ming was the work of Dr. Bogert and Marvin Johnson.

We are indebted to our collaborators at the Brookhaven National Laboratory, Dr. D. Berley and Dr. P. Yamin for aid in utilization of the 6600 computer.

Finally, we wish to thank the technical staff of our laboratory, headed by Peter Martin and Walter Lund. All of our measuring apparatus was built under their supervision.

---

*Work supported in part by the U.S. Atomic Energy Commission, AEC Contract No. AT(11-1) 3075.

†Present address: Department of Physics, The Johns Hopkins University, Baltimore, Maryland, 21218.

[1]Terry S. Mast, Lawrence K. Gershwin, Margaret Alston-Garnjost, Roger O. Bangerter, Angela Barbaro-Galtieri, Joseph J. Murray, Frank T. Solmitz, and Robert D. Tripp, Phys. Rev. **183**, 1200 (1969).

[2]W. T. Ford, P. A. Piroue, R. S. Remmel, A. J. S. Smith, and P. A. Souder, Phys. Lett. 38B, 335 (1972). See also W. T. Ford, P. A. Piroue, R. S. Remmel, A. J. S. Smith, and P. A. Souder, Phys. Rev. Lett. **25**, 1370 (1970), and R. S. Remmel, thesis, Princeton University, 1971 (unpublished).

[3]William J. Heintzelman, thesis, Yale University, 1968 (unpublished).

[4]D. Bogert, P. Lucas, H. Taft, W. Willis, D. Berley, P. Yamin, R. Kofler, G. Meissner, and S. Yamamoto, Phys. Rev. D **2**, 6 (1970).

[5]D. Bogert, M. Johnson, P. Lucas, T. Ludlam, J. Sandweiss, J. Slaughter, and W. Willis, Nucl. Instr. Meth. **107**, 337 (1973).

[6]What is described here is a modification of a 0C fit technique presented in A. Firestone, J. Kim, J. Lach, J. Sandweiss, H. Taft, and P. Guidoni, Phys. Rev. Lett. **18**, 176 (1967).

[7]Versions of this method which did not work, and a discussion of mathematical details are given in P. Lucas, Ph.D thesis, Yale University, 1972 (unpublished).

---

PHYSICAL REVIEW D VOLUME 8, NUMBER 3 1 AUGUST 1973

# Study of $K^-$-Meson Decay Properties Using Dalitz Pairs for $\pi^0$ Detection*

P. W. Lucas,† H. D. Taft, and W. J. Willis

*Physics Department, Yale University, New Haven, Connecticut 06520*

(Received 27 March 1973)

A sample of about 5000 $K^-$ decays in flight, each accompanied by a Dalitz pair, has been observed in a hydrogen bubble chamber. These events consist of 3564 ± (3.1%) $K_{\pi 2}$ events, 786 ± (4.6%) $K_{e3}$ events, 554 ± (7.6%) $K_{\mu 3}$ events, and 574 ± (5.9%) $\tau'$ events. Ratios among these numbers give results in agreement with accepted $K^+$ branching ratios. The separation of the events has involved a sophisticated Monte Carlo program, ionization, kinematics of the $K^-$ decay, and kinematics of the Dalitz decay. The $\pi^-$ energy spectrum of the $\tau'$ decays may be written as: (phase space) $\times [1+(2A/M_\pi{}^2)(s_3-s_0)]$, where $s_i$ is the square of the 4-momentum transfer to particle $i$, $s_0=\frac{1}{3}(s_1+s_2+s_3)$, and $A$ is a slope to be determined. We find $A=0.242\pm0.042$.

## I. INTRODUCTION

Most bubble-chamber experiments studying the properties of decays of charged $K$ mesons have done so by stopping $K^+$ in either heavy liquid or liquid hydrogen. We have performed a study of these properties different in several essential ways from this norm: We have studied $K^-$ instead of $K^+$, and since $K^-$ do not decay at rest but are absorbed, we have of necessity studied them in flight. Furthermore, we have performed this experiment using a new event recognition procedure, studying only three-prong decays specifically including all events in which a Dalitz decay of a $\pi^0$,

$$\pi^0 \to e^+e^-\gamma,$$

has been the source of two of the three prongs. Thus we have been able to study

$$\begin{aligned} K^- &\to \pi^-\pi^0, && K_{\pi 2} \\ &\to e^-\nu\pi^0, && K_{e3} \\ &\to \mu^-\nu\pi^0, && K_{\mu 3} \\ &\to \pi^-\pi^0\pi^0, && \tau' \end{aligned}$$

as well as

$$K^- \to \pi^-\pi^+\pi^+, \quad \tau$$

which is by far the dominant three-prong decay mode. The results on these $\tau$'s, together with descriptions of the beam, measuring procedure, etc., have been reported elsewhere,[1] and our purpose here is to discuss only the Dalitz-pair events. Suffice it to say that we have studied $1.1\times 10^6$ exposures to a beam of 400-MeV/$c$ $K^-$ mesons in the BNL 30-in. liquid-hydrogen bubble chamber.

This is not an experiment designed to be statistically competitive with other $K$-decay studies. The fact that the beam is in flight, combined with the low branching fraction of Dalitz pairs from $\pi^0$ decay, about 1/80, makes the amassing of a large sample of data difficult. With approximately fifteen beam tracks per picture these events are observed within an acceptable fiducial volume at the rate of one per 225 frames. However, with a more-than-one-million-picture exposure we have observed several thousand examples, one of which is shown in Fig. 1.

The experiment, on the other hand, is designed to minimize the possibility of systematic errors or data contaminations which might be present in other studies. In particular, $K_{\mu 2}$ events, which make up more than half of all charged $K$ decays, are eliminated as they do not lead to a three-prong topology. Also $\pi^0$'s are detected with an efficiency which is well known, the Dalitz-pair branching fraction, and is not dependent upon the observation of conversion pairs or showers from $\pi^0$ decay $\gamma$'s. Finally the kinematics of the Dalitz decay itself has been useful in sorting events among these various types.

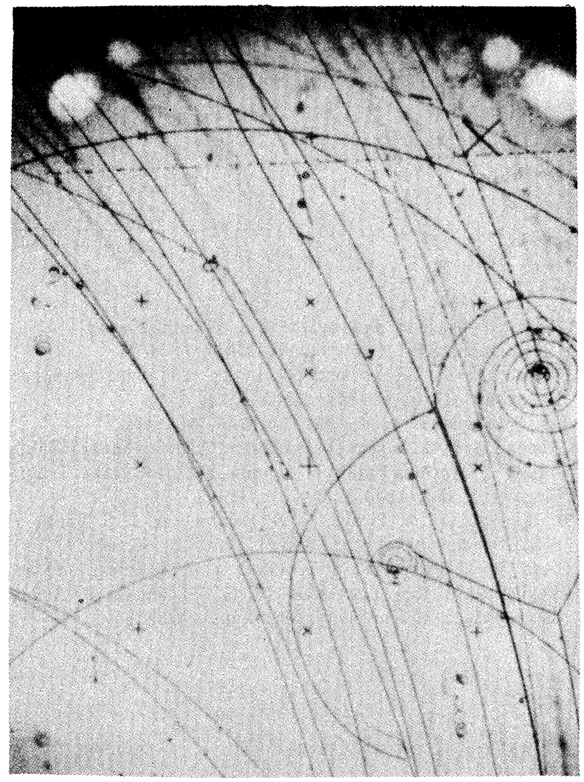

FIG. 1. A portion of a bubble-chamber picture showing a typical Dalitz-pair event.

