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New Determination of the Dalitz-Plot Distribution for τ^- Decays*

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We have measured the X and Y projections of the Dalitz plot of 81 000 τ^- decays. The decays were observed in a beam of 400-MeV/c K^-'s in the BNL 30-in. liquid-hydrogen bubble chamber. For the X distribution we find no deviation from phase space. For Y we find a distribution of (phase space) $\times(1+AY)$ and have determined for A the values 0.252 $\pm$ 0.011 with Coulomb corrections to the distribution, 0.219$\pm$ 0.011 without Coulomb corrections. The results have been obtained using a method which involves measured rather than fitted quantities.

I. INTRODUCTION

The distribution of the Dalitz plot for charged K mesons decaying via the τ mode,

$$K^{\pm}\to\pi^{\pm}\pi^{\pm}\pi^{\mp},$$

should be a property of the weak interaction measurable with a minimum of experimental difficulty. The decay can be observed in a bubble chamber or in a counter system, with all tracks, both incoming and outgoing, charged and observable. For K^+ the decays can be observed for particles at rest, while for K^- they must be observed in flight to avoid nuclear absorption. The distribution is experimentally well represented by three-body phase space (flat Dalitz plot) multiplied by a term linear in the kinetic energy of the odd pion. There is some evidence stated for more involved structure,[1,2] but none has been convincingly proved.

Despite the apparent simplicity of the problem, the precise measurement of the slope of the linear term, the only measured quantity needed to describe the distribution within the accuracy of current experiments, has been difficult and has given inconsistent results previously. It was in an attempt to resolve this inconsistency that the experiment reported here was performed.

II. DALITZ PLOT

We have chosen to parametrize the Dalitz plot in terms of the variables X and Y defined by

$$X=\frac{\sqrt{3}}{Q}\,|T_1-T_2|,\qquad Y=\frac{1}{Q}(3T_3-Q),$$

where T_1 and T_2 are the kinetic energies in the K^- center-of-mass system of the two π^-, T_3 is the kinetic energy of the π^+, and Q is the energy release of the decay, given by

$$Q=M_K-3M_\pi$$
$$=75.09\pm0.12\text{ MeV}.$$

In terms of these variables the Dalitz plot is bounded by the closed curve formed by the Y axis and the curve

$$X=+\left[\frac{2M_KQY^3+(4M_\pi^{\,2}-2QM_K+Q^2)Y^2-(2M_K-Q)^2}{Q^2Y^2+2Q(3M_\pi-Q)Y-4M_\pi^{\,2}+2QM_\pi-Q^2}\right]^{1/2}.$$

The distribution of events within this boundary is then given by

$$\text{const} \times (1 + AY),$$

where A is known as the slope of the distribution on the Dalitz plot. The two highest-statistics experiments measuring A have obtained

$A = 0.247 \pm 0.009$, Mast[1]

$A = 0.2752 \pm 0.0033$ Ford[2].

III. DATA COLLECTION

The data for this experiment come from an exposure consisting of 1.1 million pictures from the BNL 30-in. liquid-hydrogen bubble chamber. The beam consisted of fifteen K^-'s per picture of average momentum 400 MeV/c. The details of the exposure have been described previously.[3]

The pictures were scanned for all three-prong decays. In addition to the τ's, all modes consisting of one charged prong plus a π^0, the π^0 decaying via the Dalitz mode $\pi^0 \to e^+ + e^- + \gamma$, were included. These other event types make up about 7% of the total and have been separated by kinematics and by individual examination. The mechanics of the scan included recording the vertex position so that events could be measured on the Yale PEPR (Precision Encoding and Pattern Recognition) automatic system in the zone guidance mode.[4]

The PEPR hardware and software were not changed in any essential way from those used in the preceding PEPR experiment.[4] A brief description of the method will be given, however, as it is essential to the analysis of the data. The input to PEPR, in addition to the usual scan information of roll and frame numbers, event type, etc., was for each view the zone of the vertex, as recorded by the scanners on a grid whose smallest subdivision was about 1.25×1.25 cm^2 on the scan table or about 1.25×1.25 mm^2 on the film. The grid was positioned relative to the fiducial marks so that the event could easily be found, and within this region pattern recognition was attempted. All data found within an area twice as large as this box were linked together by PEPR into tracks. The tracks were followed an additional distance of 5 mm on the film to ensure sufficient length to determine sense of curvature, and then classified as to sign, whether consistent with the beam direction, and whether ending in the scanner zone or through track. A routine was then entered which attempted to make up the event—one beam track, one positive prong, and two negative prongs—out of the track segments present. Failing this, it cut in two any track which passed through a suspected vertex point and then tried to use the two new tracks created, plus the tracks previously present, to form the event. It was not uncommon in the case of τ's to have one of the negative prongs look like an extension of the beam for the short track lengths present at this stage. If the event were identified according to this scheme, the four tracks were followed either to their endpoints or to the edge of the picture and suitable points chosen for output.

