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PHYSICAL REVIEW D VOLUME 8, NUMBER 2 15 JULY 1973

$\Delta I=\frac{1}{2}$ Rule in a Gauge Model*

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We present a formulation of the schizon scheme of Lee and Yang which is within the framework of a renormalizable gauge theory and which yields the full octet rule at the SU(3) level. The model contains six massive gauge fields, two charged and four neutral.

I. INTRODUCTION

The purpose of this note is to present an updated version of the schizon scheme devised by Lee and Yang[1] to incorporate the $\Delta I=\frac{1}{2}$ and $\Delta S\neq 2$ selection rules in the weak-interaction Lagrangian. We require that (a) the scheme be within the framework of a modern renormalizable gauge theory[2] and (b) that the $\Delta I=\frac{1}{2}$ rule be associated with an octet rule on the SU(3) level.[3] These requirements can be implemented by enlarging the gauge group U(1)⊗SU(2), as used in a recent note[4] (hereinafter called BZ) to a group U(1)⊗SU(2)⊗SU(2). The theory therefore contains three massive vector bosons in addition to the three in BZ, making a total of six in contrast to the four used by Lee and Yang. These extra bosons are all electrically neutral and their coupling to leptons vanishes automatically if one requires the separate conservation of electron number and muon number, again in contrast to the Lee-Yang theory in which the removal of unwanted neutral-lepton couplings could not be associated with any general principle. In the interest of simplicity we shall assume that the physical modes of the Higgs fields are so massive that their contribution to nonleptonic decay amplitudes can be safely ignored in comparison to gauge-field contributions; our model is, therefore, diametrically opposed to the model of Lee and Treiman[5,6] in which the $\Delta I = \frac{1}{2}$ rule emerges only if the Higgs-field contributions somehow overwhelm the gauge-field contributions. Also in the interest of simplicity, we shall assume in this note that CP is conserved.

We emphasize that our considerations are logically independent of, and without prejudice to, dynamical theories of octet enhancement. In the present state of our ignorance of hadron dynamics it seems prudent to keep all options open.

II. THE GAUGE GROUP

As in BZ, our model for hadron structure is the three-triplet model with fractional charges [the "red, white, and blue" quark (RWB) model]. In the present context, unlike the situation in BZ, the RWB model is the only available three-triplet model; our constructions do not go through for the Han-Nambu model. We label the quarks as $\mathcal{P}_i$, $\mathcal{N}_i$, λ_i $(i=1,2,3)$ and require that they transform according to the $(3, 3^*)$ representation of the group $SU(3)\otimes SU(3)'$; all hitherto known physical particles are presumed to be $SU(3)'$ singlets to an accuracy of about one part in 10^3.

The charge-raising weak current in BZ is

$$J_\rho^{(+)} = (g/\sqrt{2})\big([\bar{\mathcal{P}}_1\gamma_\rho\mathcal{N}_1(\theta) + \bar{\mathcal{P}}_2\gamma_\rho\lambda_1(\theta)]_L + (\bar{\mathcal{P}}_3\gamma_\rho\mathcal{N}_2 + \bar{\mathcal{P}}_2\gamma_\rho\lambda_3)_R + \{[\tfrac{1}{3}\bar{\nu}_e - (\sqrt{8}/3)\bar{E}_0]\gamma_\rho e^- + [\tfrac{1}{3}\bar{\nu}_\mu - (\sqrt{8}/3)\bar{M}_0]\gamma_\rho\mu^-\}_L + (\bar{E}_0\gamma_\rho e^- + \bar{M}_0\gamma_\rho\mu^-)_R\big)\,, \tag{1}$$

where L and R denote the left and right chiral projections, respectively, $\mathcal{N}_1(\theta) \equiv \mathcal{N}_1\cos\theta + \lambda_1\sin\theta$, $\lambda_1(\theta) \equiv -\mathcal{N}_1\sin\theta + \lambda_1\cos\theta$, θ being the Cabibbo angle, E_0 and M_0 are the neutral heavy leptons required to maintain universality and render the theory anomaly free. The neutral current in BZ is not relevant in the present context since it does not give rise to change of strangeness.

