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Current Algebra and Gauge Theories. I*

Steven Weinberg
Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
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The methods of current algebra are applied to the problem of calculating corrections to the symmetries of the strong interactions in renormalizable gauge theories of the weak and electromagnetic interactions. The strong interactions are described by a neutral-vector-gluon model, so that their symmetries are just the symmetries of the quark mass matrix, and are determined by the vacuum expectation values of the weakly coupled scalar fields. Corrections to these symmetries are calculated to all orders in the gluon coupling, but only to second order in the gauge coupling e. After putting the results in a gauge-invariant form, it is found that all divergences cancel in the corrections to "natural" symmetries of the strong interactions. The weak interactions can produce corrections to strong-interaction symmetries of the same order of magnitude as the electromagnetic corrections, but such "order α" effects occur only as corrections to the quark mass matrix, and therefore necessarily conserve parity, strangeness, charm, etc., and may produce only isovector corrections to isotopic-spin conservation. It is suggested that these weak-interaction effects of order α are responsible for the nonelectromagnetic corrections to isotopic-spin conservation which seem to be needed in calculations of mass differences and η decay.

I. INTRODUCTION AND SUMMARY

The gauge theory of weak and electromagnetic interactions has appropriated many of the physical problems, mathematical methods, and personnel that had earlier been associated with current algebra. However, there is still lacking a unified theoretical framework, which would incorporate the best features of current algebra and gauge theories. Our need for such a synthesis is increasingly urgent, for a number of reasons:

(1) It is essential to be able to apply the gauge theories to processes involving hadrons, especially because the greater part of our experimental information about the weak interactions comes to us from *semi*leptonic processes. Most of the work along this line so far has simply used lowest-order perturbation theory, ignoring the strong interactions altogether. Eventually, if we want to do reliable quantitative calculations, we are going to have to include effects of the strong interactions, and it appears that current algebra is the only tool available for this purpose.

(2) There appears to be a conflict between the assumptions made in current algebra and the gauge theories as to the origin of the broken symmetries of the strong interactions. In current algebra, it is assumed that the strong interactions obey a group of *approximate global* symmetries, such as chiral SU(2)⊗SU(2) or SU(3)⊗SU(3), which are spontaneously broken down to isospin and hypercharge conservation; in consequence, the pion and perhaps other spin-zero mesons must have low mass and satisfy approximate low-energy theorems, more or less like Goldstone bosons. In contrast, in the gauge theories it is usual to suppose that the only fundamental symmetries of the Lagrangian are a set of *exact local* symmetries are spontaneously broken down to electromagnetic gauge invariance, but the Goldstone bosons associated with this symmetry breakdown are unphysical particles, which can be removed by a gauge transformation. If we accept the viewpoint of the gauge theories, then what is the pion, and why should it satisfy low-energy theorems?

(3) Many of the successful predictions of current algebra are based on an *ad hoc* identification of the weak and electromagnetic hadron currents with the conserved or approximately conserved currents of the underlying group of approximate strong-interaction symmetries. On the other hand hand, the gauge theories prescribe the nature of the weak-interaction currents almost uniquely, so that we have a chance of either explaining or contradicting the assumptions made in current algebra.

(4) It is a general feature of the unified gauge theories that the exchange of virtual intermediate bosons can produce "radiative" corrections of the same order of magnitude as produced by exchange of an equal number of virtual photons. This immediately raises the question of why parity and strangeness conservation are not violated in order α, a question that must be answered using the tools of current algebra within the context of specific assumptions as to the nature of the strong interactions. In addition, we must now modify the various current-algebra calculations of isotopic-

spin breaking so as to include intermediate-vector-boson exchange as well as photon exchange.

This paper is the first of a series which aims at the incorporation of current algebra into the general framework of the renormalizable gauge theories of weak and electromagnetic interactions. The material presented here is an outgrowth of recent work[1-5] on the problem of approximate symmetries in perturbation theory, and, as in this recent work, our concern here is not with specific models, but with the general features of renormalizable gauge theories.

Before descending into technicalities, it may be useful to run through an outline of the general ideas which underlie both this paper and future papers of this series.

The first step is to decide on a specific model of the strong interactions. Clearly, one necessary condition is that the asymptotic behavior of hadronic matrix elements of the currents should be mild enough at large momenta so as not to spoil the renormalizability of the theory. One way to guarantee this, and the only way to be considered here, is to describe the strong interactions themselves in terms of a renormalizable field theory. This requires that the strongly interacting fields appearing in the Lagrangian must be limited to a set of elementary spin-$\frac{1}{2}$ baryon or quark fields, *plus* either

(a) a massive neutral vector gluon strongly coupled to an absolutely conserved quantity such as baryon number;

(b) a set of strongly interacting gauge fields,[6] with masses produced by spontaneous symmetry breaking;

(c) a set of spin-zero fields with strong Yukawa couplings to the fermions[7]; or

(d) any combinations of (a), (b), and (c).

It turns out that our work is very much simplified if we adopt case (a), and in the present paper it will be explicitly assumed that the only hadron fields in the Lagrangian are spin-$\frac{1}{2}$ baryons or quarks plus a neutral vector gluon. We shall see that the inclusion of other strongly interacting fields [as in cases (b), (c), or (d)] would raise serious obstacles to the resolution of some of the four problems listed above.

In addition to the strongly interacting fermion and gluon fields, the Lagrangian must contain a set of spin-1 gauge fields and a set of spin-0 fields ϕ_i, all with couplings nominally of order e. The gauge group is spontaneously broken through the appearance of vacuum expectation values of the scalar fields. In consequence, all the vector particles except the photon pick up large masses, and the fermion mass matrix becomes, in zeroth order,

$$m = m_0 + \Gamma_i \lambda_i \ . \tag{1.1}$$

Here m_0 is the bare mass, which must be absent in theories with chiral gauge groups, Γ_i is the matrix describing the Yukawa coupling of ϕ_i to the fermions, and λ_i is the lowest-order value of the vacuum expectation value of ϕ_i. Note that the term $\Gamma_i \lambda_i$ *is* of zeroth order, because Γ_i is of order e and λ_i is of order $1/e$.

Now, it is crucial at this point to realize that the symmetries of the strong interactions in the limit $e \to 0$ are precisely the symmetries of the zeroth-order fermion mass matrix (1.1). The fermion-gluon coupling is invariant under *all* unitary global transformations on the fermion fields, including chiral transformations, so the invariance group of the strong interactions just consists of those unitary transformations which commute with m. It is well known that by a suitable redefinition of the fermion dields, any mass matrix can be transformed into a real diagonal matrix free of γ_5 terms,[8] so *the strong interactions must necessarily conserve parity*, as well as all quantum numbers such as strangeness, charm, etc. which can be expressed in terms of the numbers of different types of quarks. However, the strong interactions do not necessarily obey the underlying gauge symmetry of the theory, even in a global sense, because, as shown in Eq. (1.1), the fermion mass matrix may receive a contribution of zeroth-order in e from the vacuum expectation values of the nonhadronic scalar fields. Nevertheless, even though the zeroth-order fermion mass matrix need not enjoy the full gauge invariance of the Lagrangian, it often turns out that it is invariant under some natural symmetry group of global unitary transformations on the fermion fields.[1-5] By a "natural" zeroth-order symmetry is meant here an exact symmetry of the zeroth-order mass matrix which does not depend on some particular choice of parameters in the Lagrangian, but which arises for all renormalizable gauge invariant models based on a given set of elementary fields. Such natural zeroth-order symmetries can arise either because not all of the scalar fields which could participate in Yukawa interactions actually appear in the theory (these are called "type 1" symmetries), or because λ_i is subject to certain constraints (these are called symmetries of "type 2" and "type 3").[9] As frequently emphasized, natural zeroth-order symmetries are attractive not only on aesthetic grounds, but also because the weak and electromagnetic corrections to such symmetries are necessarily finite and calculable.[10]

It is proposed here that isotopic-spin conservation is just such a natural zeroth-order symmetry. It presumably arises because the gauge invariance and renormalizability of the Lagrangian force some eigenvalues of m, say, the zeroth-order masses of the $\mathcal{P}$ and $\mathfrak{N}$ quarks, to be equal. Models of this sort are by now well known.[11]

It is also possible that SU(3) or chiral SU(2)⊗SU(2) or SU(3)⊗SU(3) are natural zeroth-order symmetries of m. They could arise if three eigenvalues of m were forced to be equal, or if two or three were forced to vanish.[12] However, the status of these badly broken symmetries seems much more in doubt.[13]

Any sort of natural zeroth-order symmetry of m can be broken in either or both of two different ways: Higher-order effects of the weak and electromagnetic interactions will introduce corrections which are at most of order α, and in addition there may be a gross spontaneous breakdown of the symmetry due to the strong interactions themselves.[14] In the limit $e \to 0$, a spontaneous breakdown intrinsic to the strong interactions will be manifested through the failure of physical matrix elements (including physical masses) to obey the symmetries of the zeroth-order mass matrix (1.1), and will be accompanied by the appearance of massless spin-0 bosons, one for each broken zeroth-order symmetry.[15] These are "pseudo-Goldstone" bosons[16] rather than Goldstone bosons, in the sense that the broken symmetries are not gauge symmetries of the whole Lagrangian, but are merely global symmetries of the zeroth-order mass matrix, even though these symmetries are supposedly forced on us by gauge invariance and renormalizability. In consequence, the pseudo-Goldstone bosons are not eliminated by the Higgs mechanism[17] when we turn on the gauge interactions,[18] but instead pick up masses of order e, in units of a typical hadronic mass.

As already mentioned, isotopic-spin conservation is presumed to be a natural zeroth-order symmetry. There is no reason to suspect that it will be spontaneously broken by the strong interactions in zeroth order, so we do not expect pseudo-Goldstone bosons here. However, both the weak and electromagnetic interactions produce corrections to isotopic spin conservation of order α.

On the other hand, if chiral SU(2)⊗SU(2) is a natural symmetry of m, then on experimental grounds it must be spontaneously broken, giving rise to a massless pion, which must obey all the familiar low-energy theorems. The weak and electromagnetic interactions would then break SU(2)⊗SU(2) and produce a squared pion mass of order α. It should be recalled that m_π^2/m_{A1}^2 is in fact equal to 2.3α, so this sort of calculation may turn out to give the right answer. However, it is not clear whether a similar mechanism could account for the masses of the K and η.

The present paper deals only with the "intrinsic" breaking of natural symmetries by the weak and electromagnetic interactions. In future papers the spontaneous symmetry breaking due to the strong interactions will be taken into account, with special attention to the weak and electromagnetic contributions to the pseudo-Goldstone boson mass matrix. It will be shown there that the possibility of spontaneous symmetry breaking necessitates no changes in the conclusions of the present paper.

Our assumptions are spelled out in detail in Sec. II. Then in Sec. III the methods of current algebra are used to express the corrections to a general hadronic S-matrix element of order e^2 as a sum of gauge invariant integrals. The asymptotic behavior of the integrands is estimated in Sec. IV, and the results are used in Sec. V to show that each integral contributes a finite correction to any natural symmetry.[19] Finally in Sec. VI we isolate those parts of the weak-interaction corrections to natural symmetries which are of the same order of magnitude as the electromagnetic corrections. The corrections to the S matrix due to exchange of a photon or an intermediate vector boson are of the form

$$e^2 \int d^4k\, F_{\alpha\beta}(k^2)(k^2+\mu^2)^{-1}{}_{\alpha\beta}\,, \qquad (1.2)$$

where F is a matrix element of two vector currents and μ^2 is the vector-boson mass matrix. The photon appears as an eigenvector of μ^2 with eigenvalue zero, and gives a correction of order α. On the other hand, if $F(k^2)$ vanished as $k^2 \to \infty$ faster than $1/k^2$, then the contribution of an intermediate vector boson of very large mass should be suppressed by a factor $1/\mu_W^2$, so that instead of a correction of order α, we would get a contribution of order $G_F \sim e^2/\mu_W^2$. The only corrections produced by a heavy intermediate vector boson which would be of the same order of magnitude as the electromagnetic corrections are those which arise from terms in $F(k^2)$ which behave like $1/k^2$ (times powers of $\ln k^2$) as $k \to \infty$. According to the analysis presented in Sec. IV, the only such terms are those arising from diagrams in which the two currents are separated from the external hadron line by a "bridge," consisting of either two fermion lines, two gluon lines, or two fermion lines and a gluon line. There is no way that corrections to a gluon self-energy can affect any symmetry of the strong interactions, and the corrections to the gluon-fermion vertex are related by gluon gauge invariance to the corrections to the fermion self-energy. We conclude that *the corrections to natural zeroth-order symmetries of order* α *(as op-*

posed to G_F) consist solely of one-photon-exchange terms plus a weak correction δm to the fermion mass matrix.

