Li, Phys. Rev. D 7, 3815 (1973).
[28]The gauge used here was introduced by K. Fujikawa, B. W. Lee, and A. I. Sanda, Phys. Rev. D 6, 2923 (1972). Also see Y.-P. Yao, Phys. Rev. D 7, 1647 (1973); G. 't Hooft and M. Veltman, Nucl. Phys. B50, 318 (1972); T. Appelquist, J. Carazzone, T. Goldman, and H. R. Quinn, Phys. Rev. D, to be published. The Feynman rules for general theories are derived, in notation used here, in Appendix A of Ref. 5.
[29]B. W. Lee, Nucl. Phys. B9, 649 (1969); J.-L. Gervais and B. W. Lee, Nucl. Phys. B12, 627 (1969).
[30]S. Weinberg, Phys. Rev. 118, 838 (1960). A more readable exposition is given by J. D. Bjorken and S. D. Drell, *Relativistic Quantum Fields* (McGraw-Hill, New York, 1965), Sections 19.10, 19.11, and 19.14.
[31]K. Wilson, unpublished report; and Ref. 24. Also see W. Zimmermann, in *Lectures on Elementary Particles and Quantum Field Theory* (M.I.T. Press, Cambridge, Mass., 1970). The relation between the operator-product expansion and the bridge analysis of Ref. 30 was pointed out by C. Callan, Phys. Rev. D 5, 3202 (1972).
[32]These singularities were brought to my attention by R. Jackiw and K. Johnson.
[33]M. Gell-Mann and F. E. Low, Phys. Rev. 95, 1300 (1954). Also see N. N. Bogoliubov and D. V. Shirkov, *Introduction to the Theory of Quantized Fields* (Interscience, New York, 1959).
[34]The possibility of "superheavy" fermions is mentioned in Ref. 1.
[35]An example of "type 2" zeroth-order symmetry which does not depend on the quartic nature of the polynomial is given in Ref. 3. The fact that $T1$ contributions to δm vanish in such cases was discovered in the course of calculations by A. Duncan and P. Schattner.
[36]H. Georgi and S. L. Glashow, Phys. Rev. Lett. 26, 1494 (1972).

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A Class of Gauge Theories with Superweak CP Violation*

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A class of gauge theories is discussed in which the $\Delta S = 0$ and the $\Delta S = 1$ semileptonic decays are mediated by distinct intermediate bosons, whose mass ratio is related to the Cabibbo angle θ. *Common* features of the models are as follows: θ is well defined only as the result of spontaneous symmetry breaking; μ decay and the semileptonic $\Delta S = 0$, $\Delta S = 1$ decays are in the ratio $1 : \cos^2\theta : \sin^2\theta$ only if CP is maximally violated in the lepton sector; a breakdown of μe universality related to CP violation; a superweak impact of CP violation on K decays; the mediation through a neutral vector boson of nonleptonic decays which obey $|\Delta I| = \frac{1}{2}$; and an amplitude $\ll O(G)$ for $\nu_\mu e$ scattering. Two *distinct* types of theories are discussed in detail. (a) The gauge group O(4) reported before. Here the CP-violating parameter needs a renormalization. To $O(G)$, ν_μ-nucleon reactions are possible only if a heavy lepton is produced. (b) $O(4) \times \mathcal{G}$, where left- [right-] handed fermions are in O(4) [$\mathcal{G}$] but scalar with respect to $\mathcal{G}$ [O(4)]. Here the CP-violating parameter can be made finite if a constraint between electron and muon multiplets is satisfied. Further consequences for case (b) are: the $\bar{\nu}_e$-e and ν_e-e elastic cross sections are $(1 + \sin 2|\theta|)$ times their respective $V - A$ values, and ν_μ-nucleon reactions are possible with or without production of a heavy lepton. However, the final hadronic state is necessarily "charmed." The example $\mathcal{G} = U(1)$ is discussed in detail. The role of discrete symmetries is emphasized.

I. INTRODUCTION

A surge of theoretical activity has been generated by the discovery of a new class of renormalizable theories in which the notion of spontaneous breakdown of a local symmetry plays a key role. This development opens the strongly attractive prospect of unifying weak interactions with electromagnetism. Current investigations are proceeding on two main fronts. First and foremost, work is going on to clarify further some difficult and obscure technical aspects of this new renormalization program. Secondly (and hopefully not too early) attempts are under way to close in on the local symmetry that is chosen by nature, and on the representations of the symmetry to be assigned to the particles.[1]

Features common to all these investigations are (1) the occurrence of a number of vector mesons with masses that appear to hover invariably in a region well over 10 GeV/c^2, (2) the appearance of scalar mesons mainly needed for the mass generation of particles by the Higgs mechanism,[2] but about whose own mass relatively little can be said, (3) the insight that the field-theoretical stability of the phenomenological weak-interaction selection rules and the classification of weak processes in "allowed" and "forbidden" categories[3] is a consequence of the fact that the fine-structure constant α is small. Beyond these traits every single proposal has its own much more specific properties which embody one or more of the following novelties: (1) the occurrence of "new currents," (2) the occurrence of heavy leptons, (3) the occurrence of new hadronic states which will demand an extension of the generally accepted internal- (global) symmetry quantum-number labeling of hadrons. All this taken together provides a rich diet, perhaps too rich. Yet it is hard to escape the impression that, if the whole strategy makes sense, some new experimental insights ought to be beckoning of a kind stranger than what has been seen so far.

A condition most commonly imposed is that the theory shall contain an interaction of the form

$$if[\bar{\mathcal{P}}\gamma_\mu(1+\gamma_5)\mathfrak{N}_C+\bar{\nu}_e\gamma_\mu(1+\gamma_5)e+\bar{\nu}_\mu\gamma_\mu(1+\gamma_5)\mu+\cdots]W_\mu+\text{H.c.}, \qquad (1.1)$$

where W is a charged vector field with mass M, related to the Fermi constant G and to f by $G=f^2M^{-2}\sqrt{2}$, and where $\mathfrak{N}_C=\mathfrak{N}\cos\theta+\lambda\sin\theta$. $\mathcal{P}$, $\mathfrak{N}$, λ are the conventional quark fields, θ is the Cabibbo angle. (In what follows, θ will actually denote the absolute value of this angle.) Here the assumption is made that (at least to a good approximation) one and the same θ appears in both the V and the A part of the interaction. This assumption is also made in what is to follow. The $\cdots$ in Eq. (1.1) stands for contributions due to less-conventional particles, as they may appear. If Eq. (1.1) is adopted, then one task consists in finding representations for the group at hand which at least contain the left-handed parts of $\mathfrak{N}_C$ and of $\lambda_C=\mathfrak{N}\sin\theta-\lambda\cos\theta$ as members. Thus θ enters the theory via the states, so that this angle already appears before the spontaneous-symmetry mechanism sets in or, as we shall say hereafter, "in the symmetry limit." In such a strategy, θ plays the role of a variational parameter, the value of which is to be fixed or at least delimited by some physical constraints.[4] This is somewhat in contrast to a strategy which has been pursued starting from the different direction of (global) hadronic-type symmetries namely that θ is defined only after such symmetries are broken.[5] This is not to say that this contrast is necessarily a conflict. However, it led me to ask the question: Is it likewise possible to express θ in terms of the parameters of the spontaneous-breakdown mechanism, hence that θ does not enter via the states? It was in the study of this question that I came upon a possible view on the origin of CP violation. It is the purpose of this paper to report on this work in detail. A communication of these attempts has already appeared.[6] The present paper is an elaboration and also an extension of the results quoted earlier, in that not only the gauge group O(4) but also certain extensions thereof (already briefly mentioned at the end of PRL) are treated.

The starting point consists in an attempt to take an alternative road to the ones which lead to Eq. (1.1), namely to see what happens if Eq. (1.1) is replaced by

$$if_1[\bar{\mathcal{P}}\gamma_\mu(1+\gamma_5)\mathfrak{N}+\bar{\nu}_e\gamma_\mu(1+\gamma_5)e+\bar{\nu}_\mu\gamma_\mu(1+\gamma_5)\mu+\cdots]W^1 +if_2\{\bar{\mathcal{P}}\gamma_\mu(1+\gamma_5)\lambda+a[e^{i\phi}\bar{\nu}_e\gamma_\mu(1+\gamma_5)e+e^{i\psi}\bar{\nu}_\mu\gamma_\mu(1+\gamma_5)\mu]+\cdots\}W^2+\text{H.c.}, \qquad (1.2)$$

where W^1, W^2 are a pair of charged vector mesons and f_1, f_2 a pair of semiweak coupling constants. Clearly, Eq. (1.2) satisfies μe universality to $O(G)$. The extra parameters[7] a, ϕ, ψ are introduced in order to satisfy the "Cabibbo condition"

$$|\mathfrak{M}_\mu|:|\mathfrak{M}_\mathcal{P}|:|\mathfrak{M}_\lambda|=1:\cos\theta:\sin\theta, \qquad (1.3)$$

where the $\mathfrak{M}$'s in order of appearance denote the matrix elements for μ decay and $\mathfrak{N}\to\mathcal{P}$ and $\lambda\to\mathcal{P}$ β decays. Equation (1.3) implies that

$$(1-a^2)\tan\theta=2a\cos(\psi-\phi), \qquad (1.4)$$

where

$$\tan\theta=\frac{f_2^2}{f_1^2}\frac{M_1^2}{M_2^2}. \qquad (1.5)$$

One can look upon Eq. (1.4) in essentially two distinct ways. If $a\neq1$, it can be considered as an equation for θ in terms of a, ψ, and ϕ. In fact for $a\neq1$ there would be no need to introduce ψ and ϕ at all. Note that for $\psi=\phi=0$ one has a CP- and T-conserving scheme[8]; and μe universality holds strictly. If, on the other hand $a=1$, then Eq. (1.4) demands that $\psi=\phi\pm\frac{1}{2}\pi$ and we have a CP- and T-violating scheme. As will be explained in detail in this paper, for all semileptonic and nonleptonic processes CP remains strictly conserved to all orders in the strong and electromagnetic interactions and to $O(G)$ in the weak interactions. Also for purely leptonic processes CP remains conserved to all electromagnetic and strong orders and to $O(G)$ in the weak interactions as long as (momentum transfers)^2 and (lepton masses)^2 are neglected relative to $M_1{}^2$ and $M_2{}^2$. But to higher order in the weak interactions *CP* breaks down. It will also be shown that *CP* violation delimits the validity of μe universality.

It is this $a=1$ version which will be pursued here. It should be stressed that, from the point of view of a gauge theory, a is in essence a Clebsch-Gordan coefficient, so that once a group is picked and representations are chosen, the quantity a is fixed. However, before entering into such group considerations we must pause and ask: Whatever happened to nonleptonic decays? To the usual g^2 order Eq. (1.2) does not allow such processes at all.

Let us digress briefly on the odd position which nonleptonic decays have occupied throughout in the current-current picture. As is familiar, Eq. (1.1) implies on the face of it a large $|\Delta I|=\frac{3}{2}$ component along with the desired $|\Delta I|=\frac{1}{2}$ part. Attempts to ameliorate this by adding suitably adjusted neutral currents to Eq. (1.1) run into problems which are again familiar. As an alternative, it has been argued that a hadronic "octet dominance" mechanism may relatively suppress $|\Delta I|=\frac{3}{2}$ contributions. There is no stringent argument against this assumption. However, this author, for one, believes that an important clue may be missed by adopting this device, especially because of a fairly satisfactory understanding of the deviations of $|\Delta I|=\frac{1}{2}$ which does not appear to necessitate to any appreciable extent the $|\Delta I|=\frac{3}{2}$ mechanism just mentioned. This situation has been reviewed recently by Lee and Treiman[9] who make a new suggestion in this regard which may be denoted as "scalar-field dominance." In the context of a specific model, they demonstrate the possibility of having $|\Delta I|=\frac{1}{2}$ dominantly mediated by the scalar fields which are inherent in this gauge theory. The dominance would emerge due to a favorably chosen scalar to vector-boson mass ratio.

Not to change the subject, a look at Eq. (1.2) shows that one needs at least four gauge fields. The electromagnetic field makes five which is not a pretty number from the point of view of groups. So I decided to begin with six and study O(4). This introduces a sixth vector meson which is neutral and thus coupled to a neutral current. It was an encouragement to find that this coupling precisely mediates nonleptonic decays to order G. The coupling is $V-A$. The quark structure of the familiar hadrons is such that the interaction is $\Delta I=\frac{1}{2}$; see Sec. IV and Appendix C.

The three main and interlocking themes of the present study have now been indicated: The origin of θ is directly linked to spontaneous-symmetry breaking; the dynamical origin of *CP* violation is directly associated with the leptonic sector (supported, as we shall see, by an appropriate Higgs system); and nonleptonic decays have, to leading order, nothing to do with charge-raising or -lowering currents. In all three respects this approach differs from other gauge theories proposed so far. The smallest gauge group I know of which implements these ideas is O(4). However, O(4) is not unique in this respect. As will be developed in this paper, there is a *class* of gauge theories which can serve this purpose. The class as a whole has a common trait in the way the left-handed fermions are treated. This is why Secs. III and IV deal with these states separately. The treatment of the right-handed fermion states (and therefore of the Higgs system) is distinct from case to case. On the phenomenological level, the cases differ in their predictions concerning neutral-current events.

I shall now outline the plan of this paper on the more technical level. In Sec. II the general aspects of the gauge group O(4) are discussed. The covariant derivative is given by ($\partial_\mu=\partial/\partial x_\mu$)

$$D_\mu=\partial_\mu-i(g_1\vec{A}_\mu\cdot\vec{t}+g_2\vec{C}_\mu\cdot\vec{p}) . \tag{1.6}$$

$\vec{A}_\mu, \vec{C}_\mu$ are the six Hermitian gauge fields. The $\vec{t}, \vec{p}$ satisfy

$$\vec{t}\times\vec{t}=i\vec{t}, \quad \vec{p}\times\vec{p}=i\vec{p}, \quad [\vec{t},\vec{p}]=0 \tag{1.7}$$

characteristic for SU(2)×SU(2). More specifically we shall take

$$g_1=g_2\equiv g, \tag{1.8}$$

so that D_μ is also invariant under the reflection operation $R: \vec{A}_\mu\leftrightarrow\vec{C}_\mu$. The discussion of spontaneous breakdown is begun in Sec. II and it is shown that

$$\tan\theta=\frac{M_1{}^2}{M_2{}^2} , \tag{1.9}$$

corresponding to Eq. (1.5) with $f_2=f_1$. Since the vector mesons are massless in the symmetry limit, and acquire mass due to the spontaneous-breakdown mechanism, it is clear from Eq. (1.9) that θ is not defined in the symmetry limit but only becomes meaningful after the symmetry is spontaneously broken. It is noted that O(4) contains a second mass ratio:

$$\xi=\frac{M_0{}^2}{M_1{}^2+M_2{}^2} , \tag{1.10}$$

where M_0 is the mass of the neutral heavy vector meson which appears in the model. It is further shown that there are two versions of O(4) depending on whether W^1 is R-odd or R-even and that this distinction is not manifest in the structure of the currents but affects the expression of θ in terms of the symmetry-breaking parameters.

In Sec. III the left-handed lepton multiplets are discussed. These are two quartets each with electric charge (+, 0, 0, −). Two pairs of heavy leptons appear, one (x^+, x^0) of the e type and one (y^+, y^0) of the μ type. It is explained that the possibility of having CP phases enter the theory [as exemplified in Eq. (1.2)] is intimately related to the fact that the representations chosen contain two particles with the same electric charge.[10] It is shown that these phases can be rigorously transformed away from all vector currents in the limit where all neutral lepton masses are set equal to zero. Thus there exists an intimate relation between CP violation and the mechanism of mass generation. It is then demonstrated that the impact of the CP violation on purely leptonic processes is superweak.

At this point it should be explained in what precise sense the term "superweak" is employed in this paper. Obviously it is an adaptation of its usage in neutral K decays[11] where we have learned[12] to distinguish, phenomenologically, two possible origins for CP violation in decays like $K_L \to 2\pi$, namely on-shell or $(\Delta S = 1)$ transition amplitude effects and off-shell or $(\Delta S = 2)$ mass mixing effects. In general, let $\mathfrak{a}$ be the CP-conserving amplitude for a weak process to the conventional leading order: $O(G)$. Let $\mathfrak{a}'$ be a small CP-violating amplitude contribution coherent with $\mathfrak{a}$. Then if

$$\mathfrak{a}' \ll O(G\alpha), \tag{1.11}$$

we shall call this CP-violating amplitude a superweak amplitude. As is well known,[12] if a theoretical scheme yields Eq. (1.11) for all on-shell $\Delta S = 1$ decay amplitudes—a view which is currently favored—then the CP-violating mechanism in K decays is to a high degree of approximation due to a $\Delta S = 2$ mass mixing term—which is what superweak means in practice. To return to Sec. III, Eq. (1.11) is shown to hold there for purely leptonic processes. With reference to Eq. (1.2), note that the smallness of CP violation has in no way necessitated the introduction of a minuscule new parameter, but is due to the fact that at one point the $\bar{\nu}_e e$ and the $\bar{\nu}_\mu \mu$ currents are maximally out of phase.

However, we shall encounter in Sec. III a subtle question of renormalization. As is already clear from the prototype Eq. (1.2), effective CP violation can only arise if the currents coupled to W^1 and W^2 interfere, in particular if the W^1, W^2 mesons are subject to mass mixing due to virtual lepton loops. This is a perturbative effect in a highly nondegenerate system. Two parameters appear, ρ and σ [defined by Eqs. (3.16) and (3.17) below], which respectively correspond to real and to imaginary mass mixing. We shall find (at this stage) that ρ is finite, σ logarithmically divergent. This raises two problems. (a) To understand "why" ρ is finite. Observe that such loop effects are potentially quadratically divergent. The general proof of finiteness of ρ is an application of "Weinberg's lemma."[13] Briefly, the point is this: First write down the most general Lagrangian (in the symmetry limit) compatible with the precepts of *strict* renormalizability (for the precise definition of which see Refs. 14, 15, and 16). Consider a quantity which is potentially divergent, such as a correction to a zeroth-order mass relation. Try to locate a counterterm in the Lagrangian which can act as a renormalizer if the potential divergence were an actual divergence. If such a counterterm does not exist then what is potentially divergent is actually convergent. This key remark is due to Weinberg[13] and was phrased more broadly by Lee.[1] With the help of this lemma the finiteness of ρ can be demonstrated. (b) The divergence of σ can be proved not to be incompatible with renormalizability because a counterterm can be located. The reason why σ, potentially quadratically divergent, is actually logarithmically divergent will be stated shortly. In PRL we proceeded by making the assumption that the renormalization of the logarithmic divergence will not change the order of magnitude of the effect, a type of reasoning with which we have lived for a long time for quantities like mass differences. Be that as it may, it is obvious that one would wish to have both ρ and σ finite. To this question we return in Sec. VII.

Meanwhile, we introduce the left-handed quarks in Sec. IV, again quartets, again two of them. It is shown that no $\Delta S = 1$ neutral currents appear and that $\Delta S = 2$ hadronic transitions and the amplitude for $K_L \to \bar{\mu}\mu$ are sufficiently suppressed. The semileptonic CP violations are shown to be superweak. The $|\Delta I| = \frac{1}{2}$ semileptonic decay mechanism is exhibited. As in other versions of eight-quark models, we assume that all charmed quarks are SU(3)-singlets. It can then be shown, further, that it is compatible with the "$\pi^0 \to 2\gamma$ condition"[17]

$$2\sum_h I_3^h (Q^h)^2 = 1, \tag{1.12}$$

where I_3^h is the isospin of a quark, Q^h its charge and the summation goes over all quarks which couple to π^0. In Sec. IV we also discuss the baryon model of Ref. 9 and note that it does not lend itself readily to the interpretation of static SU(6) properties. However (see Appendix C) one may conceive of alternative baryon models within the eight-quark scheme in which SU(6) may become more transparent.

In Sec. V the assumption is examined that the right-handed fermions are O(4) representations of the triplet type. Here we encounter a tight connection between the choice of *CP* phases [like ϕ and ψ in Eq. (1.2)] and the structure of the neutral current. It is explained why the assumption that right-handed leptons are O(4) triplets leads one to choose what amounts to $\phi=0$, $\psi=\frac{1}{2}\pi$ in Eq. (1.2). The reason is that this forbids to $O(G)$ the reactions

$$\nu_\mu+e\to\nu_\mu+e\ . \tag{1.13}$$

Indications are that this reaction is indeed suppressed,[18] though it is much too early to say whether the amplitude is $O(G\alpha)$ as several models,[19,20] including the present one, imply. With this *CP*-phase choice, not only is $\nu_\mu e$ scattering suppressed but also $\nu_\mu+p\to\nu_\mu+p$ and $\nu_\mu+p\to\nu_\mu$ +anything (again as in certain other models[19,20]). The only neutral reaction which $\nu_\mu+p$ can make to $O(G)$ is $\nu_\mu+p\to y^0+\cdots$, that is (inelastic) y^0 production. ν_e-e scattering within O(4) is also discussed; see Eqs. (5.6), (5.7), (5.8), (5.20), and (5.21). In any event, what emerges is an intimate connection between the *CP* structure of the model and the magnitude of the neutral current processes.

A more complete discussion of ρ and σ is also given in this section. Here the importance of discrete symmetries begins to emerge. Three cases have to be distinguished. (a) The *bare*-coupling-constant relation $g_1=g_2$, Eq. (1.8), is part of a *strict R* invariance in the symmetry limit. Then ρ is finite. (b) $g_1=g_2$ but the R invariance is approximate in the sense that the scalar field couplings to fermions are not R-invariant. Then ρ is logarithmically divergent. The counterterm problem for this case is solved in Appendix A; here $g_1=g_2$ is a "zeroth-order coupling-constant relation" only. (c) $g_1\neq g_2$. Now ρ is quadratically divergent. (In all cases σ is logarithmically divergent.) All three schemes are renormalizable. But what begins to be evident is that the imposition of discrete symmetries can make one renormalizable theory more convergent than another. I believe that this phenomenon may well transcend the particular model discussed here. Section V concludes with the full description of the Higgs mechanism for O(4) and it is found that the scalar-field system needed is rather complicated (full details are given in Appendix B).