It often occurred that the PEPR system was unable to find an event consisting of four tracks. Confusion due to large numbers of background beam tracks, "self-confusing" events, and occasionally poor film quality led to this problem. The decision was made to output three of the four tracks of the event if only three could be found. Often the missing track would be measured in the remaining two stereo views and still be reconstructed correctly. Failing this, the event would go into the kinematic fitting stage with only three of the four tracks. These are sufficient to attempt a one constraint fit to a normal τ decay, instead of the four-constraint fit attempted if all tracks are present.

IV. ANALYSIS PROGRAMS

The standard geometrical reconstruction and kinematic fitting programs TVGP and YACK were used in the analysis of this data. The only change of any substance made to TVGP was the inclusion of a section to match properly the negative prongs measured in different views. PEPR simply measured them right to left in each view as a first guess. However, this sometimes led to errors most easily corrected at the geometrical reconstruction stage. Furthermore, if PEPR had, as a last resort, measured only three of the necessary four tracks and if the missing track were a negative prong, then the problem would have been compounded. With a negative prong missing it is impossible to know with which prongs of the other views to associate that one which is present. Therefore, if TVGP detected errors in one of the negative prongs sufficient to fail the track, the program attempted swaps of the negative prongs present, or swaps of track numbers of present prongs and missing ones, to improve the situation. This resulted in success on many events which would otherwise have been lost.

The program YACK attempted fits of the decay of the beam either into a normal τ event or into the combination

$$K^- \to \pi^- + \pi^0, \quad \pi^0 \to e^+ + e^- + \gamma.$$

The fit to a normal τ decay was either a four- or one-constraint fit. If all tracks were measured and the event truly a normal τ, the four-constraint fit should have been satisfactory. If it was not, either through a YACK reject or high χ^2 (>20.0), the program attempted one-constraint fits dropping each of the four tracks in turn, treating them as if they had been missed. The particular problem which often generated this situation was that of short tracks. τ's produce many short stopping tracks whose momentum must be determined from their range in the liquid hydrogen. However, since PEPR was not equipped with an endpoint measuring program, the accurate range was not known, and these tracks generated many failures.

The $K_{\pi 2}$ plus Dalitz pair fits consisted of two steps with two over-all constraints. All tracks must have been present and correctly measured for these fits to be satisfactory.

Finally, a general tape reading, event classification, and histogramming program called PROCES was written to study the geometry and kinematic output. Its primary job was to classify the events in the following scheme:

(1) 4-constraint τ event. 4C χ^2 less than 20, and satisfying certain tests on missing masses of various pion combinations to eliminate the possibility of the fitted χ^2 being small only because the errors were large.

(2) 1-constraint τ event. 1C χ^2 less than five and satisfying whichever of the missing mass tests were possible for the combination of tracks present. Also the fitted beam track was forced to have momentum and direction consistent with the known values.

(3) 2-constraint $K_{\pi 2}$ event. The χ^2 of the two-step fit, taken as the sum of the χ^2's of the two steps, was less than ten and satisfied missing-mass and beam-direction tests as above.

(4) Ambiguous. χ^2 did not clearly distinguish between τ and $K_{\pi 2}$ hypotheses.

(5) Failure. Events with unsatisfactory measurement or no satisfactory fits.

(6) Fiducial volume reject. Events determined by PEPR to be too near the edge of the picture to be satisfactorily measurable.

V. REMEASUREMENT

All events in the ambiguous and failure categories were remeasured until satisfactorily disposed of by the program or inspected by a physicist. These remeasurements, consisting for the first pass of about 30% of the sample and substantially less on succeeding passes, were performed on the Yale Wire Measuring System. This system is described in Ref. 5. It consisted, for this experiment, of four image plane digitization devices whose principle of encoding is the timing of an acoustic wave in a piece of magnetostrictive wire. The pulse travels at the speed of sound and can be timed with acceptable accuracy by a 40 MHz scaler commercially available. These four machines, as well as three others measuring film from a different bubble chamber, were all controlled by an on-line PDP-1 computer. The computer sorted the data from the several machines, made format checks, gave feedback to the operators, calibrated the data to remove geometrical distortions, and wrote a FORTRAN compatible magnetic tape with points sorted into complete events.