## II. SCANNING AND MEASURING

The first scan of the film for these events was made in conjunction with a collection of all the normal $\tau$ events, and for some of the exposure, other event types as well. Unfortunately these rare events were found with poor efficiency, probably as a result of a preliminary scan for $\tau$ events where Dalitz events were not desired. About half the film was double scanned on this pass and the calculated single-scan efficiency did not lead to a $(K^-\to \text{all Dalitz pair})/(K^-\to\tau)$ branching ratio consistent with $K^+$ values or that reported previously for $K^-$ by Mast *et al.*[2]

It was found during the studies of these efficiencies that Dalitz-pair events could almost always be distinguished from normal $\tau$ events by eye, without kinematic analysis. The scanners were taught to recognize this event type only, with the possibility of some contamination from the normal $\tau$ events. The film was totally rescanned for Dalitz pairs only, and all new events as well as old ones not making satisfactory multiconstraint fits were examined by a physicist and measured under his supervision. This sample of events, together with those from the original data making a satisfactory two-constraint fit to the hypothesis

$$K^-\to\pi^-\pi^0 \searrow e^+e^-\gamma,$$

comprise the data of this study. The over-all scanning efficiency is discussed in detail below, but is found to be above 90% for the final sample.

The contamination of the $\tau$ events with Dalitz-pair events is estimated as 75 and these are handled in such a way as to be considered part of the Dalitz-pair scanning inefficiency. The contamination of Dalitz pairs with $\tau$ events is small, as the Dalitz-pair events have almost all been scrutinized by physicists, and is estimated as 20 in the total sample of over 5000.

All measurements were performed on the Yale Wire Measuring System, and the data were analyzed using the standard geometry and kinematics programs TVGP and YACK.

## III. RESULTS OF VISUAL ANALYSIS

The visual observation of these events by physicists has yielded two important pieces of information. The first is the aforementioned selection of the Dalitz-pair events, and as well the determination of which tracks of the event form the pair. The second is an estimate of the ionization density of that prong which is not a part of the pair. This information can be used in particular to identify $K_{e3}$ events. It is felt that an important result of this work has been the realization that essentially all of the Dalitz pairs produced by particles in our energy range are recognizable by eye. The data on which this statement is based follow.

The kinematics of Dalitz decay are specified by two variables $x$ and $y$ given by[3]:

$$x = [(E_+ + E_-)^2 - (\vec{P}_+ + \vec{P}_-)^2]^{1/2},$$

$$y = |E_+ - E_-|/|\vec{P}_+ + \vec{P}_-|,$$

where $E_\pm$ are the energies of the positron and electron in the $\pi^0$ c.m. system, and $\vec{P}_\pm$ are the corresponding momenta. Theoretical expressions[4] exist for the distributions of these two quantities and show that while $y$, the energy partition, is fairly uniform over its allowed range, $x$, the pair mass, is sharply peaked at its minimum allowed value of $2m_e$. This peaking at low mass induces a peaking at 0° opening angle, as seen in the example given in Fig. 1. Figure 2 shows the distribution in $x$ for the Dalitz-pair data plotted together with the theoretical curve. For comparison the same quantity is plotted in Figs. 3 and 4, with electron interpretation of all tracks, for the

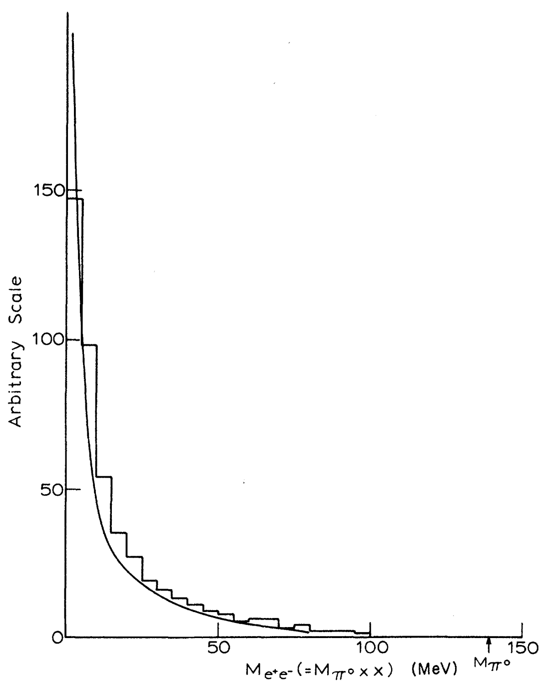


FIG. 2. The distribution of $x$ multiplied by $M_{\pi^0}$, i.e., the pair mass, for Dalitz-pair events using the correct pair as determined by visual observation.

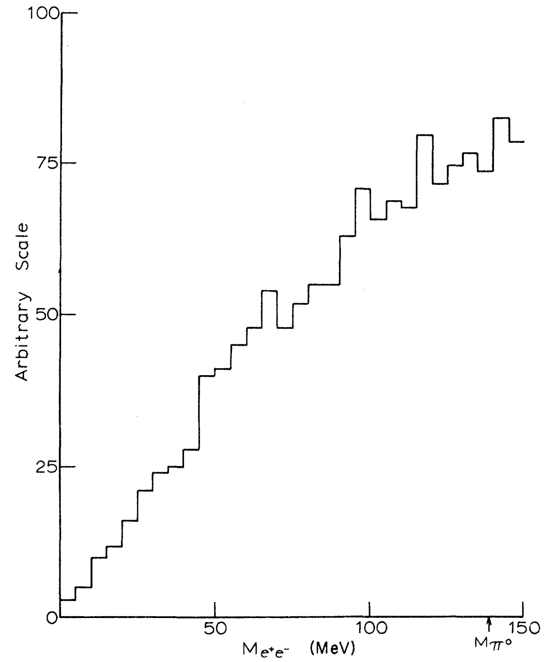


FIG. 3. Pair-mass distribution for Dalitz-pair events using the incorrect pair.

positron of each Dalitz-pair event matched with the wrong negative track, and for the two plus-minus pairs possible for each normal $\tau$ event.

Searches for possible biases in the Dalitz pairs of these events have been carried out. In particular the laboratory energy and angle distributions have been plotted. The energy distribution is shown in Fig. 5 together with a Monte Carlo calculation for the same quantity. The particular $\pi^0$'s which give these pairs come from $K_{\pi 2}$ decays, the only Dalitz-pair events making a kinematic fit. There is no apparent bias against either low- or high-momentum tracks although there is some distortion of events among the three lowest bins of this plot; however, the Monte Carlo and real data are in agreement as to the total of the three. It is felt that this juggling represents the difficulty of simulating by our Monte Carlo techniques the particular errors in measurement and reconstruction of such low-momentum tracks.