Assuming that strong interactions are gentle enough to permit us to contemplate the nonleptonic decay amplitudes in the local limit,[7] the effective nonleptonic decay Hamiltonian may be displayed in the form:

$$H_{\text{eff}} = (18G_F/\sqrt{2})\sin 2\theta(\bar{\mathcal{P}}_1\gamma_\rho\mathcal{P}_1 - \bar{\mathcal{P}}_2\gamma_\rho\mathcal{P}_2)_L \times (\bar{\mathcal{N}}_1\gamma^\rho\lambda_1 + \bar{\lambda}_1\gamma^\rho\mathcal{N}_1)_L\,. \tag{2}$$

In writing Eq. (2) we have dropped terms which do not transform as $SU(3)'$ singlets and used a Fierz transformation to effect the charge-retention ordering.

H_{eff} contains $\Delta I = \frac{1}{2}$ and octet terms as well as $\Delta I = \frac{3}{2}$ and 27-plet terms. In order to cancel the latter one must introduce at least two neutral bosons *with opposite CP properties*. (With only one neutral boson one can achieve the $\Delta I = \frac{1}{2}$ rule only at the expense of the $\Delta S \neq 2$ rule. Furthermore, cancellation of $\Delta S = 2$ transitions is possible only if the bosons have opposite CP properties; with identical CP properties, the $\Delta S = 2$ amplitudes generated by the two bosons would interfere constructively.) The simplest Lie group which provides us with bosons with the requisite properties is SU(2); accordingly we enlarge the $U(1)\otimes SU(2)_J$ gauge group used in BZ to $U(1)\otimes SU(2)_J\otimes SU(2)_K$, where we have introduced subscripts J and K to differentiate between the two SU(2) groups.

III. PARTICLE ASSIGNMENTS AND NONLEPTONIC HAMILTONIAN

We note first that since two of the gauge bosons furnished by $SU(2)_K$ are electrically neutral, the Lie algebra requires that all three be neutral.

The requirement that $SU(2)_K$ commute with $U(1)\otimes SU(2)_J$ forces us to assign leptons to this group in one of four ways:

(a) all singlets;

(b) right-handed leptons as singlets and left-handed leptons as doublets,

$$\begin{pmatrix} \frac{1}{3}\nu_e - \frac{\sqrt{8}}{3}E_0 \\ \frac{1}{3}\nu_\mu - \frac{\sqrt{8}}{3}M_0 \end{pmatrix}_L, \quad \begin{pmatrix} e^- \\ \mu^- \end{pmatrix}_L;$$

(c) left-handed leptons as singlets and right-handed leptons as doublets,

$$\begin{pmatrix} E_0 \\ M_0 \end{pmatrix}_R, \quad \begin{pmatrix} e^- \\ \mu^- \end{pmatrix}_R;$$

(d) left-handed leptons as in (b) and right-handed leptons as in (c).

It is evident that the principle of separate conservation of electronic and muonic numbers rules out all assignments except (a); the leptons therefore decouple completely.

In the hadronic sector, we note first that the quarks used to construct the J currents can also be used to construct K currents with $(\mathcal{P}_1, \mathcal{P}_2)_L$ and

$(\mathfrak{N}_1(\theta), \lambda_1(\theta))_L$ regarded as K-spin doublets. However the nonleptonic weak Hamiltonian which results from the self-coupling of these currents is *identical*, within a constant factor, to the Hamiltonian in Eq. (2); consequently, these assignments are useless for our purpose. We therefore construct the K spin out of the quarks $\mathfrak{N}_2$, $\mathfrak{N}_3$, λ_2, and λ_3. It is convenient to introduce the notation

$$A = \frac{1}{\sqrt{2}}(\mathfrak{N}_2 + \lambda_2), \quad B = \frac{1}{\sqrt{2}}(-\mathfrak{N}_2 + \lambda_2),$$
$$C = \frac{1}{\sqrt{2}}(\mathfrak{N}_3 + \lambda_3), \quad D = \frac{1}{\sqrt{2}}(-\mathfrak{N}_3 + \lambda_3). \tag{3}$$

By considering various possible assignments, subject to the proviso that the $\Delta S \neq 2$ rule be sacrosanct in leading order, we are led to the following identification (modulo trivial rotations):