This concludes the discussion of O(4). In Sec. VI the following questions are raised. To what extent are the results obtained uniquely characteristic for O(4)? Is it possible to eliminate the divergence of the *CP* parameter σ if another group is used? These motivations are spelled out in more detail in Sec. VIA. The starting point is the recognition that most of the results of physical interest hinge by and large only on the way in which left-handed fermions are treated, as was already stressed in PRL. New insights emerge if one extends O(4) to O(4)×$\mathcal{G}$ in such a way that all the representations for the left-handed fermions are taken over bodily from O(4), in the sense that they are scalar with respect to $\mathcal{G}$, while the right-handed fermions are scalar with respect to O(4) and are further distinguished by their assignments within $\mathcal{G}$.

A main reason why this affects the *CP* problem is, once more, the tight connection between the *CP* phases and the structure of neutral currents. We shall show that if O(4)×$\mathcal{G}$ is used it is no longer necessary to restrict oneself to the choice $\phi=0$, $\psi=\frac{1}{2}\pi$, in the language of Eq. (1.2), while yet one can maintain the forbiddenness of the reaction Eq. (1.13) (supposing this to be desirable). This opens a new view on the *CP* problem.

The rest of this paper is devoted to a special *example* of the choice of $\mathcal{G}$, namely $\mathcal{G}=$ U(1). Section VIB is devoted to the description of O(4) ×U(1). Left-handed fermions as well as all spin-0 particles emerge as quartets. A further neutral current appears but one which is "neutrino free." No right-handed neutral fermions of any kind appear in any current. The Higgs mechanism is relatively simple (perhaps too simple). It is observed that any gauge theory of the type O(4)×$\mathcal{G}$ contains a parameter-free prediction for the cross sections $\sigma(\bar\nu_e e)$, $\sigma(\nu_e e)$ for elastic $\bar\nu_e e$ and $\nu_e e$ scattering:

$$\sigma(\bar\nu_e e)=\frac{4}{3}\,\frac{G^2m(e)}{\pi}E_\nu(1+\sin2\theta)\,, \tag{1.14}$$

$$\sigma(\nu_e e)=3\sigma(\bar\nu_e e)\,, \tag{1.15}$$

where E_ν is the neutrino lab energy in GeV. These are $(1+\sin2\theta)\approx1.4$ times the respective $V-A$ values.

Section VII contains the general parametrization of the *CP* problem. It is shown that even for general ϕ,ψ [in the language of Eq. (1.2)] the *CP*-mixing parameter σ cannot be finite *unless* there exists a constraint between the electronic and muonic multiplets. This constraint is given for the $O(\alpha)$ contributions to the mixing. From this second-order calculation a condition for finiteness of σ is abstracted which is then studied further on its implications to all orders. The condition reads

$$m(x^0)=m(y^0)\,, \tag{1.16}$$

and it is shown to follow that then [always in the language of Eq. (1.2)] $\phi=45°$, $\psi=-45°$. A consequence of these values for ϕ, ψ turns out to be that *every* current appearing in the theory *separately* satisfies μe universality, yet that this universality property breaks down when the currents interfere, that is to $O(\alpha^2)$. The validity of Eq. (1.16) would imply that there is not just the familiar single mass degeneracy between electron and muon multiplets, the *simultaneous* vanishing of their neutrino members; but a double mass degeneracy namely the mass equality of their respective neutral members. The finite expressions for ρ and σ are given in Eqs. (7.23) and (7.24).

In this way of dealing with CP, the neutral current which induces neutrino reactions gives rise to *inelastic* processes of the type

$$\nu_\mu + \text{nucleon} \to \nu_\mu + \text{hadrons}. \qquad (1.17)$$

The reactions (1.17) have to be inelastic since the hadrons can only be produced if a change of "charm" takes place. The hadronic term in the current which conspires with the $\bar{\nu}_\mu \nu_\mu$ term to give the reactions (1.17) to $O(G)$ is precisely the one responsible for the $|\Delta I| = \frac{1}{2}$ decay mechanism. Another class of inelastic reactions is ν_μ + nucleon $\to y^0$ + hadrons. Again the final hadron system must carry charm.

Section VII concludes with the discussion of the question: Can the CP mechanism studied here account quantitatively for the values of the CP parameters in K_L decay? Of course by the very nature of the mechanism, $|\Delta I| = \frac{1}{2}$ for $K_L \to 2\pi$ is implied. However it is altogether another and dynamically very complicated matter to show that a quantity like η_{+-} has the right value. A very crude argument is presented which shows that, at least, the required magnitude does not appear to do any violence to the orders of magnitude of the parameters in the theory.

The attempt made here to incorporate superweak CP violation in gauge theories demands a close interweaving of arguments which stem from phenomenology, from field theory (especially renormalization questions) and from group theory (especially discrete symmetries). It may be helpful to conclude the Introduction by providing a brief road map of what follows. Since the groups considered all have a common quartet structure for left-handed fermions, the latter are treated separately (Secs. III and IV). The union with right-handed fermions leads to their assignments as triplets in O(4) (Sec. V), as singlets in O(4) $\times$U(1) (Sec. VI), in each case with distinct properties of the scalar field system. The main reasons for presenting a class of models rather than a single one are threefold: (1) to exhibit the renormalization problem attendant on particle mixing, which in these models is at the root of CP violation (Secs. III and VII); (2) to exhibit the connection between CP and the structure of neutral currents (Secs. V and VI); (3) to exemplify the role of discrete symmetries (Secs. V and VII).

As a further guide to the two groups studied in detail in this paper, the following equations provide the principal tools.

(a) Group O(4). The representations are: left-handed leptons, Eqs. (3.1)–(3.4); left-handed quarks, Eqs. (4.1) and (4.2); right-handed leptons, Eq. (5.1); right-handed quarks, Eq. (5.9). The currents $J^{(1)}, J^{(2)}, J^{(0)}$ coupled respectively to W^1, W^2, Z are each divided in four parts. These parts are found in Eqs. (3.5), (4.3), (5.2), and (5.10) for $J^{(1)}$; in Eqs. (3.6), (4.4), (5.3), and (5.10) for $J^{(2)}$; in Eqs. (3.7), (4.6), (5.4), and (5.10) for $J^{(0)}$.

(b) Group O(4) × U(1). The representations are: left-handed leptons, Eqs. (3.1), (3.2), (7.7), (7.8), (7.9), and (7.19); left-handed quarks, Eqs. (4.1) and (4.2). The right-handed fermions are scalars. The currents $J^{(1)}$, $J^{(2)}$, $J^{(0)}$, and $J^{(v)}$ coupled respectively to W^1, W^2, Z and a further neutral vector meson V are: for $J^{(1)}$, Eqs. (6.22) and (7.10); for $J^{(2)}$, Eqs. (6.23) and (7.20); for $J^{(0)}$, Eqs. (6.24) and (7.22); for $J^{(v)}$, Eq. (6.20).

The models presented here have the following physical characteristics:

(1) No $\Delta S = 1$ neutral currents (Sec. IV).

(2) No unwanted magnitude for $\Delta S = 2$ amplitudes via 2-meson exchange (Sec. IV).

(3) Suppression of $K_L \to \mu\bar{\mu}$ (Sec. IV).

(4) A superweak CP-violating mechanism (leptonic, Sec. III; semileptonic, Sec. IV; nonleptonic, Sec. VII).

(5) A $|\Delta I| = \frac{1}{2}$ nonleptonic mechanism without need for $|\Delta I| = \frac{3}{2}$ suppression in charge-changing currents (Sec. IV).

(6) The $\pi^0 \to 2\gamma$ condition can be satisfied (Sec. IV and Appendix C).

(7) A prediction for elastic $\nu_e e$ scattering to $O(G)$: an inequality for O(4) (Sec. V), a parameter free equality for O(4)×$\mathcal{G}$ (Sec. VII).

(8) A suppression of $\nu_\mu e$ scattering (Secs. V and VII).

(9) A suppression for $\nu_\mu + p \to \nu_\mu + p$ or $\nu_\mu + \Delta$. Inelastic reactions are allowed to $O(G)$, with y^0 production for O(4) (Sec. V); with charmed hadron production for O(4)×$\mathcal{G}$, with or without y^0 production (Sec. VII).

(10) θ is defined only upon spontaneous symmetry breaking (Sec. II); so is CP violation (Sec. III).

(11) The models are anomaly-free[21] (Sec. VI).

(12) It is impossible to introduce in a consistent way maximal CP violation in the hadron sector, as was done here in the lepton sector (Sec. IV).

It may finally be noted that the groups O(4)×$\mathcal{G}$ are not exhausted by the choice $\mathcal{G}$ = U(1). As was stated in PRL, O(4)×O(4) is another candidate.

The only two morals I would like to draw from this investigation is that one should not settle for less than the inclusion in gauge models of all we know (or believe we know[1]) about the phenomenology of weak interactions at low energies; and that we are in crying need for more experimental information in the developing field of neutrino-induced reactions.

II. GENERAL FEATURES OF THE GAUGE GROUP O(4)

We start from Eqs. (1.6)–(1.8) and define the electric charge operator eQ by

$$Q=(t_3+\rho_3)\,. \tag{2.1}$$

It follows that e and the electromagnetic field A_μ are given by

$$g=e\sqrt{2}\,, \tag{2.2}$$

$$A_\mu=\frac{A_\mu^3+C_\mu^3}{\sqrt{2}}\,. \tag{2.3}$$

Remark: Quite generally, A_μ and e are fixed for any gauge group once D_μ and Q are specified, independently of the details of the spontaneous-symmetry breaking.[22] Let $D_\mu=\partial_\mu-i\sum_n g_n F_n A_\mu^n$, where the g_n are real constants, the F_n are the Hermitian group generators and the A_μ^n are the Hermitian gauge fields. Further, let $Q=\sum\alpha_n F_n$, where the α_n are real constants. Then

$$e=\Big(\sum_n \alpha_n{}^2 g_n{}^{-2}\Big)^{-1/2}\,, \tag{2.4}$$

$$A_\mu=e\sum_n \alpha_n g_n{}^{-1}A_\mu^n\,. \tag{2.5}$$

Equations (2.4) and (2.5) will again be used later on in Sec. VI B.

Next introduce the definitions

$$W_\mu^1=\tfrac{1}{2}[A_\mu^1-C_\mu^1-i(A_\mu^2-C_\mu^2)]\,, \tag{2.6}$$

$$W_\mu^2=\tfrac{1}{2}[A_\mu^1+C_\mu^1-i(A_\mu^2+C_\mu^2)]\,, \tag{2.7}$$

$$Z_\mu=\frac{A_\mu^3-C_\mu^3}{\sqrt{2}}\,. \tag{2.8}$$

The D_μ becomes

$$D_\mu=\partial_\mu-ieQA_\mu-\frac{ig}{\sqrt{2}}(t_3-\rho_3)Z_\mu - \tfrac{1}{2}ig[W_\mu^1(t_+-\rho_+)+W_\mu^2(t_++\rho_+)+\text{H.c.}]\,, \tag{2.9}$$

$$t_\pm=t_1\pm it_2\,,\quad \rho_\pm=\rho_1\pm i\rho_2\,. \tag{2.10}$$

This form of D_μ is given in anticipation of the introduction of a spontaneous-breakdown mechanism designed such that W^1, W^2, Z are normal modes with respective masses M_1, M_2, M_0 and so that at least $M_1\neq M_2$.

In order to start the description of this mechanism, consider a quartet H of scalar mesons which transforms as $(\frac{1}{2},\frac{1}{2})$ under O(4). H is short for (H_1,H_2,H_3,H_4). Here and below, quartet members are always ordered such that the respective charges are $(+,0,0,-)e$, corresponding to the respective eigenvalue pairs $(t_3,\rho_3)=(\frac{1}{2},\frac{1}{2})$, $(-\frac{1}{2},\frac{1}{2})$, $(\frac{1}{2},-\frac{1}{2})$, $(-\frac{1}{2},-\frac{1}{2})$. One has also to specify the Hermiticity properties of H. If H corresponds to a real representation, then it can be expressed in terms of four self-adjoint field operators ξ,η,ζ,χ. We work in the spherical basis:

$$H=(1/\sqrt{2})(\xi-i\eta,\zeta-i\chi,\zeta+i\chi,\xi+i\eta)\,. \tag{2.11}$$

Such an H will be called a real H. Let $(\xi',\eta',\zeta',\chi')$ be a second set of self-adjoint scalar field operators. Then if

$$H=\tfrac{1}{2}(\xi-i\eta+i(\xi'-i\eta'),\zeta-i\chi+i(\zeta'-i\chi'),\zeta+i\chi+i(\zeta'+i\chi'),\xi+i\eta+i(\xi'+i\eta'))\,, \tag{2.12}$$

H will be called a complex H. In the limit of unbroken symmetry, the real and imaginary parts of such an H transform separately as irreducible representations of O(4). [For scalar fields, the use of such complex representations of a local gauge group is of course quite familiar for the standard U(1) case of electromagnetism.]

Consider the coupling of a generally complex H to the vector fields, generated by the gauge-invariant expression[23] $-|D_\mu H|^2$. This expression contains the following terms, among others.

$$\mathfrak{L}_{00}=-\tfrac{1}{2}g^2Z_\mu{}^2(H_2^\dagger H_2+H_3^\dagger H_3)\,, \tag{2.13}$$

$$\mathfrak{L}_{11}=-\tfrac{1}{2}g^2W_\mu^{1\dagger}W_\mu^1[H_1^\dagger H_1+(H_2-H_3)^\dagger(H_2-H_3)+H_4^\dagger H_4]\,, \tag{2.14}$$

$$\mathfrak{L}_{22}=-\tfrac{1}{2}g^2W_\mu^{2\dagger}W_\mu^2[H_1^\dagger H_1+(H_2+H_3)^\dagger(H_2+H_3)+H_4^\dagger H_4]\,, \tag{2.15}$$

$$\mathfrak{L}_{12}=\tfrac{1}{2}g^2(W_\mu^{1\dagger}W_\mu^2-W_\mu^{2\dagger}W_\mu^1)(H_2^\dagger H_3-H_3^\dagger H_2)\,. \tag{2.16}$$

In the derivation of these expressions, the following representation for $\vec{t},\vec{\rho}$ has been adopted.

$$\vec{t}=\tfrac{1}{2}(\vec{\tau}\otimes 1)\,,\quad \vec{\rho}=\tfrac{1}{2}(1\otimes\vec{\tau})\,, \tag{2.17}$$

where $1,\vec{\tau}$ are Pauli matrices.

Consider now the asymmetric vacuum in which the neutral fields H_2, H_3 have nonzero vacuum expectation values $\langle H_2\rangle=a$, $\langle H_3\rangle=a'$. We denote such a quartet by $H(a,a')$. The contribution of these vacuum parts of H to $\mathfrak{L}_{00}, \mathfrak{L}_{11}, \mathfrak{L}_{22}$ yield the vector-meson mass terms. Since we are interested in W^1 and W^2 as zeroth-order particle states, it is necessary that the H-vacuum part of $\mathfrak{L}_{12}$ be zero. It suffices to have for each quartet separately that

$\langle H_2\rangle$, $\langle H_3\rangle$ are relatively real. (2.18)

In the discussion of Eq. (2.18) the following cases must be considered. (a) One single real H. Here axes may be chosen such that $\langle H_2\rangle$ is real, without loss of generality. With reference to Eq. (2.11) this corresponds to $\langle\zeta\rangle = a$, $\langle\chi\rangle = 0$. Equation (2.18) is now satisfied. However for a real H we have more specifically $\langle H_2\rangle = \langle H_3\rangle$, see Eq. (2.11). In the notation just introduced the quartet is $H(a, a)$. But then W^1 remains massless, according to Eq. (2.14): One $H(a, a)$ is not enough. (b) One complex quartet $H(a, a')$, $a \neq a'$ and a, a' real. This corresponds to $\langle\zeta\rangle = \frac{1}{2}(a+a')$, $\langle\chi'\rangle = \frac{1}{2}(a-a')$, see Eq. (2.12). Equation (2.18) is satisfied while, moreover, M_1 and M_2 are nonzero and distinct. However, it will turn out that a single complex H is inadequate for the treatment of fermion mass problems. For ease of presentation, the latter problem will be deferred to Sec. V for the group O(4) and to Secs. VI B and VII for O(4)×U(1). There it will become apparent that at least two H's will be necessary. In PRL I chose these[24] to be $H(a, a')$ (with $a = a'$ permitted) and $H(0, b)$. However there is a way which is more economical in the sense that fewer degrees of freedom are needed for the Higgs system. Namely take *two real* H's: $H(a, a)$ and $H(ib, -ib)$, a, b real. They each satisfy Eq. (2.18). After axes have been chosen such that a is real, the direction of the second H relative to the first has meaning (one H has $\langle\zeta\rangle \neq 0$, the other $\langle\chi\rangle \neq 0$).

It is instructive to discuss for a moment the *incomplete* expressions for M_1^2, M_2^2 and ξ, Eq. (1.10), which follow if the contributions of $H(a, a)$ and $H(ib, -ib)$ only are considered:

$$M_1^2 = 2g^2b^2, \tag{2.19}$$

$$M_2^2 = 2g^2a^2, \tag{2.20}$$

$$\tan\theta = \frac{b^2}{a^2}, \tag{2.21}$$

$$\xi = 1. \tag{2.22}$$

The purpose of displaying these expressions at this point is to make a comment on the structure of the coupling of the vector mesons to left-handed fermions. We shall use the symbol f^L to denote any left-handed fermion.

As was stated in Sec. I it will be assumed throughout that the f^L are quartets with respect to O(4), and scalar with respect to any possible extension $\mathcal{G}$. Let one such quartet be $(f_1, f_2, -f_3, -f_4)_L$. For a specific particle symbol a, the meaning of a_L is

$$a_L = \tfrac{1}{2}(1+\gamma_5)a. \tag{2.23}$$

Let $J^{(1)}, J^{(2)}, J^{(0)}$ be the currents coupled to W^1, W^2, Z, respectively. Here we suppress the four-vector subscripts μ for ease of writing. From Eq. (2.17) one finds that the f^L contribute

$$\text{to } J^{(1)}:\ -\tfrac{1}{2}ig[\bar f_1(f_2+f_3)+(\bar f_2+\bar f_3)f_4]_L, \tag{2.24}$$

$$\text{to } J^{(2)}:\ -\tfrac{1}{2}ig[\bar f_1(f_2-f_3)-(\bar f_2-\bar f_3)f_4]_L, \tag{2.25}$$

$$\text{to } J^{(0)}:\ -(ig/\sqrt{2})(\bar f_2 f_2-\bar f_3 f_3)_L, \tag{2.26}$$

with the following definition:

$$(\bar a b)_L \equiv \bar a_L\gamma_\mu b_L = \tfrac{1}{2}\bar a\gamma_\mu(1+\gamma_5)b. \tag{2.27}$$

In order that θ in Eq. (2.21) shall be the physical Cabibbo angle, it is of course necessary that the baryon multiplets are chosen such that $J^{(1)}$ shall contain $(\bar{\mathcal{P}}\mathfrak{N})_L$ and also $J^{(2)}$ contain $(\bar{\mathcal{P}}\lambda)_L$. In order that these respective currents shall correspond to the $\Delta S = 0$ and 1 transitions it is furthermore necessary that the asymmetry $m_{\mathfrak{N}} \neq m_\lambda$ be part of the scheme, since without this distinction one could exchange the symbols $\mathfrak{N}$ and λ so that the physical interpretation of θ would remain unspecified.

Let us refer to Eqs. (2.6), (2.7), and (2.21) as describing the "type I" version of O(4) in order to make a contradistinction with the "type II" version of O(4) in which the definitions for W^1 and W^2 are interchanged:

$$\left.\begin{aligned} W_\mu^1 &= \tfrac{1}{2}[A_\mu^1 + C_\mu^1 - i(A_\mu^2 + C_\mu^2)] \quad (2.28)\\ W_\mu^2 &= \tfrac{1}{2}[A_\mu^1 - C_\mu^1 - i(A_\mu^2 - C_\mu^2)] \quad (2.29)\end{aligned}\right\}\ \text{"type II"}.$$

with Eqs. (2.1)–(2.3) and (2.8) held fixed. Let us furthermore change the phase *convention* in the definition of the f^L quartet just introduced by replacing $(f_1, f_2, -f_3, -f_4)_L$ by $(f_1, f_2, f_3, f_4)_L$. Then the expressions (2.24)–(2.26) remain equally true for type II as they were for type I. Let us, on the other hand, *hold fixed* the vacuum parameters a, b. Then type I has the θ formula given in Eq. (2.21), but type II has

$$\tan\theta = a^2/b^2:\ \text{type II}, \tag{2.30}$$

since the alternative definitions Eqs. (2.28), (2.29) imply the interchange of the right-hand sides of Eqs. (2.14) and (2.15) [and a change of sign for the right-hand side of Eq. (2.16)].

This point is belabored here at some length in order to emphasize a feature common to all gauge theories but perhaps more pronounced in the present case, namely that the physical content of the theory is not defined until the symmetry breaking is fully specified. In the present instance, the meaning of θ is not defined unless and until one has ascertained that $m_{\mathfrak{N}}$ and m_λ are distinct. Moreover, the structure of the currents $J^{(1)}$ and $J^{(2)}$ by itself does not specify the value of θ; the latter is only fully determined in terms of the parameters of the theory if one simultaneously considers the fermion couplings to vector *and* to scalar mesons. For definiteness[25] we shall stick with the type I description throughout the sequel of this paper.

The distinction just mentioned is related to the behavior of the various quantities under the transformation R: $\vec{t} \to \vec{p}$. R is the reflection operation of the group O(4). Inasfar as it acts on a $(\frac{1}{2}, \frac{1}{2})$ representation, R is realized by

$$R = \frac{1 + 4\vec{t} \cdot \vec{p}}{2} . \quad (2.31)$$

Thus

$$R(f_1, f_2, f_3, f_4) = (f_1, f_3, f_2, f_4), \quad (2.32)$$

so that R interchanges the neutral members in a quartet. With reference to Eq. (2.11), R keeps ξ, ζ, η fixed but changes the sign of χ, which illustrates the interpretation of R as the parity operator in O(4). Returning for one last time to the distinction between types I and II, the vector mesons transform under R as

$$W^1 \to -W^1,\ W^2 \to W^2,\ Z \to -Z,\ A \to A: \text{ type I}, \quad (2.33)$$

$$W^1 \to W^1,\ W^2 \to -W^2,\ Z \to -Z,\ A \to A: \text{ type II}, \quad (2.34)$$

so that for type I $(W^2, W^{2\dagger}, A)$ are associated with the even-parity subalgebra O(3) of O(4), while for type II the association is with $(W^1, W^{1\dagger}, A)$.

