Dependence upon a limited common pool of polarization data from p-carbon scattering is intrinsic to experiments using this reaction as an analyzer. The data relevant to the present experiment are from sources referenced in McNeely's report above. Kretschmar associated with each interpolated p-carbon analyzing power determination a quantitative uncertainty which reflected the statistical accuracy and interconsistency of the contributing measurements. The small ultimate effect of these uncertainties is discussed in Sec. IV, above, and is illustrated in Fig. 2. These tests do not rule out the possibility that the original results on p-carbon scattering may simply be wrong. This contingency would have a more severe effect on experiments in which large proton polarizations are observed than on the present one which demonstrates an apparent zero-crossing.

[6] Polarizations were obtained by maximizing the likelihood function

$$\mathcal{L}(P)=\prod_i\,[1+A(T_i,\,\theta_i,\,\Delta T_i)P\cos\varphi_i\,]\,,$$

where φ_i is the azimuthal p-carbon scattering angle for each event and A is the corresponding analyzing power as a function of energy, polar angle, and inelasticity.

[7] E. D. Bloom, C. A. Heusch, C. Y. Prescott, and L. S. Rochester, Phys. Rev. Lett. 19, 671 (1967); E. D. Bloom, Ph.D. Thesis, California Institute of Technology, 1967 (unpublished), Table IV, p. 39.

[8] S. Cheng, Ph.D. Thesis, California Institute of Technology, 1970 (unpublished), Table 4.3, p. 69.

[9] M. Deutsch, L. Golub, P. Kijewski, D. Potter, D. J. Quinn, and J. Rutherfoord, Phys. Rev. Lett. 29, 1752 (1972); 30, 249(E) (1973).

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[12] R. L. Walker, Phys. Rev. 182, 1729 (1969). (For definitions of $H_1, \ldots, H_4$.)

[13] P. S. L. Booth *et al.*, Daresbury Nuclear Physics Laboratory Report No. DNPL/P-95, 1971 (unpublished); also P. Spillantini and V. Valente, CERN Report No. CERN-HERA 70-1, 1970 (unpublished).

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[15] Reference 8, page 69.

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Cosmic-Ray Muon Integral Intensity Measurement Under Water*

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(Received 14 August 1972)

The cosmic-ray muon integral intensity ($E_{\mu\text{-threshold}} \gtrsim 1$ GeV/c) was measured under water with a detector of small angular aperture (6.5° full width at half maximum). The motivation of the work was to check the range-energy relation for muons under a medium, water, whose properties are very well known and whose atomic properties (Z, Z^2/A) are significantly different from "rock" used in underground measurements. The ratio of these measured relative muon intensities, converted to equivalent depths of standard rock, to those measured under rock and converted to standard rock, for 50 hg cm$^{-2} \lesssim d \lesssim 1000$ hg cm^{-2} of equivalent standard rock, is 1.09 ± 0.06. This measurement shows that the range-energy interactions of muons are consistent between "rock" and water for $E_\mu \lesssim 240$ GeV.

I. INTRODUCTION

This experiment measured the relative integral intensity of single cosmic-ray muons with a small-angle aperture (the angle for full width at half maximum is $\theta_{\text{FWHM}} = 6.5°$) for water depths of 244 m and for slant-angle (zenith) detector orientations up to 75°. The purpose of this experiment was to establish accurately the consistency between muon range-energy interactions in water and rock.[1-3] The significant differences between water and "standard rock" are partially reflected in their atomic properties (Z/A, Z^2/A) and their specific densities $[(0.50, 3.67)_W,\ (0.50, 5.5)_R,\ (1.0)_W,\ (2.65)_R]$.

A simple two-element small-angle-aperture detector telescope was constructed and used as the principal detector after a series of different configurations, generally of wider angular aperture, were experimented with in an early phase of the work.[4] Relatively hard muons, $E_\mu \gtrsim 1$ GeV, were necessary to trigger an event in the telescope. This directionally sensitive telescope registered the number of muons, in fixed time periods, while positioned at different water depths (30 m $\lesssim d_W \lesssim$ 244 m) and at different slant angles ($0° \lesssim \theta \lesssim 75°$) in Lake Chelan.[5] The telescope was aligned with its axis of symmetry fixed, relative to the zenith, for a set of different depths. Section II describes the experiment, the environment, and the telescope instrument. The procedure for collecting the data and the nature of backgrounds is discussed in Sec. III. The final results of these underwater data, in comparison to the well-known data from under rock, are the subject of Sec. IV.