K triplet,

$$\begin{pmatrix} A \\ \dfrac{-D+B}{\sqrt{2}} \\ C \end{pmatrix};$$

K singlet,

$$\frac{D+B}{\sqrt{2}}.$$

The relevant part of the resulting interaction may be written in the form

$$\mathfrak{L}_I = \tfrac{1}{2} g_K (\vec{\mathrm{K}}_\mu^{(1)} + \vec{\mathrm{K}}_\mu^{(2)}) \cdot \vec{\mathrm{X}}^\mu, \tag{4}$$

where $\vec{\mathrm{X}}^\mu$ are the $\mathrm{SU}(2)_K$ gauge fields and

$$\vec{\mathrm{K}}_\mu^{(1)} \equiv \begin{bmatrix} (\bar{\mathfrak{N}}_2\gamma_\mu\mathfrak{N}_3 + \bar{\mathfrak{N}}_3\gamma_\mu\mathfrak{N}_2)_L - (\mathfrak{N} \to \lambda) \\ (-i)(\bar{\mathfrak{N}}_2\gamma_\mu\mathfrak{N}_3 - \bar{\mathfrak{N}}_3\gamma_\mu\mathfrak{N}_2)_L - (\mathfrak{N} \to \lambda) \\ (\bar{\mathfrak{N}}_2\gamma_\mu\mathfrak{N}_2 - \bar{\mathfrak{N}}_3\gamma_\mu\mathfrak{N}_3)_L + (\mathfrak{N} \to \lambda) \end{bmatrix}, \tag{5}$$

$$\vec{\mathrm{K}}_\mu^{(2)} \equiv \begin{bmatrix} (-\bar{\mathfrak{N}}_2\gamma_\mu\mathfrak{N}_2 + \bar{\lambda}_2\gamma_\mu\lambda_2)_L + (2 \to 3) \\ (-i)(\bar{\mathfrak{N}}_2\gamma_\mu\lambda_2 - \bar{\lambda}_2\gamma_\mu\mathfrak{N}_2)_L - (2 \to 3) \\ (\bar{\mathfrak{N}}_2\gamma_\mu\lambda_2 + \bar{\lambda}_2\gamma_\mu\mathfrak{N}_2)_L - (2 \to 3) \end{bmatrix}. \tag{6}$$

(The splitting of the current into two pieces has no particular significance in the present context.)

It is evident that with judicious choice of parameters $[(g_K/m_{X_3})^2 = (g/m_W)^2 \sin\theta\cos\theta]$, the full weak Hamiltonian in the nonleptonic sector, and within the manifold of low-lying states, may be written in the form

$$H_{\text{eff}} = (18 G_F \sin 2\theta/\sqrt{2})[(\bar{\mathcal{P}}_1\gamma_\mu\mathcal{P}_1 + \bar{\mathfrak{N}}_1\gamma_\mu\mathfrak{N}_1 + \bar{\lambda}_1\gamma_\mu\lambda_1)_L - (1 \to 2)](\bar{\mathfrak{N}}_1\gamma^\mu\lambda_1 + \bar{\lambda}_1\gamma^\mu\mathfrak{N}_1)_L \tag{7}$$

which embodies the $\Delta I = \frac{1}{2}$ rule on the SU(2) level and the octet rule on the SU(3) level. In writing H_{eff} in the form (7) we have explicitly used the fact that all low-lying states belong *either* to the identity *or* the alternating representation of S_3', the group of permutations of the SU(3)$'$ indices 1, 2, and 3. In the SU(3)$'$ limit, the identity representation of S_3' corresponds to mesons, the alternating representation to baryons and transitions between the two are, of course, strictly forbidden.