In the following sections the contributions to the currents $J^{(0,1,2)}$ will be discussed. It is convenient to decompose the contributions to each of these currents as follows.

$$J = J^{lL} + J^{lR} + J^{hL} + J^{hR}, \quad (2.35)$$

where l (h) refer to leptonic (hadronic) contributions and $L(R)$ further subdivides each such contribution inasfar as it stems from left- (right-) handed states. We shall also use the notation f for any fermion, and f^{lL} for any left-handed lepton; and similarly for f^{lR}, f^{hL}, f^{hR}.

III. THE f^{lL}; A FIRST LOOK AT THE *CP* PROBLEM

The f^{lL} are assumed to belong to two quartets, one (E^L) of the electron type, the other (M^L) of the muon type:

$$E^L = (x^+, a_1 x^0 + a_2 \nu_e, a_2^* x^0 - a_1^* \nu_e, -e)_L, \quad (3.1)$$

$$M^L = (y^+, b_1 y^0 + b_2 \nu_\mu, b_2^* y^0 - b_1^* \nu_\mu, -\mu)_L. \quad (3.2)$$

Thus two pairs of heavy leptons (and their antiparticles) appear: x^+, x^0 with e number $=1$, μ number $=0$; and y^+, y^0 with e number $=0$, μ number $=1$. Proper normalization demands that the complex numbers a_1, a_2, b_1, b_2 satisfy

$$|a_1|^2 + |a_2|^2 = 1, \quad |b_1|^2 + |b_2|^2 = 1. \quad (3.3)$$

This general parametrization will be discussed in all detail in Sec. VII. However, in this section only the case

$$a_1 = 1, \quad a_2 = 0, \qquad b_1 = b_2 = e^{i\pi/4}/\sqrt{2} \quad (3.4)$$

will be considered. This was the special choice made in PRL. It will be explained in Sec. V why this choice seems to be the most appealing one on experimental grounds if the gauge group is O(4). On the other hand, if the gauge group is O(4)×$\mathcal{G}$ it will be seen in Sec. VI A that the motivation for the specialization to Eq. (3.4) no longer applies. This will then lead us to reconsider in Sec. VII the general case of Eqs. (3.1)–(3.3).

From Eqs. (2.24)–(2.26) and (3.1), (3.2), (3.4) it follows that

$$J^{(1)lL} = -\tfrac{1}{2} ig[\bar{x}^+(x^0 + \nu_e) + (\bar{\nu}_e + \bar{x}^0)e + \bar{y}^+(\nu_\mu + iy^0) + (\bar{\nu}_\mu - i\bar{y}^0)\mu]_L, \quad (3.5)$$

$$J^{(2)lL} = -\tfrac{1}{2} ig[\bar{x}^+(x^0 - \nu_e) + (\bar{\nu}_e - \bar{x}^0)e + \bar{y}^+(y^0 + i\nu_\mu) + (i\bar{\nu}_\mu - \bar{y}^0)\mu]_L, \quad (3.6)$$

$$J^{(0)lL} = -(ig/\sqrt{2})(\bar{\nu}_e \nu_e - \bar{x}^0 x^0 - \bar{\nu}_\mu y^0 - \bar{y}^0 \nu_\mu)_L . \quad (3.7)$$

The particular asymmetry of the E^L—as compared to the M^L—contributions in the neutral current is a consequence of the special choice Eq. (3.4). For example it will be evident that if we would interchange the values for (a_1, a_2) with those for (b_1, b_2) in Eq. (3.4), $J^{(0)lL}$ would contain $(\bar{\nu}_\mu \nu_\mu)_L$. This would then give rise to neutral current events induced by μ neutrinos of a kind which appears to be undesired, as will become more clear in Sec. V.

For low q^2, $J^{(1)lL}$ and $J^{(2)lL}$ contribute to the effective μ-decay interaction to $O(g^2)$:

$$(G/\sqrt{2})\bar{\mu}\gamma_\mu(1+\gamma_5)\nu_\mu \bar{\nu}_e \gamma_\mu(1+\gamma_5)e + \text{H.c.} \quad (3.8)$$

in such a way that

$$\frac{G}{\sqrt{2}} = \frac{g^2}{16}\left(\frac{1}{M_1^4} + \frac{1}{M_2^4}\right)^{1/2} \exp\left(-i \arctan\frac{M_1^2}{M_2^2}\right), \quad (3.9)$$

where the phase factor has been included for completeness, not for relevance. Thus it follows from Eq. (1.9) and from Eq. (2.2) that

$$M_1 = \frac{37.2}{\sqrt{\cos\theta}}, \quad M_2 = \frac{37.2}{\sqrt{\sin\theta}}, \quad (3.10)$$

in units GeV/c^2.

Let us next start the discussion of CP-violating effects. These are due, potentially, to the occurrence of the phase factors i in Eqs. (3.5) and (3.6). Note that such factors cannot be transformed away in general by redefining phases (rephasing) for lepton and/or for vector mesons. It is this impossibility of eliminating all CP- and T-violating phases by rephasing which is at the root of all violation effects of the kind which interest us here. However there are three special situations in which no CP violations are induced in spite of the occurrence of these phases.

(i) If to $O(G)$ the contributions from $J^{(1)}$ and from $J^{(2)}$ cannot interfere, then one can separately rephase $iy^0 \to y^0$ in Eq. (3.5) and $\nu_\mu \to i\nu_\mu$ in Eq. (3.6) and leave Eq. (3.7) as is. Then there is no CP violation to order G, and not only that. As will become clear in the sequel, the plan of the present work is to examine what happens, if *no other CP*-violating mechanism is introduced anywhere in the currents except for the one exhibited above in the lepton sector. Specifically, no further CP phases will be introduced in the hadronic contributions to the currents $J^{(0,1,2)}$ while, in addition, the strong and electromagnetic interactions will be assumed throughout to have the conventional C- and P-conserving properties. The argument just given can therefore be stated more generally as follows. To all orders in the strong and electromagnetic interactions and to $O(g^2)=O(G)$ in the weak interactions CP is conserved, where $J^{(1)}$ and $J^{(2)}$ do not interfere. We shall see that this situation applies to all semileptonic and nonleptonic processes, whether they are decays or neutrino induced reactions.

(ii) Consider higher-order corrections to the amplitude Eq. (3.8) of such a nature that G is corrected to $G(1+\epsilon)$. Here ϵ is at most of order g^2. ϵ may be complex, due to the occurrence of the CP phases. Even so, no CP violation is induced, the only result of the correction being to rescale G to $G|1+\epsilon|^{1/2}$. More generally, any over-all complex scale-factor correction to any scattering or decay amplitude is a CP-conserving correction.

(iii) Consider the limit in which the y^0 has zero mass, all other lepton masses being whatever they are [but of course $m(\nu_e)=m(\nu_\mu)=0$]. Then one can define new orthonormal particle states u, v:

$$u\sqrt{2}=\nu_\mu+iy^0, \quad v\sqrt{2}=y^0+i\nu_\mu . \tag{3.11}$$

In this u, v description, all CP violation vanishes everywhere; note that

$$\bar{\nu}_\mu y^0+\bar{y}^0\nu_\mu=\bar{v}u+\bar{u}v .$$

This is very useful if ν_μ, y^0 appear in *virtual* states. The three arguments just presented will serve as diagnostics for pinpointing where and how true CP violation can in fact appear.

Returning to μ decay, Eq. (3.8) stems from the sum of single W^1 and single W^2 exchange. Let us insert into these two graphs all possible virtual-photon corrections. This can lead to a CP-violating contribution to the amplitude at most of order $G\alpha m^2(\mu)M^{-2}$ ($M=M_1$ or M_2), which is a superweak order. Here argument (ii) is used combined with dimensional reasoning: If we neglect $m^2(\mu)$, $m^2(e)$, q^2 relative to M^2, then the M dependence of the resultant radiative corrections (*including* induced terms) is $(M_1^{-2}-iM_2^{-2})$ which gives them the same *over-all* phase as appears in Eq. (3.9).

The strategy to prove Eq. (1.11) for μ decay should now be clear. Any correction which maintains $V-A$ as in Eq. (3.8) also conserves CP, by (ii). (This includes all $V-A$ conserving renormalizations.) A CP-violating correction can at most appear if the $V-A$ ratio is unbalanced and/or if induced terms appear with other tensor structure. I have not found any contributions which do not satisfy Eq. (1.11). (Estimates are made most conveniently in the $\xi=1$ gauge.[1])

As a further example, consider the box graph displayed in Fig. 1. It contributes to μ decay; and with trivial changes, also to $\nu_\mu+e\to\nu_e+\mu$. The mechanism is a WZ exchange where $W=W^1$ or W^2. With the help of Eqs. (3.5)–(3.7) one sees that a factor i appears in the (W, y^0, μ) vertex if $W=W^2$. No such factor appears for W^1Z exchange.

In order to assess the order of magnitude of the CP violation one needs to evaluate the box graph. With minor modifications in the particle labels this graph has been analyzed in Refs. 19 and 20. (For consistency, the box graphs have to be treated jointly with the electromagnetic corrections graphs of the same order.) The important feature of the graph of Fig. 1 is that all four vertices are of the $\gamma_\mu(1+\gamma_5)$ type. The result of the authors is that the amplitude is again of the $V-A$ form as in Eq. (3.8) as long as

$$\frac{q^2}{M^2}\ll 1, \tag{3.12}$$

where M stands indifferently for either M_1 or M_2.

In the present context this means that there is

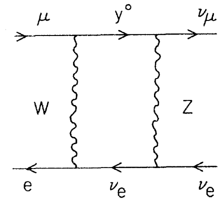

FIG. 1. Example of graph which induces true CP violation, but only for $q^2 \gtrsim M^2$.

effectively no CP-violation due to WZ exchange, in the domain given by Eq. (3.12). Here argument (ii) has been invoked.

The calculations of Refs. 19 and 20 were performed under the simplifying assumption that

$$\delta \equiv \frac{m_l}{M} \ll 1\,, \qquad (3.13)$$

where m_l denotes any lepton mass, and as before $M = M_1$ or M_2. Equation (3.13) is assumed in most papers on gauge models and will be adopted here as well.[26]

It is qualitatively evident that electromagnetic corrections to the box graph of Fig. 1 will in general unbalance the $V - A$ ratio and will also generate induced terms with other structures. Under such circumstances one will obtain CP-violating effects even in the q^2 domain given by Eq. (3.12). But these effects will at most be of order $G\alpha^2$, that is, in the nomenclature adopted in Sec. I, they are superweak.

While the restriction Eq. (3.12) will cover all of the accessible experimental domains for a long time to come, it is nevertheless conceptually interesting to note that CP-violating effects may turn out to be enhanced at extremely large momentum transfers.

At this point it should be emphasized that the argument just given for the magnitude of CP violation due to WZ exchange will have to be reconsidered when we also incorporate the f^R couplings to vector mesons. Indeed, contributions to such graphs as the one in Fig. 1 are not in general additive in the contributions due to left-handed and to right-handed fermions. The stepwise discussion of the problem will nevertheless turn out to be quite helpful, especially because it will serve to underline the distinctions between the gauge groups O(4) and $O(4)\times\mathcal{G}$ insofar as the CP discussion is concerned. These remarks apply equally to the CP effects due to W^1-W^2 mixing, to be considered next.

Due to virtual emission and absorption of lepton pairs, an effective interaction $\sum_{i,j} W_\mu^{\dagger i} W_\nu^j \theta_{\mu\nu}^{ij}$ +H.c. will be generated, where $i, j = 1, 2$ run over the labels which distinguish the two kinds of charged vector mesons.[27] To zeroth order, $\theta_{\mu\nu}^{ij}$ is of course diagonal in (i,j), due to the choice of the initial normal modes. Corrections to the diagonal elements $i=j$ correspond to mass and wave-function renormalization of the respective W fields. We are particularly interested in the off-diagonal elements which are generally of the form

$$\theta_{\mu\nu}^{12} = -i[A^{12}(q^2)(q^2\delta_{\mu\nu} - q_\mu q_\nu) + B^{12}(q^2)\delta_{\mu\nu}] \qquad (3.14)$$

and likewise for "21." The A's and B's are of leading order g^2. For the region $q^2 \ll M^2$ of prime interest we focus on the quantities $B^{12}(0)$ and $B^{21}(0)$ which give rise to an effective term $\mathcal{H}$ in the Hamiltonian:

$$\mathcal{H} = \mathcal{H}^{\rm re} + \mathcal{H}^{\rm im}\,, \qquad (3.15)$$

$$\mathcal{H}^{\rm re} = \rho(W_\mu^{\dagger 1} W_\mu^2 + W_\mu^{\dagger 2} W_\mu^1)\,, \qquad (3.16)$$

$$\mathcal{H}^{\rm im} = i\sigma(W_\mu^{\dagger 1} W_\mu^2 - W_\mu^{\dagger 2} W_\mu^1)\,, \qquad (3.17)$$

where ρ and σ are $O(g^2)$ and have dimensions $(\text{mass})^2$. $\mathcal{H}$ is the off-diagonal correction to the (W^1, W^2) mass matrix. Similar to the familiar 2×2 K-meson matrix, ρ (σ) correspond to CP-conserving (-violating) mixing. However, unlike what happens in the K-meson complex, the mass mixing occurs in a system which is strongly nondegenerate to zeroth order so that $\mathcal{H}$ may be treated as a small perturbation of the zeroth-order normal modes.

From Eqs. (3.5) and (3.6) one finds[28] for ρ and σ, to $O(g^2)$:

$$\rho = -\frac{\alpha}{2\pi} m^2(x^0)[m^2(x^+)\phi(x^0, x^+) - m^2(e)\phi(x^0, e)]\,, \qquad (3.18)$$

$$\sigma = -\frac{\alpha}{2\pi} m^2(y^0)\ln\Lambda^2\,, \qquad (3.19)$$

with the definition

$$\phi(a, b) = \frac{1}{m_a^2 - m_b^2}\ln\frac{m_a^2}{m_b^2}\;. \qquad (3.20)$$

Λ is a cutoff in mass units.[29] Note that the expressions (3.18) and (3.19) for ρ and σ are intimately tied to the special choice made in Eq. (3.4). This will become fully clear in Sec. VII where ρ and σ will be discussed in terms of the general parametrization given in Eqs. (3.1) and (3.2). Preparatory to this more general case to follow, let us make here the following comments regarding Eqs. (3.18) and (3.19).

(a) As was seen above, CP-violating effects must vanish for $m(y^0)\to 0$. This is verified here since $\sigma \to 0$ (at least formally) in this limit.

(b) The finiteness of ρ provides an application of Weinberg's lemma. To see this, note that counterterms for W^1-W^2 mixing are provided by $\mathcal{L}_{12}$, Eq. (2.16). Let us repeat that the vacuum expectation values for the H-fields were chosen to obey Eq. (2.18) in order that $\mathcal{L}_{12}$ be zero in zeroth order (tree approximation). When radiative corrections are included the following can happen.

(α) The vacuum expectation values a, b may need to be rescaled by a real factor.[30] This does not affect $\mathcal{L}_{12}$ in the sense that it still gives zero in the so corrected tree approximation.

(β) We are here in a situation, anticipated by

Weinberg,[31] where the structure of the H multiplets is such that not only the scale but also the direction of the vacuum expectation values can change. The logical consistency of such a redirection was proved in Ref. 22. With reference to the discussion of complex H's [see Eq. (2.12)] it is clear that such a change of direction can result in the development of an imaginary part of an expectation value which was initially real in the tree approximation. But even if *this* happens, one still cannot attain a counterterm with a W^1, W^2 dependence which is as in Eq. (3.16). Rather, one finds a counterterm with a structure as in Eq. (3.17). It follows that the ρ parameter is finite, not only to $O(g^2)$ but to all orders. On the other hand, since a counterterm for $\mathcal{H}^{\rm im}$ does exist, σ need not necessarily be finite, as indeed it is not.

At this point we note that this discussion of the properties of ρ and σ will need a modification when the f^{lR} are introduced within O(4) (see Sec. V), while the f^{lR} do not affect ρ, σ for $O(4)\times\mathcal{G}$ (see Sec. VII). In the latter case we shall argue that while σ *may* diverge on the grounds of the above $\mathcal{L}_{12}$ argument, it actually *need* not do so.

(c) A chirality argument. Insofar as we have considered only the couplings of the left-handed f^l up to this point, ρ and σ depend only on even powers of the lepton masses. This can be seen to follow from the chirality structure of the interactions, in the following way. In all gauge theories, fermion mass effects can be described as tadpole couplings linking a fermion line to the asymmetric vacuum.[32] For the present case, such couplings are exemplified in Fig. 2(a). If the f coming into the tadpole vertex is f^L (f^R) then the outgoing f is f^R (f^L). Since at this stage only f^L appear in the $W^{1,2}$ vertices, the number of tadpole couplings on any fermion line segment with specified charge and other quantum numbers must be even, whence the dependence on m_l^2 only. In Sec. VII we shall make ample use of this property in conjunction with discrete invariance arguments.

(d) The case $g_1 \neq g_2$. All interactions written down so far are R-invariant in the unbroken-symmetry limit. No such invariance is defined for $g_1 \neq g_2$ in Eq. (1.6). It is instructive to ask what happens to charged vector-meson mixing in this more general case. This question is answered in Appendix A, where two main results are established. First, it is shown that for $g_1 \neq g_2$ counterterms appear which can serve to renormalize the quantity ρ in Eq. (3.16) if such renormalization were needed. Secondly, it is shown that ρ indeed does need renormalization. In fact, whereas ρ is finite for $g_1 = g_2$, it becomes quadratically divergent for $g_1 \neq g_2$. This demonstrates that the special choice $g_1 = g_2$ made in PRL actually corresponds to the "smoothest" version of SU(2) $\times$SU(2).

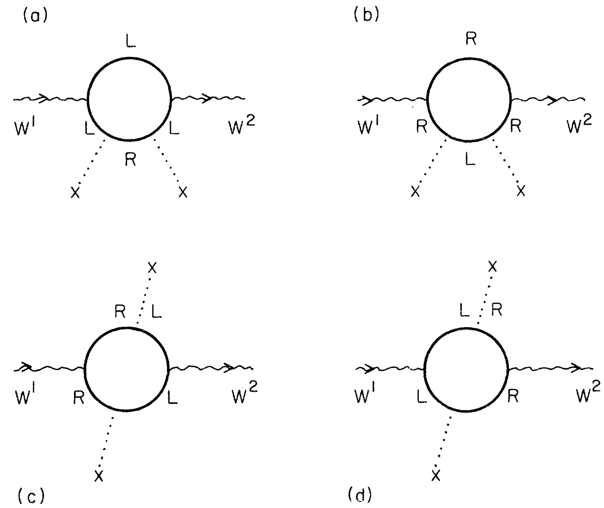

FIG. 2. Samples of tadpole couplings to fermion lines in W^1-W^2 mass mixing graphs. (a) is the only one allowed for $O(4)\times\mathcal{G}$, see Sec. VII.

(e) Scalar-field loops. Since the theory contains (WHH) couplings, one must also ask what are the contributions to the $\theta_{\mu\nu}^{ij}$ due to loops of H pairs. As a consequence of mass degeneracies within each H multiplet one finds that, at least to $O(g^2)$, no W^1-W^2 mixing is induced by H pairs.

Let us turn to the physical aspects of the W^1, W^2 mass mixing. Of course, such a discussion is marred by the occurrence of the logarithmic singularity in Eq. (3.19). For the time being, we shall proceed as in PRL. Namely we shall assume (as is commonly done for "weak," i.e., logarithmic singularities) that the renormalization needed does not change the order of magnitude of the coefficient σ. Moreover, anticipating the results of Sec. VII, the order of magnitude of ρ is indeed the same as that of σ, namely αm_l^2 in the finite version presented there. This then will be the order of magnitude adopted in what follows.

Consider the graphs of Fig. 3. They refer to μ decay and, with a change of arrows, also to $\nu_\mu + e \to \nu_e + \mu$. Figure 3(a) corresponds to the usual $O(g^2)$ contribution. Figure 3(b) gives the leading mass mixing effect, where the dot on the vector-meson propagator denotes the "mass mixing vertex" with value $\simeq \alpha\delta^2/\pi$; δ was defined in Eq. (3.13). For μ decay Eq. (3.12) holds true and we consider the same q^2 domain for lepton-lepton scattering. Then by the CP argument (ii), the interference between Fig. 3(a) and 3(b) is CP-conserving. Just as for the mechanism described by Fig. 1, CP violation can begin to occur only to the order that electromagnetic effects come into play, such as for example the interference between Figs. 3(b) and 3(c); and between 3(a) and 3(d). Again we find that the CP effect is superweak in the sense defined in Sec. I. Note furthermore that the δ^2 factor suppresses these on-shell effects even more.

This concludes our discussion of leptonic CP effects in first go around. We conclude this section with a comment on trilinear vector-meson couplings. Since the continuous group structure at hand is $SU(2)\times SU(2)$ there are no triangle graph anomalies. Finite corrections to such couplings do of course occur. The only such coupling which appears in the initial Lagrangian is of type (W^1W^2Z), because of R parity. It is readily verified that the finite correction to this vertex, due to the couplings given in Eqs. (3.2), (3.6), and (3.7) is CP-conserving.

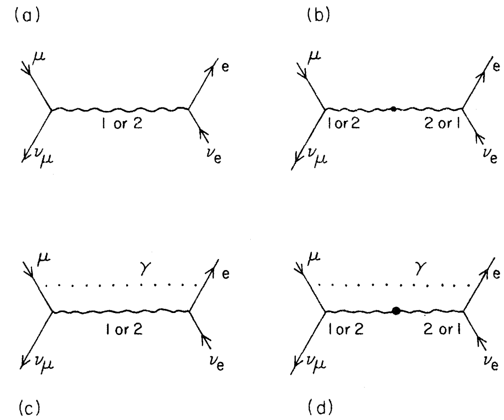

FIG. 3. A sample of graphs which demonstrates the superweak impact of CP violation on leptonic processes. "1" and "2" refer to W^1 and W^2.

IV. THE f^{hL}; NONLEPTONIC DECAYS; MORE ABOUT CP

The customary method is followed here to construct the hadronic parts of the currents in terms of quark fields and to generate effective interaction operators in terms of these currents. These effective operators are supposed to mediate the various transitions involving the usual meson and baryon states. These transitions are assumed further to be dominated by rather low quark momentum transfers q^2 such that in particular the inequality Eq. (3.12) applies.

