following reaction should have a vanishing cross section at high energies:

$$\eta+N\to\eta'+N. \quad (3.1)$$

It is not clear that this kind of selection rule should hold going from SU(3) to SU(6). What SU(6) means for particles in motion which cannot be brought simultaneously to rest is not known. Maybe it would be more proper to talk about selection rules in terms of the vertex symmetry $SU(6)_W$ which we know how to deal with for relativistic particles.

Let us now consider elastic πN scattering in terms of our model amplitude. The nucleon vector matrix elements can be written as in Eq. (2.10):

$$\langle N_2|\bar{V}_\mu|N_1\rangle=3\bar{u}(p_2)[\gamma_\mu f_1(q^2)+\sigma_{\mu\nu}q_\nu f_2(q^2)]u(p_1). \quad (3.2)$$

Here q is the momentum transfer $q=p_2-p_1$ and f_1 and f_2 are form factors. Instead of these, it is more convenient to use the Sachs form factors:

$$g_E(q^2)=f_1+\frac{q^2}{2m}f_2, \qquad g_M(q^2)=f_1+2mf_2, \quad (3.3)$$

where m is the nucleon mass. These form factors we can easily find in terms of the proton and neutron electromagnetic form factors:

$$g_E(q^2) = G_E^p(q^2) + G_E^n(q^2),$$
$$g_M(q^2) = G_M^p(q^2) + G_M^n(q^2), \tag{3.4}$$

Experimentally, the so-called scaling law[26] seems to work pretty well, at least for $-q^2 < 2$ GeV^2:

$$G_E^p(q^2) = \frac{1}{\mu_P} G_M^p(q^2) = \frac{1}{\mu_n} G_M^n(q^2) = G_D(q^2) \tag{3.5}$$

and

$$G_E^n(q^2) \simeq 0,$$

where

$$G_D(q^2) = \left(1 - \frac{q^2}{0.71}\right)^{-2}. \tag{3.6}$$

Combining Eqs. (3.3), (3.4), and (3.5) we get

$$\left(2m - \frac{q^2}{2m}\right) f_2(q^2) = (\mu_p + \mu_n - 1) G_D(q^2) = (2.79 - 1.91 - 1) G_D(q^2)$$

or

$$f_2(q^2) = \frac{-0.12}{2m} \frac{G_D(q^2)}{1 - q^2/4m^2} . \tag{3.7}$$

Also

$$g_M(q^2) = +0.88 G_D(q^2) . \tag{3.8}$$

We find f_2 much smaller than g_M and will from now on set it equal to zero. This corresponds to a proton magnetic moment of +3 and −2 for the neutron which is very close to the real values. In other words, from now on

$$f_2(q^2) = 0 \text{ and } g_M(q^2) = G_E^p(q^2) . \tag{3.9}$$

Equation (3.2) then takes the form

$$\langle N_2 | \overline{V}_\mu | N_1 \rangle = 3\,\overline{u}(p_2)\gamma_\mu u(p_1) g_M(q^2) . \tag{3.10}$$

Similarly, we have for the pion vector matrix element

$$\langle \pi_2 | \overline{V}_\mu | \pi_1 \rangle = 2(k_{1\mu} + k_{2\mu}) f_\pi(q^2), \tag{3.11}$$

where $f_\pi(q^2)$ is the pion electromagnetic form factor.

In this way we get for the full πN diffractive amplitude

$$\mathrm{Im}\, T(\pi N \to \pi N) = 6P(t)\overline{u}_2(\not k_1 + \not k_2) u_1 g_M(t) f_\pi(t) . \tag{3.12}$$

It is clear that this result implies s-channel helicity conservation at large energies where this model should apply. This property of elastic πN scattering was first discussed by Gilman *et al.*[27] and is now experimentally verified by Barger and Halzen.[18]

The differential cross section resulting from Eq. (3.12) is

$$\frac{d\sigma}{dt}(\pi N \to \pi N) = 36 \frac{P^2(t)}{4\pi} g_M{}^2(t) f_\pi{}^2(t), \tag{3.13}$$

while for elastic nucleon-nucleon scattering we get

$$\frac{d\sigma}{dt}(NN \to NN) = 81 \frac{P^2(t)}{4\pi} g_M{}^2(t) g_M{}^2(t) . \tag{3.14}$$

Here $P(t)$ is an unknown function of t. Setting it equal to a constant, we recover the original proposal by Wu and Yang[11] for elastic scattering. This is in fair agreement with experiment.

Why should P be a constant? We can make the following argument. In a quark or parton model of the hadrons, $P(t)$ is related to the t dependence of quark-quark or parton-parton scattering. If we consider these collisions as point interactions the only t dependence would result from the form factors of these constituents. But we have learned from the parton model applied to deep-inelastic electroproduction that the partons act as without any structure. All this suggests that we should not be too surprised by finding P to be approximately constant. A parton model for diffractive processes along these lines has been developed by Landshoff and Polkinghorne.[28] They consider parton-parton collisions mediated by a point vector-vector coupling and thereby also obtain s-channel helicity conservation and differential cross sections given by the electromagnetic form factors of the hadrons involved.