Since the electron spectrum peaks at a low value, there is the possibility of contamination of the events by two-prong topologies with fast positive tracks not readily distinguishable from minimally ionizing, and large δ-rays near the vertex. An example from our film might be:

$$K^- + p \to \Lambda^0 \pi^- \pi^+ \searrow \delta \text{ ray}.$$

If this contamination were sizable the low-momentum portions of the electron and positron spectra

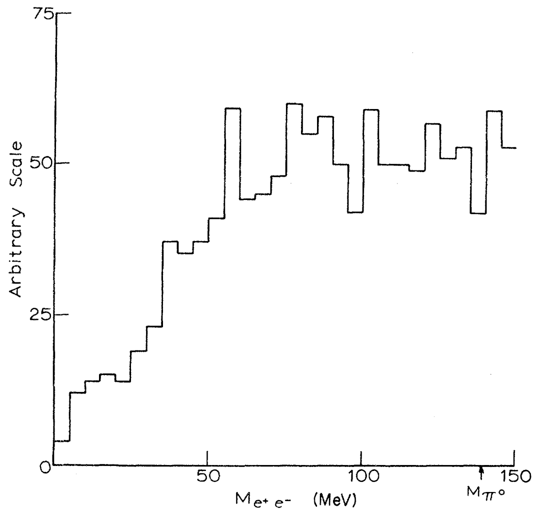


FIG. 4. Pair-mass distribution for normal $\tau$ events interpreted as Dalitz pairs.

should be visibly different at very low momenta. But Fig. 6 shows that there is no such effect. The data used in this plot are all Dalitz pairs not part of events making $K_{\pi 2}$ fits.

There is one small source of contamination which is unavoidable. It is those events with $\gamma$ conversion pairs, external pairs, with vertex close to that of the primary decay. If the external and internal pairs are assumed indistinguishable within 1 mm of the primary vertex, a typical bubble gap length, then there is a contamination of 0.6% in the number of all Dalitz-pair events present.

The other feature of the visual examination of the event was the possible identification of the

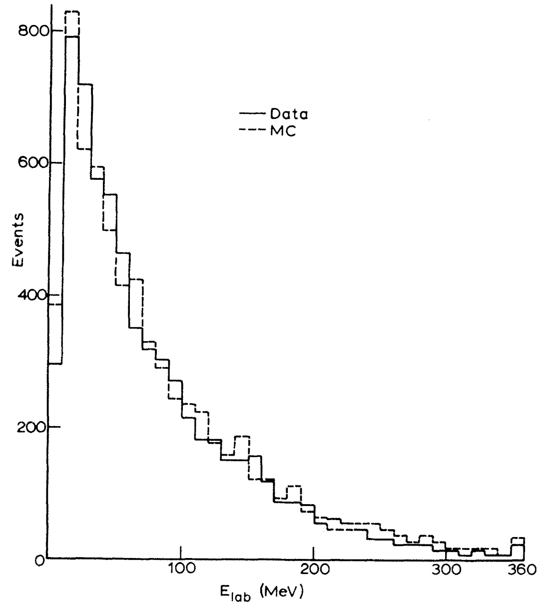


FIG. 5. Laboratory distribution of the energy $E_{lab}$, of Dalitz electrons from $K_{\pi 2}$ events, real data, and Monte Carlo.

primary decay track based on ionization or other features. All possible information concerning the nature of these tracks was recorded: ionization, kinks, scatters, behavior on stopping, etc. However, these data could not be used in the analysis unless the probability of making any particular decision could be clearly stated. For the ionization data the determination of this probability is particularly difficult. This is because apparent

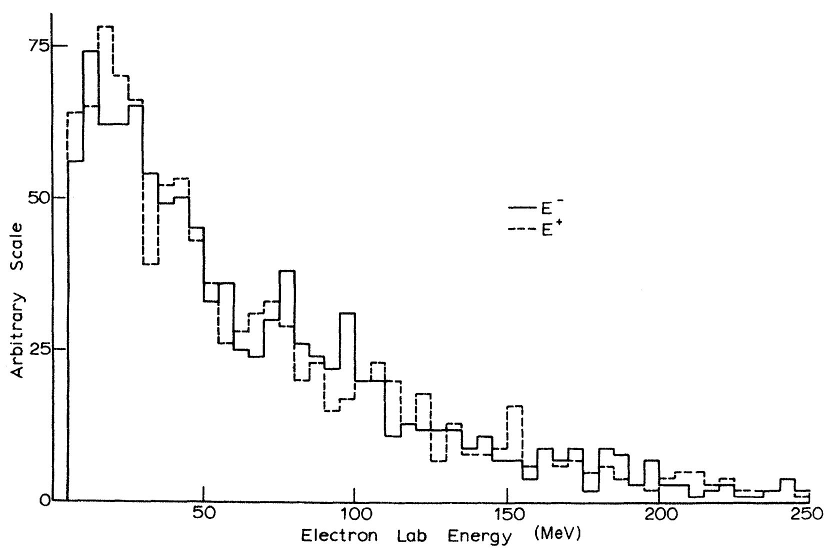


FIG. 6. Laboratory distribution of the energies $E^+$ and $E^-$, of Dalitz-pair positrons and electrons in unfitted Dalitz-pair events.

ionization density is a function not only of the particle mass, but of momentum, dip angle, and depth in the chamber, and one's ability to determine it is a function of track length and picture quality.

One clear and useful determination was made; tracks of momentum less than 200 MeV/$c$ were all categorized as to whether or not they were electrons. Those which were served to identify the event as a $K_{e3}$. Those which were not were either $K_{\pi2}$, $K_{\mu3}$, or $\tau'$. Since most $K_{\pi2}$ events made satisfactory two constraint fits, the only serious ambiguity in the low-momentum sample was between $K_{\mu3}$ events and $\tau'$ events. Attempts to separate these visually were not successful.

## IV. MONTE CARLO PROGRAM

The analysis of these data has depended heavily upon a Monte Carlo program which generated events of the four types under consideration. A serious effort was made to have these events be as similar to the real ones as reasonably possible. A summary of the steps involved follows; the $K_{\mu3}$ generation is described in detail, the other types following a similar pattern.

(1) Generate an event on the $K_{\mu3}$ Dalitz plot. Density is assumed given by vector interaction with form factors $\xi(0) = -0.85$, $\lambda_+ = 0.03$, $\lambda_- = 0$. Radiative corrections can be neglected.

(2) Pick a muon direction in the $K^-$ c.m. frame. Muon is isotropic in this frame.

(3) Choose an appropriate beam track. Actual tracks from the real data are used for this purpose. Thus the beam momentum spectrum and the distribution of decay points are automatically the same for the Monte Carlo and real data.

(4) Lorentz transform the muon 4-vector momentum to the laboratory frame. For this purpose we do not use the beam track of step 3, but instead use one whose momentum is the average of that of step 3 weighted inversely as its error and the central momentum of the beam weighted inversely as the beam width. This procedure seems to simulate the effects of mismeasurement of the beam. The track of step 3 is still considered the beam track of this event.

(5) The resulting lab 4-vector momentum is used to generate a track as it appears in the bubble chamber. In particular the canonical track parameters are calculated and a first-order trajectory (circular helix) is calculated to determine the length of the track in the bubble chamber.

(6) Errors on the track parameters are calculated incorporating setting error, track length, and multiple Coulomb scattering.

(7) The original parameters are smeared in a Gaussian manner as determined by these errors. The results of this smearing and the errors calculated in step 6 constitute the muon interpretation of this track.

(8) The track is also interpreted as an electron and a pion, as it would have been in the real data. For this purpose the angles are assumed unchanged, but the momentum at the vertex differs due to different energy loss rates for the other interpretations. Pion and electron interpretations are also assigned errors.

(9) The $\pi^0$ of the $K_{\mu3}$ is generated in the $K^-$ c.m. system. Its momentum is read directly from the generated Dalitz plot point, its angle with respect to the muon is calculated by kinematics, and its azimuth about the $\mu^-$ is generated assuming uniformity of this distribution. A problem arises later in incorrect combinations of Lorentz transformation if this $\pi^0$ momentum is not taken to the laboratory frame. This transformation is now performed using the same beam parameters as used to transform the $\mu^-$.