The final event totals after remeasurement from which the Dalitz-plot study was made are:

50 527 4C fits

30 968 1C fits

379 events with no fit but determined not to be leptonic or Dalitz-pair events.

The scanning efficiency on these events alone, as determined from a double scan of about half the sample, is 96.5%.

VI. DATA CONTAMINATIONS AND BIASES

The events used in the analysis of this experiment are those 81 495 which make either a four-constraint or a one-constraint fit. The numbers of these which are not in fact normal τ decays are known to be small. The first possible contaminant is radiative τ's:

$$K^- \to \pi^+ + \pi^- + \pi^- + \gamma .$$

From K^+ branching ratios our data would be expected to contain about 145 such events with γ energy greater than 10 MeV in the K^- c.m. system. Some fraction of these will doubtless satisfy the criteria of either a 4C or 1C τ event. For γ energy less than 10 MeV the events are generally inseparable from τ's. The radiative events which are present in the final sample are assumed not to affect the Dalitz-plot distribution appreciably.

The second possible contaminant is Dalitz-pair events. With these there is far greater cause for concern than with the radiative events. Firstly, there are far more of them, about 7% of the total three prongs, and secondly, they might be expected to have a peculiar Dalitz-plot distribution if incorrectly interpreted as τ's. However, an experiment has been performed using these events and they have been carefully separated. An extra scan of the film was made looking just for Dalitz-pair events, and any satisfying the conditions for a 1C τ have been examined by physicists. Monte

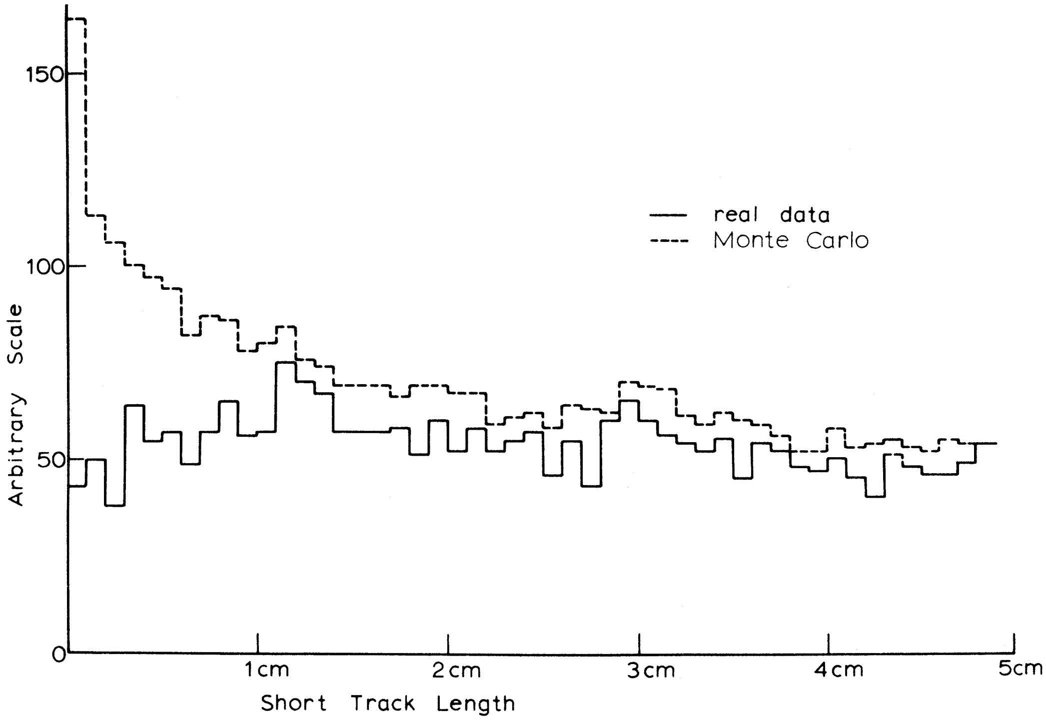

FIG. 1. Length distribution for short tracks. Comparison of real data and Monte Carlo data.

Carlo calculations show that fewer than 20 of the "4C τ's" would be expected to be in fact Dalitz-pair events. From the numbers of events found and the calculated efficiencies of the two scans, the contamination of normal τ's by Dalitz pairs is estimated as 75 events. Searches for these in apparently overpopulated regions of the Dalitz plot have not turned up more than the number expected from statistics, and thus they do not have appreciable effect on the distribution.