From now on we neglect any t dependence in $P(t)$ and determine its constant value from elastic πN scattering, Eq. (3.13). We find

$$\frac{P^2}{4\pi} = 0.82 \text{ mb/GeV}^2 \text{ or } P = 2 \text{ mb}. \tag{3.15}$$

This is the value we will use in all subsequent calculations. In terms of total cross sections it means

$$\sigma_{\text{tot}}(\pi N) = 12P = 24 \text{ mb},$$
$$\sigma_{\text{tot}}(NN) = 18P = 36 \text{ mb}. \tag{3.16}$$

The result

$$\frac{\sigma_{\text{tot}}(\pi N)}{\sigma_{\text{tot}}(NN)} = \frac{2}{3} \tag{3.17}$$

was first derived in the quark model by Levin and

Frankfurt.[29]

Next we will consider the single diffraction excitation process

$$\pi + N \to \pi + N^* . \tag{3.18}$$

The pion vector matrix element will be the same as in Eq. (3.11). For the matrix element of the nucleon vector transition operator we write

$$\langle N^* | \bar{V}_\mu | N \rangle = 2 M v_\mu , \tag{3.19}$$

where M is the mass of the produced nucleon resonance N^*. We find the cross section in the same way as for the similar process of electroproduction as done by Bjorken and Walecka.[30] At energies large compared to the particle masses and the momentum transfer we get

$$\frac{d\sigma}{dt}(\pi N \to \pi N^*) = 4\frac{P^2}{4\pi}\left(\frac{-q^2}{Q^{*2}}\right)\left[\left(\frac{-q^2}{Q^{*2}}\right)|v_0|^2 + \tfrac{1}{2}(|v_+|^2 + |v_-|^2)\right] f_\pi{}^2(t) . \tag{3.20}$$

Here v_0 is the matrix element of the time component $\bar{V}_0/2M$ evaluated in the N^* rest frame, v_+ and v_- are similarly the matrix elements of the $(x \pm iy)/\sqrt{2}$ components. Q^* is the three-momentum of the initial nucleon moving along the negative z axis in the rest system of the resonance,

$$Q^{*2} = [(M+m)^2 - q^2][(M-m)^2 - q^2]/4M^2 . \tag{3.21}$$

If we in reaction (3.18) had incident nucleons instead of pions we would get the relation

$$\frac{d\sigma(NN \to NN^*)}{d\sigma(\pi N \to \pi N^*)} = \frac{d\sigma(NN \to NN)}{d\sigma(\pi N \to \pi N)} = \left(\frac{3 g_M(t)}{2 f_\pi(t)}\right)^2 . \tag{3.22}$$

It is obvious from the formula Eq. (3.20) that we predict zero cross sections in the forward direction for all inelastic diffractive processes. This is a direct consequence of the condition Eq. (2.20). These forward zeros would also have appeared in electroproduction cross sections had it not been for the photon propagator $1/q^2$ which blows up at that point. This is a very powerful prediction and is pivotal to this model. Its experimental verification would be strong evidence for this description tying diffractive excitation of hadrons to electromagnetic excitation. We will discuss the experimental situation in the following section.

Should this prediction be considered a disturbing consequence of this model? We do not think so. It is well known and easy to show that all inelastic diffractive reactions in nuclear physics where the mass changes are negligible, will have zero cross sections in the forward direction. How to decribe hadronic diffraction reactions is obviously not known. Byers and Frautschi[31] have used the same picture as for nuclear reactions and argue for nonvanishing cross sections at $t=0$ because of the mass differences caused by the excitation. On the other hand, Feynman[32] has developed a description which gives forward zeros and Kislinger[33] argues for it by orthogonality of initial and final states.

Another result which follows from Eq. (3.20) is that in all diffraction excitations where the quark spin of the produced particle is different from the quark spin of the initial particle, only t-channel helicity flip will contribute.

This prediction has some immediate consequences when we write down the wave functions of resonances in terms of orbital and quark spin states. The $D_{13}(1520)$ and $S_{11}(1535)$ are believed to belong to octets with quark spin $S=\frac{1}{2}$ in the $[\underline{70}, 1^-]$. Clebsch-Gordan coefficients now give

$$\begin{aligned} D_{13}(1520)\colon\ |\tfrac{3}{2}, +\tfrac{1}{2}\rangle &= (\tfrac{1}{3})^{1/2}|L_z=+1\rangle|S_z=-\tfrac{1}{2}\rangle + (\tfrac{2}{3})^{1/2}|L_z=0\rangle|S_z=+\tfrac{1}{2}\rangle , \\ S_{11}(1535)\colon\ |\tfrac{1}{2}, +\tfrac{1}{2}\rangle &= (\tfrac{2}{3})^{1/2}|L_z=+1\rangle|S_z=-\tfrac{1}{2}\rangle - (\tfrac{1}{3})^{1/2}|L_z=0\rangle|S_z=+\tfrac{1}{2}\rangle . \end{aligned}$$

We therefore get the relation

$$\frac{d\sigma}{dt}(N \to D_{13}(1520)) = 2\frac{d\sigma}{dt}(N \to S_{11}(1535)) . \tag{3.23}$$

If the three nucleon resonances $D_{15}(1670)$, $D_{13}(1700)$, and $S_{11}(1700)$ are assigned to octets with quark spin $S=\frac{3}{2}$ in the $[\underline{70}, 1^-]$, then we get for their cross sections in the same way as above

$$\begin{aligned} \frac{d\sigma}{dt}(N \to D_{15}(1670)) &= \frac{27}{5}\frac{d\sigma}{dt}(N \to S_{11}(1700)) , \\ \frac{d\sigma}{dt}(N \to D_{13}(1700)) &= \frac{28}{5}\frac{d\sigma}{dt}(N \to S_{11}(1700)) . \end{aligned} \tag{3.24}$$

