This gives one the additional infrared-type behavior and is model-dependent.

Therefore we conclude that to first order of pion interactions renormalization does not change the nonanalyticity of the S-matrix elements, while there is a change in such behavior for the current-matrix elements.

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Precise Hybrid Formula for Electromagnetic Mass Differences

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We present the derivation of a mass formula which relates electromagnetic mass differences of baryons and pseudoscalar mesons and is experimentally accurate within ~ 2%. The derivation reveals that the new formula is the appropriate hybrid analog to the Coleman-Glashow formula.

It has been repeatedly emphasized[1,2] that there is no analog in the pseudoscalar-meson system to the successful Coleman-Glashow formula for baryon electromagnetic mass differences.[3] In a recent letter[2] Cicogna, Strocchi, and Vergara-Caffarelli have derived a formula for electromagnetic mass differences of pseudoscalar mesons which contains also quantities expressing SU(3) and SU(2) noninvariance of the vacuum, namely F_K, F_π $(F_K \neq F_\pi)$, and $\lambda_3 \equiv \langle 0|u_3|0\rangle$. Moreover, η-η' mixing also appears in their formula. With suitable values[4] for the various additional parameters their formula is shown to hold to a high precision (1–1.5%).

In this paper we present a formula relating electromagnetic mass differences of pseudoscalar mesons and baryons and show through its derivation that its physical content is partially the same as that which leads to the Coleman-Glashow formula. Our expression, unlike that of Ref. 2, involves only observable masses and is of the type called "hybrid" by Coleman and Glashow.[5] The formula reads

$$\frac{5(m_{K^0}^2 - m_{K^+}^2) + 2(m_{\pi^+}^2 - m_{\pi^0}^2)}{2m_K^2 + m_\eta^2 - 3m_\pi^2} = \frac{3}{2}\,\frac{m_{\Sigma^-} - m_{\Sigma^+}}{m_\Xi - m_N}, \tag{1}$$

and the high accuracy to which it holds (~2%) is most probably a manifestation of the physical assumptions necessary for its derivation (barring the possibility of an "accidental agreement," which we consider improbable).[6] Throughout this paper we use the common wording of "electromagnetic mass differences." Nonetheless, this is not meant to imply a commitment as to the origin of the mass differences within isotopic-spin multiplets, as our derivation cannot exclude the presence of an SU(2)-breaking interaction in addition to electromagnetism if it fulfills all our requirements.

We outline now the derivation in a way which makes transparent the similarity to the Coleman-Glashow formula for baryon electromagnetic mass differences. Let us express the Hamiltonian to first order in the SU(3)-breaking couplings (g_i) and the SU(2)-breaking couplings $(\alpha_i,\ \beta_i,\ \gamma_i)$ as follows:

$$H_{\rm SB}=g_8H^{(8)}_{(000)}+g_{27}H^{(27)}_{(000)}+\alpha_8\overline{H}^{(8)}_{(000)}+\alpha_{27}H^{(27)}_{(000)}+\beta_8\overline{H}^{(8)}_{(010)}+\beta_{27}\overline{H}^{(27)}_{(010)}+\gamma_{27}\overline{H}^{(27)}_{(020)}\,. \quad (2)$$

The baryon electromagnetic mass differences are given by

$$m_p-m_n=-(\tfrac{3}{10})^{1/2}\beta_8\overline{D}^8_S+(\tfrac{1}{6})^{1/2}\beta_8\overline{D}^8_A+(\tfrac{1}{5})^{1/2}\beta_{27}\overline{D}^{27}\,, \quad (3a)$$

$$m_{\Xi^-}-m_{\Xi^0}=-(\tfrac{3}{10})^{1/2}\beta_8\overline{D}^8_S-(\tfrac{1}{6})^{1/2}\beta_8\overline{D}^8_A+(\tfrac{1}{5})^{1/2}\beta_{27}\overline{D}^{27}\,, \quad (3b)$$

$$\sqrt{2}\,(m_{\Sigma^-}-m_{\Sigma^0})=-(\tfrac{1}{3})^{1/2}\beta_8\overline{D}^8_A+(\tfrac{3}{2})^{1/2}\gamma_{27}\overline{D}^{27}\,, \quad (3c)$$

$$\sqrt{2}\,(m_{\Sigma^+}-m_{\Sigma^0})=(\tfrac{1}{3})^{1/2}\beta_8\overline{D}^8_A+(\tfrac{3}{2})^{1/2}\gamma_{27}\overline{D}^{27}\,, \quad (3d)$$

where $\overline{D}^i$ are the appropriately normalized reduced matrix elements with the convention of de Swart,[8]

$$D^N_{S,A}\equiv\xi(\tfrac{1}{8}N)^{1/2}\langle 8\|N\|8\rangle_{S,A}\,. \quad (4)$$

