[14]We have used the normalization as $\bar{u}u = m^{2S}/E$ for any spin in Refs. 3 and 4 and $\bar{u}_\lambda u_\lambda = m'^3/2E'$. The differential cross section which appeared in Refs. 3 and 4 should be divided by $\frac{1}{2}m'^2$.

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# Electron-Deuteron Scattering and Two-Photon Exchange*

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The general expression for the electron-deuteron scattering amplitude is derived within the framework of Glauber theory. An approximation to this expression, used earlier, is shown to be valid at large, but not at small, momentum transfers. Two-photon exchange effects are somewhat smaller than previously thought. A simple extrapolation of the data does *not* indicate two-photon exchange dominance at large momentum transfers. Existing data indicate that because of interferences, two-photon exchange effects can change the cross section by $\sim 10\%$ for $-t \approx 1$ $(\text{GeV}/c)^2$.

It has been known for some time that double scatterings dominate in collisions of high-energy hadrons with deuterons at large momentum transfers.[1] The methods used to analyze such collisions are usually based upon Glauber theory.[2] The amplitude $F_2$ for double scattering is a two-dimensional integral, over momentum transfers $\hbar\vec{q}$, involving the hadron-proton and hadron-neutron strong-interaction elastic scattering amplitudes $f_p(\vec{q})$ and $f_n(\vec{q})$ and the deuteron wave function. It takes the form[3]

$$F_2(\vec{q}) = \frac{i}{2\pi k}\int f_n(\tfrac{1}{2}\vec{q}+\vec{q}\,')f_p(\tfrac{1}{2}\vec{q}-\vec{q}\,')S(\vec{q}\,')d^2q', \quad (1)$$

where $S(\vec{q})$ is the deuteron form factor and $\hbar\vec{k}$ is the incident momentum. Since the deuteron is considerably larger than the range of the hadron-nucleon strong interaction, $S(q')$ decreases much more rapidly with increasing $q'$ near $q'=0$ than do $f_{n,p}(\frac{1}{2}\vec{q} \pm\vec{q}\,')$. Hence it is usually a good approximation to replace the amplitudes $f_n$ and $f_p$ in Eq. (1) by their values at $q'=0$. This leads to an approximation to $F_2$ given by

$$F_2(q) \approx i f_n(\tfrac{1}{2}q)f_p(\tfrac{1}{2}q)\langle r^{-2}\rangle/k, \quad (2)$$

where $\langle r^{-2}\rangle$ is the expectation value, in the deuteron ground state, of the inverse-square neutron-proton separation. The intensity for double scattering is then given by

$$|F_2(q)|^2 \approx \frac{1}{k^2}\frac{d\sigma_n(\frac{1}{2}q)}{d\Omega}\frac{d\sigma_p(\frac{1}{2}q)}{d\Omega}\langle r^{-2}\rangle^2. \quad (3)$$

Double scattering is typically smaller than single scattering near the forward direction. However the single-scattering intensity contains a factor $S^2(\frac{1}{2}q)$ which decreases rapidly with $q$ near $q=0$. We see that in Eq. (3) the structure of the deuteron appears only via $\langle r^{-2}\rangle$, a constant. Consequently, double scattering does not decrease so rapidly with $q$, and eventually dominates single scattering. Its $q$ dependence is insensitive to the structure of the deuteron.

Equation (3) has been recently used by Gunion and Stodolsky[4] to describe electron-deuteron scattering, an electromagnetic interaction. Since Eqs. (1)–(3) were derived for strong interactions, it is necessary to derive an equivalent expression for the electromagnetic case. We see, for example, that for $e$-$d$ scattering the integral in Eq. (1) diverges since $f_p(q)\propto q^{-2}$ for small $q$. Hence Eqs. (2) and (3) will clearly not be valid for small $q$. The need for a special derivation was recognized in Ref. 4.

Let the $e$-$p$ and $e$-$n$ scattering amplitudes, $f_p$ and $f_n$, be written as

$$f_{p,n}(\vec{q}) = \frac{ik}{2\pi}\int [1-e^{i\chi_{n,p}(\vec{b})}]e^{i\vec{q}\cdot\vec{b}}d^2b, \quad (4)$$

where $\vec{b}$ is an impact-parameter vector and $\chi_{n,p}(\vec{b})$ are phase-shift functions. In Glauber theory, the $e$-$d$ elastic scattering amplitude will take the form[2,3]

$$F(\vec{q})=\frac{ik}{2\pi}\int\{1-e^{i[\chi_n(\vec{b}-\vec{s}/2)+\chi_p(\vec{b}+\vec{s}/2)]}\}$$
$$\times e^{i\vec{q}\cdot\vec{b}}|\varphi(\vec{r})|^2d^2b\,d\vec{r}, \qquad (5)$$

where $\varphi(\vec{r})$ is the deuteron ground-state wave function and $\vec{s}$ is the projection of $\vec{r}$ onto the plane of impact parameters. We note the identity

$$1-e^{i(\chi_p+\chi_n)}\equiv 1-e^{i\chi_p}+e^{i\chi_p}(1-e^{i\chi_n}). \qquad (6)$$