For the pion resonance $A_2(1300)$, $A_1(1070)$, and $\delta(962)$ we can derive similar relations by combining their quark spin $S=1$ with their internal, orbital angular momentum $L=1$. In this way we obtain

$$\frac{d\sigma}{dt}(\pi \to A_2) = \frac{d\sigma}{dt}(\pi \to A_1) \tag{3.25}$$

and

$$\frac{d\sigma}{dt}(\pi \to \delta) = 0 . \tag{3.26}$$

These results have been previously obtained by Levin and Frankfurt[34] who could show that these relations should hold not only for the Pomeranchukon part of the reaction, but for any normal-parity Regge exchange. The main assumption going into their derivation is that additivity of the two-body quark scattering amplitudes is valid. In this way it is not surprising we arrive at the same result. Very recently Le Yaouanc *et al.*[35] have also derived Eqs. (3.25) and (3.26) in their quark model.

According to the present model the amplitudes for diffractive excitation of nucleon resonances are given directly in terms of the amplitudes for electroproduction of proton and neutron resonances:

$$\frac{\mathrm{Im}T_\lambda(NN\to NN^*)}{\mathrm{Im}T_\lambda(NN\to NN)}=\frac{F_\lambda(ep\to ep^*)+F_\lambda(en\to en^*)}{F_\lambda(ep\to ep)+F_\lambda(en\to en)}\ . \tag{3.27}$$

The lower index λ designates the different helicities occurring. This relation is an obvious generalization of Eq. (3.4). If we now make the assumption that the SU(3) difference between the proton and neutron does not change the q^2 dependence of their resonance electroproduction cross sections very much, we get

$$\frac{d\sigma(NN\to NN^*)}{d\sigma(NN\to NN)}\sim\frac{d\sigma(eN\to eN^*)}{d\sigma(eN\to eN)}\ . \tag{3.28}$$

Experimentally,[36] the right-hand side of this relation goes to a constant when $-q^2>2$ GeV^2. This is also predicted to be the case in the parton model.[37]

If for some reason our model can be used at large momentum transfers, then according to (3.28) this should mean that the cross sections for diffractive excitation of nucleons should have the same t dependence when $-t>2$ GeV^2. This is in very good agreement with the experiments of Amaldi *et al.*[38]

We conclude that there is some good evidence for the validity of the extension of the Wu-Yang conjecture for elastic scattering to inelastic scattering as proposed in this phenomenological model. Similar relations between electroproduction and diffraction production have also been explored by Berman and Jacob[39] and Elitzur.[40] In the next section we will use a specific quark model to test the predictions of our diffractive mechanism in a quantitative way.

IV. SOME SPECIFIC QUARK-MODEL RESULTS

To calculate the matrix elements entering the expression for the cross section, Eq. (3.20), we will use the relativistic quark model of Feynman, Kislinger, and Ravndal.[12] It has been applied to electroproduction of nucleon resonances by Ravndal[41] and Copley, Karl, and Obryk.[42] The results obtained are apparently in good agreement with existing data.

In this model the diffractive matrix elements for nucleon excitation can be written as

$$\begin{aligned} v_0 &= 9G\langle N(+\tfrac{1}{2})|e_a Se^{-\lambda a_z}|N^*(+\tfrac{1}{2})\rangle\,,\\ v_- &= 9G\langle N(+\tfrac{1}{2})|e_a(Ta_+ +R\sigma_{a-})e^{-\lambda a_z}|N^*(+\tfrac{3}{2})\rangle\,, \\ v_+ &= 9G\langle N(+\tfrac{1}{2})|e_a(Ta_- +R\sigma_{a+})e^{-\lambda a_z}|N^*(-\tfrac{1}{2})\rangle\,, \end{aligned} \tag{4.1}$$

Here a_z, a_+, and a_- are harmonic-oscillator lowering operator for the internal quark wave functions and $\vec{\sigma}_a$ is the spin of quark a with SU(3) charge e_a. The matrix elements of these charges are for nucleons equal to $3F-D$ where F and D can be found in Ref. 12. Furthermore we have

$$\begin{aligned} &\lambda=\left(\frac{2}{\Omega}\right)^{1/2}Q^*\,,\quad T=\frac{1}{3M}\left(\frac{\Omega}{2}\right)^{1/2},\\ &S=(3Mm+q^2-m^2)/6M^2\,,\\ &R=\sqrt{2}\,Q^*\frac{M+m}{(M+m)^2-q^2}\,, \end{aligned} \tag{4.2}$$

where Q^* is given in Eq. (3.21) and $\Omega=1.05$ GeV^2 is the harmonic-oscillator constant. Notice that for elastic scattering, $M=m$, S in (4.2) is inconsistent with Eq. (3.10). That expression corresponds to

$$S(M=m)=\tfrac{1}{3}\ . \tag{4.3}$$

However, when $-q^2<2m^2$ the difference between Eq. (4.3) and S in (4.2) is small.

Since we will only consider resonances with mass $M<1750$ MeV, we will use for the form factor G:

$$G(M,t)=e^{2.5t}\left(1-\frac{t}{4M^2}\right)^{(1-N)/2}\ . \tag{4.4}$$

N is here the number of excitations in the N^*. With this choice the differential cross section for elastic NN scattering in Eq. (3.14) takes the form

$$\frac{d\sigma}{dt}(NN\to NN)=81\frac{P^2}{4\pi}e^{10t}\ , \tag{4.5}$$

which is close to experiment when $-t<1$ GeV^2.