(10) The decay of the $\pi^0$ itself is now generated. Values for $x$ and $y$ are picked from the theoretical distributions and these specify a Dalitz-plot position up to a decision as to whether the $e^+$ or $e^-$ has higher energy. Thus one coin flip finishes this process.

(11) The $e^+$ is generated first. Its momentum is calculated from the Dalitz-plot position given above, its angles by assuming isotropy in the $\pi^0$ c.m. frame. It is now taken to the lab frame and length, errors, smearing, and interpretation as $\pi^+$ and $\mu^+$ are handled as above.

(12) The Dalitz $e^-$ is handled in a similar manner. The only difference is that its polar angle with the $e^+$ is calculated by kinematics, and its azimuth about the $e^+$ by assuming uniformity.

4000 $K_{\mu3}$ events were generated in this way, nearly ten times the number of real data events expected so that statistical errors on the Monte Carlo could be neglected for most purposes. Similarly 4000 $K_{e3}$ events were generated. For these, radiative corrections to the Dalitz plot were made using the prescription of Ginsberg.[5] For $\tau'$ events a quite different Dalitz-plot structure was needed. It was assumed that the distribution is governed by a term linear in the kinetic energy of the odd pion, the $\pi^-$ in this case, and the slope was that of Davison *et al.*,[6] which is the most precise experiment measuring this quantity for $K^+$. Otherwise the $\tau'$ generation proceeded as for the leptonics, again yielding 4000 events. Finally for the $K_{\pi2}$ generation the Dalitz plot consisted of only one point, this being a two-body decay, and the rest of the program remained as for the three-body cases. Only 1500 of these events were generated as the computer time they occupied in succeeding programs was prohibitive. Later enough were generated to again make the statistical errors small compared with those of the data, which consisted of over 3000 $K_{\pi 2}$ events. However, these latter Monte Carlo events did not include the Dalitz pair, but only the beam and primary $\pi^-$ tracks.

Actually the $K_{\pi 2}$ events were generated first since they were fitable. Tests were performed and adjustments made such that if the smearing were included the $\chi^2$ and missing mass widths agreed with those of the real data. The setting error used in the Monte Carlo program was determined in this way.

The result of the lengthy Monte Carlo generation was data which looked like the result of a geometry program, and a tape in TVGP output format was written. From this point all programs run on the real data could also be run on the Monte Carlo data, and comparisons made.

The first thing done with this Monte Carlo data was to run it through YACK, the kinematics program. The fits attempted here were four-constraint $\tau$ and two different two-constraint $K_{\pi 2}$'s with each negative track being tried as the primary decay $\pi^-$. Acceptable fits were taken as $\chi^2$ less than five times the number of constraints, as with the real data. Although it seemed unlikely that many of these events would make fits as $\tau$ events, an assumption proven correct, the same could not be said for fits to $K_{\pi 2}$'s. This was especially true of the leptonic decay events where there is actual kinematic overlap with $K_{\pi 2}$'s near the top of the spectrum if only the primary decay track is considered. When the Dalitz pair is included momenta can look like those of $K_{\pi 2}$'s but with slightly different angles. Thus it was not clear how many leptonics would fit $K_{\pi 2}$'s with our criteria without actually running YACK. There is no kinematic overlap of $\tau'$ events with $K_{\pi 2}$'s, and it was assumed that almost none of these would make a fit.

The results of the fitting procedure are as follows.

| Event Type | % $K_{\pi 2}$ Fits |
|---|---|
| $K_{\pi 2}$ | 96% |
| $K_{e3}$ | 10% |
| $K_{\mu 3}$ | 9% |
| $\tau'$ | 2% |

It was found that each of the three-body channels has about 2% of its events fitting $K_{\pi 2}$ through the mechanism of Dalitz electron mistaken as the primary decay track and vice versa. This interpretation seems to allow in a random fashion some kinematical overlap not present in a more logical interpretation of the event. Essentially all of the $\tau'$ events fitting $K_{\pi 2}$ do so in this way. Fewer than 1% of the events in each class make 4C fits as $\tau$'s.

We now present results on the numbers of $K_{\pi 2}$, $K_{e3}$, $K_{\mu 3}$, and $\tau'$ events in our data, together with the slope of the $\tau'$ Dalitz plot.

## V. NUMBER OF $K_{\pi 2}$ EVENTS

The data contain 3370 events fitting $K_{\pi 2}$ within our criteria. Corrections to this number to obtain the actual events are for other types of events making this fit, correct events failing to make it, a scanning bias in the angular distribution, and imperfect efficiency.

We first recapitulate the numbers of events of other types making these fits. First of all, as was stated above, most Dalitz-pair events were observed by physicists and ruled out as normal $\tau$ events. However events already making good $K_{\pi 2}$ fits from the first measuring pass were not examined and if any of these were not actual $K_{\pi 2}$'s they would have been misclassified. The number is estimated as 20. The number of contaminating leptonics and $\tau'$ events can be determined from the above percentages of events making this fit and current values of branching ratios taken from $K^+$. The contamination is 120 events.

The $K_{\pi 2}$ events lost to the $\tau$ events, estimated as 50, can be seen to be lost as part of the scanning inefficiency and no account is made for them here. The question of whether $K_{\pi 2}$ events are lost to leptonics is a more difficult one. The Monte Carlo results above indicate that 4% of the $K_{\pi 2}$'s would fail to fit within our criteria. This number is more than would be expected in the tail of a properly distributed $\chi^2$ curve, but during the writing of the Monte Carlo program it was found that the errors on Dalitz electrons were abnormally large. There certainly is an indication of a leakage into leptonics, but not a proof. The problem is solved by studying the leptonics themselves and looking for a bias. Plotted in Fig. 7 is the c.m. kinetic energy of primary prongs, with Lorentz transformations to the $K^-$ rest frame performed assuming the pion mass for the primary decay particle. The events used are any in which this prong is a possible pion from the ionization information and related cuts as discussed above. The Monte Carlo curve has the same cuts and has the leptonic and $\tau'$ events combined according to $K^+$ branching ratios. It can be seen that there is indeed an excess of events around the unique value of $T_\pi$ for $K_{\pi 2}$'s. The region of excess is taken to be $T_\pi$ greater than 90 MeV. The Monte

Carlo is renormalized below this value and then extrapolated over the entire range. There are now 118 events in excess of the Monte Carlo curve in the high region. This is taken as the number of "leaked" $K_{\pi 2}$'s and is, fortuitously or not, in quite good agreement with the 4% predicted from $K_{\pi 2}$ Monte Carlo. Clearly when looked at from the point of view of the leptonic events, the region above $T_\pi$ of 90 MeV is seriously contaminated with $K_{\pi 2}$'s and cannot be used.

In a search for a possible bias against short tracks, the analog of something clearly seen in normal $\tau$ events, we plotted the cosine of the angle between the beam direction and primary decay track, as taken in the $K^-$ c.m. system. For a monoenergetic beam this plot is equivalent to one of track length and is easier to study as it should be flat and requires no comparison with a Monte Carlo curve. The result is given in Fig. 8 and is surprising. The bias seen is against forward tracks, the ones fast in the lab, instead of the backward slow ones. The same angle, but taken in the laboratory frame is plotted in Fig. 9. The Monte Carlo in this figure is normalized to the first 18 bins, which are assumed bias free. It is here seen as a bias against very forward tracks. These have outgoing prong momenta and directions similar to those of the beam (although different ionization) and apparently did not show up well in crowed regions of the film. Examination of lab and c.m. plots yields different values for the correction to be made for this effect. It is taken as $156 \pm 50$ events, the 50 being a possible systematic error.