This is by far the most important result derived in this paper. One immediate consequence is that parity, strangeness, charm, etc. are automatically conserved in order α, because by a unitary transformation on the fermion fields we can reduce the total fermion mass matrix $m+\delta m$ to a diagonal matrix free of γ_5 terms.[20] In addition, the weak interactions now appear as the source of the "tadpoles" which have been needed in theories of isospin breaking since the work of Coleman and Glashow.[21] If the elementary fermions of the theory all have isospin 0 or $\frac{1}{2}$, then δm consists purely of $\Delta I=0$ and $\Delta I=1$ terms, in agreement with the well-known observation that photon exchange can explain $\Delta I=2$ effects such as the pion mass difference and the quadratic part of the Σ mass difference, but not $\Delta I=1$ effects such as the nucleon or kaon mass differences.[22] In particular, the Sutherland theorem[23] may indicate that the decay $\eta\to 3\pi$ arises almost entirely from the *weak* interactions.[24]

We can now see why a neutral vector gluon theory of strong interactions is so attractive. In general, when we introduce other strongly interacting boson fields of spin 0 or 1 into the Lagrangian, parity, strangeness, etc. may not be natural zeroth-order symmetries. Even if parity and strangeness *are* natural zeroth-order symmetries, we would expect the weak interactions to produce corrections to the trilinear and quadrilinear boson self-interactions and to the boson-fermion Yukawa interaction which would be comparable in magnitude to the electromagnetic corrections, but which would not necessarily conserve parity and strangeness. No "diagonalization" could be expected to remove such order α violations of parity and strangeness. Finally, even if the weak interactions did not produce any corrections of order α anywhere but in the fermion mass matrix, the unitary transformation which diagonalizes this matrix would produce violations of strangeness and parity in the Yukawa coupling between the fermions and any boson except a neutral vector gluon.

Although the presence of strong interactions prevents us from being able to calculate even the weak corrections of order α, we can still say much about their matrix structure. In general, δm has the form

$$\delta m=\gamma_4\{t_\alpha t_\beta,\gamma_4 m\}C_{\alpha\beta}+\Gamma_i\Delta_i\,, \tag{1.3}$$

where the t_α are the matrices (perhaps involving γ_5 terms) which represent the algebra of the gauge group, Γ_i are the Yukawa coupling matrices of the scalar fields, C is a series of powers of the matrix $\ln\mu'^2$, and Δ_i is a set of fairly complicated real coefficients. (The prime on μ indicates that the photon mass is taken to be Λ rather than zero, where Λ is the cutoff used in calculating the ordinary photon exchange term.) Under assumptions related to Bjorken scaling,[25] the matrix $C_{\alpha\beta}$ would be simply

$$C_{\alpha\beta}=\mathcal{C}(\ln(\mu'^2/\Lambda^2))_{\alpha\beta}\,, \tag{1.4}$$

with $\mathcal{C}$ a dimensionless real number, perhaps just given by the Born approximation.[26] In addition, for natural symmetries of type 1, the second term in (1.3) may be dropped, so if (1.4) is correct, the whole "tadpole" would then be uniquely determined by the intermediate-vector-boson mass matrix and the single constant $\mathcal{C}$. Applications of these results will be discussed elsewhere.

II. STATEMENT OF THE THEORY

We assume that the only elementary strongly interacting particles are a set of spin-$\frac{1}{2}$ baryons or quarks with fields $\psi_n(x)$, and a single massive neutral vactor gluon with field $B_\mu(x)$. The gluon is assumed to be coupled to quark or baryon number, taken here to be absolutely conserved. In addition to the hadron fields, the Lagrangian also contains a set of gauge fields $A_{\alpha\mu}(x)$ and real scalar fields $\phi_i(x)$, coupled in such a way as to make the Lagrangian invariant under a gauge group G. The gauge couplings of $A_{\alpha\mu}$ are of order e, and the Yukawa couplings of ϕ_i are no stronger, and perhaps much weaker.

With these assumptions, the Lagrangian takes the form[27]

$$\mathcal{L}=-\tfrac{1}{4}G_{\mu\nu}G^{\mu\nu}-\tfrac{1}{2}\mu_B{}^2B_\mu B^\mu-\tfrac{1}{4}F_{\alpha\mu\nu}F_\alpha^{\mu\nu}-\tfrac{1}{2}(D_\mu\phi)_i(D^\mu\phi)_i$$
$$-\bar\psi\gamma^\mu D_\mu\psi-\bar\psi m_0\psi-P(\phi)-\bar\psi\Gamma_i\psi\phi_i\,, \tag{2.1}$$

with

$$G_{\mu\nu}\equiv\partial_\mu B_\nu-\partial_\nu B_\mu\,, \tag{2.2}$$

$$F_{\alpha\mu\nu}\equiv\partial_\mu A_{\alpha\nu}-\partial_\nu A_{\alpha\mu}-C_{\alpha\beta\gamma}A_{\beta\mu}A_{\gamma\nu}\,, \tag{2.3}$$

$$(D_\mu\phi)_i\equiv\partial_\mu\phi_i+i(\theta_\alpha)_{ij}\phi_j A_{\alpha\mu}\,, \tag{2.4}$$

$$(D_\mu\psi)_n\equiv\partial_\mu\psi_n+i(t_\alpha)_{nm}\psi_m A_{\alpha\mu}+ig_B\psi_n B_\mu\,. \tag{2.5}$$

Here $C_{\alpha\beta\gamma}$ are the structure constants of G, and θ_α and t_α are the matrices representing the Lie algebra of G on the ϕ_i and ψ_n fields, respectively, with

$$[\theta_\alpha,\theta_\beta]=iC_{\alpha\beta\gamma}\theta_\gamma\,, \tag{2.6}$$

$$[t_\alpha,t_\beta]=iC_{\alpha\beta\gamma}t_\gamma\,, \tag{2.7}$$

$$(\theta_\alpha)^*_{ij}=(\theta_\alpha)_{ji}=-(\theta_\alpha)_{ij}\,, \tag{2.8}$$

$$t_\alpha^\dagger=t_\alpha\,. \tag{2.9}$$

Also, m_0 and Γ_i are G-covariant and Hermitian matrices, in the sense that

$$[t_\alpha, \gamma_4 m_0] = 0 \ , \tag{2.10}$$

$$[t_\alpha, \gamma_4 \Gamma_i] = -(\theta_\alpha)_{ij} \gamma_4 \Gamma_j \ , \tag{2.11}$$

$$m_0^\dagger = \gamma_4 m_0 \gamma_4 \ , \tag{2.12}$$

$$\Gamma_i^\dagger = \gamma_4 \Gamma_i \gamma_4 \ . \tag{2.13}$$

Both Γ_i and t_α may contain terms proportional to γ_5. The polynomial $P(\phi)$ is real and G-invariant, in the sense that

$$\frac{\partial P(\phi)}{\partial \phi_i} (\theta_\alpha)_{ij} \phi_j = 0 \ . \tag{2.14}$$

The quantities $C_{\alpha\beta\gamma}$, t_α, and θ_α are all of order e, while Γ_i is of order e or less, and g_B is a real coupling constant of order unity.

The gauge symmetry group G is broken by allowing ϕ_i to have a nonvanishing vacuum expectation value, which in zeroth order is a quantity λ_i satisfying the symmetry-breaking condition

$$\frac{\partial P(\phi)}{\partial \phi_i} = 0 \quad \text{at } \phi = \lambda \ . \tag{2.15}$$

The zeroth-order mass matrices of the spin-0, spin-$\frac{1}{2}$, and gauge fields are then

$$M^2{}_{ij} = \left. \frac{\partial^2 P(\phi)}{\partial \phi_i \partial \phi_j} \right|_{\phi=\lambda} , \tag{2.16}$$

$$m = m_0 + \Gamma_i \lambda_i \ , \tag{2.17}$$

$$(\mu^2)_{\alpha\beta} = \lambda_i (\theta_\alpha \theta_\beta \lambda)_i \ . \tag{2.18}$$

The effective Lagrangian is obtained by shifting the scalar field

$$\phi_i = \phi_i' + \lambda_i \tag{2.19}$$

and introducing a complex spin-0 fermion "ghost" field ω_α. This gives an effective interaction[5]:

$$\begin{aligned} \mathcal{L}' = & -i g_B \bar\psi \gamma^\mu \psi B_\mu - i \bar\psi \gamma^\mu t_\alpha \psi A_{\alpha\mu} + \tfrac{1}{2}(\partial_\mu A_{\alpha\nu} - \partial_\nu A_{\alpha\mu}) C_{\alpha\beta\gamma} A_\beta^\mu A_\gamma^\nu - \tfrac{1}{4} C_{\alpha\beta\gamma} C_{\alpha\delta\epsilon} A_{\beta\mu} A_{\gamma\nu} A_\delta^\mu A_\epsilon^\nu - i \partial_\mu \phi_i' (\theta_\alpha)_{ij} \phi_j' A_{\alpha\mu} \\ & - (\theta_\beta \theta_\alpha \lambda)_i \phi_i' A_{\alpha\mu} A_\beta^\mu - \tfrac{1}{2}(\theta_\beta \theta_\alpha)_{ij} \phi_i' \phi_j' A_\alpha^\mu A_{\beta\mu} - \bar\psi \Gamma_i \psi \phi_i' - \tfrac{1}{6} f_{ijk} \phi_i' \phi_j' \phi_k' - \tfrac{1}{24} f_{ijkl} \phi_i' \phi_j' \phi_k' \phi_l' \\ & - \partial_\mu \omega_\alpha^* C_{\alpha\beta\gamma} \omega_\beta A_\gamma^\mu - \xi^{-1} \omega_\alpha^* \omega_\beta (\theta_\beta \theta_\alpha \lambda)_i \phi_i' \ , \end{aligned} \tag{2.20}$$

where

$$f_{ijk} \equiv \left. \frac{\partial^3 P(\phi)}{\partial \phi_i \partial \phi_j \partial \phi_k} \right|_{\phi=\lambda} , \tag{2.21}$$

$$f_{ijkl} \equiv \left. \frac{\partial^4 P(\phi)}{\partial \phi_i \partial \phi_j \partial \phi_k \partial \phi_l} \right|_{\phi=\lambda} , \tag{2.22}$$

and ξ is a free parameter which depends on our choice of gauge.[28] The correct Feynman rules are generated by using (2.20) as if $-\mathcal{L}'$ were the interaction Hamiltonian, but taking the propagators as

$$\begin{aligned} \Delta^A_{\alpha\mu,\beta\nu}(k) = & \ \eta_{\mu\nu} (k^2 + \mu^2)^{-1}{}_{\alpha\beta} \\ & + (1-\xi) k_\mu k_\nu ((k^2 + \mu^2)^{-1} (\xi k^2 + \mu^2)^{-1})_{\alpha\beta} \ , \end{aligned} \tag{2.23}$$

$$\begin{aligned} \Delta^\phi_{ij}(k) = & \ (k^2 + M^2)^{-1}{}_{ij} + (\theta_\alpha \lambda)_i (\theta_\beta \lambda)_j (k^2)^{-1} \\ & \times (\xi k^2 + \mu^2)^{-1}{}_{\alpha\beta} \ , \end{aligned} \tag{2.24}$$

$$\Delta^\psi_{nm}(k) = (i \gamma^\mu k_\mu + m)^{-1}{}_{nm} \ , \tag{2.25}$$

$$\Delta^\omega_{\alpha\beta}(k) = \xi (\xi k^2 + \mu^2)^{-1}{}_{\alpha\beta} \ , \tag{2.26}$$

$$\Delta^B_{\mu\nu}(k) = \eta_{\mu\nu} (k^2 + \mu_B{}^2)^{-1} \ , \tag{2.27}$$

with propagators of derivatives of fields taken as the corresponding derivatives of the propagators of the fields. For $\xi \neq 0$ this theory is manifestly renormalizable, while for $\xi = 0$ it is just what we should get by applying the canonical quantization procedure to (2.1) in a "unitarity" gauge in which all true Goldstone bosons are absent.[27] In what follows, we shall keep ξ arbitrary, and will check explicitly that all observable results are ξ-independent.

The strong interaction part of the Lagrangian in this theory may be simply identified as that part of the Lagrangian which survives when we neglect all quantities of order e or smaller:

$$\begin{aligned} \mathcal{L}_{\text{strong}} = & -\bar\psi \gamma^\mu (\partial_\mu + i g_B B_\mu) \psi - \bar\psi m \psi - \tfrac{1}{4} G_{\mu\nu} G^{\mu\nu} \\ & - \tfrac{1}{2} \mu_B{}^2 B_\mu B^\mu , \end{aligned} \tag{2.28}$$

with m given by (2.17). [Recall that (2.18) requires λ to be of order $1/e$, while Γ is nominally of order e, so the term $\Gamma_i \lambda_i$ in m is of *zeroth* order in e, and must be included in the strong interaction Lagrangian.] The Hermiticity conditions (2.12) and (2.13) allow us to redefine the fermion fields so that m is a real diagonal matrix free of γ_5 terms,[8] and we shall assume that this has been done. Parity is thus automatically conserved by the strong interactions.

Aside from the fermion mass term, the Lagrangian (2.28) is invariant under the full group of unitary transformations on the fermion fields, including unitary transformations which involve γ_5 matrices. Hence, *the actual symmetry group of the strong interaction consists precisely of those unitary transformations* $\psi \to U\psi$ *which leave the fermion mass matrix* m *invariant,* in the sense that

$$U^{\dagger}\gamma_4 m U = \gamma_4 m \ , \qquad (2.29)$$

it being understood that U may contain terms proportional to γ_5 as well as 1. For instance, if there are n_0 fermion fields of zero mass, n_1 fermion fields of equal nonzero mass, n_2 fermion fields with a different nonzero mass, and so on, then the symmetry group of the strong interactions is[12]

$$\mathrm{U}_L(n_0)\otimes \mathrm{U}_R(n_0)\otimes \mathrm{U}(n_1)\otimes \mathrm{U}(n_2)\otimes \cdots \ . \qquad (2.30)$$

It is clear that this strong-interaction symmetry group may have a structure entirely different from that of the underlying gauge group G. However, the zeroth-order mass relations of interest here, whether they take the form of a vanishing or an equality of masses, are supposed to arise solely because of the G invariance and renormalizability of the Lagrangian (and our choice of elementary particle fields), and not because of any special choice of parameters in the Lagrangian. There are various ways that this can come about, which have been extensively discussed in the recent literature.[1-5,9] However, until the end of Sec. VI we will not need to specify just how the zeroth-order mass relations arise. For the present, we shall simply assume that there are "natural" zeroth-order symmetry relations which are obeyed by the fermion masses for all values (in at least a finite range) of the parameters in the Lagrangian. Our problem then is to calculate the weak and electromagnetic corrections to such natural symmetries.