Using now the tables in Ref. 12 it is easy to obtain all the diffractive transition amplitudes for the resonances of interest. These are given in Table I. Here we have also included the not-yet-established $P_{13}(1700)$ which is predicted to exist in the quark model as the quark spin partner of the $F_{15}(1688)$.

We are now in the position to calculate the differential cross sections according to Eqs. (3.20) and (3.22). In Fig. 2 we present the result for the Roper resonance, $P_{11}(1470)$. The next resonance bump $N^*(1525)$ should consist of the sum of $D_{13}(1520)$ and $S_{11}(1535)$ as shown in Fig. 3, while

TABLE I. Classification of the lowest nucleon resonance and their diffractive transition amplitudes.

Nucleon resonance	SU(6) ⊗ O(3)	$^{2S+1}(\mathrm{SU}(3))_J$	$v_0/3G$	$v_+/3G$	$v_-/3G$
$D_{13}(1520)$	$[\underline{70}, 1^-]$	$^2(\underline{8})_{3/2}$	0	$-(\frac{4}{3})^{1/2}\lambda R$	0
$S_{11}(1535)$	$[\underline{70}, 1^-]$	$^2(\underline{8})_{1/2}$	0	$+(\frac{2}{3})^{1/2}\lambda R$	
$D_{15}(1670)$	$[\underline{70}, 1^-]$	$^4(\underline{8})_{5/2}$	0	$+(\frac{3}{10})^{1/2}\lambda R$	$-(\frac{3}{5})^{1/2}\lambda R$
$D_{13}(1700)$	$[\underline{70}, 1^-]$	$^4(\underline{8})_{3/2}$	0	$-(\frac{1}{30})^{1/2}\lambda R$	$-(\frac{9}{10})^{1/2}\lambda R$
$S_{11}(1700)$	$[\underline{70}, 1^-]$	$^4(\underline{8})_{1/2}$	0	$-(\frac{1}{6})^{1/2}\lambda R$	
$P_{11}(1470)$	$[\underline{56}, 0^+]$	$^2(\underline{8})_{1/2}$	$-(\frac{3}{4})^{1/2}\lambda^2 S$	$-(\frac{1}{12})^{1/2}\lambda^2 R$	
$F_{15}(1688)$	$[\underline{56}, 2^+]$	$^2(\underline{8})_{5/2}$	$+(\frac{9}{10})^{1/2}\lambda^2 S$	$-(\frac{9}{5})^{1/2}\lambda T+(\frac{1}{10})^{1/2}\lambda^2 R$	$-(\frac{18}{5})^{1/2}\lambda T$
$P_{13}(1700)$	$[\underline{56}, 2^+]$	$^2(\underline{8})_{3/2}$	$-(\frac{3}{5})^{1/2}\lambda^2 S$	$+(\frac{27}{10})^{1/2}\lambda T+(\frac{1}{15})^{1/2}\lambda^2 R$	$+(\frac{9}{10})^{1/2}\lambda T$

the third bump, $N^*(1690)$, in Fig. 4 consists of $S_{11}(1700)$, $D_{13}(1700)$, $D_{15}(1670)$, $P_{13}(1700)$, and $F_{15}(1688)$. In these figures we have also plotted data from Amaldi *et al.*[38]

Except for in the very near forward direction where there is no data, the agreement with experiment is very good for the $N^*(1525)$ and the $N^*(1690)$. More data at smaller $-t$ are necessary in order to check the forward dip predicted by this model. However, it looks like the present

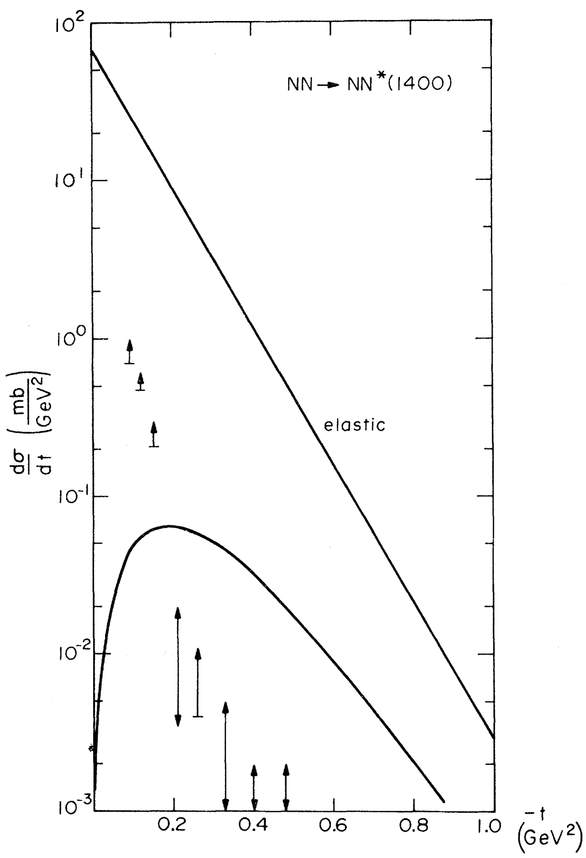

FIG. 2. Calculated diffractive differential cross section for the $P_{11}(1470)$ and measured cross sections for the $N^*(1400)$ from Ref. 38.