Finally the scanning efficiency has been determined. This determination has been made using

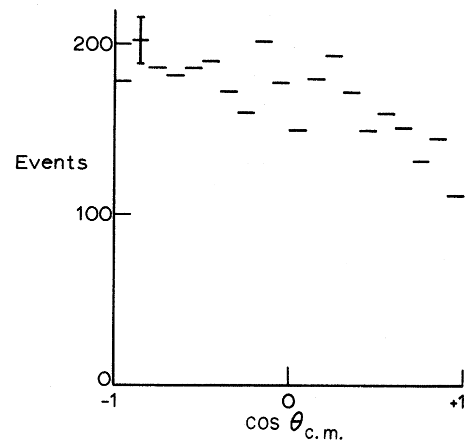


FIG. 8. The distribution of the pion decay angle, in the c.m. frame, taken with respect to the beam direction for $K_{\pi 2}$ events.

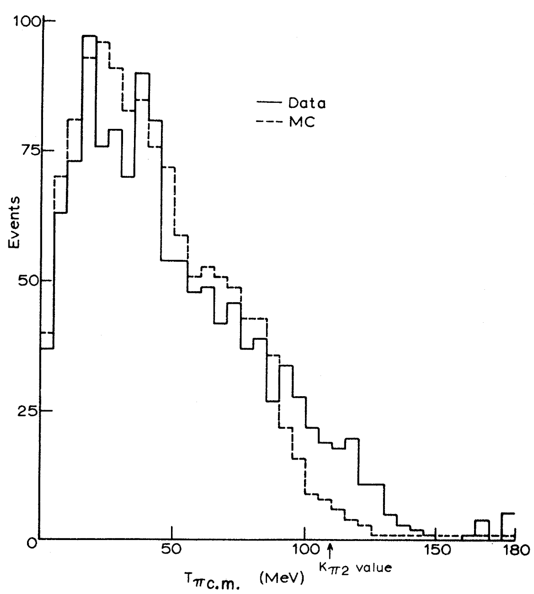


FIG. 7. Distribution of $T_{\pi c.m.}$, the c.m. kinetic energy of the primary $K^-$ decay prong, pion interpretation, for all unfitted Dalitz-pair events both real data and Monte Carlo. A $K_{\pi 2}$ contamination is evident.

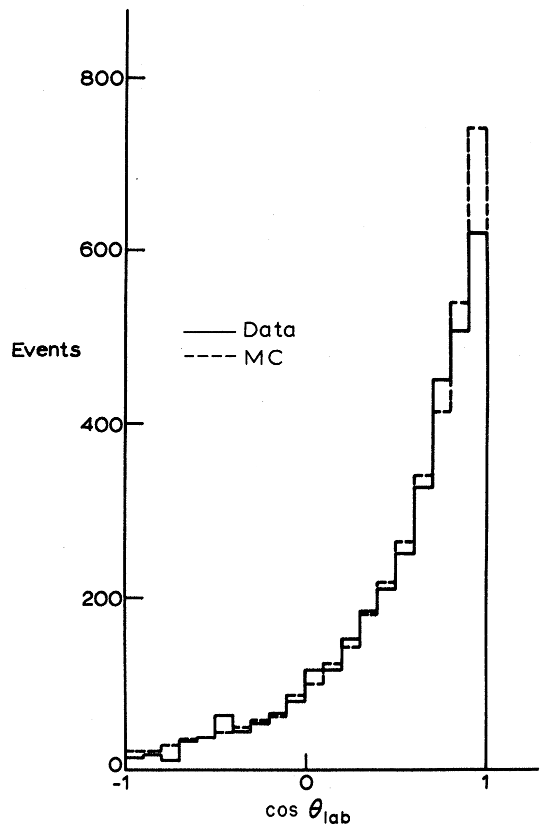


FIG. 9. The distribution of the pion decay angle in the laboratory frame taken with respect to the beam direction for $K_{\pi 2}$ events, both real data and Monte Carlo.

the fact that all of the film has been at least double scanned, although one of the scans had low efficiency for Dalitz-pair (DP) events. The over-all double-scan efficiency is given as 98.0% as determined on those events actually comprising the $K_{\pi 2}$ sample. Correction for external pairs and minor biases subtracts 11 events. The total $K_{\pi 2}$ + DP events present is thus given as:

$$\left(\frac{3370-20-120+118+(156\pm 50)-11}{0.980}\right)\pm 3.1\%$$

$$= 3564 \pm 110 \text{ events or } 3564 \text{ events} \pm 3.1\%.$$

The statistical error included is based on the number of events found in the first 18 bins of the cos $\theta$ lab plot, since the final number is based only on these.

## VI. NUMBER OF $K_{e3}$ EVENTS

There is a difficulty with the leptonic and $\tau'$ events which is not present for the $K_{\pi 2}$ events. The latter, when observed with Dalitz pairs, lead to two-constraint fits which give great simplification in the data analysis. In the former cases the fits are underconstrained. A second problem is that no ionization information can be used in the region of laboratory momentum greater than 200 MeV/$c$. Thus there are three event types confused in this region with neither fits nor visual information available to effect a separation. These data have not been used, correction for the cut being made through the Monte Carlo.

In the region less than 200 MeV/$c$ the $K_{e3}$ events are uniquely determined by the Dalitz-pair topology plus the identification of the primary decay track as an electron by ionization. The number of such events, automatically excluding any which incorrectly makes fits, is 474. The Monte Carlo run with the 200 MeV/$c$ cut and fitted events removed shows that the detection probability by this procedure is 0.615. It also estimates a small correction of three muons so short as to be missed and whose decay electrons presumably result in a $K_{e3}$ classification. The scanning efficiency is 96.8% on these events alone. Including finally the 0.6% external pairs contamination we find a total number of $K_{e3}$ + DP events of:

$$\left(0.994\frac{474-3}{0.968\times 0.615}\right)\pm 4.6\%$$

$$= 786 \pm 36 \text{ events or } 786 \text{ events} \pm 4.6\%.$$

The errors quoted are purely statistical in this case as we know of no comparable systematic problem.

## VII. NUMBER OF $K_{\mu 3}$ EVENTS

It is difficult to study muons in an experiment of this type. The low end of the spectrum is overwhelmed by $\tau'$ events, the high end by $K_{\pi 2}$'s, and there seems to be no satisfactory means of visually separating them, even though $K_{e3}$'s have been eliminated. Although many events were noted as having a stopping muon, this sample seemed to be contaminated with muons from $\pi$-$\mu$ decays from $\tau'$ pions.