Before entering on our analysis of the weak and electromagnetic effects, it will be necessary to say a word about the renormalization technique to be used in dealing with the *strong* interactions. Normally, as in quantum electrodynamics, one would define a renormalized mass m_R as the position of the pole in the fermion propagator, use m_R in place of the zeroth-order mass m in the free-field Lagrangian, and compensate by adding to the interaction a counterterm

$$\delta\mathcal{L} = -\bar{\psi}(m - m_R)\psi \ , \qquad (2.31)$$

with m defined by the condition that the complete self-energy insertion, including $\delta\mathcal{L}$, should vanish at the pole. For instance, to second order in $g_B^{\,2}$, and neglecting all effects of the weak and electromagnetic interactions (except of course for their zeroth-order contributions to m) the zeroth-order and renormalized mass matrices are related by

$$m = m_R\left\{1 + \frac{g_B^{\,2}}{8\pi^2}\left[-\frac{7}{4} + \int_0^1 \ln\left(\frac{\Lambda^2}{\mu_B^{\,2}(1-x) + m_R^{\,2}x^2}\right)\times(1+x)dx\right]\right\} , \qquad (2.32)$$

with Λ being an ultraviolet cutoff.

The difficulty with this conventional renormalization approach arises when the zeroth-order mass matrix m is subject to *constraints.* Simple constraints on m, which require some of its eigenvalues to be equal or to vanish, cause no particular problem, as they merely require the corresponding eigenvalues of m_R to be equal or to vanish. However, more complicated linear constraints on m may force the renormalized masses to be subject to nonlinear constraints *which depend on the coupling constant* g_B. For example, if the gauge group is SU(3), and the fermions form an octet whose masses arise from the vacuum expectation values of an octet and a singlet of scalar fields, then the bare masses will be subject to the Gell-Mann–Okubo rule:

$$3m_\Lambda + m_\Sigma - 2m_N - 2m_\Xi = 0 \ .$$

In consequence, the renormalized masses will be subject to a g_B-dependent constraint:

$$3m_{R\Lambda} + m_{R\Sigma} - 2m_{RN} - 2m_{R\Xi} = \frac{g_B^{\,2}}{8\pi^2}[3A(m_{R\Lambda}) + A(m_{R\Sigma}) - 2A(m_{RN}) - 2A(m_{R\Xi})] + O(g_B^{\,4}) \ ,$$

where

$$A(m) \equiv \int_0^1 m \ln[\mu_B^{\,2}(1-x) + m^2x^2](1+x)dx \ .$$

When the renormalized masses are subject to constraints of this type, it is incorrect to treat the m_R as parameters of zeroth order in generating a perturbation series. (The same problem arises if we define m_R according to Bogoliubov, as the value of the reciprocal fermion propagator at zero momentum.)

This problem is essentially the same as encountered by Lee and Gervais[29] in their study of the σ model, and the solution here is much the same. We must define an "intermediate renormalized mass" m_I as

$$m_I \equiv mZ \ , \qquad (2.33)$$

with

$$Z \equiv \lim_{m\to 0}\left(\frac{m_R}{m}\right) \ . \qquad (2.34)$$

For instance, (2.32) gives, to order $g_B^{\,2}$,

$$Z = 1 - \frac{g_B^{\,2}}{8\pi^2}\left[-\frac{7}{4} + \int_0^1 \ln\left(\frac{\Lambda^2}{\mu_B^{\,2}(1-x)}\right)(1+x)dx\right] . \qquad (2.35)$$

The intermediate renormalized mass is finite; for instance, (2.32)–(2.35) give, to order $g_B^{\,2}$,

$$m_I = m_R \left\{1 + \frac{g_B^2}{8\pi^2}\int_0^1 \ln\left(\frac{\mu_B^2(1-x)}{\mu_B^2(1-x)+m_R^2x^2}\right)(1+x)dx\right\} . \tag{2.36}$$

However, since Z is manifestly m-independent, any linear constraint on m immediately imposes the same linear g_B-independent constraint on m_I. There is therefore no difficulty in regarding the m_I as parameters of zeroth order, using m_I in place m in the unperturbed Lagrangian, and compensating by adding to the interaction a counterterm

$$\delta\mathcal{L} = -\bar\psi(m - m_I)\psi . \tag{2.37}$$

We shall see that there are advantages in applying the intermediate renormalization technique, even apart from the possibility of mass constraints. Therefore it will be tacitly assumed from now on that subtractions in vertices and gluon self-energy parts, as well as fermion self-energies, are defined at zero fermion mass.

III. SYMMETRY BREAKING IN ORDER e^2

We wish to calculate the weak and electromagnetic shift δS_{FI} in the S matrix for a transition from a general hadron "in" state I to a general hadron "out" state F. The strong interactions will be taken into account to all orders in the coupling constant g_B, but the weak and electromagnetic interactions will be included only to order e^2.

In counting powers of e, it will be assumed that θ_α, t_α, and $C_{\alpha\beta\gamma}$ are all of order e, while Γ_i, f_{ijk}, and f_{ijkl} are at most of order e. However, we want the nonvanishing elements of the zeroth-order mass matrices M^2, m, and μ^2 to be of zeroth order in e, so λ must be taken of order $1/e$.

With this understanding, the only weak and electromagnetic terms in (2.20) that concern us here are the hadron terms of first order in e:

$$J^\mu_\alpha A_{\alpha\mu} + S_i\,\phi_i , \tag{3.1}$$

where J^μ_α and S_i are the hadronic "currents"

$$J^\mu_\alpha \equiv -i\bar\psi\gamma^\mu t_\alpha\psi , \tag{3.2}$$

$$S_i \equiv -\bar\psi\Gamma_i\psi , \tag{3.3}$$

plus the first-order terms involving ϕ', which can produce ϕ' tadpoles:

$$-(\theta_\beta\theta_\alpha\lambda)_i\,\phi'_i A_{\alpha\mu}A^\mu_\beta - \tfrac{1}{6}f_{ijk}\,\phi'_i\phi'_j\phi'_k - \xi^{-1}\omega^*_\alpha\,\omega_\beta(\theta_\beta\theta_\alpha\lambda)_i\,\phi'_i . \tag{3.4}$$

The strong interactions will be taken into account by using a "strong Heisenberg representation," in which the hadron fields ψ and B_μ obey equations of motion and commutation relations dictated by the strong-interaction part of (2.1):

$$\mathcal{L}_{\rm strong} = -\bar\psi\gamma^\mu(\partial_\mu + ig_B B_\mu)\psi - \bar\psi m\psi - \tfrac{1}{4}G_{\mu\nu}G^{\mu\nu} - \tfrac{1}{2}m_B^2 B_\mu B^\mu . \tag{3.5}$$

In this representation, the currents (3.2) and (3.3) obey the conservation law (assuming the cancellation of all Adler-Bell-Jackiw anomalies)

$$\partial_\mu J^\mu_\alpha = -i(\theta_\alpha\lambda)_i S_i \tag{3.6}$$

and the commutation relations

$$[J^0_\alpha(\vec x, t), J^\nu_\beta(\vec y, t)] = -i\,\delta^3(\vec x - \vec y)C_{\alpha\beta\gamma}J^\nu_\gamma(\vec x, t) + \text{S.T.} , \tag{3.7}$$

$$[J^0_\alpha(\vec x, t), S_i(\vec y, t)] = \delta^3(\vec x - \vec y)(\theta_\alpha)_{ij}S_j(\vec y, t) , \tag{3.8}$$

where "S.T." denotes Schwinger terms, assumed here to be c-numbers. (In theories with strongly interacting spin-0 fields, there are q-number Schwinger terms and seagull terms, and in addition the T products below must be replaced with T^* products. I have checked that gauge and Lorentz invariance are achieved in such theories through cancellations among these three complications.)

The diagrams for δS_{FI} fall in three general classes, shown in Fig. 1. Those diagrams in which a gauge boson is exchanged between two currents give a contribution

$$\delta_A S_{FI} = (2\pi)^4\delta^4(P_F - P_I)\int d^4k\, \mathfrak{F}^{FI}_{\alpha\mu,\beta\nu}(k)\Delta^{A\mu}_{\alpha}{}_{,\beta}{}^{\nu}(k) , \tag{3.9}$$

where

$$\mathfrak{F}^{FI}_{\alpha\mu,\beta\nu}(k) \equiv \frac{i}{2(2\pi)^4}\int d^4x\langle F|T\{J_{\alpha\mu}(x)J_{\beta\nu}(y)\}|I\rangle e^{ik\cdot x} . \tag{3.10}$$

Those diagrams in which a spin-0 boson is exchanged between a pair of currents give a contribution

$$\delta_\phi S_{FI} = (2\pi)^4 \delta^4(P_F - P_I) \int d^4k \, \mathfrak{F}_{ij}^{FI}(k) \Delta_{ij}^\phi(k) \, , \tag{3.11}$$

where

$$\mathfrak{F}_{ij}^{FI}(k) = \frac{i}{2(2\pi)^4} \int d^4x \langle F | T\{S_i(x) S_j(0)\} | I \rangle_C \, e^{ik\cdot x} \, . \tag{3.12}$$

(The subscript C implies that we are to leave out disconnected graphs proportional to the vacuum expectation value of S, because such graphs are included in the tadpole contribution below.) Finally, those diagrams in which a single spin-0 boson tadpole is attached to a single current give a contribution

$$\delta_T S_{FI} = (2\pi)^4 \delta^4(P_F - P_I) \mathfrak{F}_i^{FI} M^{-2}{}_{ij} \, T_j \, , \tag{3.13}$$

where

$$\mathfrak{F}_i^{FI} \equiv \frac{1}{2(2\pi)^4} \langle F | S_i(0) | I \rangle \tag{3.14}$$

and T_j is the sum of all first-order tadpole graphs:

$$T_i = -\tfrac{1}{2} f_{ijk} \int d^4k \, \Delta_{jk}^\phi(k) + \tfrac{1}{2} i \, (2\pi)^4 \langle S_i(0) \rangle_0 - (\theta_\alpha \theta_\beta \lambda)_i \eta^{\mu\nu} \int d^4k \, \Delta_{\alpha\mu,\beta\nu}^A(k) + \xi^{-1} (\theta_\beta \theta_\alpha \lambda)_i \int d^4k \, \Delta_{\alpha\beta}^\omega(k) \, . \tag{3.15}$$

We must now combine (3.9), (3.11), and (3.13) into a ξ-independent formula for δS_{FI}. To this end, we note that the conservation and commutation relations (3.6)–(3.8) impose a relation on the matrix elements (3.10), (3.12), and (3.14):

$$k^\mu k^\nu \mathfrak{F}_{\alpha\mu,\beta\nu}^{FI}(k) = (\theta_\alpha \lambda)_i (\theta_\beta \lambda)_j \mathfrak{F}_{ij}^{FI}(k) - (\theta_\beta \theta_\alpha \lambda)_i \mathfrak{F}_i^{FI} - i k^\nu C_{\alpha\beta\gamma} \mathfrak{F}_{\gamma\nu}^{FI} \, , \tag{3.16}$$

where

$$\mathfrak{F}_{\gamma\nu}^{FI} \equiv \frac{1}{2(2\pi)^4} \langle F | J_{\gamma\nu}(0) | I \rangle \, . \tag{3.17}$$

We can use this result to rewrite (3.9) as a sum of three terms:

$$\delta_A S_{FI} = \delta_{A1} S_{FI} + \delta_{A2} S_{FI} + \delta_{A3} S_{FI} \, , \tag{3.18}$$

of which the first is just the contribution of the $g_{\mu\nu}$ part of Δ^A:

$$\delta_{A1} S_{FI} = (2\pi)^4 \delta^4(P_F - P_I) \eta^{\mu\nu} \int d^4k \, \mathfrak{F}_{\alpha\mu,\beta\nu}^{FI}(k) (k^2 + \mu^2)^{-1}{}_{\alpha\beta} \, , \tag{3.19}$$

and the second and third are the contributions of the $k_\mu k_\nu$ part of Δ^A, arising from the first and second terms in (3.16):

$$\delta_{A2} S_{FI} = -(2\pi)^4 (1-\xi) \delta^4(P_F - P_I) (\theta_\alpha \lambda)_i (\theta_\beta \lambda)_j \int d^4k \, \mathfrak{F}_{ij}^{FI}(k) \big((k^2 + \mu^2)^{-1} (\xi k^2 + \mu^2)^{-1}\big)_{\alpha\beta} \, , \tag{3.20}$$

$$\delta_{A3} S_{FI} = -(2\pi)^4 (1-\xi) \delta^4(P_F - P_I) (\theta_\beta \theta_\alpha \lambda)_i \mathfrak{F}_i^{FI} \int d^4k \big((k^2 + \mu^2)^{-1} (\xi k^2 + \mu^2)^{-1}\big)_{\alpha\beta} \, . \tag{3.21}$$