The choice of quark representations within the gauge group at hand is further delimited by a number of physical constraints (common to all models) which have been discussed recently in detail by Lee, Primack, and Treiman.[33] In what follows next it will be seen that many of the arguments of these authors hold with rather minor modifications in the present work. The constraints in question encompass the following.

(I) There shall be no $|\Delta S|=1$ neutral hadron currents to $O(G)$.

(II) $|\Delta S|=2$ transitions generated by vector-meson pair exchange shall have amplitudes $\mathfrak{A}$ such that $\mathfrak{A}\ll O(G\alpha)$, in order that the real part of the K_L-K_S mass difference not be too large.

(III) The amplitude difference $\mathfrak{A}(\bar{\lambda}n\to\mu\bar{\mu}) - \mathfrak{A}(\lambda\bar{n}\to\mu\bar{\mu})$ shall be $\ll O(G\alpha)$ in order that the rate for $K_L\to\mu\bar{\mu}$ not be too large.

In the present scheme three vector-meson masses M_i, $i=0,1,2$ appear. Bearing in mind relations of the type Eqs. (1.10), (2.22), and (3.10), we see that the following requirement must also hold.

(IV) In implementing the above constraints one must treat $(M_i^2-M_j^2)/(\text{any vector-meson mass})^2$ as of order unity, that is, no cancellations between graphs with different M_i dependences may be made use of. As we shall see, this makes each of the constraints (II), (III) to multiple constraints.

The analyses of Ref. 33 focus on the SO(3) model of Georgi and Glashow.[34] It was found that it takes a set of eight quark states to satisfy the mentioned constraints by means of a cancellation mechanism devised by Glashow, Iliopoulos, and Maiani.[35] This mechanism is used here too and the same number of quarks appears as well. As in Refs. 9 and 33, we assume that the quarks additional to $\mathcal{P}, \mathfrak{N}, \lambda$ carry no isospin and hypercharge. As will presently be obvious, this guarantees that properties like CVC are maintained for the conventional hadron states. The vector current in question will be contained in $J^{(1)}$.

The eight f^{hL} are assigned to two quartets Q^{1L} and Q^{2L}, as follows.

$$Q^{1L}=(\mathcal{P}, \tfrac{1}{2}(\mathfrak{N}+\lambda+q^0\sqrt{2}\,), -\tfrac{1}{2}(\mathfrak{N}-\lambda+r^0\sqrt{2}\,), -q^-)_L\,, \tag{4.1}$$

$$Q^{2L}=(q^+, \tfrac{1}{2}(\mathfrak{N}-\lambda-r^0\sqrt{2}\,), -\tfrac{1}{2}(\mathfrak{N}+\lambda-q^0\sqrt{2}\,), -r^-)_L\,. \tag{4.2}$$

The $\mathcal{P}, \mathfrak{N}, \lambda$ states are conventional. The other states, $q^{\pm,0}$, $r^{-,0}$ are supposed to carry one or more further quantum numbers collectively denoted by C. At least one of these shall be additive. Appendix C contains a few additional comments on the implications of C for hadron symmetries.

From Eqs. (2.24), (2.25), (2.26) and (4.1), (4.2) one finds

$$J^{(1)hL}=-\tfrac{1}{2}ig[\bar{\mathcal{P}}(\mathfrak{N}+A_1^0)+(\bar{\mathfrak{N}}+\bar{A}_1^0)q^- + \bar{q}^+(\mathfrak{N}-A_1^0)+(\bar{\mathfrak{N}}-\bar{A}_1^0)r^-]_L\,, \tag{4.3}$$

$$J^{(2)hL} = -\tfrac{1}{2}ig[\overline{\mathcal{P}}(\lambda + A_0^2) - (\overline{\lambda} + \overline{A}_2^0)q^- - \overline{q}^+(\lambda - A_2^0) + (\overline{\lambda} - \overline{A}_2^0)r^-]_L\,, \qquad (4.4)$$

where the following convenient abbreviations have been introduced.

$$\sqrt{2}\,A_1^0 = q^0 + r^0\,,$$
$$\sqrt{2}\,A_2^0 = q^0 - r^0\,. \qquad (4.5)$$

Further,

$$J^{(0)hL} = (ig/\sqrt{2}\,)[(\overline{\mathfrak{N}} + \overline{\lambda})q^0 - (\overline{\mathfrak{N}} - \overline{\lambda})r^0 + \text{H.c.}]_L\,. \qquad (4.6)$$

Let us now turn to the constraints stated at the beginning of this section.

(I) Equation (4.6) shows that it is satisfied, of course with the understanding that "no $|\Delta S| = 1$ current" shall mean, more specifically, "no $|\Delta S| = 1$, $\Delta C = 0$ current" in models of this kind.

(II) The $|\Delta S| = 2$ contributions in question stem from box graphs of the general structure encountered in Fig. 1, with the appropriate particle labels and with crossing properly taken into account. The types of graphs involved are much like those of Ref. 33, Fig. 3. In accordance with (IV) one must now examine separately the contributions which stem from the exchange of (W^1W^1), (W^2W^2), (W^1W^2), and (ZZ) vector-meson pairs. Since W^1 only couples to $\mathfrak{N}$ (not λ), and W^2 only to λ (not $\mathfrak{N}$), there is no contribution from (W^1W^1) and (W^2W^2) exchange. (W^1W^2) contributes to $\mathfrak{A}(\lambda\lambda \to \mathfrak{N}\mathfrak{N})$, to the order $O(\alpha^2\Delta^+\Delta^-)$,

$$\Delta^+ = \frac{m^2(\mathcal{P}) - m^2(q^+)}{M^2}\,,$$
$$\Delta^- = \frac{m^2(q^-) - m^2(r^-)}{M^2}\,, \qquad (4.7)$$

while (ZZ) contributes to the order $O(\alpha^2(\Delta^0)^2)$,

$$\Delta^0 = \frac{m^2(q^0) - m^2(r^0)}{M^2}\,. \qquad (4.8)$$

Then (II) is satisfied provided that

$$\Delta^+, \Delta^-, \Delta^0 \text{ are } \ll 1\,, \qquad (4.9)$$

that is to say, mass splittings within the quark multiplets are small compared to the vector-meson masses. See further Ref. 33 for more detailed estimates of the K_L-K_S real mass difference which bear out inequalities of the type Eq. (4.9).

(III) Again, and as in Ref. 33, the leading contributions stem from box graphs. Here we must use Eqs. (4.3)–(4.6) in conjunction with Eqs. (3.5)–(3.7). One verifies that the only contributions come from (W^1W^2) exchange. Two graph structures appear, related by crossing and with respective magnitudes

$$O(\alpha^2\Delta^+\delta^2)\,, \quad O(\alpha^2\Delta^-\delta^2)\,, \qquad (4.10)$$

so that (III) is satisfied if once again the inequalities Eqs. (3.13) and (4.9) are invoked. (For $K_S \to \mu\overline{\mu}$ the effect is zero even to this order.)

Next, we discuss nonleptonic $|\Delta S| = 1$, $\Delta C = 0$ transitions. To $O(G)$ no such processes are generated via the currents $J^{(1)}$ and $J^{(2)}$. On the other hand, the interaction $J_\mu^{(0)hL} Z_\mu$ does provide us with a nonleptonic decay mechanism to this order. The effective interaction is

$$\frac{g^2}{8(q^2 + M_0^2)}[\,\overline{\lambda}\gamma_\mu(1+\gamma_5)q^0\overline{q}^0\gamma_\mu(1+\gamma_5)\mathfrak{N} - (q^0 \to r^0)] + \text{H.c.} \qquad (4.11)$$

Once again we go to the domain $q^2 \ll M_0^2$. Since the local $V - A$ combination is invariant under Fierz transformations, one can then write Eq. (4.11) approximately as

$$\frac{G}{\sqrt{2}}\,\frac{1}{\xi}\,\frac{\sin 2\theta}{(1+\sin 2\theta)^{1/2}} \times \overline{\lambda}\gamma_\mu(1+\gamma_5)\mathfrak{N}[\,\overline{q}^0\gamma_\mu(1+\gamma_5)q^0 - \overline{r}^0\gamma_\mu(1+\gamma_5)r^0] + \text{H.c.} \qquad (4.12)$$

Equation (4.12) gives the $|\Delta S| = 1$, $\Delta C = 0$ interaction in its local $V - A$ form with all the implications thereof.[36] The only constraint on the validity of Eq. (4.12) is $q^2 \ll$ (vector-meson mass)^2. No constraints on the quark and on the scalar field masses are involved of the kinds which appear in Ref. 9.

Indeed, throughout this work there appears no reason that I am aware of for not taking the H masses sufficiently heavy, so that the influence of the H field on estimates of decay and transition amplitudes may be considered as small.

The next question [which applies equally to Eqs. (4.11) and (4.12)] is: Does the interaction also satisfy $|\Delta I| = \frac{1}{2}$ for the familiar nonleptonic processes? The answer depends on two main factors: (a) the assumed structure of the familiar meson and baryon states in terms of the quarks introduced here and (b) the isospin properties of the q^0 and r^0 which appear in Eqs. (4.11) and (4.12).

First of all, it is evident that Eq. (4.12) would not give nonleptonic K and hyperon decays at all unless the quark content of these states would embrace the presence of $\overline{q}^0q^0$ pairs *and/or* $\overline{r}^0r^0$ pairs. If both kinds of pairs occur then one will assign $I = 0$ to both q^0 and r^0. If only one kind of pair occurs, say q^0, then one needs only to assume that $I(q^0) = 0$. Let us first consider the model for mesons and baryons given in Ref. 9 in a somewhat similar discussion of the $|\Delta I| = \frac{1}{2}$ rule. Here the mesons are: $\mathcal{P}\overline{\mathfrak{N}}$ for π^+, $\mathcal{P}\overline{\lambda}$ for K^+ etc., with, in addition, a "sea" of quark pairs which has $I = Y$

$=C=0$ and which contains q^0 and r^0 pairs. The baryon states are obtained by adding a single[37] q^0 where q^0 itself has $I=Y=C=0$. Thus $\mathcal{P}\overline{\mathfrak{N}}q^0$ corresponds to Σ^+, $\mathcal{P}\overline{\lambda}q^0$ to the proton, etc. As noted in Ref. 9 one can also recover further consequences for nonleptonic decays implied by approximate SU(3) symmetry if one assumes, more specifically, that the "sea" behaves like an SU(3)-singlet.

Thus it is indeed possible to realize the $|\Delta I|=\frac{1}{2}$ rule for nonleptonic decays from Eq. (4.11). It is of some interest that the additional quarks which have been introduced in this as in many other gauge models for the purpose of suppression of unwanted processes, now begin to play a more positive role. It should be emphasized that the particular realization of the $|\Delta I|=\frac{1}{2}$ rule by means of the meson-baryon model mentioned above is but one example of how this can be done. This problem demands further investigations in terms of the embedding of the present model within hadronic symmetries, for example with regard to static SU(6) properties, as is discussed further in Appendix C. The model mentioned above should therefore be taken only as an illustration of the fact that $|\Delta I|=\frac{1}{2}$ can be implemented.

It may further be noted that both the above model of the constitution of mesons and baryons as well as the alternative model of Appendix C are in accordance with the requirements concerning the sign and magnitude of the $\pi^0\to 2\gamma$ amplitudes as stated in Sec. I, since it satisfies Eq. (1.12).

The last main topic of this section is the further discussion of CP questions. If we confine ourselves to the contributions from $J^{(0,1,2)hL}$ only then clearly there is no CP violation at all, since the structure Eqs. (4.1) and (4.2) of Q^{1L} and Q^{2L} does not include any CP-violating phases. One may well ask the following question. Since such phases were introduced in the lepton multiplets, why not introduce such phases in the Q's as well? I have examined this problem by allowing for a more general definition of Q^{2L} (holding Q^{1L} fixed by phase *conventions* suitably chosen) in which the two neutral members of Q^{2L} each are given (independent) over all extra phase factors. Then CP violation enters because of the occurrence of relative phases between the neutral members of Q^{1L} as compared to those of Q^{2L}. But now something has to be reconsidered which has no leptonic analog, namely the implementation of the constraints (II) and (III) stated at the beginning of this section. It turns out that these constraints are so severe that the extra phases just mentioned must be extremely close to zero. This still allows for the possibility of tiny on-shell CP-violating mass effects, but the introduction of effects in this way seems rather artificial; nor is there any obvious need for such a procedure. Thus the situation is quite different for the lepton and for the hadron sector. In the former case, given the group and given the choice of lepton representations one is forced to consider CP-violating phases. In the latter case no such argument exists and the constraints (II) and (III) lead to CP-conserving structures for the quark multiplets as the most natural choice.

The combined action of J^{lL} and J^{hL} in semileptonic decays gives rise to the following effects, exemplified for the transition $\lambda\to\mathcal{P}+l+\overline{\nu}_l$. To $O(G)$ there is no CP effect, the transition takes place via W^2 exchange only. To $O(g^4)$, W_1Z exchange gives a potential CP effect $O(G\alpha)$, but a further suppression takes place just as for the graph in Fig. 1 discussed for μ decay. The main effect of W^1, W^2 mixing is displayed in Fig. 4(b). The $(\overline{\lambda}\mathcal{P}W^2)$ vertex is drawn as a larger blackened circle to symbolize the influence of the strong interactions. It is crucial that these hadronic modifications *are the same in Figs. 4(a) and 4(b)*. Just as was seen in Sec. III, the influence of the mixing is to generate an *over-all* complex scale factor in the decay amplitude. This complex rescaling *is independent of strong interaction effects*. Thus it applies equally well when we use this effective interaction operator as the transition operator for K_{l2} decays as for K_{l3} and K_{l4} or any other mode. Therefore there is no on-shell CP-effect to $O(G\alpha)$ for any semileptonic K (or hyperon) decays. On-shell CP violation becomes manifest only when electromagnetic corrections are also introduced, as was discussed for Fig. 3. Thus we have shown that also in semileptonic decays the CP effects are superweak. The same is also true for nonleptonic decays, but the proof thereof will be deferred till the end of Sec. VII.

Finally, note that the ρ parameter in Eq. (3.18) is modified due to the influence of $\overline{Q}Q$ pairs. The modification must be finite by the arguments given in Sec. III. The σ parameter, Eq. (3.19), remains unaltered. From Eqs. (4.3) and (4.4) it follows that the leading correction to ρ receives no contributions from λ or $\mathfrak{N}$ states and that the effect

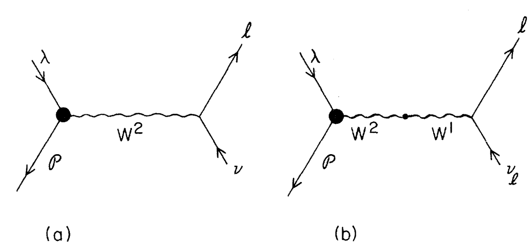

FIG. 4. $\lambda\to\mathcal{P}+l+\overline{\nu}_l$ via W^2 exchange (a); and via W^2-W^1 mixing (b).

due to q^0 and r^0 states must vanish in the limit that these two states are mass degenerate. The resulting addition to the right-hand side of Eq. (3.18) is

$$-\frac{\alpha}{4\pi}[m^2(q^0)\Phi(\mathcal{P},q^+,q^-,r^-,q^0) - m^2(r^0)\Phi(\mathcal{P},q^+,q^-,r^-,r^0)], \quad (4.13)$$

$$\Phi(\mathcal{P},q^+,q^-,r^-,q^0)=m^2(\mathcal{P})\phi(q^0,\mathcal{P})-m^2(q^+)\phi(q^0,q^+) - m^2(q^-)\phi(q^0,q^-) + m^2(r^-)\phi(q^0,r^-), \quad (4.14)$$

where Eq. (3.20) has been used.

V. A FIRST ATTEMPT TO INCORPORATE THE f^R

A. Phenomenological Aspects

In this section we ask for the consequences of the assumption that the f^R are representations of O(4), labeled by (m,n) where m,n are integer or half integer. If one tries to treat the f^R as $(\frac{1}{2},\frac{1}{2})$ representations, one runs into numerous difficulties. For example, it is impossible to avoid unwanted lepton contributions of the type $(\bar{\nu}_e e)_R$ or $(\bar{\nu}_\mu \mu)_R$ to the currents $J^{(1,2)}$. In regard to the Q^R, considered as $(\frac{1}{2},\frac{1}{2})$, a further analysis of the fermion mass problem shows that one can in no way avoid the occurrence of some zero-mass quarks.

Given the group structure considered here, there is an alternative procedure which at least circumvents these problems. In regard to the leptons one simply avoids the bad current contributions just mentioned by taking the states x_R^+, x_R^0, e_R to belong to a triplet representation, and likewise for y_R^+, y_R^0, μ_R. For the f^{hR} a somewhat similar method may be followed, namely to use two triplets and two singlets (0, 0). In fact, in order to continue to satisfy the constraints (II) and (III) stated at the beginning of Sec. IV, it suffices[38] to choose the two singlets to be $\mathfrak{N}_R$ and λ_R. As we shall see in a moment the constraint (I) will be satisfied in any event.

The group allows for two kinds of triplets, namely (1, 0), a $\vec{t}$ triplet not acted on by $\vec{\rho}$; and (0, 1), a $\vec{\rho}$ triplet not acted on by $\vec{t}$. Thus we must decide in each case which triplet to choose. Before we come to this, it is important to note the R properties of triplet assignments. Neither triplet is separately an eigenstate of R, since the action of R on (1, 0)→(0, 1), and on (0, 1)→(1,0). The implications of this behavior under R is one of the main topics of this section.

For reasons to be explained we shall next attempt to assign the E^R to (1, 0) and the M^R to (0, 1) as follows (a spherical basis is used):

$$E^R=(x^+,-x^0,-e)_R, \quad M^R=(\beta_+y^+,\beta_0y^0,\beta_-\mu)_R, \quad (5.1)$$

where the β's are phase factors to be determined from the conditions that the lepton mass matrix shall be diagonal and have positive[39] eigenvalues. The contributions to the currents are

$$J^{(1)lR}=-(ig/\sqrt{2})(\bar{x}^0e+\bar{x}^+x^0)_R + (ig/\sqrt{2})(\beta_0^*\beta_-\bar{y}^0\mu-\beta_+^*\beta_0\bar{y}^+y^0)_R, \quad (5.2)$$

$$J^{(2)lR}=-(ig/\sqrt{2})(\bar{x}^0e+\bar{x}^+x^0)_R - (ig/\sqrt{2})(\beta_0^*\beta_-\bar{y}^0\mu-\beta_+^*\beta_0\bar{y}^+y^0)_R, \quad (5.3)$$

$$J^{(0)lR}=-(ig/\sqrt{2})(\bar{x}^+x^+-\bar{e}e)_R + (ig/\sqrt{2})(\bar{y}^+y^+-\bar{\mu}\mu)_R. \quad (5.4)$$

Remark: Since E^R is (1, 0) it contributes equally to $J^{(1)}$ and $J^{(2)}$ since W^1 and W^2 are coupled equally to t; see Eq. (2.9). M^R contributes with opposite sign to $J^{(1)}$ as compared to $J^{(2)}$ since W^1 is coupled to $(-\rho)$, and W^2 to $(+\rho)$. The relative sign difference in Eq. (5.4) has a similar origin.

Let us now first examine the full neutral lepton current

$$J^{(0)l}=J^{(0)lL}+J^{(0)lR}, \quad (5.5)$$

where $J^{(0)lL}$ is given by Eq. (3.7). It is clear that the R part of $J^{(0)l}$ presents an electron target to a neutrino beam. We are now in a position to explain why the special choice Eq. (3.4) was made. Since $\nu_\mu e$ scattering appears to be suppressed, we have made use of the phases in Eqs. (3.1) and (3.2) to arrange things so that elastic $\nu_\mu e$ scattering is entirely forbidden to $O(G)$. Instead, the inelastic process $\nu_\mu+e\rightarrow y^0+e$ occurs. Any other choice of the phases in Eqs. (3.1) and (3.2) would generate some $\nu_\mu e$ scattering to $O(G)$. While the present status[18] of the $\nu_\mu e$ experiments leaves room for play here, it would at least seem most simple to avoid the $\nu_\mu e$ problem altogether. In that case $\nu_\mu e$ scattering does of course still occur, but with an amplitude[40] $O(G\alpha)$.

On the other hand, the neutral current does contribute to elastic $\bar{\nu}_e e$ and $\nu_e e$ scattering. So do the charged currents and the combined result is [cf. Eq. (1.10)]

$$\sigma(\bar{\nu}_e e)=\sigma_0\left(1+\sin 2\theta+\frac{\sin^2 2\theta}{1+\sin 2\theta}\frac{3}{\xi^2}\right), \quad (5.6)$$

$$\sigma(\nu_e e)=3\sigma_0\left(1+\sin 2\theta+\frac{\sin^2 2\theta}{1+\sin 2\theta}\frac{1}{3\xi^2}\right), \quad (5.7)$$

where

$$\sigma_0=\frac{4}{3}\frac{G^2m(e)}{\pi}E_\nu = 0.54\times10^{-41}E_\nu\,\text{cm}^2/\text{electron} \quad (5.8)$$

is the $V-A$ value for the total $\bar{\nu}_e e$ elastic scattering cross section (E_ν = neutrino lab energy in GeV). With $\sin 2\theta \simeq 0.4$ and if $\xi = 1$ [cf. Eq. (2.22)] this yields a value $\simeq 1.7\sigma_0$ for $\sigma(\nu_e e)$, which is acceptable[41] as far as the extremely difficult experiments[18] permit us to conclude. If $\xi < 1$ then the value 1.7 becomes a lower bound [see Eqs. (5.20) and (5.21) below].

Consider next the f^{hR}. In accordance with what was stated before, we introduce two triplets which (for example, see Ref. 38) can be written as

$$Q^{1R} = (\mathcal{P}, r^0, r^-)_R, \quad Q^{2R} = (q^+, q^0, q^-)_R. \tag{5.9}$$

For definiteness, let Q^{1R} be (1, 0) and Q^{2R} be (0, 1). Then

$$J^{(1)hR} = -(ig/\sqrt{2})(\bar{\mathcal{P}}r^0 - \bar{r}^0 r^- - \bar{q}^+ q^0 + \bar{q}^0 q^-)_R,$$

$$J^{(2)hR} = -(ig/\sqrt{2})(\bar{\mathcal{P}}r^0 - \bar{r}^0 r^- + \bar{q}^+ q^0 - \bar{q}^0 q^-)_R, \tag{5.10}$$

$$J^{(0)hR} = -(ig/\sqrt{2})(\bar{\mathcal{P}}\mathcal{P} - \bar{r}^- r^- - \bar{q}^+ q^+ + \bar{q}^- q^-)_R.$$

The full neutral current $J^{(0)}$ is the sum of the contributions given in Eqs. (3.7), (4.6), (5.4), and (5.10). It is obvious from Eq. (5.10) that the constraint (I) given in Sec. IV remains satisfied.