IV. $\Delta S=2$ TRANSITIONS

Next, we consider $\Delta S = 2$ transitions in our model. Our construction guarantees the absence of these transitions in leading order provided the masses endowed to X_2^μ and X_3^μ by the Higgs mechanism are equal in the tree approximation. It is impossible to make any firm statements in higher orders because of the intractability of strong interactions. If one is willing to assume that strong-interaction effects will not have a dramatic effect on orders of magnitude,[8] an assumption that may well be quite wrong, one can estimate the contribution of one loop graphs to $\Delta S = 2$ amplitudes. These contributions are easily seen to fall into three classes, with amplitudes proportional to $G_F\alpha\delta(m_X^2\sin^2\theta/m_W^2)$, $G_F\alpha\delta\sin^2\theta$ and $G_F\alpha\sin^2\theta \times (m_X^2/m_W^2)$. Note that the last-mentioned contribution does not contain the SU(3)$'$ suppression factor δ; it may be necessary, therefore, to suppress this contribution by making the X particles rather light; $m_X \sim 1-2$ GeV. Since X_2^μ and X_3^μ decouple completely from leptons and couple with infinitesimal strength ($\sim 10^{-3}e$) to low-lying hadrons on their mass shells, they are hard to produce and hard to detect; endowing them with a low mass does not lead to any difficulty with present-day experiments. We hasten to emphasize, however, that this guess on X mass should be treated as pure speculation; as stated earlier, strong interactions could change the picture completely.

V. REMARKS

(i) While we have given our construction for the octet rule in the local limit, passage to this limit is not really necessary; if $m_X = m_W$ in the tree approximation, Eq. (7) can be trivially modified so as to be valid at arbitrary momentum transfers. The price one pays is a possible loss of control over one-loop $\Delta S = 2$ amplitudes, as discussed above.

(ii) The question may naturally be asked: How much faith should one put in a scheme in which even one mass or coupling constant has to be adjusted "by hand" to guarantee a selection rule? We have no satisfactory answer, but hazard the guess that the relationship imposed in our model may emerge naturally when our gauge group is embedded in a larger gauge group that might be *the* gauge group chosen by nature.

(iii) The theoretical and experimental possibilities suggested by very weakly coupled neutral bosons of relatively light mass, as well as the question of *CP* violation in this model will be discussed elsewhere.

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[6]For an alternate approach to the problem, see A. Pais, Phys. Rev. Lett. 29, 1712 (1972); 30, 114(E) (1973), and this issue, Phys. Rev. D 8, 625 (1973). Pais's work is based on an O(4) group and differs from ours in several respects; for example, it does not start from the Lee-Yang scheme, does not have an automatic decoupling mechanism for removal of unwanted neutral lepton couplings, does not involve a cancellation of 27-plet amplitudes generated by *charged and neutral* gauge fields and does not make use of either three-triplet models or the hadronic group $SU(3) \otimes SU(3)'$ or the gauge group $U(1) \otimes SU(2)_J \otimes SU(2)_K$.

[7]The local limit would clearly be meaningless if the decay amplitude received sizable contributions from hadronic intermediate states as massive as the intermediate vector bosons. (The intermediate states of concern to us here are of the class-II type, in the usual terminology.)

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PHYSICAL REVIEW D VOLUME 8, NUMBER 2 15 JULY 1973

Further Investigations on the Consistency Problem with Spin-3/2 Interactions*

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Kimel and Nath's work on quantization of the charged spin-$\frac{3}{2}$ field is extended to fourth order in the charge. The anticommutator is found to agree with that of Johnson and Sudarshan to this order. It is inferred that Kimel and Nath's quantization is also inconsistent.

After Pauli and Fierz's pioneer work, the massive higher-spin field interacting with the electromagnetic field has been discussed repeatedly.[1-5] In particular, Johnson and Sudarshan[2] pointed out that the quantization of the Rarita-Schwinger spin-$\frac{3}{2}$ field interacting with the external electromagnetic field is inconsistent when it is quantized in terms of Schwinger's action principle. It was not clear whether the inconsistency is inherent in Schwinger's action principle or stems from the peculiar propagation character of the fundamental field equation.[3] Recently, Kimel and Nath[4] reexamined the problem with a quantized electromagnetic field by using the Yang-Feldman formalism, and claim that the quantization can be carried out consistently to second order (at least) *provided* that Q^{-1} (the operator which relates the Heisenberg field to the asymptotic fields) exists and can be expanded in a power series.

We would like to point out in this note that the result of Kimel and Nath concerning the anticommutation relation of the spin-$\frac{3}{2}$ Heisenberg field *agrees* with that of Johnson and Sudarshan, when expanded in powers of the coupling constant (at least up to fourth order in the charge). Hence, the inconsistency anticipated by Kimel and Nath [see the conclusion of Ref. 4; we quote: "Internal inconsistency can arise, within the Yang-Feldman