[The last term in (3.16) drops out in a symmetric integration.] Also, the ϕ-exchange term (3.11) may be written as a sum of two terms arising from the two parts of Δ^ϕ:

$$\delta_\phi S_{FI} = \delta_{\phi 1} S_{FI} + \delta_{\phi 2} S_{FI} \, , \tag{3.22}$$

where

$$\delta_{\phi 1} S_{FI} = (2\pi)^4 \delta^4(P_F - P_I) \int d^4k \, \mathfrak{F}_{ij}^{FI}(k) (k^2 + M^2)^{-1}{}_{ij} \, , \tag{3.23}$$

$$\delta_{\phi 2} S_{FI} = (2\pi)^4 \delta^4(P_F - P_I) (\theta_\alpha \lambda)_i (\theta_\beta \lambda)_j \int d^4k \, \mathfrak{F}_{ij}^{FI}(k) (k^2)^{-1} (\xi k^2 + \mu^2)^{-1}{}_{\alpha\beta} \, . \tag{3.24}$$

Finally, the tadpole term (3.13) may be written

$$\delta_T S_{FI} = \delta_{T1} S_{FI} + \delta_{T2} S_{FI} \, , \tag{3.25}$$

where

$$\delta_{T1}S_{FI}=(2\pi)^4\delta^4(P_F-P_I)\mathfrak{F}_i^{FI}M^{-2}{}_{ij}\left[-f_{jkl}\int d^4k\,(k^2+M^2)^{-1}{}_{kl}+i(2\pi)^4\langle S_j(0)\rangle_0-6(\theta_\beta\theta_\alpha\lambda)_j\int d^4k\,(k^2+\mu^2)^{-1}{}_{\alpha\beta}\right], \tag{3.26}$$

$$\delta_{T2}S_{FI}=(2\pi)^4\delta^4(P_F-P_I)\,\mathfrak{F}_i^{FI}(\theta_\beta\theta_\alpha\lambda)_i\int d^4k(k^2)^{-1}(\xi k^2+\mu^2)^{-1}{}_{\alpha\beta}\,. \tag{3.27}$$

We can now put together the separate parts of δS into gauge-invariant combinations, using the identity

$$-\frac{(1-\xi)}{(k^2+\mu^2)(\xi k^2+\mu^2)}+\frac{1}{k^2(\xi k^2+\mu^2)}=\frac{1}{k^2(k^2+\mu^2)}\,.$$

In this way, (3.20) and (3.24) combine to give

$$\delta_{A\phi}S_{FI}\equiv\delta_{A2}S_{FI}+\delta_{\phi2}S_{FI}=(2\pi)^4\delta^4(P_F-P_I)(\theta_\alpha\lambda)_i(\theta_\beta\lambda)_j\int d^4k\,\mathfrak{F}_{ij}^{FI}(k)(k^2)^{-1}(k^2+\mu^2)^{-1}{}_{\alpha\beta}\,, \tag{3.28}$$

while (3.21) and (3.27) combine to give

$$\delta_{AT}S_{FI}\equiv\delta_{A3}S_{FI}+\delta_{T2}S_{FI}=(2\pi)^4\delta^4(P_F-P_I)\,\mathfrak{F}_i^{FI}(\theta_\beta\theta_\alpha\lambda)_i\int d^4k\,(k^2)^{-1}(k^2+\mu^2)^{-1}{}_{\alpha\beta}\,. \tag{3.29}$$

Thus δS now appears as the sum of terms

$$\delta S_{FI}=\delta_{A1}S_{FI}+\delta_{\phi1}S_{FI}+\delta_{A\phi}S_{FI}+\delta_{AT}S_{FI}+\delta_{T1}S_{FI}\,, \tag{3.30}$$

every one of which is separately ξ-independent. [Readers of Ref. 5 will note that the manipulations leading to Eq. (3.30) stand in a one-to-one correspondence with the manipulations which put the one-loop contributions to the fermion mass in a gauge-invariant form.] Explicitly, we now have for the second-order change in S_{FI}

$$\begin{aligned}\delta S_{FI}=(2\pi)^4\delta^4(P_F-P_I)\Big\{&\int d^4k\,\mathfrak{F}_{\alpha\mu,\beta\nu}^{FI}(k)(k^2+\mu^2)^{-1}{}_{\alpha\beta}\eta^{\mu\nu}+\int d^4k\,\mathfrak{F}_{ij}^{FI}(k)(k^2+M^2)^{-1}{}_{ij}\\&+(\theta_\alpha\lambda)_i(\theta_\beta\lambda)_j\int d^4k\,\mathfrak{F}_{ij}^{FI}(k)(k^2)^{-1}(k^2+\mu^2)^{-1}{}_{\alpha\beta}+\mathfrak{F}_i^{FI}(\theta_\beta\theta_\alpha\lambda)_i\int d^4k(k^2)^{-1}(k^2+\mu^2)^{-1}{}_{\alpha\beta}\\&+\mathfrak{F}_i^{FI}M^{-2}{}_{ij}\left[-f_{jkl}\int d^4k\,(k^2+M^2)^{-1}{}_{kl}+i(2\pi)^4\langle S_j(0)\rangle_0-6(\theta_\beta\theta_\alpha\lambda)_i\int d^4k\,(k^2+\mu^2)^{-1}{}_{\alpha\beta}\right]\Big\}.\end{aligned} \tag{3.31}$$

IV. ASYMPTOTIC BEHAVIOR OF THE MATRIX ELEMENTS

We shall now analyze the asymptotic behavior as $k\to\infty$ of the matrix elements $\mathfrak{F}_{\alpha\mu,\beta\nu}^{FI}(k)$ and $\mathfrak{F}_{ij}^{FI}(k)$. The results will be used in Sec. V to show explicitly how the divergences cancel in the second-order corrections to "natural" zeroth-order symmetry relations, and in Sec. VI to pick out the parts of the finite weak-interaction corrections which are comparable in strength to the electromagnetic corrections.

We are eventually going to do a Wick rotation before integrating over k, so it is sufficient for our purposes to consider the asymptotic behavior of the $\mathfrak{F}(k)$ with k^0 imaginary:

$$k^0=ik_4\,. \tag{4.1}$$

We shall let the Euclidean modulus κ go to infinity, where

$$\kappa=[(k_1)^2+(k_2)^2+(k_3)^2+(k_4)^2]^{1/2}\,, \tag{4.2}$$

with fixed direction k^μ/κ.

According to a general theorem,[30] which was proved long ago in order to supply a missing element in the renormalization program, the $\mathfrak{F}$-matrix elements are given for $\kappa\to\infty$ by a sum of terms, in each of which the two J or S currents are connected to the hadrons in the states F, I by a "bridge," consisting of a number of elementary particle lines. (Figure 2 presents examples of this structure; the bridges shown are those which will eventually turn out to dominate the asymptotic behavior for $\kappa\to\infty$.) For a given bridge consisting of F fermion lines and G gluon lines, the asymptotic behavior of the corresponding term in $\mathfrak{F}(k)$ is at most

$$\kappa^\alpha\times(\text{powers of }\ln\kappa)\,, \tag{4.3}$$

where α is just the "naive" dimensionality of the bridge-current matrix element (*not* the whole matrix element):

$$\alpha=4-2-\tfrac{3}{2}F-G\,. \tag{4.4}$$

The term -2 here accounts for the two J or S currents.

For our present purposes, we will need to employ a refined version of the above bridge analysis, provided by Wilson's operator-product expansion.[31] In order to simplify our calculations, it is convenient first to integrate over the directions of k, and also to contract the vector-current indices. The operator-product expansion then gives, as $\kappa \to \infty$,

$$\eta^{\mu\nu}\int d\Omega_k\, \mathcal{F}^{FI}_{\alpha\mu,\beta\nu}(k) \sim \sum_N \langle F|O_N|I\rangle U^{(N)}_{\alpha\beta}(\kappa)\ , \quad (4.5)$$

$$\int d\Omega_k\, \mathcal{F}^{FI}_{ij}(k) \sim \sum_N \langle F|O_N|I\rangle\, V^{(N)}_{ij}(\kappa)\ . \quad (4.6)$$

Here $d\Omega_k$ is the element of solid angle in four dimensions (with $\int d\Omega_k$ equal to $2\pi^2$), defined by setting the four-dimensional volume element equal to

$$d^4k = i\kappa^3 d\kappa\, d\Omega_k\ . \quad (4.7)$$

The operators O_N are a set of local operator products, each containing definite numbers F_N, G_N, and D_N of fermion fields, gluon fields, and derivatives acting in these fields, with all fields evaluated at the same space-time point, say, $x=0$. Because of our integration over directions of κ and our contraction of vector current indices, the O_N are Lorentz scalar operators, a restriction which greatly simplifies these expansions. The $U^{(N)}$ and $V^{(N)}$ are c-number functions of κ, with asymptotic behavior

$$U^{(N)}(\kappa) = O(\kappa^{\alpha_N} \times (\text{powers of } \ln\kappa))\ , \quad (4.8)$$

$$V^{(N)}(\kappa) = O(\kappa^{\alpha_N} \times (\text{powers of } \ln\kappa))\ , \quad (4.9)$$

with α_N given by (4.4), except that it must be lowered by one unit for each derivative appearing in O_N:

$$\alpha_N = 2 - \tfrac{3}{2}F_N - G_N - D_N\ . \quad (4.10)$$

The more complicated an operator O_N, the more negative will be the corresponding asymptotic power α_N, so the expressions (4.5) and (4.6) provide asymptotic expansions for the $\mathcal{F}$'s, with only a finite number of terms contributing for any given rate of decrease at large κ. Finally, the $U^{(N)}$ and $V^{(N)}$ are *finite* functions, except that they contain Z factors [like (2.34)] which are needed to cancel the divergences which arise when we compute the matrix elements $\langle F|O_N|I\rangle$.

Inspection of Eq. (3.31) shows that the "dangerous" bridges or O operators, which can produce individually divergent terms in the second-order correction δS_{FI}, are those with $\alpha \geq -2$. Also, we shall see in Sec. VI that all terms arising from the exchange of heavy intermediate vector bosons are suppressed by factors of order μ_W^{-2}, except for those terms with $\alpha \geq -2$. Therefore, we shall restrict our attention here to the terms in the asymptotic expansions (4.5), (4.6) with $\alpha_N \geq -2$, i.e., with

$$4 - \tfrac{3}{2}F_N - G_N - D_N \geq 0\ . \quad (4.11)$$

In addition to being Lorentz invariant, the O_N operators must be gluon-gauge invariant, because gluons couple to baryon number, while all $J_{\alpha\mu}$ and S_i currents are baryonically neutral operators. Taking into account all possible Lorentz- and gluon-gauge-invariant O_N operators satisfying (4.11), the expansions (4.5) and (4.6) become, for $\kappa \to \infty$,

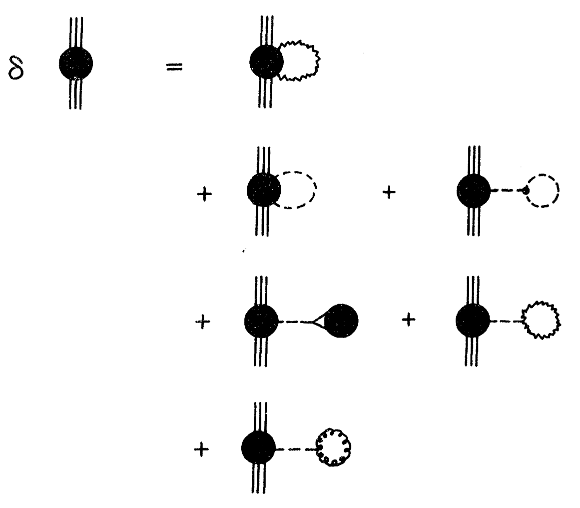

FIG. 1. Feynman graphs for the change in the S matrix due to second-order weak and electromagnetic effects. The process shown for illustration is a three-quark reaction. Here straight lines denote quarks, wavy lines denote intermediate vector mesons, dashed lines denote weakly coupled scalar particles, looped lines denote "ghost loops," and dark circles indicate sums over strong-interaction graphs.