Since the $\mathcal{P}$-quark is a nucleon constituent, $J^{(0)hR}$ provides a nucleon target for neutrinos. The phase choice Eq. (3.4) is such that no $\bar{\nu}_\mu \nu_\mu$ current enters in $J^{(0)}$. Therefore processes like $\nu_\mu + p \to \nu_\mu + p$, $\nu_\mu + p \to \nu_\mu + \Delta$ do not enter to $O(G)$. ν_μ does appear in $J^{(0)}$ via $\bar{\nu}_\mu y^0$ and $\bar{y}^0 \nu_\mu$. Therefore inelastic heavy lepton production has an amplitude $O(G)$, such as for ν_μ + proton $\to y^0$ + hadrons. All these statements bear on the current types of experiments in which the high-energy neutrino beams are to a high degree of the muonic variety.[42]

On the other hand, the phase choice Eq. (3.4) does lead to $\bar{\nu}_e \nu_e$ terms in $J^{(0)}$, see Eq. (3.7). Thus reactions like ν_e + proton $\to \nu_e$ + hadrons appear to $O(G)$ without the presence of heavy leptons in the final state.

For the group in hand we have come at this point to the conclusion of the phenomenological aspects, but for one question. We must ask how the inclusion of the f^R affects the CP problem inasfar as it has been discussed in Secs. III and IV. It was already stated in PRL that the modifications which arise do not change in any qualitative way what was found earlier in this paper. As before, logarithmic divergences will remain in the W^1-W^2 mixing. However, some rather important theoretical issues appear which bear on the meaning of the discrete symmetry R. For this reason, if for no other, it is illuminating to spell out the full CP-problem in O(4) in some detail. To what was found in Secs. III and IV we shall have to add the contributions (RR) which are solely due to the f^R and those (RL) which arise from right-left interference.

For the box graph, Fig. 1, Sec. III, the modifications are negligible. Obviously there is no RR contribution here at all, since neutrinos appear aplenty. There is a RL interference term which is $\sim\delta^2$ smaller [see Eq. (3.13)] than the $O(G\alpha)$ found for the LL term.

More interesting is the change in the W^1, W^2 mixing parameters ρ and σ defined in Eqs. (3.16) and (3.17). We begin with the RR contributions to $O(g^2)$. Note first that such terms do not contribute to σ at all. The reason for this is that (up to a $\pm$ sign) all lepton pairs couple equally to W^1 as they do to W^2 so that, for any lepton loop, the phases in Eqs. (5.2) and (5.3) cancel since we deal with one emission and one absorption vertex. Thus only the parameter ρ receives RR contributions which are found to be:

$$-\frac{\alpha}{\pi}\lim_{\Lambda\to\infty}\int_0^{\Lambda^2} z^2 dz\left[\frac{1}{z+m^2(x^0)}\left(\frac{1}{z+m^2(x^+)}+\frac{1}{z+m^2(e)}\right)-\frac{1}{z+m^2(y^0)}\left(\frac{1}{z+m^2(y^+)}+\frac{1}{z+m^2(\mu)}\right)\right], \tag{5.11}$$

where the integration variable z stands for the (Euclidean) (momentum)^2 in the loop. By the chirality argument given in Sec. III the result is again a function of (lepton mass)^2 only [see Fig. 2(b)]. The integral is logarithmically divergent.[43] We have now a seeming contradiction. In Sec. III it was stated that ρ had to be finite by Weinberg's lemma. Then how can we meet a divergent answer as in Eq. (5.11)?

The resolution of this question lies in the R-invariance properties of the theory. The behavior of the vector-meson fields under R was specified in Eq. (2.33). If we adjoin to this transformation the further relation

$$E^R \leftrightarrow M^R, \tag{5.12}$$

then Eqs. (5.2)–(5.4) show that the *vector*-meson couplings of the f^{lR} are R-invariant. On the other hand, R invariance dictates that the only counterterm mechanism for $W^{1,2}$ mixing is given by $\mathcal{L}_{12}$, see Eq. (2.16) which cannot counteract a divergence in the ρ parameter. But the validity of Eq. (5.12) implies a mass degeneracy between the electron and muon multiplet. And, indeed, *insofar as* such a degeneracy were to exist there is no paradox since, in this degeneracy limit, Eq. (5.11) gives the answer zero.

A special case of this degeneracy is provided by the limit in which all lepton masses are put equal to zero. Since the H couplings to fermions have coupling constants which are (fermion mass)-proportional, this means that (1) we have an R-invariant theory to the extent that we neglect the $H\bar{f}f$ couplings, and (2) these couplings cannot possibly be R-invariant since they would then imply an unwanted degeneracy of the electron- and muon-type leptons. This last statement needs explicit verification, to be given presently.

While we have now answered the question about ρ in a limiting case of degeneracy, this answer is still incomplete since we yet have to find the counterterm which can renormalize the loop effect of Eq. (5.11) in the realistic case that there is no E-M mass degeneracy. This counterterm is readily located by the following argument. We are perfectly entitled to postulate the relation $g_1 = g_2$, Eq. (1.8) as we have done at the outset. This is a relation between bare coupling constants. If the theory is not R-invariant, however, there is no reason why the equality of g_1 and g_2 should be respected by the radiative corrections. Moreover since the theory is not R-invariant we have, by the rules of strict renormalizability, to write down all terms that are compatible with the group SU(2) $\times$SU(2), not just O(4). This means that we must first consider the formalism for $g_1 \neq g_2$, then take $g_1 = g_2$ in the sense of a bare coupling constant relation. If [as for Eq. (5.11)] the need for renormalization arises we have to revert to SU(2)$\times$SU(2) in order to find such counterterms which we would miss if we had made the unwarranted assumption that $g_1 = g_2$ remains true in the presence of radiative corrections.

The theory for $g_1 \neq g_2$ is given in Appendix A. It contains a generalized expression, Eq. (A15) for the quantity $\mathcal{L}_{12}$ ($g_1 = g_2$) given in Eq. (2.16). It is found that Eq. (A15) does indeed provide the counterterm necessary to renormalize Eq. (5.11) so that the seeming contradiction has been eliminated. Note that this "contradiction" could not arise for the f^{iL} because these were assigned to $(\frac{1}{2}, \frac{1}{2})$. It is a property of O(4) that any representation (m,n) of SO(4) is also a representation[44] of O(4) if $m = n$.

It should be emphasized that it was only possible to make use of Eq. (5.12) because of the assignments[45] E^R, (1, 0), and M^R, (0, 1). Indeed this enables one, in the R-symmetry limit, to consider the right-handed leptons as the direct sum of (0, 1) and (1, 0), that is as a "six vector" with well-defined behavior under R (just like the electromagnetic field under the Lorentz group including parity). In order to underline this point further, consider the case where both E^R and M^R would be (1, 0). This implies the following changes: the over-all sign of the M terms in Eqs. (5.2) and (5.4) [but not in Eq. (5.3)] has to be reversed. As a consequence, the minus sign in front of the M terms in Eq. (5.11) becomes a plus sign. But this means that the ρ parameter now becomes quadratically divergent and that this divergence persists in the zero mass limit for the leptons. No wonder, since even for $m_l = 0$ the theory is not R-invariant.

There is a lesson in all this which, I believe, may be pertinent in any situation where discrete symmetries are brought to bear on gauge theories. In the present instance we have learnt that three cases should be distinguished.

(a) Strict R invariance. This is the case when all terms in $\mathcal{L}$ are R-invariant in the unbroken symmetry limit. As we have seen, this case has not been realized up to this point. But it will be realized when we come to O(4)$\times\mathcal{G}$.

(b) Approximate R-invariance. This is defined by the requirement that the vector-meson–fermion interaction is R-invariant but the H-fermion interaction is not. These conditions imply $g_1 = g_2$ as a bare coupling constant relation.

(c) SU(2)$\times$SU(2) without even approximate R invariance. An example of this has just been given which shows that this version is the "most divergent" (though still renormalizable) of the lot. A further example of this case is given in Appendix A: Start with $g_1 \neq g_2$ as a bare coupling constant inequality. Then even the left-handed fermion contribution to $W^{1,2}$ mixing becomes quadratically divergent.

The remaining LR contributions to the $W^{1,2}$ mixing teach us nothing substantially new. These contributions can inherently be at most logarithmically divergent. Such divergences indeed appear both for ρ and for σ. The terms are proportional to mm', where m and m' are distinct lepton masses, in accordance with the corresponding tadpole graphs Figs. 2(c) and 2(d).

Thus we find that an approximate discrete invariance has led to a suppression of a potentially quadratic divergence for the mixing parameters to a logarithmic one. We shall make use of several such approximate invariances in Sec. VI. But first we must consider the $H\bar{f}f$-interactions for the present group.

B. Fermion Mass Questions

We start by verifying the statement made in Sec. VA to the effect that the H couplings to fermions cannot be fully R-invariant. For notational purposes put

$$x_R^+\sqrt{2}=E_1^R-iE_2^R,\quad e\sqrt{2}=E_1^R+iE_2^R,\quad x^0=E_3^R.$$

Then $\vec{E}^R=(E_1^R,E_2^R,E_3^R)$ is a Cartesian representation for E^R defined in Eq. (5.1). Likewise $\vec{M}^R$ will be introduced. Now note that the operator $\vec{t}\cdot\vec{E}^R+\vec{\rho}\cdot\vec{M}^R\equiv X$ is invariant under O(4) including R. Then the couplings $(\bar{E}^L XH+\text{H.c.})$ and $(\bar{M}^L XH+\text{H.c.})$ are invariant likewise. Here H is one or the other of the scalar quartets introduced[46] in Sec. II. These couplings involve (potentially dangerous) transitions between muonic and electronic states. The mass diagonalization demands at least that such cross terms shall vanish in the tree approximation. It is readily verified that this is so only for the null solution in which the corresponding coupling constants are zero, so that indeed no nontrivial R-invariant $H\bar{f}f$ couplings exist.

Continuing with the assignments $E^R=(1,0)$, $M^R=(0,1)$ let us then consider the R-noninvariant mass generating mechanisms. For the E-case we have

$$\bar{E}^L(\vec{t}\cdot\vec{E}^R)[A_E H(a,a)+B_E H(ib,-ib)]+\text{H.c.},\tag{5.13}$$

where A_E, B_E are "coupling constants." With the help of Eqs. (3.1), (3.4), and (5.9) one sees that in the static limit Eq. (5.13) gives a (ν,x^0) cross term which vanishes if

$$A_E a=iB_E b.\tag{5.14}$$

Here we have used one of the identities collected in Appendix B, see Eq. (B12), from which we also infer that the $\bar{e}e$ mass term is proportional to $(A_E a-iB_E b)$, while the x^0 and x^+ masses are both proportional to $(A_E a+iB_E b)$. The bare masses satisfy

$$m(e)=0,\quad m(x^+)=m(x^0)\sqrt{2}.\tag{5.15}$$

It can be verified that Eq. (5.15) does not depend on the particular real choice of the H quartets discussed in Sec. II. In fact, Eq. (5.15) is a consequence only of the structure Eqs. (3.1) and (3.4) for the left-handed E quartet, the structure Eq. (5.1) of the right-handed E triplet and of the assumption that the H's are quartets. In further explanation of the choice $E^R=(1,0)$, note that had we taken E^R to be $(0,1)$, one would again have obtained Eq. (5.14) but Eq. (5.15) would have to be replaced with $m(x^+)=0$, $m(e)=m(x^0)\sqrt{2}$.

Next consider the M couplings

$$\bar{M}^L\vec{\rho}\cdot\vec{M}^R[A_M H(a,a)+B_M H(ib,-ib)]+\text{H.c.}\tag{5.16}$$

With the help of Eqs. (3.2), (3.4), (5.1), and (B13) one finds that the analog of Eq. (5.14) is

$$A_M a+B_M b=0\tag{5.17}$$

and the diagonal terms are

$$Aa(\epsilon\beta_+\bar{y}^+y^+-i\beta_0\bar{y}^0y^0-\epsilon^*\beta_-\bar{\mu}\mu),\tag{5.18}$$

where $\epsilon=\exp(\frac{1}{4}i\pi)$. We now fix the phases: $\beta_+=\epsilon^*$, $\beta_0=i$, $\beta_-=-\epsilon$ and take A real, $Aa>0$. Then the mass matrix is positive[39] and

$$m(y^0)=m(y^+)=m(\mu).\tag{5.19}$$

The contradistinction between Eqs. (5.15) and (5.19) illustrates how the μe nonuniversality introduced initially to cope with the Cabibbo condition can lead to major dissymmetries in regard to bare masses for e and μ. However, we have now to face the fact that the equalities Eq. (5.19) are wholly inacceptable physically.

While one may speculate about possibly large radiative mass corrections at this point, I have no wisdom to offer in this respect. Instead, in order to avoid Eq. (5.19), I explored the only other course open, namely to examine the consequences of extending the scalar field multiplet system.

Such an extension is unique. Since $E^L=(\frac{1}{2},\frac{1}{2})$ and $E^R=(1,0)$ will be retained, the only other scalar multiplet which can affect the E-mass system is $(\frac{3}{2},\frac{1}{2})$. Likewise for the M system it is $(\frac{1}{2},\frac{3}{2})$. However one cannot introduce $(\frac{3}{2},\frac{1}{2})$ and $(\frac{1}{2},\frac{3}{2})$ in an uncorrelated way without spoiling the role of W^1 and W^2 as zeroth-order normal modes.

In order to show how one must proceed, denote $(\vec{A}\vec{t}+\vec{C}\vec{\rho})^2$ by Y and symbolize the quadrilinear coupling between a scalar $(\frac{1}{2},\frac{1}{2})$ multiplet and the vector fields by $(\frac{1}{2},\frac{1}{2})Y(\frac{1}{2},\frac{1}{2})$. Such an interaction is automatically R-invariant. This is not true for $(\frac{3}{2},\frac{1}{2})Y(\frac{3}{2},\frac{1}{2})$ since, symbolically, $R(\frac{3}{2},\frac{1}{2})\to(\frac{1}{2},\frac{3}{2})$. Thus one must use the coupling $(\frac{3}{2},\frac{1}{2})Y(\frac{3}{2},\frac{1}{2})+(\frac{1}{2},\frac{3}{2})Y(\frac{1}{2},\frac{3}{2})$. Once both $(\frac{3}{2},\frac{1}{2})$ and $(\frac{1}{2},\frac{3}{2})$ appear, the equal relative weight is automatic. Beyond this, it is necessary to correlate the vacuum expectation value of the $(\frac{3}{2},\frac{1}{2})$ and the $(\frac{1}{2},\frac{3}{2})$ in such a way that W^1 and W^2 are preserved normal modes in the tree approximation.

It is described in Appendix B how this is done. In addition two further identities are given there, Eqs. (B14) and (B15) with the help of which one can easily obtain the modifications of the fermion bare masses due to the inclusion of these new scalar multiplets. With their help it is easily shown, by identical steps as were followed for Eqs. (5.13)–(5.19) that a further splitting mechanism is generated by means of which an equation like (5.19) can be modified.

The same procedures can likewise be followed for the quark mass problem. Beyond the mass invariants which are direct counterparts of those used for the leptons, eight more appear, namely $(\bar{Q}^L H)\mathfrak{N}^R+\text{H.c.}$ and $(\bar{Q}^L H)\lambda^R+\text{H.c.}$, where Q^L is either Q^{1L} or Q^{2L} [see Eqs. (4.1) and (4.2)] and where H is either $H(a, a)$ or $H(ib, -ib)$. In all these instances it is then a matter essentially of counting to show that the mass matrices can be diagonalized and that, where unwanted, mass degeneracies do not appear. Since the procedure is straightforward, tedious and not illuminating, it will not be explicitly carried out here.

Finally, it is shown in Appendix B that the inclusion of the additional scalar multiplets modifies Eq. (2.22) to $\xi<1$, see Eq. (B10). This inequality makes Eqs. (5.6) and (5.7) to inequalities as well:

$$\sigma(\bar{\nu}_e e)>1.7\sigma_0, \tag{5.20}$$

$$\sigma(\nu_e e)>4.3\sigma_0, \tag{5.21}$$

where σ_0 was given in Eq. (5.8). From the comments made after Eq. (B10) it is seen that no reason exists why ξ should deviate much from unity, so that these inequalities could be approximate equalities.

No discussion of any gauge model is complete without a description of the potential surface which is consistent with the choice of scalar field vacuum expectation values. Strict renormalizability requires the inclusion of all products of degree ≤ 4 of such fields in the Lagrangian, even though it may not be necessary to have all these terms appear with nonzero coefficients in the tree approximation. In the present case there is the additional feature that one needs to show, in this approximation, that not only the magnitude of the vacuum expectation values can be inferred from the existence of a minimum of the surface but also the relative direction of four vectors like $H(a, a)$ and $H(ib, -ib)$. This problem of fixing directions appeared for the first time in the Lee[47]-Prentki-Zumino[17] model. The method of fixing directions, spelled out in Ref. 17, can easily be transposed to the present case. The idea is the following. Consider two four-vectors α and β of which it is desired that they shall be orthogonal in the tree approximation. Arrange the potential so that a coupling term between α and β appears of the form $\lambda(\alpha\cdot\beta)^2$, where λ is positive by decree. Then one of the minimization conditions is $\alpha\cdot\beta=0$, irrespective of the (positive) value of λ. If radiative corrections necessitate a "turning" of the vectors such that $\alpha\cdot\beta$ is no longer zero, then this term will have to be dealt with jointly with the rest of the potential in order to find the adjusted minimum. By following the method of Ref. 17 a set of ranges of the coefficients in the potential compatible with the desired vacuum expectation values is readily found. This is not to convey the view that this potential problem is elementary. As do others, this author believes on the contrary that this is an area where much still has to be learned.

VI. O(4)×$\mathcal{G}$; THE EXAMPLE $\mathcal{G}$= U(1)

A. Motivation

Let us first recapitulate what are the main features of the model discussed in the foregoing.

(I) Absence of neutral $|\Delta S|=1$ hadron currents to $O(G)$.

(II) Sufficient suppression of $|\Delta S|=2$ transitions due to vector-meson pairs.

(III) Sufficient suppression of the $K_L\to\mu\bar{\mu}$ amplitude. These three points were discussed in Sec. IV.

(IV) $\nu_\mu e$ and $\bar{\nu}_\mu e$ scattering are forbidden to $O(G)$ even though a neutral current appears.

(V) Similarly for $\nu_\mu+p\to\nu_\mu+p$, $\nu_\mu+p\to\nu_\mu+\Delta$, etc.

(VI) The neutral current is the sole source of nonleptonic decay to $O(G)$. The isospin assignments of charmed quarks are such that one can implement the $|\Delta I|=\frac{1}{2}$ nonleptonic decay rule. The rather obscure $|\Delta I|=\frac{3}{2}$ nonleptonic interaction which appears in the "single W" weak interaction theory is eliminated.

(VII) CP violation is incorporated. On-shell CP-violating amplitudes are superweak.

(VIII) A modification appears of the elastic ν_e-e and $\bar{\nu}_e-e$ scattering cross sections which involves the Cabibbo angle and the ξ parameter.

The following connected set of questions motivated the work to be reported on in the rest of this paper.

(a) To what extent are these combined features unique to the O(4) scheme? I do not know the answer in general, but would like to explain that at least one class of models exists with these characteristics.

(b) A CP-violating parameter appears in the foregoing, due to $W^{1,2}$ mixing. This parameter is logarithmically divergent and thus needs a renormalization. While the statement may seem reasonable that this renormalization does not change the order of magnitude of the effect, one will nevertheless want to ask if such renormalization can possibly be avoided.

(c) The scalar field system needed to generate masses is a fairly complicated one for O(4) as we just saw in Sec. V B. The question arises whether alternatives exist also in this respect.

It will be clear from the foregoing that the main results of some positive interest all emerged essentially from the examination of the f^L-contributions to the currents. This leads one to follow a strategy in which these contributions are qualitatively unmodified but where the f^R are treated differently. In particular we shall examine here what happens if the f^R are scalar with respect to O(4).

As we shall see in at least one example, such a treatment is possible. Furthermore in this case the *CP* problem can be treated differently, for the following reason. As we saw in the discussion of Eqs. (5.4) and (5.5), if the f^R are in O(4), then $\bar{e}e$ terms enter into the neutral current $J^{(0)}$. This in turn led to the *CP* phase choice made in Eq. (3.4). But if the f^R are scalar in O(4), then $\bar{e}e$ cannot enter into $J^{(0)}$. This then allows for the freedom to turn to the general discussion of the lepton representations Eqs. (3.1) and (3.2). It is this point which will be explored in some detail in what follows.

It was seen in Sec. V A and Appendix A that the equality $g_1 = g_2 = g$ favors the convergent traits of the theory. This equality will therefore be maintained from here on in.

B. O(4) ×U(1), General Features

The definitions of D_μ and Q are

$$D_\mu = \partial_\mu - ig(\vec{A}_\mu \cdot \vec{t} + \vec{C}_\mu \cdot \vec{\rho}) - ig' B_\mu Y, \tag{6.1}$$

where $\vec{t}, \vec{\rho}$ satisfy the same commutation relations as before, while $[\vec{t}, Y] = [\vec{\rho}, Y] = 0$; and

$$Q = t_3 + \rho_3 + Y. \tag{6.2}$$

From Eqs. (2.4) and (2.5):

$$e = \frac{gg'}{(g^2 + 2g'^2)^{1/2}} , \tag{6.3}$$

$$A_\mu = \frac{1}{(g^2+2g'^2)^{1/2}} [g'(A_\mu^3 + C_\mu^3) + gB_\mu]. \tag{6.4}$$

Thus a mixing angle γ appears:

$$g = \frac{e\sqrt{2}}{\sin\gamma}, \quad g' = \frac{e}{\cos\gamma} . \tag{6.5}$$

We wish to retain the definition of Z:

$$Z_\mu = \frac{A_\mu^3 - C_\mu^3}{\sqrt{2}} . \tag{6.6}$$

By orthogonality the further new neutral vector field, call it V_μ, is then to be

$$V_\mu\sqrt{2} = \frac{1}{(g^2+2g'^2)^{1/2}} [g(A_3 + C_3) - 2g'B]. \tag{6.7}$$

We also wish to retain the definitions Eqs. (2.6), (2.7), of W^1 and W^2. (Thus we continue to use the "type I" version; see Sec. II.) The first question is to find a scalar field system which supports the definitions of Z, V, W^1, W^2 as normal modes.