The electromagnetic mass differences in the pseudoscalar-meson system are given similarly[9] by

$$m_{K^0}{}^2-m_{K^+}{}^2=(\tfrac{3}{10})^{1/2}\beta_8\overline{d}^8-(\tfrac{1}{5})^{1/2}\beta_{27}\overline{d}^{27}\,, \quad (5a)$$

$$m_{\pi^+}{}^2-m_{\pi^0}{}^2=(\tfrac{1}{2}\sqrt{3})\gamma_{27}\overline{d}^{27}\,. \quad (5b)$$

In order to derive the Coleman-Glashow formula we isolate the $\beta_8\overline{D}^8_A$ term in (3) and thus obtain (without any further assumption)

$$-(\tfrac{2}{3})^{1/2}\beta_8\overline{D}^8_A=m_{\Sigma^-}-m_{\Sigma^+}=m_n-m_p+m_{\Xi^-}-m_{\Xi^0}\,, \quad (6)$$

the accuracy of which is within the accuracy of the measured mass differences.

In order to isolate the isovector contribution to the electromagnetic mass differences of pseudoscalar mesons we relate γ_{27} to β_{27} by assuming U-spin invariance for the $\underline{27}$ component of the SU(2)-breaking interaction, thus requiring

$$\gamma_{27}=(\tfrac{5}{3})^{1/2}\beta_{27}\,. \quad (7)$$

Then

$$\beta_8\overline{d}^8=(\tfrac{10}{3})^{1/2}[(m_{K^0}{}^2-m_{K^+}{}^2)+(\tfrac{2}{5})(m_{\pi^+}{}^2-m_{\pi^0}{}^2)]\,. \quad (8)$$

A formula involving only meson masses is unobtainable and if we wish to relate (6) to (8) we must obviously make some assumption about $\overline{d}^8$ and $\overline{D}^8_A$. To this end, neglecting the weaker SU(2)-breaking interaction, we first derive

$$g_8D^8_A=\sqrt{2}\,(m_N-m_\Xi)\,, \quad (9a)$$

$$g_8d^8=(\tfrac{2}{5})^{1/2}(2m_K{}^2+m_\eta{}^2-3m_\pi{}^2)\,. \quad (9b)$$

Making now the crucial identification

$$\overline{D}^8_A=D^8_A\,,\quad \overline{d}^8=d^8\,, \quad (10)$$

we obtain from (6), (8), (9), and (10) our new formula (1). Using for the masses in the denominators of (1) the average multiplet masses, one finds remarkably

$$\begin{aligned}&\text{left-hand side of Eq. (1)}=3.07\times10^{-2}\,,\\&\text{right-hand side of Eq. (1)}=3.13\times10^{-2}\,.\end{aligned} \quad (11)$$

We should like now to put forward some comments on the implications of (1) and its derivation:

(1) Hybrid mass formulas have been obtained previously by Coleman and Glashow with their tadpole model.[5] The difference between their hybrid formula for electromagnetic mass differences [Eq. (10) of Ref. 5] and (1) is obvious. Coleman and Glashow do not have $\underline{27}$ contributions to the mass differences (in the pure tadpole model $m_{\pi^+}=m_{\pi^0}$) and therefore their formula cannot take into account the cancellation of the β_{27} and γ_{27} contributions evident in (8) and (1). Moreover, Eq. (10), which is necessary for deriving (1), requires tadpole-type universality for the specified transitions only.