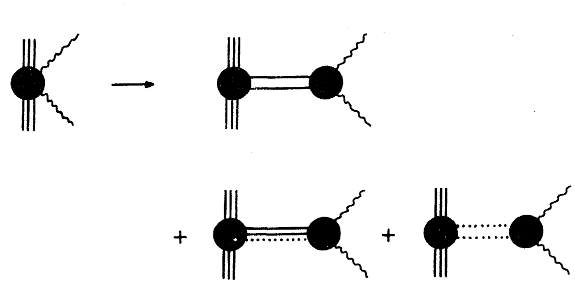

FIG. 2. Bridge graphs which dominate the asymptotic behavior of the matrix element $\mathcal{F}_{\alpha\mu,\beta\nu}(k)$. Here straight lines denote quarks, wavy lines denote vector currents carrying the large momentum k into and out of the diagram, dotted lines denote vector gluons, and dark circles indicate sums over strong-interaction graphs.

$$\eta^{\mu\nu}\int d\Omega_k\, \mathfrak{F}^{FI}_{\alpha\mu,\beta\nu}(k)$$
$$\sim \langle F|[-\bar\psi\, U^{(1)}_{\alpha\beta}(\kappa)\psi - \bar\psi\gamma^\mu U^{(2)}_{\alpha\beta}(\kappa)(\partial_\mu + ig_B B_\mu)\psi - \tfrac{1}{4}U^{(3)}_{\alpha\beta}(\kappa)G_{\mu\nu}G^{\mu\nu}]\,|I\rangle \qquad (4.12)$$

and

$$\int d\Omega_k\, \mathfrak{F}^{FI}_{ij}(k)$$
$$\sim \langle F|[-\bar\psi\, V^{(1)}_{ij}(\kappa)\psi - \bar\psi\gamma^\mu V^{(2)}_{ij}(\kappa)(\partial_\mu + ig_B B_\mu)\psi - \tfrac{1}{4}V^{(3)}_{ij}(\kappa)G_{\mu\nu}G^{\mu\nu}]\,|I\rangle. \qquad (4.13)$$

The $U^{(N)}$ and $V^{(N)}$ here are c-number functions of κ, but $U^{(1)}$, $U^{(2)}$, $V^{(1)}$, and $V^{(2)}$ now have suppressed matrix indices running over the various Dirac field types, and may include terms proportional to the Dirac matrix γ_5 as well as the unit Dirac matrix. The reality properties of the currents imply that the $U^{(N)}$ and $V^{(N)}$ obey the reality conditions

$$U^{(1)\dagger}_{\alpha\beta} = \gamma_4 U^{(1)}_{\alpha\beta}\gamma_4, \quad U^{(2)\dagger}_{\alpha\beta} = U^{(2)}_{\alpha\beta}, \quad U^{(3)*}_{\alpha\beta} = U^{(3)}_{\alpha\beta}\,, \qquad (4.14)$$
$$V^{(1)\dagger}_{ij} = \gamma_4 V^{(1)}_{ij}\gamma_4, \quad V^{(2)\dagger}_{ij} = V^{(2)}_{ij}, \quad V^{(3)*}_{ij} = V^{(3)}_{ij}\,,$$

while crossing symmetry imposes the symmetry conditions

$$U^{(N)}_{\alpha\beta} = U^{(N)}_{\beta\alpha}, \quad V^{(N)}_{ij} = V^{(N)}_{ji}\,. \qquad (4.15)$$

In writing these expansions, we have excluded operators which take the form of gradients, such as $\partial_\mu(\bar\psi\gamma^\mu U\psi)$ or $\partial_\mu(B_\nu G^{\mu\nu})$, because the states F and I have equal four-momenta, so such operators would make no contribution. The U and V functions have asymptotic behavior of the form (4.8), (4.9), with α_N given by (4.10) as

$$\alpha_1 = -1, \quad \alpha_2 = \alpha_3 = -2\,. \qquad (4.16)$$

We will also need information about the dependence of the U and V functions on the fermion mass matrix m. To be specific, we should like to argue that as long as we are only interested in terms which decrease as $\kappa\to\infty$ no faster than κ^{-2}, the functions $U^{(2)}$, $U^{(3)}$, $V^{(2)}$, and $V^{(3)}$ are independent of m, while $U^{(1)}$ and $V^{(1)}$ are at most linear in m. A persuasive but highly nonrigorous argument may be given as follows: Suppose we differentiate Eq. (4.5) n times with respect to m:

$$\eta^{\mu\nu}\int d\Omega_k \frac{\partial^n}{\partial m^n}\mathfrak{F}^{FI}_{\alpha\mu,\beta\nu}(k)$$
$$\sim \sum_N \sum_{r=1}^{n}\binom{n}{r}\left(\frac{\partial^{n-r}}{\partial m^{n-r}}\langle F|O_N|I\rangle\right)\left(\frac{\partial^r}{\partial m^r}\,U^{(N)}_{\alpha\beta}(\kappa)\right). \qquad (4.17)$$

(For the sake of notational simplicity, we are for the moment ignoring the fact that m is a matrix rather than a single variable.) Now, because of our use of the "intermediate renormalization technique" described in Sec. II, all renormalization Z factors are m-independent, so the nth derivative of $\mathfrak{F}$ with respect to m may be regarded as an $\mathfrak{F}$-matrix element with n additional fictitious zero-momentum scalar particles in the initial or final states I, F. Of these n scalar lines, any subset may be used as part of the bridge connecting the currents to the states I and F. (That is, any subset of the scalar field operators representing these n fictitious scalar particles may appear in O_N.) Furthermore, those terms in which r scalar lines appear in the bridge must have n-r scalar lines attached to the connected part of the matrix element $\langle F|O_N|I\rangle$, so such terms must involve the (n-r)th derivative of this matrix element with respect to m, and therefore may be identified with the rth term on the right-hand side of (4.17). However, when r scalar field lines appear in the bridge (or r scalar field operators appear in O_N) the asymptotic behavior of the corresponding term in $\mathfrak{F}(\kappa)$ is reduced by r factors of κ. Referring back to (4.16), we see that the only terms in (4.17) which decrease as $\kappa\to\infty$ no faster than κ^{-2} are the $N=2$ and $N=3$ terms with $r=0$ and the $N=1$ terms with $r=0$ and 1, and therefore conclude that $U^{(2)}$ and $U^{(3)}$ must be m-independent while $U^{(1)}$ can be at most linear in m. Precisely the same argument applies to $V^{(N)}$.

We can now draw immediate conclusions about the G-transformation properties of the U and V functions, for it is only the presence of the $\Gamma_i\lambda_i$ term in m that disturbs the G invariance of the strong interactions. It follows that the functions $U^{(2)}$, $U^{(3)}$, $V^{(2)}$, and $V^{(3)}$ must be G-covariant, in the sense that

$$[t_\alpha, U^{(2)}_{\beta\gamma}] = i\,C_{\alpha\beta\delta}U^{(2)}_{\delta\gamma} + i\,C_{\alpha\gamma\delta}U^{(2)}_{\beta\delta}\,, \qquad (4.18)$$
$$0 = C_{\alpha\beta\delta}U^{(3)}_{\delta\gamma} + C_{\alpha\gamma\delta}U^{(3)}_{\beta\delta}\,, \qquad (4.19)$$
$$[t_\alpha, V^{(2)}_{ij}] = -(\theta_\alpha)_{ik}V^{(2)}_{kj} - (\theta_\alpha)_{jk}V^{(2)}_{ik}\,, \qquad (4.20)$$
$$0 = (\theta_\alpha)_{ik}V^{(2)}_{kj} + (\theta_\alpha)_{jk}V^{(2)}_{ik}\,, \qquad (4.21)$$

while $U^{(1)}$ and $V^{(1)}$ may be expanded to first order in λ:

$$U^{(1)}_{\alpha\beta} = U^{(1)}_{\alpha\beta 0} + U^{(1)}_{\alpha\beta i}\lambda_i\,, \qquad (4.22)$$
$$V^{(1)}_{ij} = V^{(1)}_{ij0} + V^{(1)}_{ijk}\lambda_k\,, \qquad (4.23)$$

with G-covariant coefficients:

$$[t_\alpha, \gamma_4 U^{(1)}_{\beta\gamma 0}] = i\,C_{\alpha\beta\delta}\gamma_4 U^{(1)}_{\delta\gamma 0} + i\,C_{\alpha\gamma\delta}\gamma_4 U^{(1)}_{\beta\delta 0}\,, \qquad (4.24)$$

$$[t_\alpha, \gamma_4 U^{(1)}_{\beta\gamma k}] = i C_{\alpha\beta\delta}\gamma_4 U^{(1)}_{\delta\gamma k} + i C_{\alpha\gamma\delta}\gamma_4 U^{(1)}_{\beta\delta k} - (\theta_\alpha)_{kj}\gamma_4 U^{(1)}_{\beta\gamma j} , \quad (4.25)$$

$$[t_\alpha, \gamma_4 V^{(1)}_{ij0}] = -(\theta_\alpha)_{il}\gamma_4 V^{(1)}_{lj0} - (\theta_\alpha)_{jl}\gamma_4 V^{(1)}_{il0} , \quad (4.26)$$

$$[t_\alpha, \gamma_4 V^{(1)}_{ijk}] = -(\theta_\alpha)_{il}\gamma_4 V^{(1)}_{ljk} - (\theta_\alpha)_{jl}\gamma_4 V^{(1)}_{ilk} - (\theta_\alpha)_{kl}\gamma_4 V^{(1)}_{ijl} . \quad (4.27)$$

We must keep the γ_4 factor in the $U^{(1)}$ and $V^{(1)}$ commutation relations, because t_α may contain terms proportional to γ_5, and therefore may not commute with γ_4. However, t_α does commute with $\gamma_4\gamma^\mu$, so it is unnecessary to keep factors of $\gamma_4\gamma^\mu$ in the $U^{(2)}$ and $V^{(2)}$ commutation relations.

These G-transformation rules are all we shall need in Sec. V to prove the finiteness of corrections to "natural" zeroth-order symmetries. However, for the purposes of Sec. VI, it will be necessary to go into the structure of the U and V functions a little further.

First, note that the γ_5 invariance of the gluon-fermion interaction ensures that although $U^{(2)}$, $U^{(3)}$, $V^{(2)}$, and $V^{(3)}$ are even in m (which is consistent with their asymptotic m independence), the functions $U^{(1)}$ and $V^{(1)}$ must be *odd* in m, and therefore purely of first order in m. One consequence is that the functions $U^{(1)}_{\alpha\beta 0}$ and $V^{(1)}_{ij0}$ in Eqs. (4.20) and (4.21) are of first order in the bare fermion mass matrix m_0. Also, as indicated above, terms of first order in m have an asymptotic behavior with one less power of κ than given by Eq. (4.16), so now *all* U and V functions are seen to have the same asymptotic power: As $\kappa \to \infty$,

$$U^{(N)} \sim \kappa^{-2} \times (\text{powers of } \ln\kappa) , \quad (4.28)$$

$$V^{(N)} \sim \kappa^{-2} \times (\text{powers of } \ln\kappa) , \quad (4.29)$$

for $N = 1, 2$, or 3. (We would expect this anyway as a result of our symmetric integration.)

In addition, by examination of the various bridge-current graphs, we can show that the asymptotic U and V functions have the matrix structure

$$U^{(1)}_{\alpha\beta}(\kappa) = \kappa^{-2}[a(\kappa)(\gamma_4 t_\alpha \gamma_4 m\, t_\beta + \gamma_4 t_\beta \gamma_4 m\, t_\alpha) + b(\kappa)\gamma_4\{t_\alpha, t_\beta\}\gamma_4 m + c(\kappa) m\{t_\alpha, t_\beta\} + d(\kappa) m \operatorname{Tr}(t_\alpha t_\beta)] , \quad (4.30)$$

$$U^{(2)}_{\alpha\beta}(\kappa) = \kappa^{-2}[e(\kappa)\{t_\alpha, t_\beta\} + f(\kappa)\operatorname{Tr}(t_\alpha t_\beta)] , \quad (4.31)$$

$$U^{(3)}_{\alpha\beta}(\kappa) = \kappa^{-2} g(\kappa)\operatorname{Tr}(t_\alpha t_\beta) , \quad (4.32)$$

$$V^{(1)}_{ij}(\kappa) = \kappa^{-2}\big(p(\kappa)(\Gamma_i m \Gamma_j + \Gamma_j m \Gamma_i) + q(\kappa) m\{\gamma_4\Gamma_i, \gamma_4\Gamma_j\} + r(\kappa)\{\Gamma_i\gamma_4, \Gamma_j\gamma_4\} m + s(\kappa) m \operatorname{Tr}(\gamma_4\Gamma_i\gamma_4\Gamma_j) + t(\kappa)[\Gamma_i \operatorname{Tr}(\Gamma_j m) + \Gamma_j \operatorname{Tr}(\Gamma_i m)]\big) , \quad (4.33)$$

$$V^{(2)}_{ij}(\kappa) = \kappa^{-2}[u(\kappa)\{\gamma_4\Gamma_i, \gamma_4\Gamma_j\} + v(\kappa)\operatorname{Tr}(\gamma_4\Gamma_i\gamma_4\Gamma_j)], \quad (4.34)$$

$$V^{(3)}_{ij}(\kappa) = \kappa^{-2} w(\kappa)\operatorname{Tr}(\gamma_4\Gamma_i\gamma_4\Gamma_j) , \quad (4.35)$$

where $a, \ldots, g$ and $p, \ldots, w$ are dimensionless functions of κ (*not* matrices), all behaving like sums of powers of $\ln\kappa$ as $\kappa \to \infty$. Figures 3 and 4 show the lowest-order graphs which contribute to each of these functions. A detailed derivation of these results proceeds along the following lines.

$U^{(1)}$: The single m factor in $U^{(1)}$ must arise from one of the propagator numerators on the fermion line which forms the bridge, because γ_5 invariance ensures that each fermion closed loop is an even function of m. The two J currents can be attached to the fermion bridge, producing the a, b, and c terms, or to a closed loop, producing the d term. Diagrams in which a single $J_{\alpha\mu}$ current is attached to a closed loop give factors proportional to $\operatorname{Tr} t_\alpha$, which vanishes if as usual we exclude baryon number from the Lie algebra of the gauge group G.

$U^{(2)}$: Just as for $U^{(1)}$, the two J currents must be either attached to the fermion bridge, giving the e term, or to the same closed loop, giving the f term.

$U^{(3)}$: The two J currents here can only be attached to the same closed loop.

$V^{(1)}$: The one m factor in $V^{(1)}$ may arise either from a propagator numerator on the fermion bridge line or from a propagator numerator in a closed fermion loop. In the former case γ_5 invariance requires the two S currents to be attached either to the fermion bridge, giving the p, q, and r terms, or to the same fermion closed loop, giving the s term. In the latter case, where m arises from a fermion closed loop, γ_5 invariance requires *one* of the two S currents to be attached to the same loop, and the other can only be attached to the fermion bridge line, giving the t term.

$V^{(2)}$: Here γ_5 invariance requires the two S currents to be either both attached to the fermion bridge line, giving the u term, or to the same closed loop, giving the v term.

$V^{(3)}$: The two S currents here can only be attached to the same closed loop.