Our study of biases follows closely that of Ref. 1. The chief bias sought has been one against short tracks. As mentioned, τ's produce a number of events with quite short stopping tracks, and if the expected distribution of track lengths is appreciable near zero, a bias is likely. This distribution from a Monte Carlo program is plotted vs the data in Fig. 1. The normalizations are such that the Monte Carlo data and real data have the same total number of events if the entire distribution, of which this figure is only a fraction, is included. For the Monte Carlo program, phase space was chosen for the pion momentum distributions. In the case of the real data, range from fitted momentum is plotted, not measured length. The results are similar to those of Ref. 1, and a similar procedure has been adopted to deal with them. A cut is made at 11 mm on the length plot, which corresponds to 37 MeV/c in momentum. No event with data below these values is accepted. The fraction of such events in each bin is recorded, and the data are corrected for this fraction. The final distribution over the Dalitz plot is insensitive to the precise

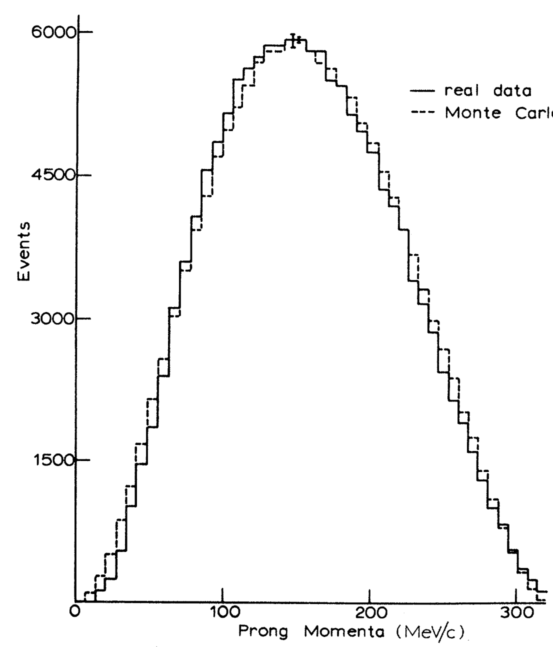

FIG. 2. Momentum spectrum of decay tracks. Comparison of real data and Monte Carlo data.

position of this cut.

In addition, the momentum distribution of all prongs is plotted against the phase-space Monte Carlo data (Fig. 2). The most striking feature is the bias against low-momentum tracks. Also there is a slight shift in the entire distribution above 60 MeV/c. This is attributed to a combination of a nonuniform matrix element for the real data and a possible slight misrepresentation of the beam spectrum. Several plots were made to test whether there were differences in momenta for 4C and 1C events. The most striking is given in Fig. 3 which shows the fitted momenta of the unmeasured prongs. The low-momentum peak (much of which is later lost to the short-track cut) is caused by the inability to measure correctly short tracks. More disturbing is the fact that the general momentum distribution of these is quite different from that of all prongs. These calculated prongs are not necessarily incorrect, but certainly care must be taken in handling them as they are clearly not an unbiased sample of tracks.

VII. ELECTROMAGNETIC CORRECTIONS

It is customary to correct the measured distribution for the effects of electromagnetic interactions and thus obtain the effects due to the weak interaction. These corrections are assumed to be dominated by the Coulomb interactions of the outgoing pions, and take the form of weighting phase space by a factor

$$C = C_{12} \times C_{23} \times C_{31},$$

where

$$C_{ij} = \frac{n_{ij}}{e^{n_{ij}} - 1}, \quad n_{ij} = \frac{2\pi q_i q_j}{\hbar v_{ij}};$$

q_i is the charge of pion i, and v_{ij} is the relative velocity of pions i and j. As there are possible theoretical objections to the use of this formula, all results are quoted both with and without it.

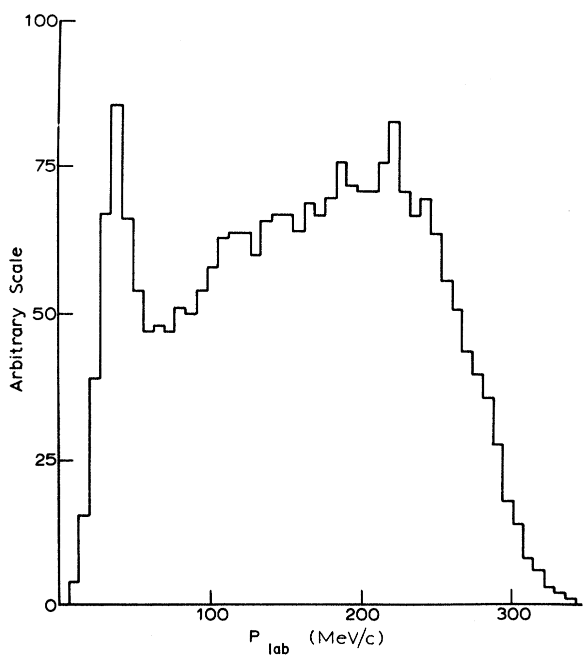

FIG. 3. Fitted momenta of unmeasured or unused prongs in 1C events.