II. EXPERIMENT

The two-element telescope detector was comprised of a flat circular disk of scintillator plastic radiator mounted coaxially, along its axis of cylindrical symmetry, with a cylindrically symmetric Čerenkov detector; the scintillator radiator was "in front" of the Čerenkov detector.[6] The detectors and the scintillation and Čerenkov counters were optically separate. An event in the whole telescope detector was registered by an electronic coincidence between bona fide events in each of the two elements. An event in one element was the result of a threefold coincidence between three photomultipliers (56 AVP), used optically in parallel, each viewing the element's radiator.

The telescope detector, Fig. 1(a), was designed to have a small angular aperture to be able to measure large slant depths with relatively modest absolute depths and to limit the need for making extensive measurements and calculations of the acceptance function of the detector. The small-angle aperture and the physical configuration of the telescope detector minimized the contribution from several kinds of backgrounds, muon knock-on electrons, and correlated muon pairs. The telescope's relative angular sensitivity was calculated (from its geometrical configuration) and measured in a set of calibration runs during the experiment. The Monte Carlo calculation and the calibrated angular sensitivity agreed very closely[7] [Fig. 1(b)].

III. DATA

The data were collected in a series of sets of runs. Each set took several days. The final series was distributed over five months. The calibration of the relative angular sensitivity was taken in the middle of these data runs. The experimental procedure for a typical run involved setting the telescope's slant angle and running for preset time intervals at different depths:

$$\theta_z(\text{detector}) = 0°,\ 60°,\ 75°;$$

$$d_W = 30.5,\ 61,\ 122,\ \text{and}\ 244\ \text{m}.$$

The element-event signals were established to be free of experimental biases; through monitoring and extensive tests it was well established that "singles" feed-through, ambient light, or chance coincidence (using coincidence resolving times $\lesssim$20 nsec) had less than 1% effect in any data run. Estimates of knock-on electron and correlated muon pair rates using other source data[1,8] showed our data to be essentially uncontaminated to the level of a negligible fraction of our error in the ratio $I_\mu(\text{water})/I_\mu(\text{rock})$. While we ran with the tele-