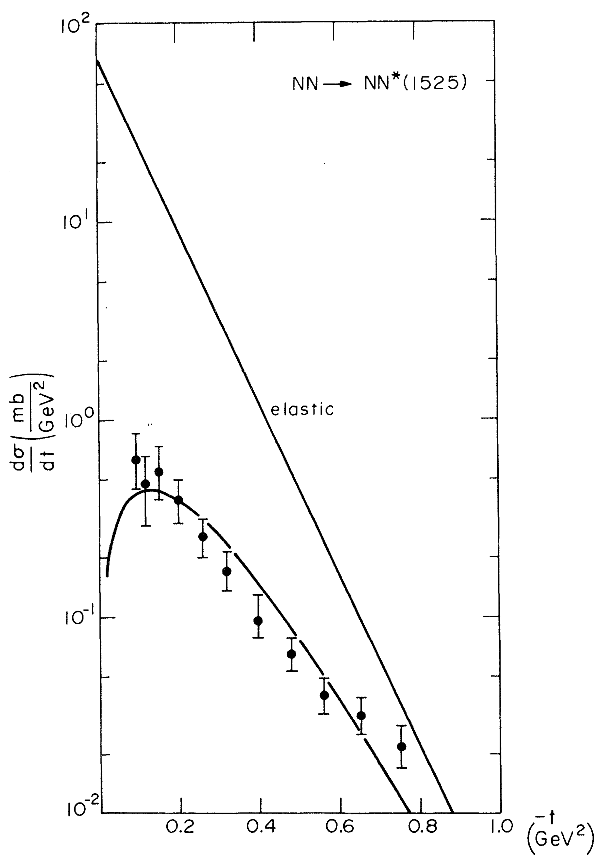

FIG. 3. Experimental diffractive differential cross section for the $N^*(1525)$ from Ref. 38 compared with the sum of the calculated contributions from $D_{13}(1520)$ and $S_{11}(1535)$.

data are flattening somewhat out around $-t = 0.1-0.2$ GeV^2. In pion-induced diffraction production experiments by Anderson *et al.*[43] this flattening is more evident. Even a forward dip in the $N^*(1525)$ can be seen.

Very bad agreement between the present theory for the $P_{11}(1470)$ and experiments for the $N^*(1400)$ is found in Fig. 2. First of all the $N^*(1400)$ is shifted in mass relative to the $P_{11}(1470)$ which is well established in pion-nucleon phase-shift analysis. Experimentally, the $N^*(1400)$ has a forward peak with slope larger than for elastic scattering, while the $P_{11}(1470)$ has a much smaller slope, similar to the $N^*(1525)$ and $N^*(1690)$.

It is possible to make a consistent explanation of these discrepancies between theory and experiment. We believe that there is no relation between the $N^*(1400)$ and $P_{11}(1470)$. The $N^*(1400)$ is a pure kinematical enhancement due to one-pion exchange, the so-called Deck effect.[44] A diagram of this process is shown in Fig. 5.

The incoming nucleon disintegrates into a virtual

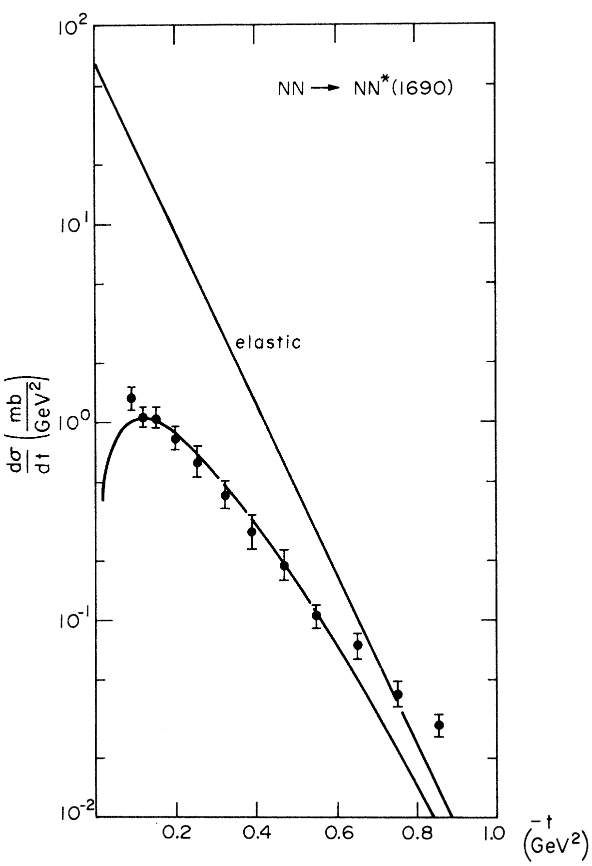

FIG. 4. Experimental diffractive differential cross section for the $N^*(1690)$ from Ref. 38 compared with the sum of the calculated contributions from $F_{15}(1688)$, $P_{13}(1700)$, $D_{15}(1670)$, $D_{13}(1700)$, and $S_{11}(1700)$.