By means of Eqs. (4) and (6), Eq. (5) becomes

$$F(\vec{q})=f_p(\vec{q})S(-\tfrac{1}{2}\vec{q})$$
$$+\frac{ik}{2\pi}\int e^{i\chi_p(\vec{b}+\vec{s}/2)}[1-e^{i\chi_n(\vec{b}-\vec{s}/2)}]$$
$$\times e^{i\vec{q}\cdot\vec{b}}|\varphi(\vec{r})|^2d^2b\,d\vec{r}. \qquad (7)$$

By inverting the two-dimensional Fourier transform, Eq. (4), we may eliminate $\chi_n$ in Eq. (7) and obtain

$$F(\vec{q})=f_p(\vec{q})S(-\tfrac{1}{2}\vec{q})$$
$$+\frac{1}{4\pi^2}\int e^{i\vec{b}\cdot(\vec{q}/2-\vec{q}')}e^{i\chi_p(\vec{b})}f_n(\tfrac{1}{2}\vec{q}+\vec{q}')$$
$$\times S(\vec{q}')d^2q'\,d^2b. \qquad (8)$$

In this expression $\chi_p$ is that operator which, when used in Eq. (4), yields the $e$-$p$ scattering amplitude. Equation (8) is the general expression for $e$-$d$ scattering in Glauber theory and contains single and double scattering effects (i.e., one- and two- photon exchange effects). Equation (8) may also be written as

$$F(\vec{q})=f_p(\vec{q})S(-\tfrac{1}{2}\vec{q})+f_n(\vec{q})S(\tfrac{1}{2}q)$$
$$+\frac{1}{4\pi^2}\int e^{i\vec{b}\cdot(\vec{q}/2-\vec{q}')}[e^{i\chi_p(\vec{b})}-1]$$
$$\times f_n(\tfrac{1}{2}\vec{q}+\vec{q}')S(\vec{q}')d^2q'\,d^2b, \qquad (9)$$

in which the last term may be identified as a double scattering term. In Eqs. (8) and (9) we do not eliminate $\chi_p$ in favor of $f_p$ since Eq. (4) cannot be inverted for a Coulomb amplitude. We now make the peaking approximation that led to Eq. (2) for the strong-interaction case. However the $e$-$p$ and $e$-$n$ amplitudes vary *quite rapidly* for small momentum transfers. Consequently the peaking approximation is not valid near the forward direction. It is a valid approximation provided that $S(q)$ decreases rapidly enough with $q$, and provided that $q^2\langle r^2\rangle\gg 1$. In that range of $q$, $f_n(\frac{1}{2}\vec{q}+\vec{q}')$ does not vary rapidly except near $\vec{q}'=-\frac{1}{2}\vec{q}$. However if $q^2\langle r^2\rangle\gg 1$, the contribution to the integral from $\vec{q}'\approx-\frac{1}{2}\vec{q}$ is small. [Note that $f_n(q)\propto q^{-1}$ for small $q$, in contrast to $f_p$, which goes as $q^{-2}$ for small $q$.] The double scattering term $F_2(\vec{q})$ is then given approximately by

$$F_2(\vec{q})\simeq(2\pi)^{-2}f_n(\tfrac{1}{2}q)\int e^{i\vec{b}\cdot\vec{q}/2}[e^{i\chi_p(\vec{b})}-1]d^2b$$
$$\times\int S(\vec{q}')d^2q' \qquad (10)$$
$$=if_n(\tfrac{1}{2}\vec{q})f_p(\tfrac{1}{2}\vec{q})\langle r^{-2}\rangle/k, \qquad (11)$$

which is identical with Eq. (2) except that it is not valid for small $q$. The cross section for double scattering is then[4]

$$\frac{d\sigma^{\text{double}}}{d\sigma^{\text{Coulomb}}}\approx\left(\frac{32\alpha\langle r^{-2}\rangle}{q^2}\right)^2\left\{(G_E^pG_E^n)^2+\frac{q^2}{24M^2}\left[(G_E^pG_M^n)^2+(G_E^nG_M^p)^2+G_E^pG_E^nG_M^pG_M^n\right]+\left(\frac{q^2}{16M^2}G_M^pG_M^n\right)^2\right\}\Big/(1+q^2/16M^2)^2, \qquad (12)$$

where the $G$'s are electromagnetic form factors and are to be taken at $\frac{1}{2}q$, $\alpha$ is the fine-structure constant, and $d\sigma^{\text{Coulomb}}=(2\alpha k/q^2)^2d\Omega$.