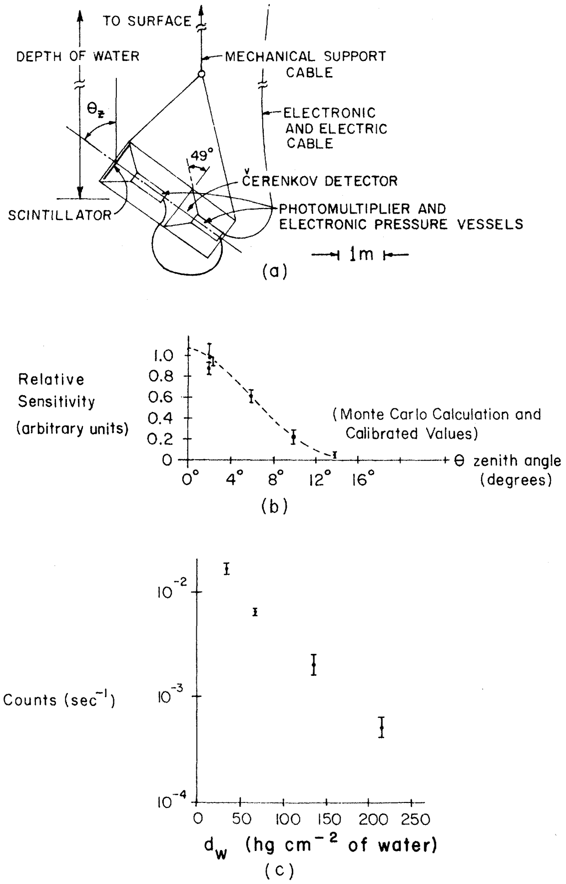

FIG. 1. (a) The muon telescope detector, as it was positioned, under the surface station (a moored raft) on Lake Chelan. The upper element of the telescope was a scintillation counter, with the set of three photomultipliers encased in the adjacent pressure vessel. The second element, the Čerenkov counter, utilized the *in situ* lake water as the Čerenkov radiator; the light-baffled cone in front of the set of three photomultipliers of the second vessel allowed the daytime use of the detector. (b) The relative angular sensitivity of the telescope $\eta(\theta)$ cm^{2}sr, times an arbitrary constant, as determined by the Monte Carlo calculation and the experimental calibration measurement. (c) A typical run of the $IM(d)_\theta$ data set at $\theta_z = 60°$.

scope slant angle at 90° the systematic errors in the operation[9] prevented the use of these data with those taken at $\theta \lesssim 75°$. Runs were also made at angles larger than 90° and showed no anomalous rate ($<10^{-3}$ of the 0° rate).

IV. RESULTS AND ANALYSIS

The final configuration of the telescope had its data grouped together in sets $IM(d_W)_{\theta=\text{fixed}}$; this reflects the procedure in which the data were collected. These rates were compared with the rates expected from standard-rock measurements[2] using the following procedure: The depth of water, d_W, was used to calculate the effective range, d, from the top of the atmosphere in "standard rock." This d was used to calculate the vertical integral muon intensity, $I(d, 0°)$, using Miyaki's empirical formula[2,10] as a model:

$$I(d, 0°) = \frac{K}{K+d}\, d^{-\alpha} e^{-\beta d}\ (\text{sec}^{-1}\,\text{cm}^{-2}\,\text{sr}^{-1}),$$

with the parameters d (hg cm^{-2} of standard rock), $K=164$, $H=400$ hg cm^{-2}, $\alpha=1.53$, $\beta=0.65\times10^{-3}$ hg^{-1} cm^{2}. The full form of the integral muon flux was calculated using the model of Barrett *et al.*,[1,2] which gives

$$I(d, \theta) = I\left(\frac{d}{\cos\theta}, 0°\right) \frac{(\epsilon+1)E + E_0}{(\epsilon+1)\cos\theta\, E + E_0},$$

with the parameters $\epsilon = 1.9$, $E_0 = 90$ GeV; E is the minimum energy for range d. The model of Barrett *et al.* has proved to be an accurate description of $I(d, \theta)$ for the muon energy range involved in this experiment.

The results of this experiment are characterized by the ratio

$$R(d)_\theta = \frac{IM(d)_\theta I(d_{\min})_\theta}{IM(d_{\min})_\theta I(d)_\theta}.$$

The effective values of the parameters d and θ are $d = (41$ hg cm^{-2}, 71 hg cm^{-2}, 132 hg cm^{-2}, 254 hg cm$^{-2})$ of "standard rock" and $\theta = (0°, 58°, 73°)$ effective mean zenith angle of detector. This $d_{\min}$ is the depth to which the data at the common angle, θ, are normalized. The value of $R(d)_\theta$ is displayed in Fig. 2. In principle $R(d)_\theta$ should show no dependence on θ; our results show no θ dependence. The mean value of $R(d)$ over the range of slant depths in this experiment is 1.09 ± 0.06, 50 hg cm^{-2} $< d <$ 1000 hg cm^{-2}; most of the error is statistical in origin.[9] This result shows that the range-energy relation of muons in water, $E_\mu < 240$ GeV, is quite consistent with our knowledge of muon range-energy in standard rock. Had there been either a d dependence or a θ dependence for $R(d)_\theta$ this might have had its origin in these $I(d, \theta)$ measurements in water, a moderating medium quite different from the usual rock[3] overburden of underground experiments.