In all cases, we use the crossing-symmetry re-

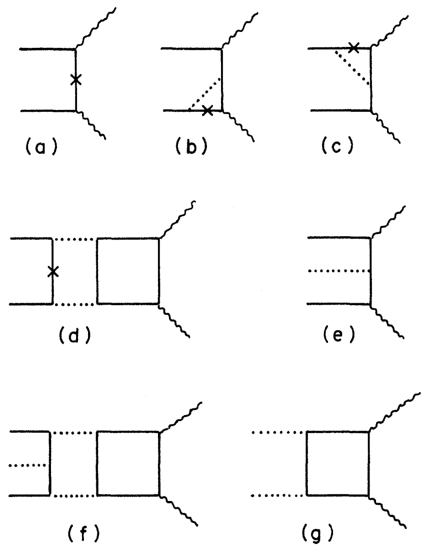

FIG. 3. Some low-order graphs which contribute to the functions $a(\kappa)$, $b(\kappa)$, etc. appearing in $U^{(N)}(\kappa)$. Here straight lines denote quarks, wavy lines denote vector currents, dotted lines denote vector gluons, and crosses denote derivatives with respect to the fermion mass.

lations (4.15) to simplify the results. Also, γ_4 matrices are inserted to keep track of the fact that t_α and Γ_i commute with an even number of γ matrices but not necessarily with an odd number. It is straightforward to check that U and V matrices of the form (4.30)–(4.35) satisfy the G-transformation rules (4.18)–(4.27). Finally, the reality properties (4.14) require that

$$a, d, e, f, g, p, s, t, u, v, w \text{ are real} \tag{4.36}$$

and

$$b^* = c, \quad q^* = r \; . \tag{4.37}$$

So far, all we know about the κ dependence of $a(\kappa)$, $b(\kappa), \ldots$ is that they behave as $\kappa \to \infty$ like a series of powers of $\ln\kappa$. Since these functions are dimensionless, and involve no dimensional parameters other than κ and μ_B, they can also be written as series in powers of $\ln(\kappa/\mu_B)$, with dimensionless coefficients independent of μ_B as well as κ. The asymptotic behavior of the $a(\kappa)$, $b(\kappa)$, etc. as $\kappa \to \infty$ can thus be studied by considering their singularities in the limit of vanishing gluon (as well as fermion) mass.

Logarithmic singularities in μ_B can arise in the U and V functions in two different ways. First, there are "reducible" graphs, like that shown in Fig. 5, which contribute the usual infrared divergences in the limit $\mu_B \to 0$. [It should be noted that although the $\ln(m/\mu_B)$ dependence arising from such graphs appears in the hadronic matrix element $\langle F|O_N|I\rangle$ rather than in $U^{(N)}(\kappa)$ or

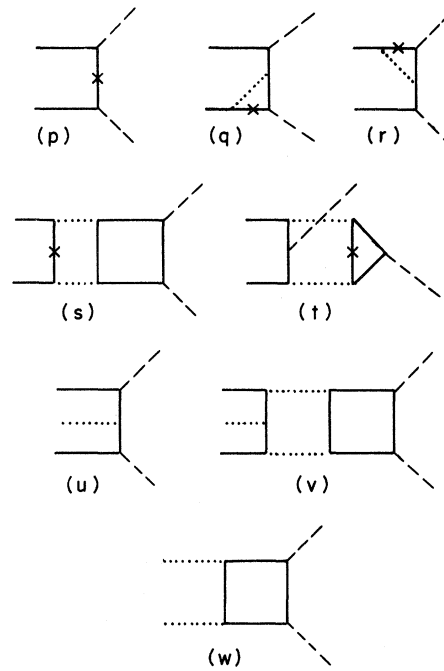

FIG. 4. Some low-order graphs which contribute to the functions $p(\kappa)$, $q(\kappa)$, etc. appearing in $V^{(N)}(\kappa)$. Here straight lines denote quarks, dashed lines denote scalar currents, dotted lines denote vector gluons, and crosses denote derivatives with respect to the fermion mass.

$V^{(N)}(\kappa)$, there is also a $\ln(\kappa/\mu_B)$ dependence which appears in $U^{(N)}(\kappa)$ and $V^{(N)}(\kappa)$ rather than in $\langle F|O_N|I\rangle$.] Second, there are also logarithmic singularities in μ_B arising from the counterterms used to renormalize the fermion self-energy, gluon fermion vertex, and gluon self-energy.[32] [For instance, see Eq. (2.35).] It might be that these singularities cancel if the gluon coupling constant g_B satisfies something like a Gell-Mann–Low eigenvalue condition.[33] However, we would still have to worry about logarithms arising from reducible graphs.

It is possible that the powers of $\ln(\kappa/\mu_B)$ in $a(\kappa)$, $b(\kappa)$, etc. sum up to a power (see Ref. 31) $(\kappa/\mu_B)^\delta$, with δ a small positive or negative exponent. This would make no difference here, provided that

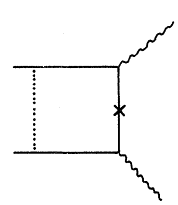

FIG. 5. A typical "reducible" graph, which contributes a term proportional to $\ln\kappa$ in $a(\kappa)$. Here straight lines denote quarks, wavy lines denote vector currents, the dotted line denotes a vector gluon, and the cross denotes a derivative with respect to the fermion mass.

$$|\delta| \lesssim \left(\ln\frac{\mu_W}{\mu_B}\right)^{-1} .$$

There are arguments based on scaling to the effect that δ is zero, so that $a(\kappa)$, $b(\kappa)$, etc. are asymptotically *constant*.[25] In this case, it appears[26] that these quantities will be simply given by their values in the Born approximation, i.e., in the limit $g_B=0$. Such problems will not be addressed in the present work. For our purposes here, it will be sufficient merely to note the special simplifications that would arise if $a(\kappa)$, $b(\kappa)$, etc. were constant.

V. CANCELLATION OF DIVERGENCES

We can now use the results of Sec. IV to check that the divergent part of each term in the second-order correction δS_{FI} is of the same form as would be produced by adding a renormalizable Hermitian gauge-invariant Lorentz-invariant term to the Lagrangian. It will then follow immediately that all such divergences may be eliminated by a renormalization of the parameters in our original Lagrangian, because our original Lagrangian is the most general possible gauge-invariant Lorentz-invariant Hermitian and renormalizable Lagrangian which can be constructed from the given field multiplets ψ, ϕ, B_μ, and A_μ. In particular, whenever we have a zeroth-order symmetry relation which is valid for all values of the parameters in the Lagrangian, the second-order corrections to this symmetry relation will receive no contribution from the divergent terms in δS_{FI}, and will therefore be finite and calculable.[19]

Let us go through Eq. (3.30) term by term, to check that the divergent part of each term is of the same form as could be produced by adding a possible renormalizable gauge-invariant Hermitian and Lorentz-invariant term to the Lagrangian. Our reasoning here will follow along precisely the same lines as the corresponding discussion in Ref. 5, which dealt with one-loop graphs in the absence of strong interactions, so an outline of the arguments will suffice here.

$A1$: Inspection of Eq. (3.19) shows that the $A1$ term contains a logarithmically divergent part, arising from the leading part (4.12) of the vector-current matrix element, which behaves asymptotically like $1/k^2$ times powers of $\ln k^2$. As long as we are only interested in the divergent part of the $A1$ term, we can make the replacement

$$(k^2+\mu^2)^{-1}{}_{\alpha\beta} \to \delta_{\alpha\beta}/k^2 .$$

The divergent part of the $A1$ term is therefore the same as would be produced by a term in the Lagrangian

$$[\Delta_{A1}\mathcal{L}]_\infty = -\bar\psi(\delta m_0+\phi_i\,\delta\Gamma_i)\psi - \bar\psi\gamma^\mu\delta Z_2(\partial_\mu+ig_BB_\mu)\psi - \tfrac{1}{4}\delta Z_3 G_{\mu\nu}G^{\mu\nu} , \tag{5.1}$$

where

$$\delta m_0 = \int_0^\infty U^{(1)}_{\alpha\alpha 0}(\kappa)\kappa d\kappa , \tag{5.2}$$

$$\delta\Gamma_i = \int_0^\infty U^{(1)}_{\alpha\alpha i}(\kappa)\kappa d\kappa , \tag{5.3}$$

$$\delta Z_2 = \int_0^\infty U^{(2)}_{\alpha\alpha}(\kappa)\kappa d\kappa , \tag{5.4}$$

$$\delta Z_3 = \int_0^\infty U^{(3)}_{\alpha\alpha}(\kappa)\kappa d\kappa . \tag{5.5}$$

[See Eqs. (4.7) and (2.19).] Contracting Eqs. (4.24), (4.25), and (4.18) gives

$$[t_\alpha, \gamma_4\delta m_0]=0 , \tag{5.6}$$

$$[t_\alpha, \gamma_4\delta\Gamma_i] = -(\theta_\alpha)_{ij}\gamma_4\delta\Gamma_j , \tag{5.7}$$

$$[t_\alpha, \delta Z_2]=0 . \tag{5.8}$$

Thus (5.1) is G-invariant, as well as manifestly Lorentz-invariant, gluon-gauge-invariant, Hermitian, and renormalizable.

$\phi 1$: Inspection of Eq. (3.23) shows that the $\phi 1$ term contains a logarithmically divergent part arising from the part (4.13) of the scalar current matrix element which behaves asymptotically like $1/k^2$ times powers of $\ln k^2$. As long as we are only interested in the divergent part of the $\phi 1$ term, we can make the replacement

$$(k^2+M^2)^{-1}{}_{ij} \to \delta_{ij}/k^2 .$$

The divergent part of the $\phi 1$ term is therefore the same as would be produced by a term in the Lagrangian

$$[\Delta_{\phi 1}\mathcal{L}]_\infty = -\bar\psi(\delta m_0'+\phi_k\,\delta\Gamma_k')\psi - \bar\psi\gamma^\mu\delta Z_2'(\partial_\mu+ig_BB_\mu)\psi - \tfrac{1}{4}\delta Z_3' G_{\mu\nu}G^{\mu\nu} , \tag{5.9}$$

where

$$\delta m_0' = \int_0^\infty V^{(1)}_{jj0}(\kappa)\kappa d\kappa , \tag{5.10}$$

$$\delta\Gamma_i' = \int_0^\infty V^{(1)}_{jji}(\kappa)\kappa d\kappa , \tag{5.11}$$

$$\delta Z_2' = \int_0^\infty V^{(2)}_{jj}(\kappa)\kappa d\kappa , \tag{5.12}$$

$$\delta Z_3' = \int_0^\infty V^{(3)}_{jj}(\kappa)\kappa d\kappa . \tag{5.13}$$

Contracting Eqs. (4.20), (4.23), (4.26), and (4.27) gives

$$[t_\alpha, \gamma_4 \delta m_0'] = 0 \ , \tag{5.14}$$

$$[t_\alpha, \gamma_4 \delta \Gamma_k'] = -(\theta_\alpha)_{kl} \gamma_4 \delta \Gamma_l' \ , \tag{5.15}$$

$$[t_\alpha, \delta Z_2'] = 0 \ . \tag{5.16}$$

Thus (5.9) is G-invariant, as well as gluon-gauge-invariant, Lorentz-invariant, Hermitian, and renormalizable.

$A\phi$: Inspection of Eq. (3.28) shows that since $\mathfrak{F}_{ij}(\kappa)$ vanishes at least as fast as $1/k^2$ times powers of $\ln k^2$, the $A\phi$ term is finite.

AT: Inspection of Eq. (3.29) shows that the AT term contains a logarithmically divergent part, which is just the same as would be produced by a term in the Lagrangian

$$[\Delta_{AT}\mathfrak{L}]_\infty = -2\pi^2\left(\int_0^\infty \frac{d\kappa}{\kappa}\right)\bar{\psi}\Gamma_i\psi(\theta_\alpha\theta_\alpha)_{ij}\,\phi_j \ . \tag{5.17}$$

This is manifestly G-invariant, gluon-gauge-invariant, Lorentz-invariant, Hermitian, and renormalizable.

$T1$: It was shown in Ref. 5 that the scalar loop and vector loop contributions in (3.26) may be written as derivatives with respect to λ:

$$f_{jkl}\int d^4k(k^2+M^2)^{-1}{}_{kl} = \frac{\partial}{\partial\lambda_i}\int d^4k\,\mathrm{Tr}\ln(k^2+M^2) \ ,$$

$$(\theta_\alpha\theta_\beta\lambda)_i\int d^4k(k^2+\mu^2)^{-1}{}_{\alpha\beta} = \tfrac{1}{2}\frac{\partial}{\partial\lambda_i}\int d^4k\,\mathrm{Tr}\ln(k^2+\mu^2).$$

In addition, the hadronic contribution to the tadpole may be written as a λ derivative

$$\langle S_i(0)\rangle_0 = \frac{\partial\Phi(\lambda)}{\partial\lambda_i} \ , \tag{5.18}$$

where Φ is the sum of all connected single-particle-irreducible vacuum fluctuation graphs produced by the strong interactions (i.e., setting $e=0$). Equation (3.26) thus may be written

$$\delta_{T1}S_{FI} = -2i\,(2\pi)^8\delta^4(P_F-P_I)\,\mathfrak{F}_i^{FI}M^{-2}{}_{ij}\frac{\partial V_1}{\partial\lambda_j} \ , \tag{5.19}$$

where V_1 is the "one-loop potential,"[5] here generalized to include effects of the strong interactions:

$$V_1(\lambda) = \frac{1}{16\pi^2}\int_0^\infty \kappa^3 d\kappa\,\mathrm{Tr}\ln(\kappa^2+M^2) + \frac{3}{16\pi^2}\int_0^\infty \kappa^3 d\kappa\,\mathrm{Tr}\ln(\kappa^2+\mu^2) - \tfrac{1}{2}\Phi(\lambda) \ . \tag{5.20}$$

This is a G-invariant function of λ. In addition, since each term is quartically divergent, and since taking a derivative with respect to λ lowers the degree of divergence of each term by one unit, the divergent part $V_{1\infty}(\lambda)$ of $V_1(\lambda)$ is a quartic polynomial in λ. Inspection of (5.19) shows that the effect of $V_{1\infty}(\lambda)$ is the same as would be produced by a shift in λ:

$$[\Delta\lambda_i]_\infty = -M^{-2}{}_{ij}\frac{\partial}{\partial\lambda_j}V_{1\infty}(\lambda) \ ,$$

and this shift in λ is just the same as would be produced by adding a term to the original Lagrangian[5]:

$$[\Delta_{T1}\mathfrak{L}]_\infty = V_{1\infty}(\phi) \ . \tag{5.21}$$

Since $V_{1\infty}(\phi)$ is quartic in ϕ, this is renormalizable, as well as G-invariant, gluon-gauge invariant, Lorentz invariant, and Hermitian.