If the primary decay track of the event is transformed to the $K^-$ c.m. with pion interpretation, the $\tau'$ spectrum runs from 0 to 53 MeV and only two events are expected to spill over into the region above 55 MeV. The cut for $K_{\pi 2}$ contamination is at 90 MeV. Thus the available part of the muon spectrum is between 55 and 90 MeV, pion interpretation, and in this region there are 93 events. We subtract two events for $\tau'$ spillovers and add one for a short muon misinterpreted as its decay electron. The Monte Carlo predicts this region to be 17.26% of the total and the scanning efficiency on these events is 97.2%. Thus the total $K_{\mu 3}$ + DP events we calculate in this way is:

$$\left(0.994\frac{93-2+1}{0.1726\times 0.972}\right)\pm 10.4\%$$

$$= 545 \pm 57 \text{ events or } 545 \text{ events} \pm 10.4\%.$$

There is in this experiment another source of clearly identified muons which arises from a study of the kinematics of the Dalitz decays themselves. For this purpose we consider those events of primary decay momentum less than 200 MeV/$c$ in the lab, nonelectron by ionization, and ambiguous by kinematics between $\tau'$ and $K_{\mu 3}$. The kinematics of the primary decay of such an event is twofold underconstrained in either the $\tau'$ or $K_{\mu 3}$ case. Consider the whole event as observed in the $K^-$ c.m. system. If we let the direction of the charged prong, transformed again as a pion, represent the positive $P_\parallel$ axis in a new coordinate system, then the two missing quantities can be taken as the component of $\pi^0$ momentum parallel to this new $P_\parallel$ axis and the azimuth of this momentum vector about the axis. Energy and momentum conservation are sufficient to calculate all other quantities as functions of these. If we project out the azimuthal dependence then we have a plot which is the parallel versus perpendicular components of $\pi^0$ momentum relative to the charged decay direction. Plots of allowed values are given in Fig. 10. The full manifolds of possible values are figures obtained by rotating these two curves about the $P_\parallel$ axis. By the fact that the third particle in a $\tau'$ decay is a $\pi^0$ while in a $K_{\mu 3}$ it is a $\nu$, the $\tau'$ curve always lies within that for $K_{\mu 3}$.

Now consider the Dalitz decay of this same $\pi^0$. If its momentum is completley specified, then the decay

$$\pi^0 \to e^+ e^- \gamma$$

makes a one-constraint fit as in the case of the $K_{\pi 2}$'s. Since the $\pi^0$ is not specified this fit cannot be attempted, but its one constraint can be used to establish one relation between the two unknowns of the previous paragraph, namely $P_{\parallel}$ and the azimuthal angle around $P_{\parallel}$. The resulting manifold of possible $\pi^0$ momentum vector values is one dimensional. It is a closed curve drawn on the surface of rotation discussed above.

The entire event can be viewed as minus-one constraint, and to understand what can be obtained from this consider the more frequently used zero-constraint fit. In this case the for equations of energy and momentum conservation can be used to calculate either zero, one, or two possible configurations. Although there are no checks it is still possible for this fit to fail, the zero-solutions case. It is possible that energy and momentum cannot be conserved for the mass interpretations given and a particular set of input data. In the case of a minus 1C fit the situation is similar. The solution is in general a one-dimensional manifold instead of discrete points, but it is still possible that no solution exists.

The general nature of the way that these data can be used is illustrated in Fig. 11. If we isolate the kinematics of $\pi^0$ decay, independent of production, we again have a two-dimensional set of points —$\pi^0$ momenta which could have given the Dalitz pair which we observed. If we project the boundaries of two such different sets of points into the same two-dimensional coordinate system used in Fig. 10, we obtain as typical cases curves DP a and DP b. The curves from $\pi^0$ decay always have this general appearance with infinity as part of the boundary. This reflects the fact that a normal-energy Dalitz pair can result from backward emission by an abnormal energy $\pi^0$. We see that for curve (a) there is an intersection on this plot of possible $\pi^0$'s from production as a $\tau'$ and from decay. In this case the one-dimensional manifold is not empty, while for curve (b) it will be empty and the $\tau'$ interpretation is not possible.

Thus the kinematics of the Dalitz pairs give a clear cut piece of information; either this event is a possible $\tau'$ or it is not, and some $K_{\mu 3}$'s can be identified. Much effort has gone into trying to find a probability estimator to choose between $\tau'$ and $K_{\mu 3}$ in a statistical way for those events which remain ambiguous. None has been found with sufficient resolving power to make its application to the data worthwhile.

Since this procedure does not take measuring errors into account, it is possible that an actual $\tau'$ event could be incorrectly classified as a definite $K_{\mu 3}$ by this procedure. This has been tested by running it on the $\tau'$ Monte Carlo data. The result is that indeed mistakes are made. They are highly energy dependent and total 6% of the $\tau'$ events. This is to be compared with 53% of the otherwise ambiguous $K_{\mu 3}$ events being rejected as $\tau'$ events.

There are 126 cases of $K_{\mu 3}$'s definitely identified in this way. Of this number 24 are estimated to be $\tau'$ events so that we have $102 \pm 11$ muons identified. The Monte Carlo predicts this to be 18.1% of the total $K_{\mu 3}$'s. Thus the number of $K_{\mu 3}$+DP predicted in this way is

$$564 \pm 62 \text{ events or } 564 \text{ events} \pm 11\%.$$

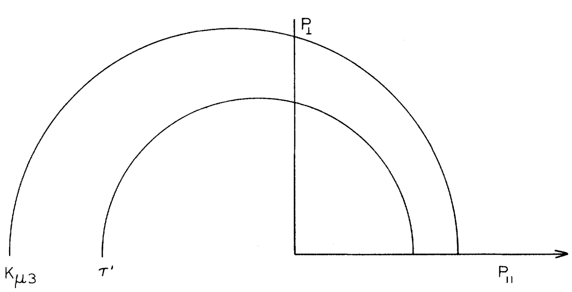


FIG. 10. Curves of allowed values of $\pi^0$ momenta for $\tau'$ and $K_{\mu 3}$ interpretations of a typical ambiguous event, using primary $K^-$ decay kinematics only. $P_{\parallel}$ is defined in the text and $P_\perp$ is any direction perpendicular to $P_{\parallel}$. The proper three-dimensional surfaces are formed by rotating these curves 360° about the $P_{\parallel}$ axis.

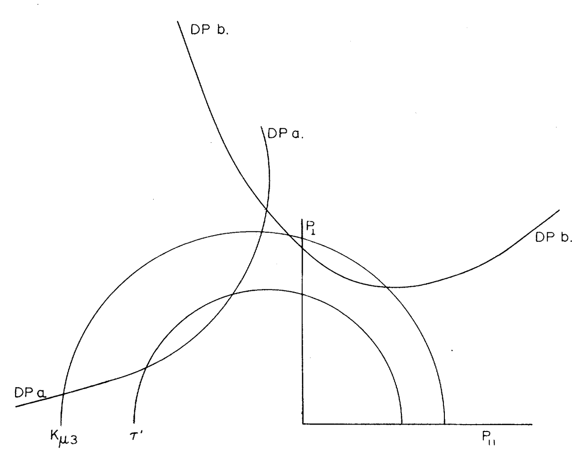


FIG. 11. Demonstration of how the information seen in Fig. 10 combined with Dalitz-pair kinematics can lead to an identification of the event. The open curves DP a and DP b represent different possible limits on $\pi^0$ momenta resulting from Dalitz decay kinematics.

These new events are statistically independent of those above so that we can combine the two sets of data and estimate a total of:

$554 \pm 42$ events or 554 events $\pm$ 7.6%.