To obtain an estimate of Eq. (12) we take the "dipole" fit

$$G_E^p(q)=(1+q^2/0.71)^{-2}$$
$$=G_M^p(q)/\mu_p$$
$$=G_M^n(q)/\mu_n, \quad \mu_p=2.79, \quad \mu_n=-1.91.$$

For $G_E^n$ we take[5] $G_E^n(q)=-\mu_n\tau G_E^p(q)/(1+5.6\tau)$, $\tau=q^2/4M^2$. The data[5,6] for $d\sigma/d\sigma^{\text{Coulomb}}$ are shown in Fig. 1. [We actually plot $A(q)$ in the usual expression $d\sigma/d\sigma^{\text{Coulomb}}=A+B\tan^2(\frac{1}{2}\theta)$ since we assume $\theta^2\ll 1$.] We also show $d\sigma^{\text{double}}/d\sigma^{\text{Coulomb}}$ for $\langle r^{-2}\rangle=0.0107\text{ GeV}^2$ (obtained[3] from the Hamada-Johnston potential) and for $\langle r^{-2}\rangle=0.015\text{ GeV}^2$ (the value used in Ref. 4).

We first remark that the effect of $G_E^n$ on the intensity is only ~2% and is consequently unimportant. Secondly, most realistic deuteron wave functions yield a value for $\langle r^{-2}\rangle$ closer to 0.0107 GeV$^2$ and hence the lower curve is a more realistic estimate of the two-photon exchange effect.

We see from Fig. 1 that the double scattering intensity is considerably smaller than the data, and that if the data continues its present trend, the double scattering intensity will remain considerably smaller. Furthermore, the two-photon exchange effect appears to have its greatest effect near $q^2 \approx 1.3\ \mathrm{GeV}^2$. There the double-scattering intensity is only ~1/400 of the total intensity. Consequently the "modulus" of the double scattering *amplitude* is roughly 5% of that of the complete scattering amplitude. Now since the single- and double-scattering amplitudes interfere, the double-scattering amplitude could change the total intensity by as much as approximately 10%. Thus the two-photon exchange effect is a 10% effect at $q^2 \approx 1.3\ \mathrm{GeV}^2$.

We point out that use of the general expression Eq. (8) requires knowledge of $\chi_p(b)$. For the "dipole" form factors we have used for $G_E^p$ and $G_M^p$, we obtain

$$e^{i\chi_p(\vec{b})} = (ab)^{2i\alpha} - 2\alpha[K_0(ab) + \tfrac{1}{2}abK_1(ab)] - 2i\alpha\vec{\sigma}^p\cdot(\hat{b}\times\hat{k})(Mb)^{-1} \times[1 - \tfrac{1}{2}a^2b^2K_2(ab)], \qquad (13)$$

where $a^2 = 0.71\ \mathrm{GeV}^2$ and $K_0$, $K_1$, $K_2$ are modified Bessel functions. This result may be used in Eq. (8) to calculate the double scattering amplitude at small or intermediate momentum transfers.

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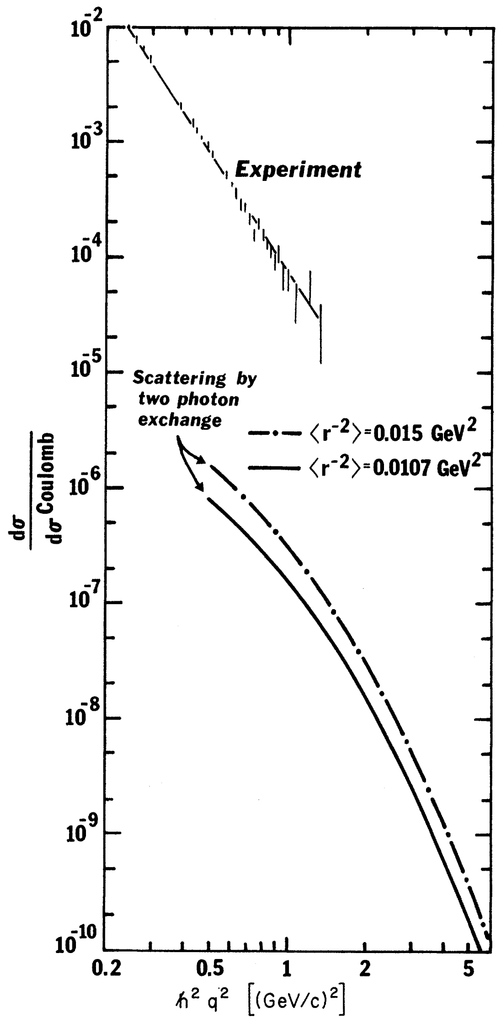


FIG. 1. Intensity for double scattering in elastic $e$-$d$ collisions. Theoretical results are shown for two values of $\langle r^{-2}\rangle$. Also shown are the data from Refs. 5 and 6 for $A(q)$ in $d\sigma = d\sigma^{\mathrm{Coulomb}} A(q)$, together with a straight line drawn through the data points.

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