VIII. DALITZ-PLOT ANALYSIS

Figure 4 shows the data corrected for the short-track cut and Coulomb interactions projected on the Y axis and divided by phase space. The strong

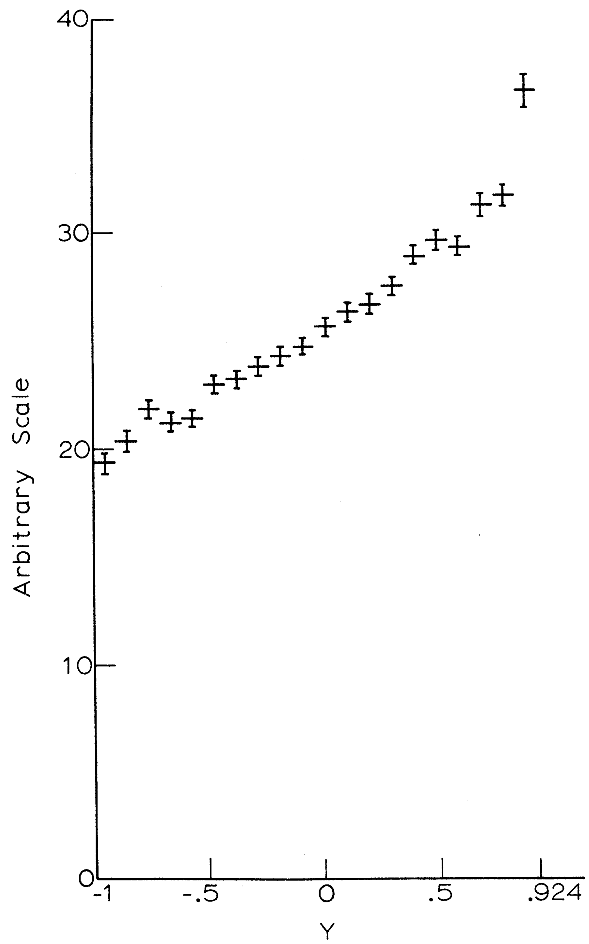

FIG. 4. Y projection of the Dalitz plot corrected for phase space, short-track cut, and Coulomb interactions. The Y scale spans the physically allowed range -1.0–0.924 and is divided into twenty bins of equal size. All data.

overenhancement of the top bin and the positive quadratic nature of the data—effects not noted by other authors—are suspect. In Fig. 5 we see the same data plotted, but now with 4C and 1C events separated. The problems seem to lie in the 1C fits alone. Careful examination has turned up no errors in the analysis, and the following interpretation of these 1C fits has been made.

There is of course some dispersion present in the measuring process, and the purpose of a fitting procedure is to use the constraints of energy and momentum conservation to obtain so called true values for the parameters of tracks. This is done by shifting the measured values within their errors to satisfy the constraints. However, values may conserve energy and momentum but not necessarily be correct in the case of the 1C fits. One constraint is not sufficient to pull measured values all the way to the correct true ones.

The problem is most simply stated in terms of data with one-constraint fits having insufficient energy resolution. Furthermore, it seems that reasonable assumptions as to the effects of this resolution can lead precisely to the distribution observed. Although plots of the Y distribution divided by phase space are approximately straight lines, the Y distribution itself is peaked in the middle. The effect of measurement is to broaden the peak and to put some data beyond the kinematic limit at the upper end. If a weakly constrained fit is made the peak is left broadened, though not so much as with measured values, and the events beyond the kinematic limit are pulled into the physical region, but tend to overpopulate the upper end. When the resulting spectrum is divided by phase space, we get just the effects noted. The broadened peak shows up as a depletion of the middle relative to the ends, i.e., positive quadratic, and the overpopulated top bin is as observed.

This problem may be dealt with by a Monte Carlo technique.[6] Events of the type under consideration are generated, the track parameters are smeared in accordance with their errors, and the shifts on the Dalitz plot are evaluated. Since for fitted quantities the values and errors are correlated, and the errors are presumably not Gaussian near the boundaries of the Dalitz plot, the procedure adopted was to use measured values instead. Each track is characterized by the three variables: ϕ the azimuthal angle, $\tan\lambda$ the tangent of the dip angle, and k the projected curvature. Errors on these three quantities are published by TVGP and are assumed Gaussian and independent. They are used to simulate the measurement process and are taken from real, not Monte Carlo, data.