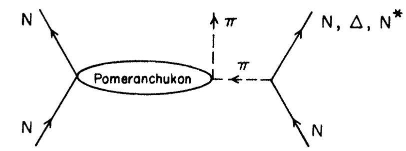

FIG. 5. OPE diagrams contributing to the background in nucleon diffraction excitation.

pion and a N, Δ, or N^*. The pion scatters off the other nucleon and will together with the N, Δ, or N^* give a broad enhancement in missing mass. Detailed calculations by Berger[45] are able to explain both the shape and absolute magnitude of the $N^*(1400)$. The $P_{11}(1470)$ itself gives a small cross section which will be hard to separate from the $N^*(1525)$.

If we believe in this picture, there should also be a sharply peaked kinematical enhancement where the $N^*(1525)$ and $N^*(1690)$ are. This would then to some degree fill in the dips we predict for the genuine resonances. Experimentally, we would then expect to see for $-t < 0.1$ GeV^2 a smooth background with no resonance structure. This prediction will hopefully be tested in the very near future. There already exists some evidence this may be true in diffraction dissociation experiments off heavy nuclei[46] where small enough momentum transfers can be more easily obtained.

We will now consider diffractive excitation of meson resonances in the same quark model as used for baryons. For the process

$$M + N \to M^* + N, \tag{4.6}$$

where the meson M in our case will be the pion and M^* the $A_1(1070)$, $A_2(1300)$, or $A_3(1650)$, the cross section can be written in the same form as in Eq. (3.20):

$$\frac{d\sigma}{dt}(\pi N \to \pi^* N) = 9\frac{P^2}{4\pi}\left(\frac{-q^2}{Q^{*2}}\right)\left[\left(\frac{-q^2}{Q^{*2}}\right)|v_0|^2 + \tfrac{1}{2}(|v_+|^2 + |v_-|^2)\right] g_M{}^2(t). \tag{4.7}$$

Here $g_M(t)$ is the nucleon form factor, which we as previously will set equal to, for $-t < 1$ GeV^2:

$$g_M(t) = e^{2.5t}. \tag{4.8}$$

The transition amplitudes in Eq. (4.7) are now given in the quark model:

$$\begin{aligned} v_0 &= 4F\langle M(0)|e_a s e^{-\delta a_z}|M^*(0)\rangle, \\ v_- &= 4F\langle M(0)|e_a(ta_+ + r\sigma_{a-})e^{-\delta a_z}|M^*(+1)\rangle, \\ v_+ &= 4F\langle M(0)|e_a(ta_- + r\sigma_{a+})e^{-\delta a_z}|M^*(-1)\rangle. \end{aligned} \tag{4.9}$$

In these expressions r, s, t, and δ correspond to

TABLE II. Classification of the lowest pion resonances and their diffractive transition amplitudes.

Pion resonance	J^{PC}	SU(6)⊗O(3)	$^{2S+1}(\mathrm{SU}(3))_J$	$v_0/2F$	$v_+/2F$	$v_-/2F$
$A_2(1300)$	2^{++}	$[\underline{35}, 1^+]$	$^3(\underline{8})_2$	0	$+\delta r$	$+\delta r$
$A_1(1070)$	1^{++}	$[\underline{35}, 1^+]$	$^3(\underline{8})_1$	0	$+\delta r$	$+\delta r$
$\delta(962)$	0^{++}	$[\underline{35}, 1^+]$	$^3(\underline{8})_0$	0		
$A_3(1650)$	2^{-+}	$[\underline{35}, 2^-]$	$^1(\underline{8})_2$	$+(\frac{4}{3})^{1/2}\delta^2 s$	$-2\delta t$	$-2\delta t$

the baryon structure functions R, S, T, and λ in (4.1):

$$\delta = \left(\frac{1}{\Omega}\right)^{1/2} Q^*, \quad t = \frac{1}{4M}(\Omega)^{1/2},$$

$$s = (4Mm + M^2 - m^2 + q^2)/8M^2, \tag{4.10}$$

and

$$r = R = \sqrt{2}\, Q^* \frac{M+m}{(M+m)^2 - q^2} \;.$$

For elastic scattering we find that s will be zero when $-q^2 = 4m^2$. In other words, the pion form factor should become zero at $-t = 0.08$ GeV^2. That is certainly a disturbing consequence of this particular quark model. To heal this disease, we impose the condition as in the nucleon case, Eq. (4.3):

$$s(M = m) = \tfrac{1}{2}\,. \tag{4.11}$$

The form factor F entering the amplitudes in (4.9) will be chosen as

$$F(M, t) = e^{-K^{*2}/\Omega} e^{1.5t}\left(1 - \frac{t}{4M^2}\right)^{1-N}. \tag{4.12}$$

This corresponds to Eq. (4.4) for nucleons except for the K^*-dependent term where

$$K^* = (M^2 - m^2)/2M. \tag{4.13}$$

The reason for this extra term is the following. In this model the matrix elements have to satisfy Eq. (2.20). For the π-A_2 transition this corresponds to (as shown in Ref. 41)

$$M^2(A_2) - m^2(\pi) = \Omega$$
$$= 1.05 \text{ GeV}^2\,. \tag{4.14}$$

Since $M(A_2) = 1300$ MeV this condition is not satisfied because of the small pion mass. We could improve the situation by using in Eq. (4.14) not the real pion mass, but the mean mass of the ground-state mesons which is around 700 MeV. This was done in Ref. 13. Here we will instead take $m_\pi = 138$ MeV together with the extra compensating term in Eq. (4.12). The net result is almost the same. This kind of form factor is also crucial when calculating decay rates of meson resonances in this quark model.[12]

Our choice for the meson form factor (4.12) gives for the differential cross section in elastic πN scattering

$$\frac{d\sigma}{dt}(\pi N \to \pi N) = 36 \frac{P^2}{4\pi} e^{8t}\,. \tag{4.15}$$

This is a good description of data for $-t < 1$ GeV^2.