To this end we continue to employ $H(a, a)$ and $H(ib, -ib)$ introduced in O(4), but must now characterize their representation as $(\frac{1}{2}, \frac{1}{2})^{(0)}$ where the superscript (0) indicates that they have $Y = 0$. Therefore we shall now call these multiplets $H^{(0)}(a, a)$, $H^{(0)}(ib, -ib)$ in order to distinguish them from the further quartet $(\frac{1}{2}, \frac{1}{2})^{(1)}$ which we will denote by $H^{(1)}(c)$ and which completes the scalar quartets to be used here. $H^{(1)}(c)$ has charge states $(++, +, +, 0)$ and

$$\langle H^{(1)}(c)\rangle = (0, 0, 0, c). \tag{6.8}$$

This set of H's does what is required and one finds

$$M_1^2 = \frac{e^2}{\sin^2\gamma}(4b^2 + c^2), \tag{6.9}$$

$$M_2^2 = \frac{e^2}{\sin^2\gamma}(4a^2 + c^2), \tag{6.10}$$

$$M_0^2 = \frac{4e^2}{\sin^2\gamma}(a^2 + b^2), \tag{6.11}$$

$$M_V^2 = \frac{8e^2}{\sin^2 2\gamma} c^2, \tag{6.12}$$

where $M_{0,1,2}$ are again the respective Z, W^1, W^2 masses and M_V is the V mass. Thus we have the sum rule

$$M_1^2 + M_2^2 = M_0^2 + M_V^2 \cos^2\gamma, \tag{6.13}$$

which expresses the mixing angle γ in terms of the vector-meson masses, while

$$\tan\theta = \frac{4b^2 + c^2}{4a^2 + c^2} , \tag{6.14}$$

$$\xi = 1 - \frac{c^2}{2a^2 + 2b^2 + c^2} . \tag{6.15}$$

We now use a result of Sec. VII which shows that (up to phase) G is given by the same Eq. (3.9) as for O(4):

$$\frac{G}{\sqrt{2}} = \frac{g^2}{16}\left(\frac{1}{M_1^4} + \frac{1}{M_2^4}\right)^{1/2}, \tag{6.16}$$

but where the definition of g is now changed to Eq. (6.5). Therefore

$$M_1 = \frac{37.2}{(\sin\gamma\cos\theta)^{1/2}} , \quad M_2 = \frac{37.2}{(\sin\gamma\sin\theta)^{1/2}} , \tag{6.17}$$

so that Eqs. (3.10) now turn into lower bounds for M_1, M_2.

D_μ becomes

$$D_\mu = \partial_\mu - ieQA_\mu - \frac{ie}{\sin\gamma}\{(t_3 - \rho_3)Z_\mu + (1/\sqrt{2})[(t_+ - \rho_+)W_\mu^1 + (t_+ + \rho_+)W_\mu{}^2 + \text{H.c.}]\} - ie[(t_3 + \rho_3)\cot\gamma - Y\tan\gamma]V_\mu . \tag{6.18}$$

The R properties of the vector mesons are extended to [see Eq. (2.33), we stay with "type I"]

$$W^1 \to -W^1,\ W^2 \to W^2,\ Z \to -Z,\ A \to A,\ V \to V. \tag{6.19}$$

Next we turn to the fermion representations and introduce the following assumption:

All left-handed fermions are quartets $(\frac{1}{2},\frac{1}{2})^{(0)}$; all right-handed fermions are $(0,0)^{(Q)}$, that is, they are singlets in O(4) and have $Y=Q$. As a first consequence, it follows that no neutral f^R whatsoever appears in any of the currents.

More specifically, the f^{lL} representations are taken to be given again by Eqs. (3.1) and (3.2) but without the restriction to Eq. (3.4), while the f^{hL} are again given by Eqs. (4.1) and (4.2). Quark-mass terms arise as follows. $\overline{Q}^{1L}H^{(1)}(c)q_R^- + \text{H.c.}$, $\overline{Q}^{2L}H^{(1)}(c)r_R^- + \text{H.c.}$ give mass to q^-, r^-. $\overline{Q}^{1L}H^{(1)*}(c)\mathcal{P}_R + \text{H.c.}$, $\overline{Q}^{2L}H^{(1)*}(c)q_R^+ + \text{H.c.}$ do likewise for $\mathcal{P}$ and q^+. To treat the neutral quarks consider a linear combination with arbitrary coefficients of the set of sixteen interactions $\overline{Q}^{iL}H^{(0)}f_R^0 + \text{H.c.}$, where $i=1$ or 2; $H^{(0)}$ is $H^{(0)}(a,a)$ or $H^{(0)}(ib,-ib)$ and f_R^0 is $\mathfrak{N}_R$, λ_R, q_R^0, or r_R^0. It is easily seen that this set of couplings diagonalizes the mass matrix. At this point one realizes the difference between the use of one complex $H^{(0)}$ field as compared with two real $H^{(0)}$'s. The degrees of freedom are the same in both cases, but the number of coupling constants is only eight for the case of one complex field. As a result one finds that several quarks remain massless if only one complex $H^{(0)}$ were used.

We discuss next the question of anomalies in the trilinear vector-meson vertices due to triangular fermion insertions.[21] For O(4), such anomalies cannot arise due to the group structure. This is not so for O(4)×U(1), yet anomalies still do not appear due to the fermion representations adopted here in which equal numbers of fermions occur with $Q=e$ and with $Q=-e$. In more detail the following three types of vertices occur: (1) the R-invariant vertex W^1W^2Z which lies entirely in O(4) and is therefore finite; (2) the R-invariants (W^1W^1V) and (W^2W^2V) which are readily shown to be finite as well due to $Q=\pm e$ cancellations, as it should be; (3) these two types appear in the (R-invariant) Lagrangian. In addition there appears the induced trilinear (W^1W^2V) which is also finite since it does not appear in $\mathcal{L}$ due to its R noninvariance. Its finiteness is then a further application of Weinberg's lemma. It should be stressed that this absence of anomalies has nothing to do with any choice of phases in Eqs. (3.1) and (3.2) since the anomaly problem is an affliction which can be diagnosed already in the zero fermion mass limit. Therefore we can treat the question with the help of the representation Eq. (3.11). In other words, the anomaly problem is disjoint from the CP problem.

Another quantity which is independent of the CP problem is the neutral current $J^{(V)}$ coupled to the V meson. Its fermion part is

$$J^{(V)} = -ie\sum[(\bar{f}^+f^+ - \bar{f}^-f^-)_L\cot\gamma - (\bar{f}^+f^+ - \bar{f}^-f^-)_R\tan\gamma], \tag{6.20}$$

where the summation is over all fermions with the appropriate electric charge. No argument will be advanced in this paper which delimits γ. Note however the curious circumstance that

$$\gamma = \tfrac{1}{4}\pi \to J^{(V)} = -ie\sum\bar{f}Q\gamma_\mu\gamma_5 f, \tag{6.21}$$

where Q is the electric charge operator and the sum goes over all fermions: This is an axial-vector current coupled to the electric charge. In any event, $J^{(V)}$ does not affect the criteria (I), (II), (III) stated in Sec. IV.

The hadronic parts of $J^{(1,2,0)}$ are

$$J^{(1)h} = \frac{-ie}{\sqrt{2}\sin\gamma}[\overline{\mathcal{P}}(\mathfrak{N}+A_1^0) + (\overline{\mathfrak{N}}+\overline{A}_1^0)q^- + \bar{q}^+(\mathfrak{N}-A_1^0) + (\overline{\mathfrak{N}}-\overline{A}_1^0)r^-]_L, \tag{6.22}$$

$$J^{(2)h} = \frac{-ie}{\sqrt{2}\sin\gamma}[\overline{\mathcal{P}}(\lambda+A_2^0) - (\bar{\lambda}+\overline{A}_2^0)q^- - \bar{q}^+(\lambda-A_2^0) + (\bar{\lambda}-\overline{A}_2^0)r^-]_L, \tag{6.23}$$

$$J^{(0)h} = \frac{ie}{\sin\gamma}[(\overline{\mathfrak{N}}+\bar{\lambda})q^0 - (\overline{\mathfrak{N}}-\bar{\lambda})r^0 + \text{H.c.}]_L, \tag{6.24}$$

where $A_{1,2}^0$ were defined in Eq. (4.5). These expressions differ in two respects from Eqs. (4.3)–(4.6): (1) the current symbols $J^{(0,1,2)}$ no longer have to carry the label L, they are the complete hadronic currents since no f^R terms can enter them; (2) As compared to O(4), the only difference is a scale factor $\sqrt{2}/\sin\gamma$, as was already noted[48] in PRL. Furthermore, this scale factor is absorbed in the definition Eq. (6.16).

It is evident, therefore, that the discussion of Sec. IV concerning the desiderata (I), (II), and (III) applies verbatim to O(4)×U(1) and is moreover complete in the sense that no f^R modifications arise. (For $K_L \to \mu\bar{\mu}$ we need the lepton currents as well. As will be shown in Sec. VII the above statement also holds for that decay.) Likewise, the results of Sec. IV for nonleptonic decays apply as well to O(4)×U(1). Equation (4.12) reappears unmodified. Finally, the hadronic contribution to $W^{1,2}$ mixing remains as in Eqs. (4.13) and (4.14) up to a further factor $2/\sin^2\gamma$.

Note added in proof. In O(4)×U(1), the quark charges are not necessarily integral. For non-integral charges, Eq. (6.20) is modified.

VII. THE CP PROBLEM IN O(4)×$\mathcal{G}$

We saw in the preceding section that the right-handed hadrons had departed from the currents $J^{(0,1,2)}$ and that, otherwise, these currents are as they were for O(4) up to a redefinition of g. Qualitatively, these properties are independent of the choice of $\mathcal{G}$. Similar comments apply to the leptonic contribution, to be considered next. We continue the exemplification with $\mathcal{G}\equiv$ U(1).

From Eqs. (2.24)–(2.26) and (3.1)–(3.3) one finds that the complete expressions for $J^{(0,1,2)}$ are

$$J^{(1)l}=-\tfrac{1}{2}ig\,[\,(a_1-a_2^*)\bar{x}^+x^0+(a_2+a_1^*)\bar{x}^+\nu_e+(a_1^*-a_2)\bar{x}^0e+(a_2^*+a_1)\bar{\nu}_e e$$
$$+(b_1-b_2^*)\bar{y}^+y^0+(b_2+b_1^*)\bar{y}^+\nu_\mu+(b_1^*-b_2)\bar{y}^0\mu+(b_2^*+b_1)\bar{\nu}_\mu\mu]_L\,, \tag{7.1}$$

$$J^{(2)l}=-\tfrac{1}{2}ig\,[\,(a_1+a_2^*)\bar{x}^+x^0+(a_2-a_1^*)\bar{x}^+\nu_e-(a_1^*+a_2)\bar{x}^0e-(a_2^*-a_1)\bar{\nu}_e e$$
$$+(b_1+b_2^*)\bar{y}^+y^0+(b_2-b_1^*)\bar{y}^+\nu_\mu-(b_1^*+b_2)\bar{y}^0\mu-(b_2^*-b_1)\bar{\nu}_\mu\mu]_L\,, \tag{7.2}$$

$$J^{(0)l}=(ig/\sqrt{2}\,)[\,(|a_1|^2-|a_2|^2)(\bar{x}^0x^0-\bar{\nu}_e\nu_e)-2a_1^*a_2\bar{x}^0\nu_e-2a_1a_2^*\bar{\nu}_e x^0$$
$$+(|b_1|^2-|b_2|^2)(\bar{y}^0y^0-\bar{\nu}_\mu\nu_\mu)-2b_1^*b_2\bar{y}^0\nu_\mu-2b_1b_2^*\bar{\nu}_\mu y^0]_L\,, \tag{7.3}$$

which reduce to Eqs. (3.5)–(3.7) if Eq. (3.4) is used. We now see $\bar{\nu}_\mu\nu_\mu$ currents emerge in $J^{(0)l}$.

One point needs to be cleared up related to the constraints (III) of Sec. IV concerning $K_L\to\bar{\mu}\mu$. It is easily shown that Eq. (4.10) is unaffected by the use of the more general currents Eqs. (7.1) and (7.2): The $K_L\to\bar{\mu}\mu$ constraint has nothing to do with any particular choice of the CP parameters.

From Eqs. (6.22), (6.23) and (7.1), (7.2) it follows that μ-e universality to $O(G)$ for $\mathcal{P}$ and for λ decays implies that

$$\mathrm{Re}a_1a_2=\mathrm{Re}b_1b_2\,. \tag{7.4}$$

The Cabibbo condition Eq. (1.3) takes the form

$$\lambda_0M_1^{-4}+\lambda_1M_2^{-4}+\lambda_2M_1^{-2}M_2^{-2}=0\,,$$

where the λ's depend on the a's and b's. One can look upon this relation as a quadratic equation for $\tan\theta$ in terms of the λ's. I choose not to do so but, instead, to satisfy the relation by putting the λ's $=0$. This gives

$$\mathrm{Re}a_1a_2=\mathrm{Re}b_1b_2=0\,, \tag{7.5}$$

$$[\,|b_2|^2-|b_1|^2][\,|a_2|^2-|a_1|^2]+4\,\mathrm{Im}a_1a_2\,\mathrm{Im}b_1b_2=0\,. \tag{7.6}$$

Put

$$a_1=\cos(\tfrac{1}{2}\vartheta)e^{i\phi_1}\,,\quad b_1=\cos(\tfrac{1}{2}\theta)e^{i\psi_1}\,,$$
$$a_2=\sin(\tfrac{1}{2}\vartheta)e^{i\phi_2}\,,\quad b_2=\sin(\tfrac{1}{2}\theta)e^{i\psi_2}\,. \tag{7.7}$$

This is an overparametrization since the a's and b's are coefficients of complex fields. We now choose the following convenient phase convention:

$$\phi_1=\psi_1=0\,. \tag{7.8}$$

Then Eqs. (7.5) and (7.6) are solved by

$$\phi_2=\psi_2=\tfrac{1}{2}\pi\,,\quad \theta=\vartheta+\tfrac{1}{2}\eta\pi\,,\quad \eta=\pm1 \tag{7.9}$$

and (absorb a factor $e^{i\vartheta/2}$ in x^0, ν_e; a factor $e^{i\theta/2}$ in y^0, ν_μ)

$$J^{(1)l}=-\tfrac{1}{2}ig\,[\bar{x}^+(x^0+\nu_e)+(\bar{x}^0+\bar{\nu}_e)e$$
$$+\bar{y}^+(y^0+\nu_\mu)+(\bar{y}^0+\bar{\nu}_\mu)\mu]_L\,, \tag{7.10}$$

$$J^{(2)l}\,-\tfrac{1}{2}ig\,[A\bar{x}^+(x^0-\nu_e)-A^*(\bar{x}^0-\bar{\nu}_e)e$$
$$+B\bar{y}^+(y^0-\nu_\mu)-B^*(\bar{y}^0-\bar{\nu}_\mu)\mu]_L\,, \tag{7.11}$$

$$J^{(0)l}=(ig/\sqrt{2}\,)\{\cos\vartheta[\,(\bar{x}^0x^0-\bar{\nu}_e\nu_e)-i\eta(\bar{\nu}_\mu y_0-\bar{y}_0\nu_\mu)]$$
$$-\sin\vartheta[-\eta(\bar{\nu}_\mu\nu_\mu-\bar{y}^0y^0)$$
$$+i(\bar{\nu}_e x^0-\bar{x}^0\nu_e)]\}_L\,, \tag{7.12}$$

$$A=e^{-i\vartheta}\,,\quad B=-i\eta e^{-i\vartheta}\,. \tag{7.13}$$

Thus ϑ emerges as a rotational angle in the "neutrino current plane" whose "axes" are $(\bar{\nu}_e\nu_e;\,\bar{\nu}_\mu\nu_\mu)$. The special form Eq. (3.5)–(3.7) previously employed for these currents corresponds to the choice

$$\vartheta=0,\quad \eta=1\,,\quad y_0\to -iy_0\,. \tag{7.14}$$

The phase convention Eq. (7.8) has been so chosen that $J^{(1)l}$ has no CP phases at all. This is convenient for computational purposes. As Eq. (7.14) shows, this implies a slight redefinition of y^0 as compared to what appeared in Eq. (3.5). The latter equation was maintained for the sake of continuity of presentation between PRL and the present paper. I beg the reader's indulgence for this midway change of conventions.

At this point we are ready to derive Eqs. (1.14) and (1.15) for $\sigma(\bar{\nu}_e e)$ and $\sigma(\nu_e e)$, independently of the remaining question how to choose the continuous parameter ϑ and the discrete parameter η.

Namely, since $J^{(V)}$ contains $\bar{e}e$ but not $\bar{\nu}_e\nu_e$ [Eq. (6.20)] and $J^{(0)}$ contains $\bar{\nu}_e\nu_e$ but not $\bar{e}e$, the scattering amplitudes in question arise to $O(G)$ exclusively from the charged currents $J^{(1)}, J^{(2)}$. With the definition Eq. (1.9) for θ, Eqs. (1.14) and (1.15) follow at once. Also, the definition of G given in Eq. (6.16) is an immediate consequence of Eqs. (6.5), (7.10), and (7.11).

We come now to the principal topic of this section, the discussion of the $W^{1,2}$ mixing [at[29] $q(W^1)=q(W^2)=0$] for the more general situation defined by Eqs. (7.10) and (7.11). This mixing is given by

$$+\frac{g^2}{8\pi^2}i\left[m^2(x^0)\,\mathrm{Im}A\lim\int_0^{\Lambda^2}\frac{dz}{z+m^2(x^0)}+m^2(y^0)\,\mathrm{Im}B\lim\int_0^{\Lambda^2}\frac{dz}{z+m^2(y^0)}\right]$$
$$-\frac{g^2}{16\pi^2}\{m^2(x^0)[Am^2(x^+)\phi(x^0x^+)-A^*m^2(e)\phi(x^0e)]+m^2(y^0)[Bm^2(y^+)\phi(y^0y^+)-B^*m^2(\mu)\phi(y^0\mu)]\}\,, \quad (7.15)$$

where the notations of Eqs. (3.20) and (5.11) have been used. ρ and σ, defined by Eqs. (3.16) and (3.17), are the respective real and imaginary part of Eq. (7.15). We note the following.

(1) The divergent part of Eq. (7.15) is contained in its first line and is purely imaginary. Thus ρ is finite independently of the choice of ϑ and η in Eqs. (7.10)–(7.13). This is, once more, a consequence of the Weinberg lemma: Since $\mathcal{L}_{12}$ remains of the form Eq. (2.16) (with redefined g), the argument given in Sec. III for the finiteness of ρ applies unaltered in the present case as well.

(2) It follows from Eq. (7.13) that $\mathrm{Im}A$ and $\mathrm{Im}B$ cannot *simultaneously* and *separately* be zero. [The case of O(4) discussed in PRL and in Sec. III corresponds to $\mathrm{Im}A=0$, $\mathrm{Im}B=-1$.] It follows that σ must remain logarithmically divergent *unless a constraint exists between the E^L and the M^L multiplet*, namely [use Eq. (7.13)]

$$m^2(x^0)\sin\vartheta+m^2(y^0)\eta\cos\vartheta=0\,. \quad (7.16)$$

I have examined further the implications of this constraint by considering a special way to satisfy it, namely

$$m(x^0)=m(y^0)\,, \quad (7.17)$$

$$\sin\vartheta+\eta\cos\vartheta=0\Rightarrow\vartheta=\begin{cases}\dfrac{3\pi}{4},\dfrac{7\pi}{4} & (\eta=1)\\[2mm] \dfrac{\pi}{4},\dfrac{5\pi}{4} & (\eta=-1)\,.\end{cases} \quad (7.18)$$

The motivation for doing so is the following. Clearly it would be desirable to have σ finite to all orders and there is no reason that I could find why Eq. (7.16) would be unmodified in the presence of higher-order effects. On the other hand, as will be discussed below, it would appear that from the more special Eqs. (7.17) and (7.18) it may be possible to abstract an invariance argument which has bearing on the general finiteness of σ.

Observe that Eq. (7.18) has a discrete set of solutions and that these can be distinguished only if one goes at least to $O(g^4)$. I have verified that none of the arguments to follow depends in any crucial way on which choice of solution of Eq. (7.18) is made. For *ease of presentation* only I shall continue with one particular solution from here on in:

$$\vartheta=\tfrac{1}{4}\pi\,,\quad \eta=-1\,. \quad (7.19)$$

Then the currents are: $J^{(1)}$ as in Eq. (7.10), and

$$J^{(2)l}=-\tfrac{1}{2}ig[\epsilon^*\bar{x}^+(x^0-\nu_e)-\epsilon(\bar{x}^0-\bar{\nu}_e)e+\epsilon\bar{y}^+(y^0-\nu_\mu)-\epsilon^*(\bar{y}^0-\bar{\nu}_\mu)\mu]_L\,, \quad (7.20)$$

$$\epsilon=e^{i\pi/4}\,, \quad (7.21)$$

$$J^{(0)l}=\tfrac{1}{2}ig[\bar{x}^0x^0+\bar{y}^0y^0-\bar{\nu}_e\nu_e-\bar{\nu}_\mu\nu_\mu-i(\bar{\nu}_ex^0-\bar{x}^0\nu_e-\bar{\nu}_\mu y^0+\bar{y}^0\nu_\mu)]_L\,. \quad (7.22)$$

The current structure we have now arrived at has the property that all four currents $J^{(0)}$, $J^{(1)}$, $J^{(2)}$, $J^{(V)}$ *satisfy separately the property of μe universality*. Thus all nonuniversality appears only in higher-order weak effects and in the fermion-mass problem.

We are now prepared to give the proof of the assertion made in Eq. (1.17) about inelastic hadron production by ν_μ beams. To $O(G)$ the effect is entirely due to $J^{(0)}$ of which the hadronic part was given in Eq. (6.24), the leptonic part in Eq. (7.22). In quark language, the reactions $\nu_\mu+\mathfrak{N}\to\nu_\mu+(q^0$ or $r^0)$ can take place. But since $\mathfrak{N}$ is a nucleon constituent, this means that charmed hadron production should occur to $O(G)$ in ν_μ-induced reactions. $\nu_\mu+\mathfrak{N}\to y^0+(q^0$ or $r^0)$ can also occur.