There is a problem in this procedure which is that we never know the true value of any measured quantity as we do for Monte Carlo generated events. The fitted value is assumed to be the best estimate of the true value. This will lead to incorrect results only insofar as the fitted value is incorrect and as the average distance an event is smeared in Y is a function of Y. Although there is some such dependence, it is weak. The fitted values, assumed correct, are now altered in accordance with the measured errors of the same track. A random number is chosen from a Gaussian distribution and multiplied by the error, and the result is added to the fitted quantity.

If an event is missing a negative prong, this procedure is applied only to the beam and π^+. If it is missing the beam or π^+ the procedure is applied to the three tracks present, and the missing track is calculated by 3-momentum conservation. It seemed best to include all data in this procedure; thus 4C events are handled like 1C's with missing π^-.

The result is a smearing matrix, which represents the measurement process. The elements of this matrix, M_{ij}, are the fractions of those events which truly lie in bin i, which are measured as in bin j. The index i runs from one to twenty, the same twenty bins as used previously to display the Y distribution, and the index j runs from one to thirty, the additional bins including those events which as measured lie outside the kinematical region of Y.

The errors on tracks as published by TVGP contain a parameter called the setting error ϵ, which is the estimated accuracy of the measuring process. To form the proper smearing matrix it was clearly necessary to have the correct errors, and an operational procedure was established to estimate ϵ based on observation of the tail of the distribution of the quantity Y beyond the kinematic

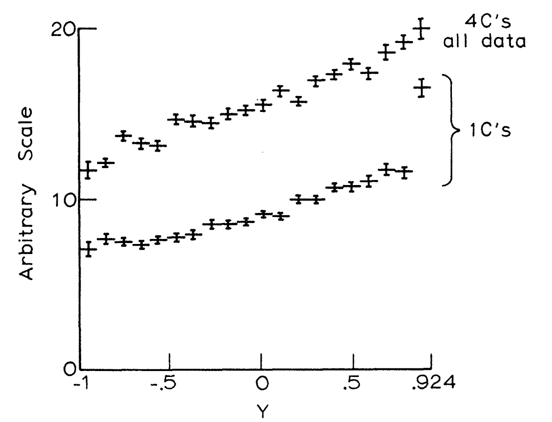

FIG. 5. Y projection of the Dalitz plot corrected for phase space, short-track cut, and Coulomb interactions. 4C and 1C events separated.

limit, rather than on the usual method of using fitted quantities. The smearing matrix, as mentioned, is not square but instead contains 20 rows by 30 columns. The smearing into the first 20 of these columns is dominated by the diagonal elements; but since there are no diagonal elements for the last ten columns, the events which show up here must do so by smearing. All errors were multiplied by a scale factor, and it was assumed that the data would match the smeared theoretical distribution in these latter bins for the correct value of this factor. The ϵ found was 35 μ on the film. It was found that the smeared theory matched the data within statistics for eight bins in Y beyond the physical or diagonal region. This is a 40% extension and gives a good indication that this method gives correct results. The procedure for determining the slope is the following:

(a) The fraction of each bin of the two-dimensional Dalitz plot which lies within the physical region is determined.

(b) Each such bin is weighted for the short-track cut.

(c) The result is projected onto the Y axis.

(d) For each of a series of values of the slope parameter A, the resulting distribution is weighted by the factor

$$1+AY.$$

(e) The result is normalized to the data.

(f) This distribution is smeared by the matrix M_{ij} as

$$\text{Bin } i \text{ (smeared)} = \sum_{j=1}^{20} M_{ij} \times \text{Bin } j \text{ (theoretical)},$$

where M_{ij} as defined maintains the normalization.

(g) A χ^2 is calculated for each slope incorporating statistical errors on both real and Monte Carlo data:

$$\chi^2 = \sum_{i=1}^{30} \frac{[\text{Bin } i \text{ (Monte Carlo)} - \text{Bin } i \text{ (data)}]^2}{[\text{Error on Bin } i \text{ (Monte Carlo)}]^2 + [\text{Error on Bin } i \text{ (data)}]^2}.$$

Steps d through g are repeated for several values of A and a χ^2 minimum and width are obtained.

(h) The Coulomb weights are applied to each bin of the Dalitz plot and steps c through g are repeated.[7]

IX. RESULTS

We obtain the values

$A = 0.219$ neglecting Coulomb corrections

$A = 0.252$ with Coulomb corrections.

The real data and smeared theory distributions for the Coulomb-corrected case and their ratio are shown in Figs. 6 and 7.