ACKNOWLEDGMENTS

We would like to express our appreciation for the continued interest shown in this work by

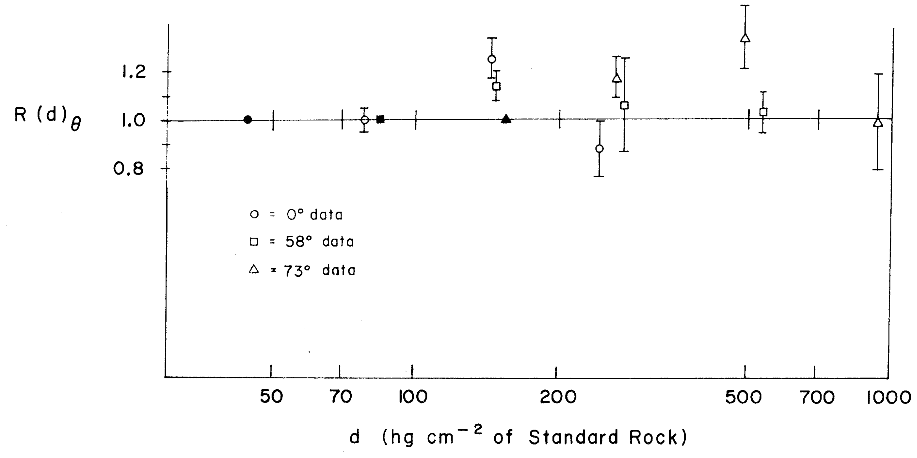

FIG. 2. The ratio of the measured to the known (as determined from rock measurements) muon flux for d (effective depth in standard rock). The filled-in symbols correspond to the shallowest depth ($d_{\min}$) measurement, to which the other points of the same angle are normalized. The measurements with zenith angle 0° are identified by the circle; the measurements with zenith angle 58° are identified by the square; the measurements with zenith angle 73° are identified by the triangle. The error flags of $R(d)$ are primarily statistical in origin. There is an increasing error in the effective depth in equivalent standard rock, d, but its largest value of a few percent for $d = 1000$ hg cm^{2} makes its effect on the interpretation of the experiment quite minimal.

Dr. R. W. Williams and Dr. J. J. Lord; the specific support for the implementation of this work by Dr. R. W. Williams is gratefully acknowledged. The assistance of Dr. R. Gustafson, Dr. J. Carlson, C. Manning, and R. Ringdahl is appreciated. W. Purcell, of the University of Washington Physics Department machine shop, and W. Rines, of the U. S. Forest Service, and their personnel, brought valuable technical assistance during the construction and operation phases of the work. The comments of Dr. F. Reines, Dr. S. M. Schindler, and Dr. W. R. Sheldon are appreciated.

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*Supported in part by the National Science Foundation and the Oregon State University Computer Center.

†Now at the University of Wisconsin, Madison, Wisconsin.

[1] P. H. Barrett, L. M. Bollinger, G. Cocconi, Y. Eisenberg, and K. Greisen, Rev. Mod. Phys. 24, 133 (1952). It is well to note that there is a natural practical advantage to this type of experiment, in which the muon range interaction is tested, to have a significantly large magnitude of the intensity depth gradient $[\partial I_\mu(d, \theta)/\partial d]$, as is offered by the "sea level" cosmic-ray muon intensity. However, if the experiment uses a zenith-angle-dependent measurement of $I_\mu(d, \theta)$ in which the flux is integrated over $d\Omega$ for a small, but macroscopic, detector receptivity for "large angles" (60° and 75°), as is done in this experiment, then care must be exercised to obtain a result that is free of systematic error (see Ref. 10).

[2] M. G. K. Menon and P. V. R. Murthy, *Progress in Elementary Particle and Cosmic Ray Physics* (Wiley, New York, 1969), Vol. IV, Chap. II, p. 163; W. R. Sheldon and N. M. Duller, Nuovo Cimento 23, 63 (1962). A recent analysis of an experiment measuring $I_\mu(d, \theta)_{\rm rock}$ using a tunnel through Mont Blanc (private communication from W. R. Sheldon) indicates close agreement with the Menon survey for depth up to 1000 $\mathrm{hg\,cm^{-2}}$, $\theta = 0°$ [W. R. Sheldon, W. G. Contrell, R. Bazer-Bachi, and A. St. Marc, in *Proceedings of the Eleventh International Conference on Cosmic Rays, Budapest, 1969,* edited by T. Gémesy *et al.* (Akademiai Kiado, Budapest, 1970)]. There is, however, a discrepancy between the rock measurements to which the water measurements of this experiment are compared (Menon survey and Sheldon *et al.*) and a recent survey of $I_\mu(d, 0°)_{\rm rock}$ completed at the University of Utah (unpublished) for depths $d \sim 1000$ $\mathrm{hg\,cm^{-2}}$.