Returning to the $W^{1,2}$-mixing problem, as a result of Eqs. (7.15) and (7.19) we now have the following finite expressions for ρ and σ:

$$\rho=-\frac{\alpha}{2\pi}\frac{1}{\sqrt{2}\,\sin^2\gamma}\{m^2(x^0)[m^2(x^+)\phi(x^0x^+)-m^2(e)\phi(x^0e)]+m^2(y^0)[m^2(y^+)\phi(y^0y^+)-m^2(\mu)\phi(y^0\mu)]\}\,, \quad (7.23)$$

$$\sigma = -\frac{\alpha}{2\pi}\frac{1}{\sqrt{2}\,\sin^2\gamma}\{m^2(x^0)[-m^2(x^+)\phi(x^0x^+) - m^2(e)\phi(x^0e)] + m^2(y^0)[m^2(y^+)\phi(y^0y^+) + m^2(\mu)\phi(y^0\mu)]\}\ , \qquad (7.24)$$

where it is understood that $m(x^0) = m(y^0)$, Eq. (7.17).

These expressions bring ρ and σ on rather an equal footing. Theoretically this is interesting since we appear to have a counterterm for σ, *if need be*, but not for ρ. Of course this is no paradox. The Weinberg lemma says neither less nor more than that if a counter structure exists, then and only then is the occurrence of a divergence compatible with renormalizability. But it is not said that the presence of such a structure necessarily implies the occurrence of divergences. This leads one to the theoretical problem whether the finiteness of ρ and σ can perhaps be understood because the theory has become, in a sense to be specified, "more invariant" as a result of the assumed Eqs. (7.17) and (7.18). I would like to present an argument to this effect which is based on a combination of (1) Weinberg's lemma, (2) discrete invariances, (3) power counting. The method will first be applied to the finite quantity ρ, in order to understand its progression to convergence in two stages. Then we shall turn to σ. It should be noted that this stage-wise progression has no analog in other applications of Weinberg's lemma (which I have seen so far) since there potential logarithmic divergences only are at stake. Three discrete invariances will be used.

(1) R invariance. From Eq. (2.32) it is seen that it is necessary for the validity of this invariance that x^0 and ν_e are mass degenerate, and likewise for y^0 and ν_μ [see also the discussion of Eq. (3.11)]. Thus the applicability of R necessitates that $m(x^0) = m(y^0) = 0$. But the *effective* interactions $\mathcal{H}^{\rm re}$ and $\mathcal{H}^{\rm im}$ [Eqs. (3.16) and (3.17)] are not invariant under R, see Eq. (6.19), while $\mathcal{L}_{12}$ [Eq. (2.16)] of course is invariant.[49] Since an invariant term cannot counter a noninvariant term, it follows that ρ and σ must vanish *insofar as R* applies. That is, they must go to zero as $m(x^0)$ and $m(y^0)$ independently $\rightarrow 0$. This is borne out by Eqs. (7.23) and (7.24) but it holds to any order in g. Furthermore, by the chirality argument of Sec. III both ρ and σ must be functions of $m^2(x^0), m^2(y^0)$. Observe that for $O(4)\times\mathcal{G}$ there are no tadpole graphs at all of the types given in Fig. 2(b), 2(c), and 2(d). Thus ρ and σ must vanish like $m^2(x^0), m^2(y^0)$. By power counting we have gained two powers of momenta in the Feynman integrals, so that a potentially quadratic divergence in either ρ or σ can now at most be logarithmic only.

(2) F invariance. The total invariance of the gauge group is fully specified by its continuous part and by R. For our discussion it is important to ask what residual invariances remain, if any, if we take $m(x^0)\neq 0$, $m(y^0)\neq 0$. We shall locate such an invariance, of the discrete variety, and will call it a partial discrete symmetry. Here "partial" connotes that it is a surviving symmetry even if the full symmetry of the gauge group is broken *to some specified extent*.

The partial discrete symmetry now to be discussed is the transformation

$$F:\ t\rightarrow -\rho^*,\quad \rho\rightarrow -t^*,\quad Y\rightarrow -Y, \qquad (7.25)$$

where the asterisk denotes complex conjugation. F is somewhat analogous to the G parity of SU(2). G involves an exchange of positive and negative charge in isotopic multiplets, which is achieved by the charge conjugation factor C contained in the usual G-parity operator. In the present problem we can also change the sign of the electric charge but without the need of the C operation. Indeed the operator corresponding to F is given by

$$F = P_Y R e^{-i\pi\rho_2 + i\pi t_2},\quad \text{so}\ [F,R]=0, \qquad (7.26)$$

where P_Y induces $Y\rightarrow -Y$. Thus F acting on the $Y=0$ fermion quartets gives

$$F(f_1,f_2,f_3,f_4) = (-f_4, f_2, f_3, -f_1), \qquad (7.27)$$

and similarly[50] for the action on the scalar quartets $H^{(0)}$. Acting on the leptons F gives $x^+\leftrightarrow -e$; $y^+\leftrightarrow -\mu$. Therefore we are entitled to operate with F even if the full gauge symmetry is broken and $m(x^0)$, $m(y^0)$ are nonzero, since F leaves f_2, f_3 in place. However, F is of use only *insofar as* $m(x^+) = m(e)$, $m(y^+) = m(\mu)$. This is not to advertise such further degeneracies as physically pertinent, but rather to find a further mathematical tool which will help us to examine the dependence of ρ and σ on the masses of the charged leptons.

F acts on the vector mesons as follows:

$$\begin{aligned} &W^1\rightarrow -W^{1\dagger},\quad W^2\rightarrow W^{2\dagger},\\ &A\rightarrow -A,\quad Z\rightarrow Z,\quad V\rightarrow -V. \end{aligned} \qquad (7.28)$$

This transformation again leaves $\mathcal{L}_{12}$ invariant. However, Eqs. (3.16) and (3.17) show that $\mathcal{H}^{\rm re}$ is F-noninvariant while $\mathcal{H}^{\rm im}$ is F-invariant. Thus *insofar as F* applies, $\mathcal{L}_{12}$ cannot counter $\mathcal{H}^{\rm re}$ but it can counter $\mathcal{H}^{\rm im}$. Continuing with $\mathcal{H}^{\rm re}$, it follows that ρ must vanish if $m^2(x^+)\rightarrow m^2(e)$, $m^2(y^+)\rightarrow m^2(\mu)$, $m(x^0), m(y^0)$ *held fixed*. But the same need not be true for σ, and indeed Eq. (7.24) shows that it is not true. Then by power counting we have now isolated four powers of mass such as

$m^2(x^0)\,[m^2(x^+)-m^2(e)]$; or $m^2(y^0)\,[m^2(y^+)-m^2(\mu)]$ which suppress the integral for ρ by four powers, so that the potentially quadratic divergence has been suppressed to gain a finite answer. But for σ we have at this stage still no argument for a suppression beyond the logarithmic stage.

(3) U invariance. This third step constitutes an attempt to make use of the special realization Eqs. (7.17), (7.18), and (7.19) of Eq. (7.16) in such a way that it generates a third partial discrete symmetry, U defined by

$$U=(E\leftrightarrow M), \tag{7.29}$$

where $E\leftrightarrow M$ is the "universality substitution" in which each member of the E quartet is exchanged with the corresponding member of the M-quartet; likewise for the right-handed lepton singlets. Thus $x^+\leftrightarrow y^+$, $x^0\leftrightarrow y^0$, $\nu_e\leftrightarrow\nu_\mu$, $e\leftrightarrow\mu$. Because of Eq. (7.17), the substitution invariance on the neutrals is inherent to the scheme in any event, even for nonzero $m(x^0)=m(y^0)$. Just as for F, the purpose of considering U is once again to judge the dependence on the charged lepton masses. U is the identity operation insofar as all other particles but leptons are concerned. Thus $\mathfrak{L}_{12}$ is invariant under U.

From Eq. (7.21) we see that $J^{(2)l}$ can be written as $(J^{(2)l}_{\rm re}+iJ^{(2)l}_{\rm im})/\sqrt{2}$, where

$$J^{(2)l}_{\rm re}=-\tfrac{1}{2}ig\,[\bar{x}^+(x^0-\nu_e)-(\bar{x}^0-\bar{\nu}_e)e+\bar{y}^+(y^0-\nu_\mu)-(\bar{y}^0-\bar{\nu}_\mu)\mu]_L\,, \tag{7.30}$$

$$J^{(2)l}_{\rm im}=-\tfrac{1}{2}ig\,[-\bar{x}^+(x^0-\nu_e)-(\bar{x}^0-\bar{\nu}_e)e+\bar{y}^+(y^0-\nu_\mu)+(\bar{y}^0-\bar{\nu}_\mu)\mu]_L\,. \tag{7.31}$$

Equations (7.10) and (7.30) show that $J^{(1)l}$ and $J^{(2)l}_{\rm re}$ are even under U while $J^{(2)l}_{\rm im}$ is odd. Thus the term

$$\mathfrak{L}'=i[(J^{(2)l}_{\rm im}W^2-\text{H.c.})-ig(\bar{\nu}_e x^0-\bar{x}^0\nu_e-\bar{\nu}_\mu y^0+\bar{y}^0\nu_\mu)_L Z/2]$$

is the only part of the Lagrangian that is odd under U. The entire CP violation stems (to any order) from an *odd* number of times in which $\mathfrak{L}'$ intervenes. The quantity $\mathcal{H}^{\rm im}$ defined in Eq. (3.17) must therefore change sign under U hence, since W^1, W^2 are unaffected, σ must be odd under U. But then *insofar as U* is applicable, the invariance $\mathfrak{L}_{12}$ cannot counter the noninvariant $\mathcal{H}^{\rm im}$. Thus σ must go to zero as $m(x^+)=m(y^+)$, $m(e)=m(\mu)$. By the same argument as for F, power counting isolates $m^2(x^0)$ times $m^2(y^+)-m^2(x^+)$ or times $m^2(e)-m^2(\mu)$ as a factor and the potentially quadratic divergence for σ has been suppressed to a finite answer.

In order to get more familiar with this new invariance, I have examined a few $O(g^4)$ contributions to σ, namely such which do not require mass and wave-function renormalizations. As is obvious, this provides a very incomplete sample, yet it is perhaps illuminating to report what happens, since each of the graphs have their own characteristic lepton mass dependence; and that is the issue. There are higher-order contributions to σ which have again a single lepton loop, such as $W_1\to W_2+Z\to W_1+Z\to W_2$ where the middle link stems from $W^{1,2}$ mixing $O(g^2)$. Such graphs also give a finite contribution to σ. Of somewhat more interest is the set of graphs whose structure is as indicated in Fig. 5, with a vector boson inside the loop. This vector boson cannot be a Z because Z couples to neutral leptons only; and it cannot be a V because V couples to charged leptons only. Thus it can only be a W^1 or W^2. This leads to a set of 32 graphs depending on which lepton combinations move in the loop. After some algebra one finds that the contribution to both ρ and σ are again finite if $m(x^0)=m(y^0)$ and if $\vartheta=\frac{1}{4}\pi$, $\eta=-1$ as in Eq. (7.19). To recapitulate, the application of discrete symmetries may perhaps bring the proofs of finiteness of ρ and of σ on an equal footing.

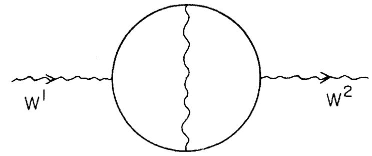

FIG. 5. A set of graphs which contribute $O(g^4)$ to σ.

We must now ask for a treatment of the lepton mass problem which in particular conforms to Eq. (7.17). It is easily seen that the charged leptons acquire mass via the couplings to $H^{(1)}(c)$ just as for the corresponding hadron problem (Sec. VI). The requirements on the neutral lepton masses are met by the interaction

$$(A\bar{E}^L x^0_R+A^*\bar{M}^L y^0_R)H^{(0)}(a,a)+(B\bar{E}^L x^0_R-B^*\bar{M}^L y^0_R)H^{(0)}(ib,-ib)+\text{H.c.}\,, \tag{7.32}$$

$$Aa=\epsilon Bb\,. \tag{7.33}$$

One finds $m(x^0)=m(y^0)=2|A|a$.

To conclude this paper we finally ask the following question. Let the $W^{1,2}$ mixing be of the order given in Eqs. (7.23) and (7.24). Does this enable one to show that the off-shell effect needed to obtain the known CP violation in K decay has the desired magnitude? Since the present theory is of the superweak variety, what we are asking is no less than to see whether one can come to the experimental value for the imaginary part of the $\bar{K}^0$-K^0 mass mixing which is $\approx i|6.10^{-15}|$ MeV/c^2.

It is obvious that the answer to our question must be no, in the rigorous sense. Any hadronic mass difference problem leads one to get involved with aspects of strong-interaction dynamics of a kind we have not yet sufficiently learned to cope with. This hadronic aspect occurs whether or not the mass-splitting mechanism is generated by weak (and/or electromagnetic) interactions. However, one can ask a more limited question: Is it possible to give a crude estimate which indicates whether or not the actual magnitude of the imaginary K_L-K_S mass difference can be obtained without doing any violence to the orders of magnitude of the parameters encountered?

In order to explore this question let us first consider two mechanisms, both of fourth order in the semiweak couplings, which contribute to the *real* mass difference m_r. (a) The box graphs for $\lambda\bar{\mathfrak{N}} \to \bar{\lambda}\mathfrak{N}$ discussed in Sec. IV. As was shown in Ref. 33, these graphs reduce to an effective current-current interaction in the off-shell $q^2=0$ limit for the quarks. This makes it possible to apply PCAC to the estimate of their contribution to the K_1-K_2 mass difference. The physical value of this difference then imposes a constraint on quark mass differences. One can proceed in a similar way in the present case, where two contributions appear: One proportional to $\alpha^2\Delta^+\Delta^-$ (W_1-W_2 exchange), one proportional to $\alpha^2(\Delta^0)^2$ ($2Z$ exchange). One can imagine the various Δ's to vary relative to each other and, as already stated in Ref. 33, this prevents one from drawing any sharp conclusions for these quantities from the consideration of m_r. (b) The foregoing mechanism goes via a connected graph. There exists another one via a pair of unconnected graphs: Consider the transition $\lambda \to q^0$ or $r^0 \to \mathfrak{N}$ via virtual emission and absorption of a Z. This induces $\lambda\mathfrak{N}$ mixing. For $q^2 \to 0$ the mixing parameter is $\sim\alpha\Delta^0 m_Q$, where m_Q is a typical mass of order $m_{\mathfrak{N}}, m_\lambda$. $\bar{\lambda}\mathfrak{N} \to \bar{\mathfrak{N}}\lambda$ can proceed via two such mixings. This results in a contribution to m_r due to the graph drawn in Fig. 6(c), where we have taken the liberty to introduce a "strong" ($K^0, \lambda, \mathfrak{N}$) vertex (of γ_5 type). Calling its strength f, we will get a contribution to $m_r \sim f^2\alpha^2(\Delta^0)^2 m_Q{}^2 m_K{}^{-1}$. We shall not dare to compare this contribution to the one of mechanism (a), since too much depends on f, to say the least. There is no reason why these contributions could not be comparable in magnitude.

Turning now to the imaginary mass difference $m_{\rm im}$, it is clear that we need to inject *CP*-violating mixing due to the leptons. We will get the leading effect by using this mixing *only once*.[51] One can easily see that such single mixing is impossible in the type (a) box graphs mentioned above. However, it is possible in the $\lambda\mathfrak{N}$-mixing contributions of type (b): Employ $\lambda \to \mathfrak{N}$ via Z once and, for *CP* mixing, use the graphs drawn in Figs. 6(a) and 6(b) which yield a contribution $\sim\alpha^2\delta^2(\Delta^+ + \Delta^-)$. The product of the two mixing parameters involved in the contribution to $m_{\rm im}$ is $\alpha^3\delta^2(\Delta^+ + \Delta^-)\Delta^0$ which is one power of α higher than for $m_{\rm re}$. Indeed *if* type (b) for m_r were dominant, the right order for $m_{\rm im}$ would follow if $\delta^2(\Delta^+ + \Delta^-) \sim \Delta^0$. In turn, this allows for ranges in δ and the Δ's. Thus for all Δ's of same order of magnitude the heavy leptons should be quite massive ($\delta \sim 1$); on the other hand δ could be considerably smaller depending on $\Delta^0/(\Delta^+ + \Delta^-)$. It would be imprudent to draw any sharp morals from this. Only then would there be trouble if type (a) were much larger than type (b). All one may conclude is that the idea that $m_{\rm im}$ arises via the lepton mechanism is not obviously outrageous.

Finally we discuss what it takes for the $\Delta S = 1$ on-shell *CP*-violating amplitudes to be superweak. As an example consider the process $\lambda \to \mathcal{P} + \bar{\mathcal{P}} + \mathfrak{N}$. This can come about via a graph which is the nonleptonic analog of Fig. 4(b) in which a mixed $W^2 \to W^1$ propagator appears [Fig. 4(a) has nonleptonic analog]. The effective *CP* violating transition operator so generated is of the type $V-A$ for $q^2 \ll M^2$ and is $O(G\alpha\delta^2)$. This operator should be added to the transition operator Eq. (4.12) which arises from Z exchange, which is also $V-A$ and which is $O(G)$. In spite of the fact that both operators are $V-A$ we may *not* argue, as for the semileptonic case, that the effect of the addition of the $W^2 \to W^1$ graph is simply an over-all multiplicative phase factor. The reason is that the $O(G)$ operator is subject to hadronic modifications which may be distinct from those of the $O(G\alpha\delta^2)$ operator. Thus the extent to which these on-shell effects are superweak depends on two factors: (1) the extent to which the hadronic modifications of the two graphs are similar. On this I shall not speculate at this time. (2) Apart there-

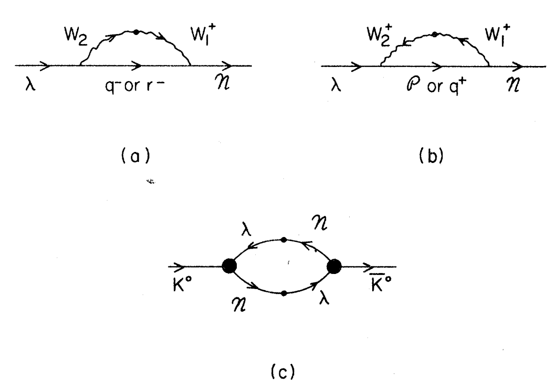

FIG. 6. (a) and (b): *CP*-violating $\lambda\mathfrak{N}$ mixing induced by W^1-W^2 mixing; (c) K^0-$\bar{K}^0$ mixing induced by $\lambda\mathfrak{N}$ mixing.

from, the on-shell *CP* effect will be suppressed the more the smaller δ is. Note that this possibly independent constraint is different from the one met for m_{im} where both δ and the Δ's appeared. All one can conclude is that it is not impossible for m_{im} to be of the right order *and* for the on-shell nonleptonic effects to be sufficiently small.

ACKNOWLEDGMENT

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APPENDIX A: THE CASE $g_1 \neq g_2$

The covariant derivative is

$$D_\mu = \partial_\mu - i(g_1 \vec{A}_\mu \cdot \vec{t} + g_2 \vec{C}_\mu \cdot \vec{\rho}), \tag{A1}$$

where $\vec{t}$ and $\vec{\rho}$ satisfy the same commutation relations as in Eq. (1.7). The expression (2.1) for Q is retained. From Eqs. (2.4) and (2.5)

$$A_\mu = \frac{g_2 A_\mu^3 + g_1 C_\mu^3}{(g_1^2 + g_2^2)^{1/2}}, \qquad e = \frac{g_1 g_2}{(g_1^2 + g_2^2)^{1/2}} \tag{A2}$$

and the neutral Z field now becomes

$$Z_\mu = \frac{g_1 A_\mu^3 - g_2 C_\mu^3}{(g_1^2 + g_2^2)^{1/2}}. \tag{A3}$$

The same H's as in Sec. II are introduced, labeled $H(a, a)$ and $H(ib, -ib)$. The mass of the Z mesons is now given by

$$M_0 = (g_1^2 + g_2^2)(a^2 + b^2). \tag{A4}$$

The charged normal modes are found by noting that the vacuum expectation values of the H's generate the terms

$$\left(\tfrac{1}{2} g_1^2 [(A_\mu^1)^2 + (A_\mu^2)^2] + \tfrac{1}{2} g_2^2 [(C_\mu^1)^2 + (C_\mu^2)^2]\right)(a^2 + b^2) + g_1 g_2 (a^2 - b^2)(A_\mu^1 C_\mu^1 + A_\mu^2 C_\mu^2). \tag{A5}$$

Write Eq. (A5) in the form

$$\alpha(A_\mu^1 \cos\gamma + C_\mu^1 \sin\gamma)^2 + \beta(-A_\mu^1 \sin\gamma + C_\mu^1 \cos\gamma)^2 + \text{"2"}, \tag{A6}$$

where "2" mean the same terms but with A_μ^1, $C_\mu^1 \to A_\mu^2$, C_μ^2. It follows that α, β, and γ satisfy

$$(\alpha - \beta)\sin 2\gamma = g_1 g_2 (a^2 - b^2),$$
$$(\alpha - \beta)\cos 2\gamma = \frac{g_1^2 - g_2^2}{2}(a^2 + b^2), \tag{A7}$$
$$\alpha + \beta = \frac{g_1^2 + g_2^2}{2}(a^2 + b^2).$$

From Eqs. (A7) one recognizes the special case $g_1 = g_2 = g$ to correspond to the special solution $\gamma = \pi/4$, $\alpha = g^2 a^2$, $\beta = g^2 b^2$ which leads to Eqs. (2.19) and (2.20). We now exclude this special case explicitly. Then

$$\tan 2\gamma = \frac{2 g_1 g_2 (a^2 - b^2)}{(g_1^2 - g_2^2)(a^2 + b^2)}, \tag{A8}$$

$$\alpha = \frac{(a^2 + b^2)}{4}\left(g_1^2 + g_2^2 + \frac{g_1^2 - g_2^2}{\cos 2\gamma}\right), \tag{A9}$$

$$\beta = \frac{(a^2 + b^2)}{4}\left(g_1^2 + g_2^2 - \frac{g_1^2 - g_2^2}{\cos 2\gamma}\right). \tag{A10}$$

Define W_μ^1 and W_μ^2 by

$$W_\mu^1 \sqrt{2} = (A_\mu^1 - iA_\mu^2)\sin\gamma - (C_\mu^1 - iC_\mu^2)\cos\gamma, \tag{A11}$$

$$W_\mu^2 \sqrt{2} = (A_\mu^1 - iA_\mu^2)\cos\gamma + (C_\mu^1 - iC_\mu^2)\sin\gamma, \tag{A12}$$

which are the generalizations to the case $g_1 \neq g_2$ of Eqs. (2.6) and (2.7). Then D_μ takes the form

$$D_\mu = \partial_\mu - ieQA_\mu - \frac{i(g_1^2 t_3 - g_2^2 \rho_3) Z_\mu}{(g_1^2 + g_2^2)^{1/2}} - \frac{i}{\sqrt{2}}[W_\mu^1 (g_1 \sin\gamma\, t_+ - g_2 \cos\gamma\, \rho_+) + W_\mu^2 (g_1 \cos\gamma\, t_+ + g_2 \sin\gamma\, \rho_+) + \text{H.c.}]. \tag{A13}$$

Let $(f_1 f_2 f_3 f_4)$ denote some fermion quartet. Then the contribution it makes to the current $J^{(0)}$ coupled to Z_μ is given by

$$\frac{-i}{(g_1^2 + g_2^2)^{1/2}}[(g_1^2 - g_2^2)(\bar{f}_1 f_1 - \bar{f}_4 f_4) - (g_1^2 + g_2^2)(\bar{f}_2 f_2 - \bar{f}_3 f_3)]. \tag{A14}$$

This shows that only for $g_1 = g_2$ does $J^{(0)}$ have the property that only neutral members of quartets contribute to it.