The obvious quadratic component seen in Fig. 7 is worthy of discussion. It is an instrumental effect which results from not fine-tuning the setting error as a function of measuring machine, fit type, and film conditions. Runs were made which, using different setting errors, yielded positive quadratic behavior far more marked than that seen here, negative quadratic behavior, and, on a fraction of the data, no obvious quadratic behavior. In no case was the best fit slope affected although the quality of the fits varied substantially.

For the results quoted, the minimum χ^2 value for the data without Coulomb corrections is 33.8 for 26 degrees of freedom, or a confidence level of 15%. For the data with Coulomb corrections it is 46.4 or a confidence level of 1%. It is higher in the Coulomb-corrected case as this correction contains a quadratic part which enhances the presumed instrumental quadratic already present.

In summary the quadratic nature of the data as seen in Fig. 4 has not been completely removed by the smearing procedure, but the slope value has been found to be insensitive to the nature of the quadratic.

The statistical error on A in both the Coulomb corrected and uncorrected cases is 0.008. We include together with this sufficient systematic error to reduce the χ^2's to values equal to the degrees of freedom. Thus for the Coulomb-corrected case we obtain

$$A = 0.252 \pm 0.011.$$

We feel that the error in the uncorrected case should be scaled similarly, yielding

$$A = 0.219 \pm 0.011.$$

We have not calculated values for A on 4C and 1C events separately as we feel that neither sample is unbiased and that the results might be misleading. Since we are able to affect so considerably the apparent quadratic term in our data through variation of the setting error, it is not appropriate for us to quote a value for the quadratic which may be in the data.

The X distribution was studied without the smearing technique. The data, with and without Coulomb corrections, are corrected for short-track cut, phase space, and reflected Y dependence. The latter is necessary here in the form of dividing the number of events in each bin by

$1 + 0.219Y$ (without Coulomb corrections)

$1 + 0.252Y$ (with Coulomb corrections)

before projecting onto X. After projection, they are fitted to a constant term only. The resulting distribution in the Coulomb corrected case is given in Fig. 8. The χ^2 in the non-Coulomb-corrected case is 21.3 for 19 degrees of freedom, confidence level of 40%, in the Coulomb-corrected case 24.6, confidence level of 20%. Given the results of our study of the Y distribution, it is of course not correct to assume that the X distribution will show no effects of the distorted fitted quantities. However, since the lowest-order structure which this distribution can have is quadratic and is known from previous work to be small, we have not tried to use the smearing technique in an attempt to observe such structure.

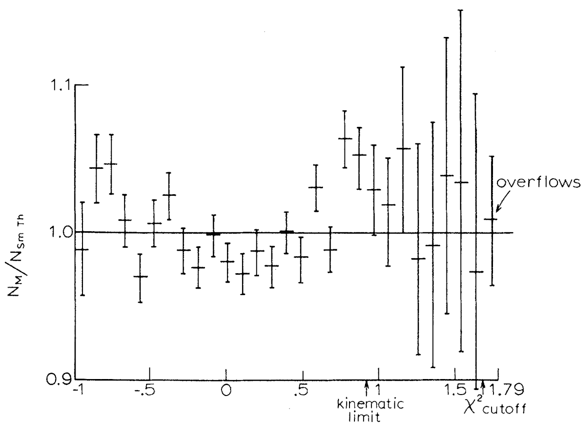

FIG. 7. Y projection of the Dalitz plot for measured values. Ratio of real data to Monte Carlo smeared theory with $A = 0.252$, both with Coulomb corrections.

X. CONCLUSION

Although the analysis of this experiment has been made difficult through the inclusion of the 1C fits in the data, we nonetheless feel that the method we have devised is valid. Our result for the Y distribution differentiates clearly between the two previous high-statistics experiments studying $\tau^{\pm}$ decays.