## VIII. NUMBER OF $\tau'$ EVENTS

Figure 12 shows the data used in the determination of the number of $\tau'$ events and their energy spectrum. The plot is again $T_{\pi\,\mathrm{c.m.}}$ for all nonfitting, nonelectron events with $P_{\mathrm{lab}}$ less than 200 MeV/$c$, and excluding the $K_{\pi 2}$ region. Also all events not possible as $\tau'$ events from Dalitz-pair kinematics are excluded. However this latter criterion is set only for events in the $\tau'$ region. The real data contain both $\tau'$ events and $K_{\mu 3}$'s; the Monte Carlo has the same cuts as the real data but contains only $K_{\mu 3}$ events. This curve is normalized to the $K_{\mu 3}$ data present outside of the $\tau'$ region, with the small corrections made as noted in Sec. VII. The number of $\tau'$ events is then

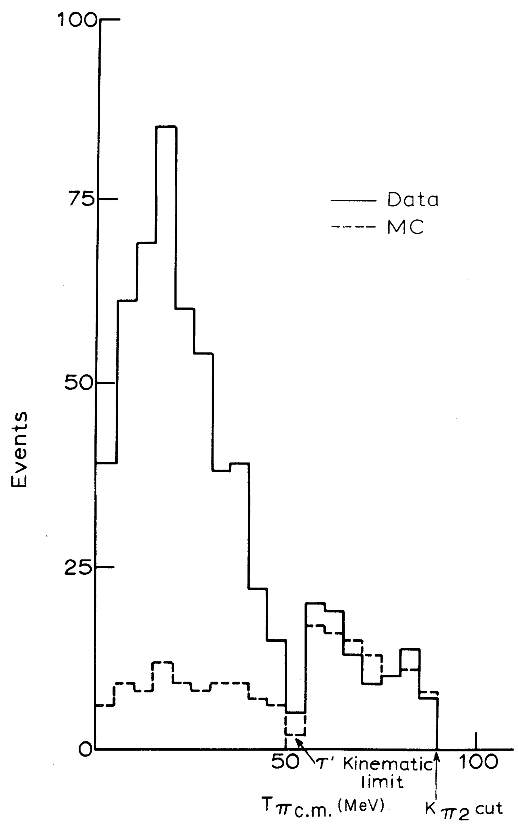


FIG. 12. The distribution of $T_{\pi\,\mathrm{c.m.}}$ for all possible $\tau'$ events. The Monte Carlo shows the expected $K_{\mu 3}$ contamination and is normalized to those events outside of the $\tau'$ kinematic region, namely $T_{\pi\,\mathrm{c.m.}}$ from 55 to 90 MeV.

the number above this background curve. There are 402 such events. They must first be corrected for the errors generated by the Dalitz-pair kinematics analysis, and increase of 5.94% or 24 events. Other corrections are for the cut at 200 MeV/$c$ lab momentum, 20.5% of the events missing the cut; spillovers out of the $\tau'$ energy region, + 2 events; − 0.6% for external pairs; and + 1.17% since there being two $\pi^0$'s, there may be two Dalitz pairs and a five-prong event which is not counted. The scanning efficiency on these events alone is 94.2%. Thus the total number is given by:

$$\left(\frac{402+24+2-3}{0.942\times 0.795}\,1.0117\right) \pm 5.9\%$$

$$= 574 \pm 34 \text{ events or } 574 \text{ events} \pm 5.9\%.$$

The error includes statistics on the background as well as on the data. If Daltiz-pair kinematics are not used, we obtain:

580 events $\pm$ 7.3%

so that their effect is to lower the error without appreciably affecting the value.

## IX. SLOPE OF THE $\tau'$ DALITZ PLOT

The $\tau'$ spectrum is assumed to be given by phase space times a factor which is linear in the c.m. kinetic energy of the odd pion, or $\pi^-$ in our case. For this study of the work of Davison *et al.*[6] has been followed. We write the spectrum divided by phase space in the form:

$$1+\frac{2A}{M_\pi^2}(s_3-s_0),$$

where

$A$ is the slope to be determined,

$s_i$ is the square of the 4-momentum transfer to particle $i$,

$s_0=\frac{1}{3}(s_1+s_2+s_3)$,

$M_\pi$ is the charged pion mass and pion number 3 is the $\pi^-$.

The quantity $s_3-s_0$ can be expanded as:

$$s_3-s_0=\tfrac{2}{3}(M_K^2-3M_KM_\pi+M_{\pi^0}^2)-2M_KT_3$$

so that linearity in this quantity is the same as linearity in $T_3$.

To determine $A$ we first state the actual distribution of the data. All the same cuts and background corrections are made as in the previous section, but they are now made one bin at a time. These totally corrected data are now compared with theoretical spectra of the form given above.

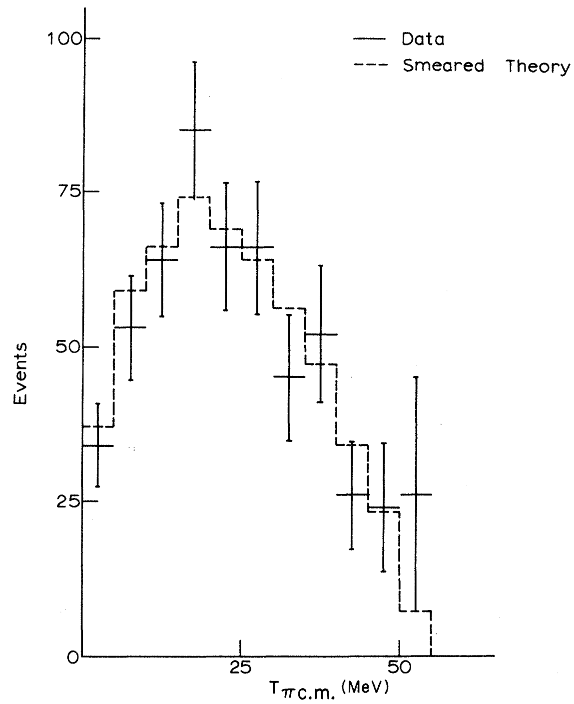


FIG. 13. The $\tau'$ spectrum with all corrections, both real data and Monte Carlo (smeared theory). The Monte Carlo includes phase space, the effects of resolution, and a linear term with our best fit value for the slope.

Kinematical phase space is modified to include the effects of resolution since the events are not fitted. From the Monte Carlo data we can find the fraction of events which truly lie in bin $i$ which will be measured as in bin $j$. The matrix $M_{ij}$ of these fractions is largely diagonal but not completely so.

We now vary $A$ and compare the resulting spectra with the data to obtain the best fit value,

$$A = 0.242 \pm 0.042$$

with $\chi^2$ value at minimum of 5.05 for 9 degrees of freedom. The data used are compared with this fit in Fig. 13.

This result compares favorably with the value $0.258 \pm 0.009$ obtained by Davison *et al.*[6] However, our data show no strong evidence of a dip at $T_\pi = 0$ as indicated by theirs. Our experiment has been performed using decays in flight which are less sensitive to systematic problems in this region. However our statistics do not rule out the effect.

## X. CONCLUSION

The results stated above are summarized as:

Number of $K_{\pi 2}$ events = 3564 ± 3.1%,

Number of $K_{e3}$ events = 786 events ± 4.6%,

Number of $K_{\mu 3}$ events = 554 events ± 7.6%,

Number of $\tau'$ events = 574 events ± 5.9%,

Slope of $\tau'$ Dalitz plot = 0.242 ± 0.042.

For formulation of branching ratios one must keep in mind that since a $\tau'$ has two $\pi^0$'s its chance of being observed is twice that for other Dalitz-pair types. The other decay mode for which we have knowledge in this exposure is $\tau$'s. With all corrections made we have observed

85 709 events ± 0.35%.