In Table II we show the diffraction transition amplitudes of interest. The calculated differential cross section for the A_1, A_2, and A_3 are shown in Fig. 6. When we use the real masses for the $A_1(1070)$ and $A_2(1300)$, the equality of their cross sections, Eq. (3.25), will be slightly broken as shown in the figure.

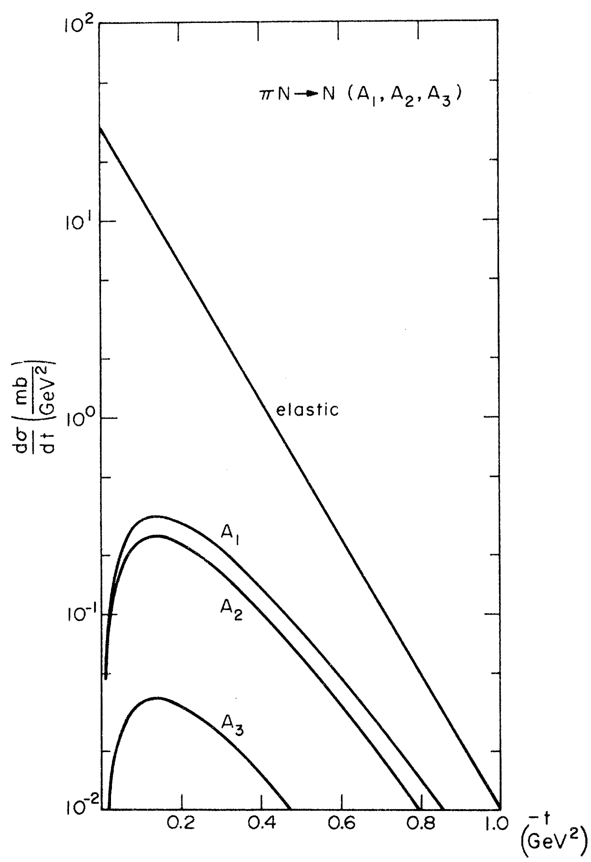

FIG. 6. Calculated diffractive differential cross sections for $A_1(1070)$, $A_2(1300)$, and $A_3(1650)$.

Let us now consider the experimental situation. Ascoli *et al.*[47] have considered all 3π events in the A_2 mass region. The data peak in the forward direction. Analyzing the spin content of the 3π mass bump, they find a nonresonant 1^+, $\rho\pi$ background which peaks in the forward direction and a 2^+, $\rho\pi$ resonance structure which dips at $t=0$. This is identified with the $A_2(1300)$. They also show that the A_2 is produced only with transverse helicity in its rest frame. This is all in agreement with our predictions. Unfortunately, these experiments are done at the rather low energies of 5 and 7.5 GeV so this evidence cannot be considered conclusive as a test of our diffractive model which should only apply at high energies where all Regge contributions have died away.

The A_2 integrated differential cross section has been found to be 180 ± 60 μb at 16 GeV by Ballam *et al.*[48] Carroll *et al.*[49] have measured 130 ± 30 μb at 7 GeV and 80 ± 40 μb at 25 GeV. According to the present theory it should be

$$\sigma(A_2)=92\ \mu\text{b}, \tag{4.16}$$

which seems to be in reasonable agreement with experiment.

Experimentally, the A_1 situation is not very clear. The cross section is peaked in the forward direction. This is similar to the 1^+ background in the A_2 case. It is now believed that this background is due to one-pion exchange (OPE) as shown in Fig. 7. This corresponds to the diagram in Fig. 5 for the nucleon case. However, the A_1 being a 1^+ resonance, it is very hard to subtract any background in its cross section.

One has also observed that this A_1 bump is produced with zero helicity in its rest frame.[50] As shown by Silver,[51] this can be understood by the OPE mechanism. According to our model, the A_1 should have purely transverse helicities. So we conclude that the real A_1 is somewhere hidden in a dominant OPE 1^+ background.

Integrated cross sections for the whole A_1 bump have been found equal to 250 ± 50 μb at 16 GeV by Ballam *et al.*[48] and 160 ± 40 μb at 25 GeV by Ascoli *et al.*[50] We get

$$\sigma(A_1)=118\ \mu\text{b}. \tag{4.17}$$

If the real A_1 is only a small part of the A_1 bump, this value is apparently somewhat too large.

The differential cross section for the A_3 bump also peaks in the forward direction.[52] However, a recent experiment by Harrison *et al.*[53] reveals that much of the A_3 structure consists of nonresonant $f\pi$ and $\rho\pi$ background. This obviously stems from the Deck effect, Fig. 7. Only the 3π mode shows a clear resonance structure at 1650 MeV. If the $\rho\pi$ and $f\pi$ background is subtracted

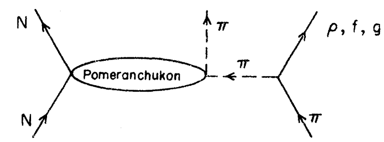

FIG. 7. OPE diagram contributing to the background in pion diffraction excitation.

from the total differential cross section, the remaining genuine A_3 resonance should dip in the forward direction. Observing this effect would be strong support for this model.