[3] There has been a long history of underwater cosmic-ray flux measurements dating to the earliest experimental work. A large-scale measurement under water [S. Higashi, F. Ohio, H. Shibata, and Y. Wantanabe, Nuovo Cimento 5, 597 (1957)] used a wide-angle-aperture detector system up to vertical water depths 1480 $\mathrm{hg\,cm^{-2}}$. The range-energy problem has been discussed again recently by K. O'Brien [Phys. Rev. D 5, 597 (1972)]. The work of L. N. Davitaev, V. M. Fedorov, Yu A. Trubkin, and Yu N. Vavilov in measuring $I_\mu(0°, d)$ undersea water is described in *Proceedings of the Eleventh International Conference on Cosmic Rays, Budapest, 1969,* edited by T. Gémesy *et al.* (Ref. 2).

[4] J. Learned, doctoral dissertation, University of Washington, 1968 (unpublished); J. G. Learned and H. F. Davis, Bull. Am. Phys. Soc. 13, 694 (1968).

[5] Lake Chelan is located in the north-central part of the state of Washington. It is about 50 miles long and averages about a mile wide. Its depth of over 1000 ft and its clearness made it a natural site for the experiment and a good Čerenkov radiator. A 40-ft raft was constructed and used at the surface as the base for the experiment; it was moored over a 1000-ft point in the lake at about the halfway point in the lake.

[6] The three photomultiplier tubes (56 AVP), mounted in the end face of the common watertight cylinder vessel (15 cm in diameter and 150 cm long), viewed the lake water through three individual Plexiglass windows. An identical vessel was used for the scintillation detector element as was used for the Čerenkov detector element. The regulated power supplies and the nanosecond coincidence logic were also placed in these vessels. DC (24 V) power was brought to the vessels through a simple power line from the raft station on the surface. There were rf switches in the signal lines in the vessels which could be actuated from the surface, which gave the single-vessel counting rate as well as the event counts corresponding to a hard muon detected by both elements of the whole telescope. The counting electronics was located on the surface station. This served as a check on the systematics of the runs. The flat face of the scintillator disk (thickness = 5 cm, diameter = 70 cm) was viewed by the three photomultipliers through the lake water. The lake water was viewed through a black-surfaced baffled cone to establish the Čerenkov-counter element signal. Both sensitive volumes were securely light-tight against the ambient light.

[7] The threshold level of the photomultiplier detectors determined the absolute sensitivity ($\mathrm{cm^2 sr}$) of each element of the telescope and thus the absolute sensitivity of the telescope itself. We did not measure the photon-trigger level of the photomultiplier devices. However, the constancy of the discrimination level was well established experimentally through the periodic measurement of single-vessel counting rates at a common shallow-depth check position during each data run. The constancy of the relative sensitivity, $\eta(\theta)$, on an inter-run basis, was well established by the good reproducibility of the shape of the $IM(d, 0°)$ data.

[8] K. H. Davis, S. M. Fall, R. B. Ingebretsen, and R. O. Stenerson, Phys. Rev. D 4, 607 (1971); J. C. Barton and C. T. Stockel, Can. J. Phys. 46, S318 (1968).

[9] There are risks in measuring $I_\mu(d, 0°)$ through the measurement of $I_\mu(d, \theta)$ for angles as large as 75° and 60° even for the relatively small instrument aperture (6.5° half width at half maximum) used in this experiment. This problem is reflected in the increasing relative contribution to the uncertainty in I_μ with zenith

angle θ, $(\partial I/\partial\theta)\Delta\theta$. We believe that the systematic checks (in the intrarun data sets and between the different runs) establish the stability of our data because there was an effective constancy of the size and the relative shape in the detector's angular sensitivity. It is important to note that the problem of uncertainty in $R(d)_\theta$ as a result of this problem is less serious than the uncertainty in the intensity determination; this is due to the data, at different fixed angles, being normalized to a common value at $d_{\min}(\theta)$. A slight change in the mean zenith angle setting has a much more significant influence on $I_\mu(d, \theta)$ than on $R(d)$. For simplicity the $R(d)_\theta$ ratio is displayed for $d = d_{\text{median}}$, with all of the error in θ being reflected in an additional uncertainty in $R(d)_\theta$; this is done rather than portraying the R results as sets of $(R, \Delta R; d, \Delta d)$.