(2) The requirement (10) is of relevance to the approach assuming[5,10] a universal tadpole to contribute to the isovector mass differences in K^+-K^0 and baryon mass differences. Our derivation can be interpreted as showing that only the f-type part of the isovector contribution to the baryon mass differences is universally related to the contribution to the K^+-K^0 mass difference. This is thus reflected in the limited success of the attempt[10] to "obtain" the tadpole contribution to K^+-K^0 from the baryon mass differences. It appears therefore that any future model for electromagnetic mass differences must embody the interesting realization of Eq. (10).

(3) Our mass formula implies that the cancellation of the β_{27} and γ_{27} contributions in (8) is quite accurate. This is a hint that models[11] which can account for the π^+-π^0 mass difference are also suitable for calculating the appropriate $\underline{27}$ part in the K^+-K^0 mass difference.[12]

(4) It is easy to trace now under which conditions the "bad" formula of Dashen[1] for pseudoscalar-meson mass differences, $m_{K^+}{}^2 - m_{K^0}{}^2 = m_{\pi^+}{}^2 - m_{\pi^0}{}^2$, would appear. A consistent treatment of the meson system to order e^2 should also take into account to this order the electromagnetic mixing in the states, i.e., η-π^0 mixing. Thus, a third equation in addition to (5a) and (5b) exists, namely,

$$(\tfrac{1}{5})^{1/2}\beta_8\bar{d}^8 + (\tfrac{3}{10})^{1/2}\beta_{27}\bar{d}^{27} = \sqrt{2}\,(\tan\phi)(m_\eta{}^2 - m_\pi{}^2)\,. \tag{5c}$$

A detailed perturbation treatment of the complete system is presented in Ref. 7. Here we only call attention to the fact that the neglect of mixing, i.e., putting $\phi = 0$, imposes the (undesirable) condition $\beta_8\bar{d}^8 = -(\tfrac{3}{2})^{1/2}\beta_{27}\bar{d}^{27}$, which when supplemented by Eq. (7) leads directly to Dashen's formula. This confirms Dashen's suspicion[1] that the "bad" formula is a result of neglecting nondiagonal terms.

(5) It is of obvious interest to speculate on the origin of Eq. (10). Firstly, (10) is typical of several classes of models like tadpole models, pole-dominance models [including the PCVC (partial conservation of vector current) formulation[6]]—which are known, however, to be of limited validity. Bypassing the possibility of a dynamic accident, a very appealing explanation for (10) would lie in a fundamental operator identity, namely $H^{(8)}$ and $\bar{H}^{(8)}$ belonging to the same octet. A realization could be with a current-current Hamiltonian, which can possibly include Ne'eman's fifth-interaction scheme.[13] Such a fundamental identification requires, however, also $D_S^8 = \bar{D}_S^8$ to hold, while the mass formula to which it leads [Eq. (4.3) of Ref. 7] fails by a factor of 2. One is thus led to contemplate a picture in which (10) supplemented by $D_s^8 = \bar{D}_s^8$ holds only at a certain level (possibly even with $D_s^8 = \bar{D}_s^8 = 0$). The latter equality then breaks down at the next level (higher-order terms?), while at the same time the equalities of (10) are practically unaffected. We have no clear idea at present how this can happen. Interestingly enough, D_s^8 and $\bar{D}_s^8$ are indeed smaller than D_A^8 and $\bar{D}_A^8$ by factors of 4 and 2.5, respectively, a hint in the direction mentioned here.

(6) Finally, we stress that our formula (1), unlike that of Ref. 2, contains only measurable quantities, and no assumption concerning η-η' mixing is necessary for its derivation. The accuracy of the formula is at the level at which higher-order contributions are expected. In this connection, it is worth emphasizing that the contribution of the $(\Delta\pi)^2$ term in the left-hand side of (1) is approximately 15% of the $(\Delta K)^2$ term, an order of magnitude larger than the precision of the formula itself. This fact underlines the unlikeliness of an accidental agreement.

Note added in proof. Radicati *et al.* [Phys. Rev. Lett. 14, 160 (1965) and Nuovo Cimento 37, 187 (1965)] have also derived hybrid mass formulas by using a perturbation formalism. Their basic assumptions and appropriately their mass formulas are generally different from ours. Their best mass formula, relating mass differences of mesons and baryons, is off experimentally by ~30%. Thus, it belongs to the domain in which $(\Delta\pi)^2$ mass differences are essentially negligible compared to $(\Delta K)^2$ mass differences [see comment (6) above].

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