This number can be combined with the ones given above if we include the branching fraction of $\pi^0$'s to Dalitz pairs. Experimentally this fraction is[3]:

0.011 66 ± 0.000 47.

The theoretical value based on quantum electrodynamics is[4] 0.0119 but is uncertain if there are large effects resulting from the strong-interaction $\pi^0$ form factor.

Unfortunately we have no absolute normalization on the number of $K^-$ from which to compute branching fractions, only ratios are possible. All our ratios seem consistent with the values published in the Particle Data Group tables[7] except those involving $\tau$'s. Combining all the Dalitz-pair data together and taking the table value for the sum of these ratios yield a $\tau$ branching fraction of (6.03 ± 0.19)% or (5.90 ± 0.30)% depending on whether the theoretical or experimental value for the Dalitz-pair fraction is taken. If these numbers are accepted, they represent a change in the $\tau$ branching fraction of +7.6% or +5.7% of its current Table value, but are within the range of values obtained by previous experiments. The difference may also be attributed to an approximately $1.5\sigma$ error in the experimental value of the Dalitz-pair branching fraction. However this would require a move of the experiment away from the theoretical value with which it is now in agreement. The last possible interpretation is that the bias against Dalitz pairs seen in the early stages of this experiment has persisted despite efforts to prevent it. It can only be said that we see no evidence for this; the point is discussed in detail in Ref. 8.

We feel on completion of this experiment that the approach of using Dalitz pairs can be a fruitful one in certain applications. These seldom used events have provided us with the most complete study of $K^-$ decays in a bubble chamber that we know of.

## ACKNOWLEDGMENTS

The scan of this entire exposure for Dalitz-pair events only was a long and difficult task. It was performed by Mrs. Karin Baum, Mrs. Ruth Childs, Miss Ana Maria Leoni, and Mrs. Sirpa Lessig. The experiment could not have been performed without their work. The endless checking and remeasuring of the film was greatly facilitated through the dedication, and good humor, of Mr. John Neves.

Finally the authors wish to thank Mr. Peter Martin and the rest of the technical staff of this laboratory.

*Research supported by the U.S. Atomic Energy Commission under contract AT(11-1)3075.

†Present address: Physics Department, The Johns Hopkins University, Baltimore, Maryland 21218.

[1]P. W. Lucas, H. D. Taft, and W. J. Willis, preceding paper, Phys. Rev. D **8**, 719 (1973).

[2]Terry S. Mast, Lawrence K. Gershwin, Margaret Alston-Garnjost, Roger O. Bangerter, Angela Barbaro-Galtieri, Joseph J. Murray, Frank T. Solmitz, and Robert D. Tripp, Phys. Rev. **183**, 1200 (1969).

[3]N. P. Samios, Phys. Rev. **121**, 275 (1961).

[4]D. W. Joseph, Nuovo Cimento **16**, 997 (1960).

[5]Edward S. Ginsberg, Phys. Rev. **162**, 1570 (1967).

[6]D. Davison, R. Bacastow, W. H. Barkas, D. A. Evans, S.-Y. Fung, L. E. Porter, R. T. Poe, and D. Greiner, Phys. Rev. **180**, 1333 (1969).

[7]Particle Data Group, Rev. Mod. Phys. **43**, S1 (1971).

[8]P. Lucas, thesis, Yale University, 1972 (unpublished).

PHYSICAL REVIEW D VOLUME 8, NUMBER 3 1 AUGUST 1973

# Study of $K^+p \to K^+p\,2\pi^+2\pi^-$ at 12.7 GeV/$c$

P. L. Jain, W. M. Labuda, Z. Ahmad, and G. Pappas

*High Energy Experimental Laboratory, Department of Physics, State University of New York at Buffalo, Buffalo, New York 14214*

(Received 16 March 1973)

We present the results of the reaction $K^+p \to K^+p2\pi^+2\pi^-$ at 12.7 GeV/$c$. These results are compared with the predictions of the model developed by Chan, Łoskiewicz, and Allison (the CŁA model) and with a modified form of this model. With the addition of resonance production to the model, adequate predictions of the experimental distributions are obtained. The data were also analyzed in terms of the quark model for hadronic structure which is revealed through the distributions of longitudinal momentum, transverse-momentum squared, and the rapidity for the negative pions.

## I. INTRODUCTION

The study of many particle processes in high-energy collisions is in a very early stage of development. Owing to their intrinsic complexity, our understanding of these processes lags far behind that of two-body reactions, which are relatively simple compared to many-body reactions. In many-body reactions the dimensionality of phase space increases rapidly so that one needs too many variables to describe all features of a reaction. In recent years there has been a growing interest among both the experimentalists and theoreticians in the study of many-body reactions. The progress in understanding high-multiplicity reactions has mainly been made through a number of theoretical models, for example (i) the bremsstrahlung model,[1] (ii) the thermodynamic model,[2] (iii) the quark model,[3] and (iv) the multi-Regge model.[4,5] The last [(iv)] model, a version of the multiperipheral model, has been extensively used to interpret the experimental data. A model of this type, aimed at describing inelastic reactions quantitatively, has been proposed by Chan, Łoskiewicz, and Allison[6] (known as the CŁA model), and has been shown by various experimental groups[7] to reproduce the main features of many body reactions.

In experiments in which pions are used as projectiles, the leading pion can be easily lost among the other pions produced. However, when the projectile is a kaon, the leading particle among the secondaries can be distinguished from the directly-produced pions on the basis of their mass difference. From the detailed analysis of multiple meson production in high-energy kaon-nucleon collisions in terms of a multi-Regge model, one can learn a great deal about the range of applicability and usefulness of the model. Hence, we present here the results for the reaction $K^+p \to K^+p2\pi^+2\pi^-$ at 12.7 GeV/$c$ and compare them with the predic-

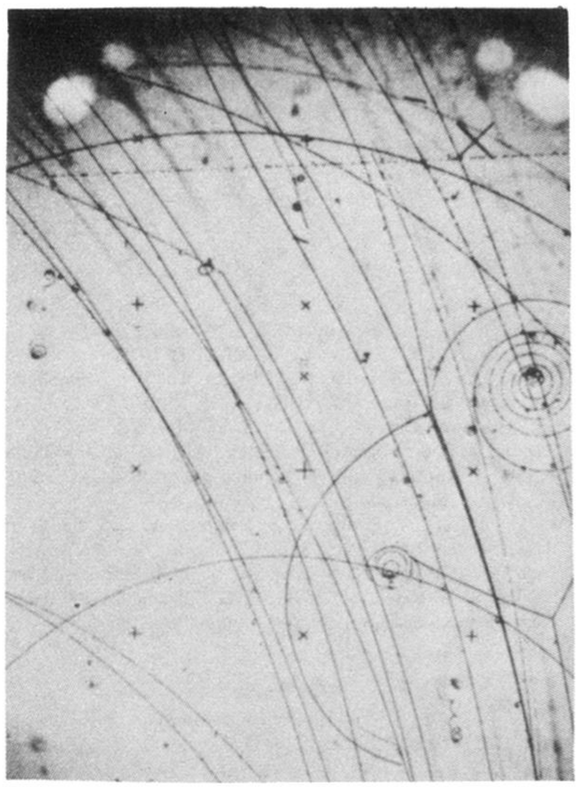

FIG. 1. A portion of a bubble-chamber picture showing a typical Dalitz-pair event.