In summary, we have shown that the divergent contributions to δS_{FI} are all of the same form as would be produced by adding G-invariant, gluon-gauge-invariant, Lorentz-invariant, Hermitian, and renormalizable terms to the original Lagrangian, and therefore such divergences can have no effect on "natural" zeroth-order symmetry relations.

VI. WEAK CORRECTIONS OF ORDER α

If all particle masses were of the same order of magnitude, then in general no further approximations would be possible, and we would have to rest content with Eq. (3.30) or (3.31) as our formula for the second-order correction δS_{FI}. However, it is at least a reasonably likely possibility that, aside from the photon, all intermediate vector boson masses are orders of magnitude larger than all baryon or quark masses.[34] In this case, the ratio m/μ_W is another small parameter of the theory, and it becomes useful to pick out those terms in δS_{FI} which are of second order in e and of *zeroth* order in m/μ_W. Such terms will typically have the order of magnitude of electromagnetic corrections, and will be referred to here as being "of order α," although as we shall see, these terms can be produced by the weak as well as the electromagnetic interactions. The other terms in δS_{FI} of second order in e are those of order

$$e^2(m/\mu_W)^2 \sim G_F m^2 \sim 10^{-5} \text{ to } 10^{-7} \ , \tag{6.1}$$

and these may be regarded as truly weak corrections.

Let us go through Eq. (3.30) term by term, to pick out those terms in δS_{FI} that are of zeroth order in m/μ_W as well as of second order in e.

$A1$: The $A1$ term is unique, in that it includes the true electromagnetic correction due to photon emission and absorption. In dealing with this contribution, it is very convenient to rewrite the vector propagator in Eq. (3.19) as

$$\left(\frac{1}{k^2+\mu^2}\right)_{\alpha\beta}=n_\alpha n_\beta\left(\frac{1}{k^2}-\frac{1}{k^2+\Lambda^2}\right)+\left(\frac{1}{k^2+\mu'^2}\right)_{\alpha\beta}, \tag{6.2}$$

where Λ is an arbitrary mass, to be taken of order μ_W, n is the eigenvector of μ^2 corresponding to the photon

$$(\mu^2)_{\alpha\beta}n_\beta=0, \quad n_\alpha n_\alpha=1 \tag{6.3}$$

and μ'^2 is the same as μ^2, except that the photon appears with mass Λ:

$$(\mu'^2)_{\alpha\beta}\equiv(\mu^2)_{\alpha\beta}+n_\alpha n_\beta\Lambda^2\ . \tag{6.4}$$

Both terms in (6.2) will make finite contributions to δS_{FI}: the first, because the photon propagator is cut off at momentum Λ, and the second, because the arguments of Sec. V only depend on the structure of the leading term as $k^2\to\infty$, which does not depend on the form of the vector mass matrix. The point of our use of Eq. (6.2) is that the first term is just the conventional photon correction with a cutoff, to which we can apply all the results of decades of work on electromagnetic corrections[22] (such as the Cottingham formula, the Sutherland theorem, the $\Delta I\leq 2$ rule, etc.), while the second term has a large denominator which suppresses all terms in the vector current matrix element which vanish rapidly as $k^2\to\infty$. To be more specific, the first term in Eq. (6.2) makes a contribution

$$\delta_{\rm em}S_{FI}=(2\pi)^4\delta^4(P_F-P_I)\times\int d^4k\,\mathcal{F}^{FI}_{\rm em}(k)\left(\frac{1}{k^2}-\frac{1}{k^2+\Lambda^2}\right), \tag{6.5}$$

where $\mathcal{F}^{FI}_{\rm em}(k)$ is the electromagnetic current matrix element:

$$\mathcal{F}^{FI}_{\rm em}(k)=\frac{i}{2(2\pi)^4}\int d^4x\langle F|\,T\{J_\mu(x)J^\mu(0)\}\,|I\rangle e^{ik\cdot x}, \tag{6.6}$$

$$J_\mu\equiv n_{\alpha\mu}J_{\alpha\mu}\ . \tag{6.7}$$

This is finite because $\mathcal{F}_{\rm em}(k)$ vanishes at least as fast as $1/k$ as $k\to\infty$, but there is nothing that can be done to simplify it without entering into a detailed analysis of the intermediate hadronic states which contribute to the time-ordered product. On the other hand, the second term in Eq. (6.2) makes a contribution

$$\delta'_{A1}S_{FI}=(2\pi)^4\delta^4(P_F-P_I)\,\eta^{\mu\nu}\times\int d^4k\,\mathcal{F}^{FI}_{\alpha\mu,\beta\nu}(k)\left(\frac{1}{k^2+\mu'^2}\right)_{\alpha\beta}. \tag{6.8}$$

Any term in $\mathcal{F}$ which vanishes at least as fast as $1/k^4$ times powers of $\ln k^2$ will make a contribution which is suppressed by a factor $1/\mu'^2$, and hence is of order G_F [see (6.1)] rather than of order α. The only terms in $\mathcal{F}$ which are *not* suppressed by factors $1/\mu'^2$ are those given by Eq. (4.12), which vanish like $1/k^2$ times powers of $\ln k^2$ as $k^2\to\infty$. Thus the part of the second-order correction (6.8) which is of order α rather than of order G_Fm^2 is just the same as would be produced by adding to the effective Lagrangian a correction term:

$$\Delta_{A1}\mathcal{L}=-\bar\psi\Upsilon_1\psi-\bar\psi\gamma^\mu\Upsilon_2(\partial_\mu+ig_BB_\mu)\psi-\tfrac{1}{4}\Upsilon_3G_{\mu\nu}G^{\mu\nu}, \tag{6.9}$$

where

$$\Upsilon_N\equiv\int_0^\infty\kappa^3d\kappa\,U^{(N)}_{\alpha\beta}(\kappa)(\kappa^2+\mu'^2)^{-1}{}_{\alpha\beta}\ . \tag{6.10}$$

Equation (4.14) gives Υ_N the reality properties

$$\Upsilon_1^\dagger=\gamma_4\Upsilon_1\gamma_4,\quad \Upsilon_2^\dagger=\Upsilon_2,\quad \Upsilon_3^*=\Upsilon_3\ . \tag{6.11}$$

The Υ_3 term may be eliminated by a renormalization of the gluon field, and obviously has no effect on any strong interaction symmetry. The Υ_2 term may also be eliminated by a renormalization of the fermion field:

$$\psi\to\psi_1\equiv(1+\tfrac{1}{2}\Upsilon_2)\psi\ , \tag{6.12}$$

but the zeroth-order mass term then becomes, to order e^2,

$$-\bar\psi m\psi=-\psi_1^\dagger(1-\tfrac{1}{2}\Upsilon_2)\gamma_4m(1-\tfrac{1}{2}\Upsilon_2)\psi_1$$

$$=-\bar\psi_1(m-\tfrac{1}{2}\gamma_4\Upsilon_2\gamma_4m-\tfrac{1}{2}m\Upsilon_2)\psi_1\ .$$

Thus the total effect of the $A1$ term on "natural" zeroth-order symmetries *in order* α is the same as a term in the Lagrangian of the form (now dropping the subscript 1)

$$\Delta_{A1}\mathcal{L}=-\bar\psi\delta_{A1}m\psi\ , \tag{6.13}$$

where

$$\delta_{A1}m=\Upsilon_1-\tfrac{1}{2}\gamma_4\Upsilon_2\gamma_4m-\tfrac{1}{2}m\Upsilon_2\ . \tag{6.14}$$

The Υ_N have a logarithmically divergent part

$$[\Upsilon_N]_\infty=\int^\infty U^{(N)}_{\alpha\alpha}(\kappa)\kappa d\kappa\ , \tag{6.15}$$

but we saw in the last section that $U^{(N)}_{\alpha\alpha}$ makes no contribution to the corrections to "natural" zeroth-order symmetries. Thus, as long as we restrict our attention to such corrections, it is permissible to eliminate the divergences in Υ_N by subtracting a similar term in which $(\mu'^2)_{\alpha\beta}$ is replaced with any matrix proportional to $\delta_{\alpha\beta}$. It is convenient to choose the mass matrix in the subtracted term as $\Lambda^2\delta_{\alpha\beta}$; then Υ_N takes the finite form

$$\Upsilon_N=\int_0^\infty \kappa d\kappa\, U^{(N)}_{\alpha\beta}(\kappa)\times[(\kappa^2+\mu'^2)^{-1}{}_{\alpha\beta}-(\kappa^2+\Lambda^2)^{-1}\delta_{\alpha\beta}], \quad (6.16)$$

so that the photon term makes no contribution here here. By using Eqs. (4.30)–(4.32) and (4.37), we can write (6.14) as

$$\delta_{A1}m=\gamma_4\{t_\alpha t_\beta,\gamma_4 m\}C_{\alpha\beta}+\gamma_4[t_\alpha,[t_\beta,\gamma_4 m]]D_{\alpha\beta}+i\gamma_4[t_\alpha t_\beta,\gamma_4 m]E_{\alpha\beta}+m\,\mathrm{Tr}(t_\alpha t_\beta)F_{\alpha\beta}\,, \quad (6.17)$$

where C, D, E, and F are the *real symmetric* matrices:

$$C_{\alpha\beta}=\int_0^\infty[b(\kappa)+c(\kappa)-a(\kappa)-e(\kappa)]\times[(\kappa^2+\mu'^2)^{-1}{}_{\alpha\beta}-(\kappa^2+\Lambda^2)^{-1}\delta_{\alpha\beta}]\kappa d\kappa\,, \quad (6.18)$$

$$D_{\alpha\beta}=-\int_0^\infty a(\kappa)[(\kappa^2+\mu'^2)^{-1}{}_{\alpha\beta}-(\kappa^2+\Lambda^2)^{-1}\delta_{\alpha\beta}]\kappa d\kappa\,, \quad (6.19)$$

$$E_{\alpha\beta}=-i\int_0^\infty[b(\kappa)-c(\kappa)]\times[(\kappa^2+\mu'^2)^{-1}{}_{\alpha\beta}-(\kappa^2+\Lambda^2)^{-1}\delta_{\alpha\beta}]\kappa d\kappa\,, \quad (6.20)$$

$$F_{\alpha\beta}=\int_0^\infty[d(\kappa)-f(\kappa)]\times[(\kappa^2+\mu'^2)^{-1}{}_{\alpha\beta}-(\kappa^2+\Lambda^2)\delta_{\alpha\beta}]\kappa d\kappa\,. \quad (6.21)$$

We can drop the F term, because it obviously cannot affect any zeroth-order symmetry relations obeyed by m. Also, we can drop the E term, because this term can be eliminated by a redefinition of the fermion field

$$\psi\to(1+it_\alpha t_\beta E_{\alpha\beta})\psi\,,$$

which leaves unchanged all terms in the strong-interaction Lagrangian (3.5) except the fermion mass term. Thus, in calculating corrections to zeroth-order symmetry relations, we may replace (6.17) with

$$\delta_{A1}m=\gamma_4\{t_\alpha t_\beta,\gamma_4 m\}C_{\alpha\beta}+\gamma_4[t_\alpha,[t_\beta,\gamma_4 m]]D_{\alpha\beta}\,. \quad (6.22)$$

We note for future reference that the D term could also be written as a sum of Yukawa coupling matrices, using the relation

$$\gamma_4[t_\alpha,[t_\beta,\gamma_4 m]]=(\theta_\alpha\theta_\beta\lambda)_i\Gamma_i\,. \quad (6.23)$$

The functions $a(\kappa)$, $b(\kappa)$, etc. are real dimensionless functions of order unity, which behave as $\kappa\to\infty$ like a series of powers of $\ln\kappa$, so the matrices C and D (as well as E and F) are real dimensionless matrices of order unity, and are given by a series of powers of the matrix $\ln\mu'^2$. [The mass shift (6.22) is of order αm because t_α is of order e.] In particular, if $a(\kappa)$, $b(\kappa)$, etc. approach *constants*, either because of a Gell-Mann–Low eigenvalue condition[33] or for some other reason connected with scaling,[25,26] then (6.18) and (6.19) become

$$C_{\alpha\beta}=\mathfrak{C}[\ln(\mu'^2/\Lambda^2)]_{\alpha\beta}\,, \quad (6.24)$$

$$D_{\alpha\beta}=\mathfrak{D}[\ln(\mu'^2/\Lambda^2)]_{\alpha\beta}\,, \quad (6.25)$$

where

$$\mathfrak{C}=\tfrac{1}{2}[b(\infty)+c(\infty)-a(\infty)-e(\infty)]\,, \quad (6.26)$$

$$\mathfrak{D}=-\tfrac{1}{2}a(\infty)\,. \quad (6.27)$$

The strong interactions then would enter into the effective mass shift only through the two real Λ-independent parameters $\mathfrak{C}$ and $\mathfrak{D}$:

$$\delta_{A1}m=(\mathfrak{C}\gamma_4\{t_\alpha t_\beta,\gamma_4 m\}+\mathfrak{D}\gamma_4[t_\alpha,[t_\beta,\gamma_4 m]])\times\left(\ln\frac{\mu'^2}{\Lambda^2}\right)_{\alpha\beta}\,. \quad (6.28)$$

The effect of the prime on the μ^2 matrix is just that we are to leave out the photon term altogether in summing over α and β.