Harrison *et al.*[53] have also measured the production cross section for the A_3 resonance. They find 36 ± 13 μb at 13 GeV and 24 ± 6 μb at 20 GeV. This is in good agreement with our calculated value,

$$\sigma(A_3)=16\ \mu\text{b}. \tag{4.18}$$

Le Yaouanc *et al.*[35] have obtained results for the $A_1(1070)$ and $A_2(1300)$ very similar to ours. However, they find a larger cross section for the A_3 which should be peaked in the forward direction in contrast to our forward dip. Also they predict that the A_3 should be produced with zero t-channel helicities which is in disagreement with our results.

V. CONCLUSION

We have proposed a phenomenological model for diffractive excitation of hadron resonances. It can be considered as a generalization of the Wu-Yang idea for elastic scattering. In this way we can relate diffraction production to electroproduction. We tried to make the model plausible by involving f and f' dominance of the Pomeranchukon, exchange degeneracy, and universality of the residues of the vector-meson Regge trajectories. This simple model combines in a unified way many apparently different ideas and models which have previously been proposed for diffractive processes and we recover most of their good results.

The selection rules proposed by Morrison and Carlitz, Frautschi, and Zweig seem to be ruled out by experiment. Instead of these qualitative ideas, our model is in accordance with most available data for diffractive excitation of nucleon and pion resonances. Using a specific quark model we are even able to obtain fair quantitative agreement with measured cross sections in a calculation involving essentially no free parameters.

The present model predicts that all inelastic diffractive cross sections for hadron resonances should be zero in the forward direction. Experimental verification of this prediction is crucial for the survival of the proposed, effective mechanism for diffractive excitation. It should be noted that these forward dips are consistent with the selection rule of Chou and Yang.

Of all the different resonances considered, only the $A_1(1070)$ was found to be in clear disagreement with experiment. If this resonance is not found in diffractive reactions (or exchange reactions), not only must this model be abandoned, but the whole quark model will be in trouble.

Our phenomenological model is in no way thought to be the final answer to the couplings of the Pomeranchukon. But we believe that the ideas presented here give a better description of the real world and have a better chance of survival than the limited understanding which follows from the old selection rules.

Obviously there are still many questions to be asked. For instance, should our amplitude be eikonalized so to partially fill in the forward dips? Is our coupling compatible with the OPE-diagrams in Figs. 5 and 7 in the sense of duality? Should the crucial condition in Eq. (2.20) be abandoned? Hopefully these and other questions will be answered in the near future.

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Rapidity and Angular Distributions of Charged Secondaries According to the Hydrodynamical Model of Particle Production*

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Recent measurements of inclusive production cross sections are analyzed in the framework of the Landau hydrodynamical model of particle production. We also give a critical analysis of recent data and the variables used in their presentation. It is concluded that the evidence for a flat rapidity distribution in the central region is not compelling. Except possibly at the very highest available ISR (CERN Intersecting Storage Rings) energy, the Landau Gaussian gives an excellent description of the rapidity distribution of the nonleading charged secondaries. The calculation of distributions in the variable $\eta = -\ln \tan(\theta/2)$ from given rapidity and transverse momentum distributions is worked out in many interesting cases. The Landau rapidity distribution is cast in a universal (energy dependent) scaling law which agrees well with available data. The angle and energy dependence of charged secondaries near 90° in the c.m. system in pp collisions agrees well with the theoretical prediction. Finally it is shown that the hydrodynamical model leads to approximate Feynman scaling except for very small values of $x = 2p_{\parallel}/\sqrt{s}$, where large deviations from scaling are predicted.

I. INTRODUCTION

The description of particle production in high-energy collisions has recently attracted a great deal of attention. Experiments at the CERN intersecting storage rings (ISR) and NAL are beginning to reveal interesting patterns in such processes. The approximate validity of scaling laws has been established for inclusive cross sections and is perhaps the most striking single result. No adequate theory yet exists which gives a satisfactory description of the data. We refer[1-3] to some recent review articles which summarize experimental results and the partial insights obtained from various phenomenological models.

The main purpose of the present paper is to resurrect Landau's hydrodynamical model[4] of particle production and particularly to elaborate its phenomenological consequences pertinent to recent experimental work. Secondly, we discuss some purely phenomenological questions having to do with the rapidity variable and the related "cosmic-ray" variable $\eta = -\ln\tan(\theta/2)$ (Secs. II and III). The hydrodynamical model, which was well regarded in the 1950's, suggests a number of interesting lines of research when cast in modern garb. Here we shall analyze the experimental consequences of the most simple version of the model, in an extension of previous work by the authors.[5] This version involves rather brutal approximations to the complicated equations of the complete theory and hence could give a distorted picture of the theory's true predictions.

The hydrodynamical model can be regarded as an extension of Fermi's statistical model.[6] One envisions a thin slab of hot hadronic matter in thermal equilibrium just after the collision; strictly speaking this is a "head-on" collision picture, but one can imagine that a fraction of the collision products are described by this initial condition while leading particles carry away a sizeable fraction (of the order of $\frac{1}{2}$) of the energy and perhaps most of the angular momentum.[1] In Landau's model the particles do not jump right out into phase space (which leads to too many heavy particles in Fermi's picture), but undergo an expansion phase before breaking up. The force responsible for the expansion is large in the longitudinal direction (the pressure gradient is mainly in the longitudinal direction because of the Lorentz contraction) and provides a natural dynamics for the well-known transverse-longitudinal asymmetry