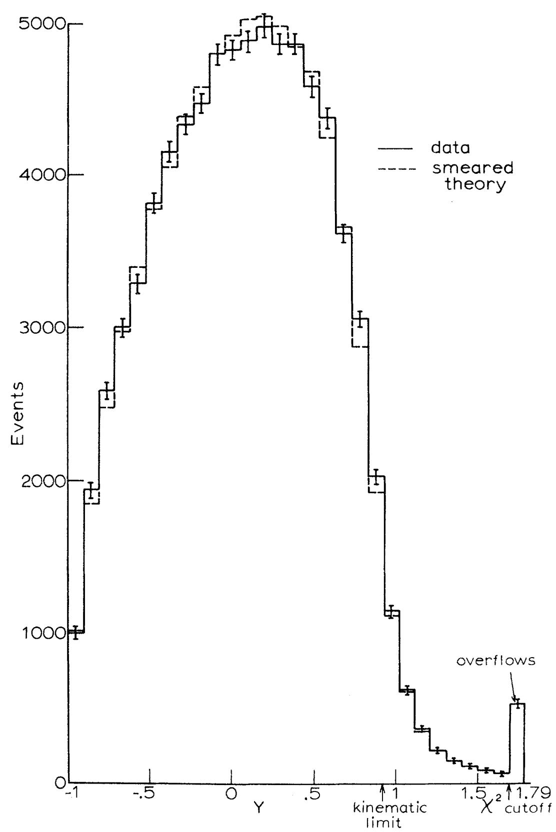

FIG. 6. Y projection of the Dalitz plot for measured values. Comparison of real data and Monte Carlo smeared theory with $A = 0.252$, both with Coulomb corrections.

ACKNOWLEDGMENTS

The experiment previous to this one on both PEPR and the wire-measuring machines was headed by Dr. Dixon Bogert. Much of the data handling for our experiment followed the lines of his work. The invaluable PDP-1 on-line program-

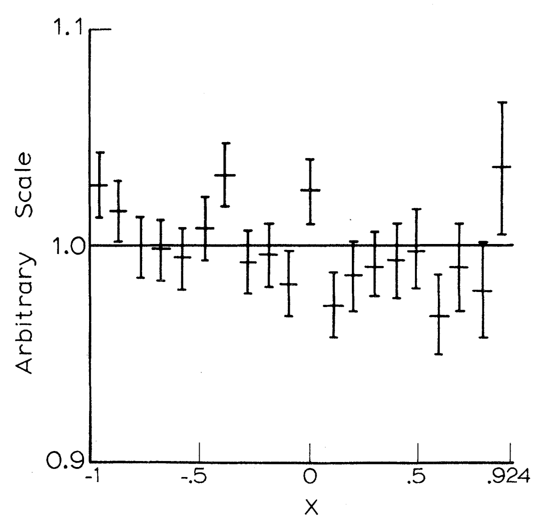

FIG. 8. X projection of the Dalitz plot corrected for phase space, short-track cut, Coulomb interactions, and reflected Y dependence. The X scale spans the physically allowed range 0.0–0.965 and is divided into twenty bins of equal size.

ming was the work of Dr. Bogert and Marvin Johnson.

We are indebted to our collaborators at the Brookhaven National Laboratory, Dr. D. Berley and Dr. P. Yamin for aid in utilization of the 6600 computer.

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Study of K^--Meson Decay Properties Using Dalitz Pairs for π^0 Detection*

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A sample of about 5000 K^- decays in flight, each accompanied by a Dalitz pair, has been observed in a hydrogen bubble chamber. These events consist of $3564 \pm (3.1\%)$ $K_{\pi2}$ events, $786 \pm (4.6\%)$ K_{e3} events, $554 \pm (7.6\%)$ $K_{\mu3}$ events, and $574 \pm (5.9\%)$ τ' events. Ratios among these numbers give results in agreement with accepted K^+ branching ratios. The separation of the events has involved a sophisticated Monte Carlo program, ionization, kinematics of the K^- decay, and kinematics of the Dalitz decay. The π^- energy spectrum of the τ' decays may be written as: (phase space) $\times [1 + (2A/M_\pi{}^2)(s_3 - s_0)]$, where s_i is the square of the 4-momentum transfer to particle i, $s_0 = \frac{1}{3}(s_1 + s_2 + s_3)$, and A is a slope to be determined. We find $A = 0.242 \pm 0.042$.

I. INTRODUCTION

Most bubble-chamber experiments studying the properties of decays of charged K mesons have done so by stopping K^+ in either heavy liquid or liquid hydrogen. We have performed a study of these properties different in several essential ways from this norm: We have studied K^- instead of K^+, and since K^- do not decay at rest but are absorbed, we have of necessity studied them in flight. Furthermore, we have performed this experiment using a new event recognition procedure, studying only three-prong decays specifically including all events in which a Dalitz decay of a π^0,

$$\pi^0 \to e^+e^-\gamma,$$

has been the source of two of the three prongs. Thus we have been able to study

$$\begin{aligned} K^- &\to \pi^-\pi^0, & K_{\pi2}\\ &\to e^-\nu\pi^0, & K_{e3}\\ &\to \mu^-\nu\pi^0, & K_{\mu3}\\ &\to \pi^-\pi^0\pi^0, & \tau' \end{aligned}$$

as well as

$$K^- \to \pi^-\pi^+\pi^+, \quad \tau$$

which is by far the dominant three-prong decay mode. The results on these τ's, together with