[10] S. Miyake, J. Phys. Soc. (Japan) 18, 1093 (1963).

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Measurement of the Σ^- Beta Decay Rate and an Improved Test of the $\Delta S = -\Delta Q$ Selection Rule*

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(Received 12 February, 1973)

Analyzing Σ^- hyperons produced by K^- stopping in the 30-in. BNL hydrogen bubble chamber, we have obtained (a) an upper limit for the ratio $\Gamma(\Sigma^+ \to e^+ n\nu)/\Gamma(\Sigma^- \to e^- n\nu)$ of 0.018 with 90% confidence and (b) a value for the branching ratio, $\Gamma(\Sigma^- \to e^- n\nu)/[\text{total } \Gamma(\Sigma^-)]$, equal to $(1.05 \pm 0.07) \times 10^{-13}$, in good agreement with existing data.

I. INTRODUCTION

The universal $V-A$ Fermi interaction modified by the Cabibbo theory[1] is still in good agreement with the existing experimental data on hyperon leptonic decays.[2] In the last few years a number of articles summarizing the theory have been published, as for example the recent report by Chounet *et al.*,[3] so we shall refrain from a repetition here.

The useful data that can be obtained from hyperon leptonic decays to test this theory are (1) data on the search for the decay $\Sigma^+ \to (e^+ \text{ or } \mu^+) + n + \nu$ [this is a $\Delta S = -\Delta Q$ decay mode, and its existence will invalidate Cabibbo's hypothesis that the hadronic current should transform as an octet of vector and axial-vector currents under the SU_3 transformations[1]], and (2) determination of branching ratios, energy spectra, and angular or polarization correlation of the decay products for Λ, Σ, and Ξ leptonic decays. From these data the sign and magnitude of the vector and axial-vector coupling constants could in principle be determined and compared with the values predicted by the theory.[2,3]

We would like to report here on a measurement of the ratio

$$\Gamma(\Sigma^- \to e^- \nu n)/[\text{total } \Gamma(\Sigma^-)]$$

and of an upper limit to the ratio

$$\Gamma(\Sigma^+ \to e^+ \nu n)/\Gamma(\Sigma^- \to e^- \nu n) .$$

Preliminary reports of this work have been reported at international conferences in 1968 and 1969.[4]

II. METHOD OF THE EXPERIMENT

The Σ^+ and Σ^- hyperons were produced, in an exposure of the 30-in. BNL hydrogen bubble chamber to a beam of K^- mesons stopping inside the chamber, by the reactions

$$K^- + p \to \Sigma^- + \pi^+ ,$$
$$K^- + p \to \Sigma^+ + \pi^- .$$

In about 10% of the cases, the $\Sigma^\pm$ hyperons were produced in a $K^- + p$ interaction in which the K^- was not at rest.[5] 490 222 pictures, containing an average of ~10 stopping-K^- per picture, have been analyzed and 858 $\Sigma^- \to e^- n\nu^-$ decays have been found. No $\Sigma^+ \to e^+ n\nu$ with e^+ momentum larger than 70 MeV/c was found. In the 90% of the cases in which the $\Sigma^\pm$ are produced by K^- particles coming to rest in the hydrogen, the momentum of the Σ at production will be 181.3 MeV/c for the Σ^+ and 173.2 MeV/c for the Σ^- particle, corresponding to a Σ range maximum of 1.27 cm and 1.06 cm, respectively.[6] With such short ranges, almost all $\Sigma^\pm$ particles decay or interact inside the bubble chamber.

Ignoring radiative decay modes, the possible $\Sigma^\pm$