For the discussion given in Sec. V it is important to have the quantity which corresponds to $\mathfrak{L}_{12}$, Eq. (2.16) but now for $g_1 \neq g_2$. One finds

$$\mathfrak{L}_{12} = (W_\mu^{1\dagger} W_\mu^2 + W_\mu^{2\dagger} W_\mu^1) \times [\tfrac{1}{4}(g_1^2 - g_2^2)\sin 2\gamma\,(H_1^\dagger H_1 + H_2^\dagger H_2 + H_3^\dagger H_3 + H_4^\dagger H_4) - \tfrac{1}{2} g_1 g_2 \cos 2\gamma\,(H_2^\dagger H_3 + H_3^\dagger H_2)] + \tfrac{1}{2} g_1 g_2 (W_\mu^{1\dagger} W_\mu^2 - W_\mu^{2\dagger} W_\mu^1)(H_2^\dagger H_3 - H_3^\dagger H_2). \tag{A15}$$

A consistency check on $\mathfrak{L}_{12}$ is that it should vanish in the tree approximation. This is indeed the case as is seen from the relative reality of $\langle H_2 \rangle$, $\langle H_3 \rangle$ and from Eq. (A8). Note further that Eq. (A15) reduces to Eq. (2.16) for $g_1 = g_2 = g$, $\gamma = \frac{1}{4}\pi$.

It is evident from Eq. (A15) that $\mathfrak{L}_{12}$ can provide a counterterm to renormalize the ρ parameter in Eq. (3.16) if such need arises in a theory which is not R invariant in the zeroth order. The counterterm will correspond to a rescaling of the quantity $(g_1{}^2-g_2{}^2)\sin 2\gamma$. The particular case where $g_1{}^2-g_2{}^2=0$ in the tree approximation but where the theory is not fully R-invariant was discussed in Sec. V. For this case the divergence encountered in ρ is linked to a coupling constant renormalization such that $g_1{}^2-g_2{}^2$ is no longer zero when radiative corrections are taken into consideration.

However, there is a qualitative difference between the case $g_1=g_2$ in the tree approximation and $g_1\neq g_2$ in this approximation. To see this, consider the example of the contribution of the multiplet E^L [given Eq. (3.1)] to the W^1W^2 mixing parameter. As we saw in Sec. III this contribution is finite for $g_1=g_2$. However for $g_1\neq g_2$ one finds a contribution to ρ of the form

$$(g_1{}^2-g_2{}^2)\sin 2\gamma I_{\text{quad}}\ , \tag{A16}$$

where I_{quad} denotes a quadratically divergent integral. I_{quad} is independent of lepton mass parameters and of any special choice of the parameters a_1, a_2 which occur in Eq. (3.1). Thus the expression Eq. (A16) has no formal zero limit value even in the zero-mass limit of all leptons involved.

APPENDIX B: THE $(\frac{1}{2},\frac{3}{2})$ and $(\frac{3}{2},\frac{1}{2})$ REPRESENTATIONS

These are 8-dimensional representations. The $(\frac{1}{2},\frac{3}{2})$ is described by $K_{i\alpha}$, $i=1,2,3$, $\alpha=1,2,3,4$ and

$$(t_i)_{\alpha\beta}K_{i\beta}=0\ . \tag{B1}$$

In the representation for t_i given in Eq. (2.17), Eq. (B1) becomes

$$\begin{aligned}&K_{12}-iK_{22}+K_{31}=0\,,\\&K_{11}+iK_{21}-K_{32}=0\,,\\&K_{14}-iK_{24}+K_{33}=0\,,\\&K_{13}+iK_{23}-K_{34}=0\,.\end{aligned} \tag{B2}$$

The neutral components are $(K_{14}-iK_{24})$, K_{32}, K_{33}, $K_{11}+iK_{21}$, only two of which are independent by Eqs. (B2). Let $\langle K_{i\alpha}\rangle$ denote the most general vacuum expectation value set. Then

$$\langle K_{i\alpha}\rangle=\begin{Bmatrix}\frac{1}{2}(c,0,0,-c')\\ \frac{1}{2}i(c,0,0,c')\\ (0,c,c',0)\end{Bmatrix}\ , \tag{B3}$$

where the rows are written in the sequence $i=1,2,3$ and where $\alpha=1,\ldots,4$ within each row. c and c' are taken to be real.

The $(\frac{3}{2},\frac{1}{2})$ is $L_{i\alpha}$, $(\rho_i)_{\alpha\beta}L_{i\beta}=0$, whence again from Eq. (2.17)

$$\begin{aligned}&L_{13}-iL_{23}+L_{31}=0\,,\\&L_{14}-iL_{24}+L_{32}=0\,,\\&L_{11}+iL_{21}-L_{33}=0\,,\\&L_{12}+iL_{22}-L_{34}=0\,.\end{aligned} \tag{B4}$$

The neutral components are $(L_{14}-iL_{24})$, L_{32}, L_{33}, $L_{11}+iL_{21}$, and

$$\langle L_{i\alpha}\rangle=\begin{Bmatrix}\frac{1}{2}(c,0,0,-c')\\ -\frac{1}{2}i(c,0,0,c')\\ (0,c',c,0)\end{Bmatrix}\ . \tag{B5}$$

Here the $\langle L_{i\alpha}\rangle$ have been chosen such that they are related by R to the $\langle K_{i\alpha}\rangle$, that is for fixed i the relation Eq. (2.32) holds; while acting on i: $K\leftrightarrow L$. It is this linking of the $\langle L_{i\alpha}\rangle$ to $\langle K_{i\alpha}\rangle$ which will maintain W^1, W^2 as normal modes.

Indeed, consider the quantity

$$-g^2K^\dagger_{i\alpha}(\vec{A}\cdot\vec{t}+\vec{C}\cdot\vec{\rho})^2{}_{ij,\alpha\beta}K_{j\beta}$$
$$-g^2L^\dagger_{i\alpha}(\vec{A}\cdot\vec{t}+\vec{C}\cdot\vec{\rho})^2{}_{ij,\alpha\beta}L_{j\beta}$$

and compute its value in the tree approximation given by Eqs. (B3) and (B4). After some calculation and with the use of techniques familiar from spin-orbit coupling problems, the L-term yields

$$-\tfrac{3}{8}g^2(c^2+c'^2)[C_1^2+C_2^2+7A_1^2+7A_2^2+(A_3-C_3)^2]$$
$$-3g^2cc'(A_1C_1+A_2C_2)\ . \tag{B6}$$

The K-term yields the same result but with $A\leftrightarrow C$. From this it follows that the completed Eqs. (2.19), (2.20), and (2.22) become

$$M_1{}^2=2g^2[b^2+3(c^2+c'^2-cc')]\,, \tag{B7}$$

$$M_2{}^2=2g^2[a^2+3(c^2+c'^2-cc')]\ , \tag{B8}$$

$$M_0{}^2=g^2[2(a^2+b^2)+3(c^2+c'^2)]\,, \tag{B9}$$

so that in general Eq. (2.22) becomes

$$\xi<1 \tag{B10}$$

(and $\xi>\frac{1}{4}$).

I have not found an argument which determines the scale of c, c' relative to the other vacuum expectation values. Thus ξ may be close to unity though not equal to unity. Nor have I found an argument which implies that c and c' need to be unequal. With $c=c'$ one can use real representations. For generality the formulas are displayed for $c\neq c'$.

Finally we write down some identities which are needed for the fermion mass problem. Let $f_L=(f^+,f^0,f^{0\prime},f^-)_L$ be a $(\frac{1}{2},\frac{1}{2})$ representation in the spherical base described in Sec. II. Let $\vec{f}_R=(f_1,f_2,f_3)_R$ be a $(1,0)$ representation in a Cartesian base, so that the spherical base [as used in

Eq. (5.1)] is

$$f_R^{\pm}\sqrt{2}=(f_1\mp if_2)_R\,,\quad f_R^0=f_R^3\,. \qquad \text{(B11)}$$

Then

$$2\bar{f}_L\vec{t}\cdot\vec{f}_R\langle H(a,a')\rangle+\text{H.c.}$$
$$=(a\bar{f}^+f^+ + a'\bar{f}^-f^-)\sqrt{2}$$
$$-a\bar{f}^0 f^0+\tfrac{1}{2}a'(\bar{f}^{0\prime}(1-\gamma_5)f^0+\text{H.c.})\,. \qquad \text{(B12)}$$

For the case that f_R is (0, 1) and with the same definitions as in Eq. (B11),

$$2\bar{f}_L\vec{p}\cdot\vec{f}_R\langle H(a,a')\rangle+\text{H.c.}$$
$$=(a'\bar{f}^+f^+ + a\bar{f}^-f^-)\sqrt{2}$$
$$+a\bar{f}^0 f^0-\tfrac{1}{2}a'(\bar{f}_0'(1-\gamma_5)f_0+\text{H.c.})\,. \qquad \text{(B13)}$$

For $\vec{f}_R=(1,0)$ we further have

$$\sqrt{2}\,\bar{f}_{L\alpha}\langle L_{i\alpha}\rangle f_{iR}+\text{H.c.}$$
$$=c\bar{f}^+f^+ - c'\bar{f}^-f^- + c'\bar{f}^0f^0\sqrt{2}$$
$$+(c/\sqrt{2})(\bar{f}^{0\prime}(1-\gamma_5)f^0+\text{H.c.})\,, \qquad \text{(B14)}$$

and for $\vec{f}_R=(0,1)$,

$$\sqrt{2}\,\bar{f}_{L\alpha}\langle K_{i\alpha}\rangle f_{iR}$$
$$=c\bar{f}^+f^+ - c'\bar{f}^-f^- + c\bar{f}^0f^0\sqrt{2}$$
$$+\frac{c'}{\sqrt{2}}[\bar{f}^{0\prime}(1-\gamma_5)f^0+\text{H.c.}]. \qquad \text{(B15)}$$

APPENDIX C: ONE-TRIPLET MODELS AND THEIR EMBEDDING

The number of quarks deemed necessary for model building has proliferated in recent years. This is true both in the discussion of purely hadronic problems (three-triplet models) and of the gauge problems. This lends a speculative touch, since at this time no evidence exists for hadronic symmetry groups larger than SU(3). However this may be, the least one must require is to show how a set of quarks introduced for gauge model building is not at variance with the requirements of hadron physics as we know them. In PRL I have called this the embedding problem. In this appendix it is noted that the eight quarks introduced in this and other gauge models, if not too many, are at least properly embeddable. From the point of view of SU(3) the quark assignments of Refs. 9 and 33 and the present paper may be called a one-triplet model since only $\mathcal{P}, \mathfrak{N}, \lambda$ are taken as *integrally charged* SU(3) triplet, the other quarks as SU(3) singlets. It has been noted in Sec. IV that such requirements as CVC and $\pi^0\to 2\gamma$ can be taken care of. The additional question arises not only of the embedding in SU(3) but also in static SU(6).

The description of the baryon octet given in Ref. 9 in which the proton is a $\mathcal{P}\bar{\lambda}q^0$ state etc. (Sec. IV) is adequate for SU(3). This model is not adaptable to SU(6) since this group demands that the octet contains three valence fermions with $I=\frac{1}{2}$. However this can be remedied by adopting a "nuclear model" for the baryons, a suggestion which has been put forward some time ago[52]: Let the proton (for example) have valence quarks $\mathcal{P}\mathcal{P}\mathfrak{N}$ and let there be a spin-zero SU(3) singlet "nucleus" with $Q=-1$. Further, let the main binding forces be due to a strong attraction between $\mathcal{P}$, $\mathcal{P}$, $\mathfrak{N}$ and this "nucleus." Then one can incorporate the $\underline{56}$ decription of SU(6) even in a one-triplet model, since it does not follow from space symmetry that this nuclear model is unstable. Since, for example, a system like $q^-\bar{q}^0$ can serve as a "nucleus," it follows that the presence of SU(3) singlet quarks obviates the strict necessity for three-triplet versions of baryon structure. The purpose of this comment is not to take any such models in a too literal sense but, to repeat, to show that one-triplet models may at least be viable.

As is evident from Refs. 9 and 33 and this paper, the *minimum* extension of SU(3) needed is SU(3) ×U(1). Since five singlets is a large number, one can speculate about groups in which the number of singlets is fewer. This is being pursued by Cheng and the author. On behalf of us both, the following simple comments are made.

(1) Extension to SU(4). This is possible if one has one quartet plus four singlets. Two quartets are out since it does not satisfy the $\pi^0\to 2\gamma$ condition Eq. (1.12), to say the least.

(2) Extension to SU(4)×SU(4)′ with the usual SU(3) as a subgroup of SU(4). This enables one to accommodate the eight quarks in (4, 1)⊕(1, 4). Here quartets in the gauge group match quartets in the hadron group.

(3) The possibility to envisage baryon states with distinct "nuclei" made up from distinct spin-zero structures allows one to speculate about the origin of states like the Roper resonance.

[Finally, note that the quark charges are necessarily integral in O(4), but not in O(4)× U(1). Also, for fractional charges one can have a nuclear model, but then Eq. (1.12) cannot be satisfied with one triplet.]

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[1] For the history of the subject and its development up to the summer of 1972 see B. W. Lee, in *Proceedings of the Sixteenth International Conference of High Energy Physics, National Accelerator Laboratory, Batavia, Ill., 1972,* edited by J. D. Jackson and A. Roberts (NAL, Batavia, Ill., 1973), Vol. 4, p. 249.

[2] P. Higgs, Phys. Lett. 12, 132 (1964). See also T. W. B. Kibble, Phys. Rev. 155, 1554 (1967).

[3] For an earlier survey of these questions see, e.g., G. Feinberg and A. Pais, Phys. Rev. 131, 2724 (1963), Sec. II and Table II; A. Pais, in *Theoretical Physics* (International Atomic Energy Agency, Vienna, 1963), p. 593.

[4] For an example of such a procedure, see H. Georgi and S. L. Glashow, Phys. Rev. D 7, 561 (1973).

[5] See N. Cabibbo, in *Proceedings of the Fifth Coral Gables Conference on Symmetry Principles at High Energies, University of Miami, 1968,* edited by A. Perlmutter, C. Angas Hurst, and B. Kurşunoğlu (Benjamin, New York, 1968), p. 339; A. Pais, Phys. Rev. 173, 1587 (1968).

[6] A. Pais, Phys. Rev. Lett. 29, 1712 (1972); 30, 114(E) (1973), hereafter referred to as PRL.

[7] More general parametrizations can be conceived of. The parameters chosen here suffice to illustrate the general idea.

[8] In all that follows, *CPT* will be conserved. The *C,P,T* properties of the strong and electromagnetic interactions will throughout be assumed to be the conventional ones.

[9] B. W. Lee and S. B. Treiman, Phys. Rev. D 7, 1211 (1973).

[10] A similar thing happens in the model discussed by R. N. Mohapatra, Phys. Rev. D 6, 2023 (1972). This model has physical implications quite distinct from the present one.

[11] L. Wolfenstein, Phys. Rev. Lett. 13, 562 (1964). For more detailed considerations of models which may be subsumed under the superweak notion as used here see M. A. B. Bég, Ann. Phys. (N.Y.) 52, 577 (1969).

[12] T. T. Wu and C. N. Yang, Phys. Rev. Lett. 13, 180 (1964).

[13] S. Weinberg, Phys. Rev. Lett. 29, 388 (1972).

[14] K. Symanzik, in *Coral Gables Conference on Fundamental Interactions at High Energies II,* edited by A. Perlmutter, G. J. Iverson, and R. M. Williams (Gordon and Breach, New York, 1970).

[15] S. Coleman, in Proceedings of the 1971 International School of Physics, Ettore Majorana (Academic, New York, to be published).

[16] T. Hagiwara and B. W. Lee, Phys. Rev. D 7, 459 (1973).

[17] S. L. Adler, Phys. Rev. 177, 2426 (1969), and in *Brandeis Summer School Lectures* (MIT Press, Cambridge, Mass. 1970). In the context of gauge theories the $\pi^0 \rightarrow 2\gamma$ question was first raised by J. Prentki and B. Zumino, Nucl. Phys. B47, 99 (1972).

[18] D. Perkins, in *Proceedings of the Sixteenth International Conference on High Energy Physics, National Accelerator Laboratory, Batavia, Ill., 1972,* edited by J. D. Jackson and A. Roberts (NAL, Batavia, Ill., 1973), Vol. 4, p. 189.

[19] K. Fujikawa, B. W. Lee, A. Sanda, and S. Treiman, Phys. Rev. Lett. 29, 682 (1972); 29, 823(E) (1972).

[20] C. Bouchiat, J. Iliopoulos, and Ph. Meyer, Phys. Lett. 42B, 91 (1972).

[21] D. Gross and R. Jackiw, Phys. Rev. D 6, 477 (1972); C. Bouchiat, J. Iliopoulos, and Ph. Meyer, Phys. Lett. 38B, 519 (1972); H. Georgi and S. L. Glashow, Phys. Rev. D 6, 429 (1972).

[22] See also S. Weinberg, Phys. Rev. D 7, 1068 (1973), Sec. VIII.

[23] Here the standard normalization for the kinetic energy of a complex field has been adopted. For a real field there is conventionally an additional factor $\frac{1}{2}$. Where the latter may arise in the sequel, this factor is supposed to be absorbed in the definitions of the fields.

[24] Here the vacuum expectation values themselves are used as a distinguishing label for the two scalar quartets.

[25] And not necessarily for preference. The issue raised here can be settled only if one gives a full theory for the value of θ. Regrettably, this is not achieved in this paper.

[26] From the point of view of *CP* violation Eq. (3.13) is of particular interest for nonleptonic processes, see the end of Sec. VII.

[27] The influence of lepton pairs on the Z propagator is of no particular concern here.

[28] At several places in PRL, terms $\sim m_e^2$ were not recorded because of their small numerical effects.

[29] $A^{12}(q^2)$ and $B^{12}(q^2) - B^{12}(0)$ are finite; likewise for "21."

[30] See for example the discussion in T. W. Appelquist, J. R. Primack, and H. R. Quinn, Phys. Rev. D 7, 2998 (1973).

[31] See Ref. 22, Sec. III.

[32] Such graphs were first introduced by B. W. Lee, Nucl. Phys. B9, 649 (1969).

[33] B. W. Lee, J. Primack, and S. Treiman, Phys. Rev. D 7, 510 (1973).

[34] H. Georgi and S. L. Glashow, Phys. Rev. Lett. 28, 1494 (1972).

[35] S. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D 2, 1285 (1970).

[36] Cf. S. L. Adler and R. F. Dashen, *Current Algebra* (Benjamin, New York, 1968), Ch. II.

[37] Or a linear combination of q^0 and r^0.

[38] Some permutations between the right-handed neutral-quark assignments produce the same result.

[39] By "positive" is meant that the mass matrix shall be of the form $\sum_i m_i \bar{\psi}_i \psi_i$ with $m_i > 0$.

[40] See Refs. 19 and 20. Up to factors ~ 1 similar estimates apply to the present case.

[41] H. S. Gurr, F. Reines, and H. W. Sobel, Phys. Rev. Lett. 28, 1406 (1972).

[42] The ν_e percentages are: $\sim 3\%$ in the Columbia experiment [R. Burns *et al.* Phys. Rev. Lett. 15, 541 (1965)]; and $\lesssim 0.6\%$ in the CERN experiment [G. Kalbfleisch, Nucl. Phys. B25, 197 (1970)].

[43] For simplicity the result has been presented here in terms of a primitive cutoff. More fanciful renormalizations do not affect anything for this $q^2 = 0$ quantity.

[44] It is in general a reducible representation under O(4) in the sense that it may decompose into the direct sum of an R-even and an R-odd part.

[45] Or, alternatively if $E^R = (0,1)$, $M^R = (1,0)$. The actual choice made was further dictated by consideration of the H couplings, see below.

[46]The implicit assumption is made that H, E^L, and M^L, all have even relative R parity. One can envisage alternatives which involve the dual of the operator X. This does not change the subsequent argument.

[47]B. W. Lee, Phys. Rev. D 6, 1188 (1972).

[48]See Ref. 6, footnote 14. The scale factor quoted there refers to the special case $g=g'$.

[49]For *all* H quartets employed, the R transformation is given by Eq. (2.32).

[50]Furthermore $F(H_1^{(1)}, H_2^{(1)}, H_3^{(1)}, H_4^{(1)}) = (-H_4^{(1)}, H_2^{(1)}, H_3^{(1)}, -H_1^{(1)})^\dagger$ where the extra Hermitian conjugation is due to the action of P_Y.

[51]In Ref. 6 this leading effect was overlooked. I found this single mixing mechanism after J. R. Primack pointed out to me that the estimate in Ref. 6 would lead to trouble for the ratio of real to imaginary mass difference in the K system.

[52]F. Gürsey, T. D. Lee, and M. Nauenberg, Phys. Rev. 135, B467 (1964); F. Gürsey and L. A. Radicati, Phys. Rev. Lett. 13, 173 (1964).