$\phi 1$: Inspection of Eq. (2.17) shows that the Yukawa coupling matrix is of order

$$\Gamma\sim m/\lambda\sim e(m/\mu_W)\,, \quad (6.29)$$

so the scalar currents S_i are smaller than the vector currents $J_{\alpha\mu}$ by a factor of order m/μ_W. The $\phi 1$ term (3.23) is therefore smaller than the $A1$ term by a factor $(m/\mu_W)^2$, and does not contribute to the symmetry-breaking corrections of order α.

$A\phi$: The integrand in (3.28) behaves like k^{-6} times powers of $\ln\kappa$ as $\kappa\to\infty$, so the final factor $(\kappa^2+\mu^2)^{-1}$ effectively contributes a factor of order μ_W^{-2}. (Note that the photon makes no contribution here, because the charge operator $n_\alpha\theta_\alpha$ annihilates λ.) The factors $\theta_\alpha\lambda$ are of order μ_W, so the $A\phi$ contribution is of the same order of the $\phi 1$ contribution to δS_{FI}, and may therefore be neglected for the same reasons.

AT: Inspection of Eq. (3.29) shows that the AT term is the same as would be produced by adding a correction to the fermion mass matrix:

$$\delta_{AT}m=\frac{1}{16\pi^2}\Gamma_i(\theta_\beta\theta_\alpha\lambda)_i\int_0^\infty(\kappa^2+\mu^2)^{-1}{}_{\alpha\beta}\kappa d\kappa\,. \quad (6.30)$$

The integral has a divergent part proportional to

$\delta_{\alpha\beta}$, but as shown in Sec. V, such terms make no contributions to the corrections to natural zeroth-order symmetry relations. Thus we may introduce a cutoff, and write

$$\delta_{AT}m = \frac{1}{16\pi^2}\Gamma_i(\theta_\beta\theta_\alpha\lambda)_i\int_0^\infty[(\kappa^2+\mu^2)^{-1}{}_{\alpha\beta} - (\kappa^2+\Lambda^2)^{-1}\delta_{\alpha\beta}]\kappa d\kappa .$$

Doing the integral and then using (6.23), we find

$$\delta_{AT}m = \frac{1}{32\pi^2}\Gamma_i(\theta_\beta\theta_\alpha\lambda)_i\left(\ln\frac{\mu^2}{\Lambda^2}\right)_{\alpha\beta}$$
$$= \frac{1}{32\pi^2}\gamma_4[t_\alpha,[t_\beta,\gamma_4 m]]\left(\ln\frac{\mu^2}{\Lambda^2}\right)_{\alpha\beta} . \quad (6.31)$$

There is no effective difference between μ^2 and μ'^2, because the charge operator $n_\alpha t_\alpha$ commutes with $\gamma_4 m$. Thus the AT term merely provides an additional contribution to the $D_{\alpha\beta}$ matrix in (6.22), of the same form as in Eq. (6.25).

*T*1: Inspection of Eq. (5.19) shows that the *T*1 term is the same as would be produced by adding a correction to the fermion mass matrix

$$\delta_{T1}m = -\Gamma_i M^{-2}{}_{ij}\frac{\partial V_1(\lambda)}{\partial\lambda_j} , \quad (6.32)$$

where $V_1(\lambda)$ is the quantity (5.20). If the scalar mass matrix M is of the same order of magnitude as the vector mass matrix μ, then as shown in Ref. 5, the *T*1 term is (like the *A*1 and *AT* terms) of order αm. The fermion term in $\partial V_1/\partial\lambda$ is smaller than the other terms by a factor of order $(m/\mu_W)^2$, and may be dropped, leaving us with

$$\delta_{T1}m = -\frac{1}{32\pi^2}\Gamma_i M^{-2}{}_{ij}[f_{jkl}(M^2\ln M^2)_{kl} + 6(\theta_\alpha\theta_\beta\lambda)_j(\mu^2\ln\mu^2)_{\alpha\beta}] . \quad (6.33)$$

Here, as for the *AT* term, the photon makes no contribution. If the scalar mass matrix is somewhat smaller than the vector mass matrix, then the second term in $\delta_{T1}m$ may dominate over all other contributions to the mass shift.

In summary, the symmetry-breaking effects of the weak and electromagnetic interactions may be divided in two parts:

(1) There is the *conventional photon contribution*, with a cutoff at an arbitrary momentum Λ of order μ_W.

(2) There is a *weak-interaction contribution*, which arises from intermediate vector bosons other than the photon, and is equivalent (to order α) to a fermion mass shift of the form

$$\delta m = \delta_{A1}m + \delta_{AT}m + \delta_{T1}m . \quad (6.34)$$

For zeroth-order symmetries of type 1,[9] which arise from the representation content of the scalar fields rather than from constraints on their vacuum expectation values, we can drop all terms in δm which have the form of a linear combination of the Yukawa coupling matrices Γ_i, because any such terms will obey the same zeroth-order type-1 mass relations as m itself.

This leaves us with only the first part of the *A*1 term (6.22), so δm here takes the remarkably simple form

$$\delta m = \gamma_4\{t_\alpha t_\beta,\gamma_4 m\}C_{\alpha\beta} , \quad (6.35)$$

or, if $a(\kappa)$, $b(\kappa)$, etc. are asymptotically constant,

$$\delta m = \mathfrak{C}\,\gamma_4\{t_\alpha t_\beta,\gamma_4 m\}\left(\ln\frac{\mu'^2}{\Lambda^2}\right)_{\alpha\beta} . \quad (6.36)$$

For zeroth-order symmetries of types 2 or 3,[9] which arise from constraints on λ, the presence or absence of tadpole contributions depends on whether the constraints apply for all G-invariant polynomials $P(\phi)$, or only for *quartic* G-invariant polynomials. In the former case,[35] the *T*1 term does not contribute, so δm here takes the form

$$\delta m = (C_{\alpha\beta}\gamma_4\{t_\alpha t_\beta,\gamma_4 m\} + D'_{\alpha\beta}\gamma_4[t_\alpha,[t_\beta,\gamma_4 m]]) , \quad (6.37)$$

where $D'_{\alpha\beta}$ is the sum of an *A*1 and an *AT* contribution. If $a(\kappa)$, $b(\kappa)$, etc. are asymptotically constant, then this simplifies to

$$\delta m = (\mathfrak{C}\,\gamma_4\{t_\alpha t_\beta,\gamma_4 m\} + \mathfrak{D}'\gamma_4[t_\alpha,[t_\beta,\gamma_4 m]])\times\left(\ln\frac{\mu'^2}{\Lambda^2}\right)_{\alpha\beta} , \quad (6.38)$$

where

$$\mathfrak{D}' = \mathfrak{D} + \frac{1}{32\pi^2} .$$

On the other hand, if the constraints on λ do depend on the quartic nature of the polynomial $P(\phi)$, then all three terms in (6.34) may contribute to δm, though the first part of the *T*1 term (6.33) may be dropped for zeroth-order symmetries of type 2.

As an example, suppose that all intermediate vector bosons except the photon have the same mass μ. [This is necessarily the case in any O(3) model of the Georgi-Glashow type.[36]] The *A*1 term given by (6.17)–(6.21) and the *AT* term given by (6.31) obviously vanish if we choose the cutoff Λ equal to μ, as we are free to do. The *T*1 term given by (6.33) also vanishes except for zeroth-order mass relations of type 3. Thus, if isospin is a zeroth-order mass relation of type 1 or 2, the corrections are purely "electromagnetic," but with a cutoff given precisely by the intermediate vector boson mass:

$$\delta S_{FI} = (2\pi)^4 \delta^4(P_F - P_I) \times \int d^4k\, \mathfrak{F}^{FI}_{\text{em}}(k)\left(\frac{1}{k^2} - \frac{1}{k^2+\mu^2}\right) . \qquad (6.39)$$

Whatever the form of δm, once we know that the "order-α effects" of the weak interactions appear only in this mass shift, then as discussed in the Introduction we can immediately conclude that such effects preserve parity and strangeness conservation. There is an interesting question: Does the exchange of *two* intermediate vector bosons conserve parity and strangeness in order α^2, and satisfy a $|\Delta S| \leq 1$ rule in order $G_F\alpha$? Such matters will be considered in future papers of this series.

Note added in proof. There are a number of reasons for being dissatisfied with the Abelian gluon theory of strong interactions discussed here. Such theories are never "asymptotically free," in the sense of D. Gross and F. Wilczek [Phys. Rev. Lett. 30, 1343 (1973)] and H. D. Politzer [Phys. Rev. Lett. 30, 1346 (1973)]. Also, if the weak-interaction gauge group contains a U(1) factor, there can arise troublesome gluon-photon mixing effects (H. S. Tsao, private communication). Fortunately, there is a large class of non-Abelian gauge theories in which the arguments of the present paper can be used to show that parity and strangeness are conserved to order α (S. Weinberg, to be published). It appears that when such theories are "asymptotically free," the results of Refs. 25 and 26 become valid in perturbation theory.

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[1]S. Weinberg, Phys. Rev. Lett. 29, 388 (1972).

[2]H. Georgi and S. L. Glashow, Phys. Rev. D 6, 2977 (1972).

[3]S. Weinberg, Phys. Rev. Lett. 29, 1698 (1972).

[4]H. Georgi and S. L. Glashow, Phys. Rev. D 7, 2457 (1973).

[5]S. Weinberg, Phys. Rev. D 7, 2887 (1973).

[6]This possibility has been particularly advocated by K. Bardakci and M. B. Halpern, Phys. Rev. D 6, 696 (1972); I. Bars, M. B. Halpern, and M. Yoshimura, Phys. Rev. Lett. 29, 969 (1972); I. Bars, M. B. Halpern, and M. Yoshimura, Phys. Rev. D 7, 1233 (1973). Also see H. Georgi, *ibid.* 7, 1258 (1973); H. Georgi and T. Goldman, Phys. Rev. Lett. 30, 514 (1973). There is an interesting possibility that baryon conservation itself could be an Abelian gauge symmetry, broken spontaneously by the vacuum expectation values of a set of complex electrically neutral scalar fields carrying nonzero baryon number. In this case, the vector particles would acquire a mass, but in the absence of fermion fields having zero baryon and lepton number, there would be no way to break the global conservation of *fermionic* baryon number. For our present purposes, there does not seem to be any important difference between such a theory and the neutral vector gluon theory of case (a), in which the gluon mass is put in by hand. [The possibility of viewing baryon conservation as a broken gauge symmetry has been suggested independently by A. Pais (to be published).]

[7]Gauge theories with strongly coupled scalar and pseudoscalar fields have been considered by S. Weinberg, Phys. Rev. Lett. 27, 1688 (1971); J. Schechter and Y. Ueda, Phys. Rev. D 5, 2846 (1972); W. F. Palmer, *ibid.* 6, 1190 (1972); M. Weinstein, *ibid.* 7, 1854 (1973); T. Hagiwara and B. W. Lee, *ibid.* 7, 459 (1973); D. A. Dicus and V. S. Mathur, *ibid.* 7, 525 (1973).

[8]See, e.g., G. Feinberg, P. Kabir, and S. Weinberg, Phys. Rev. Lett. 3, 527 (1959), especially footnote 9. The mass matrix can be made Hermitian and free of γ_5 matrices by a unitary transformation on *either* the left- or the right-handed parts of the fermion multiplet, and it can then be diagonalized by a simultaneous unitary transformation on *both* the left- and right-handed parts of the fermion multiplet.

[9]The general classification of types of natural zeroth-order symmetry, as negotiated between Georgi and Glashow and myself, is presented in Refs. 4 and 5. Also, a comprehensive paper on the varieties of natural zeroth-order symmetry is now in preparation. The possibility of natural zeroth-order symmetries of types 1, 2, and 3 was pointed out originally in Refs. 1, 3, and 2, respectively, although an example of what we would now call a natural zeroth-order symmetry of type 1 appeared in an earlier paper of G. 't Hooft, Nucl. Phys. B35, 167 (1971).

[10]See Refs. 1–5. The general proof that corrections to natural zeroth-order symmetries are finite in the one-loop approximation is given in Ref. 5. Models in which isotopic spin conservation *is not* natural, but imposed by an arbitrary choice of parameters, have been considered by Fayyazuddin and Riazuddin, Phys. Rev. D (to be published); A. Love and G. G. Ross, Nucl. Phys. (to be published). In such theories the neutron-proton mass difference, of course, turns out to be divergent.

[11]A "semi-realistic" example is given in Ref. 1. Calculations in this model have been carried out by A. Duncan and P. Schattner, Phys. Rev. D 7, 1861 (1973); D. Z. Freedman and W. Kummer, *ibid.* 7, 1829 (1973).

[12]There is a problem here, pointed out to me by H. Georgi and S. Glashow. When N fermion masses vanish, the resulting zeroth-order symmetry group is not $SU(N)\otimes SU(N)$, but $SU(N)\otimes SU(N)\otimes U(1)\otimes U(1)$. One of the extra U(1) factors is a chiral symmetry which certainly does not appear as an ordinary symmetry of physical states. It might be spontaneously broken, but then why is there not a pseudoscalar isoscalar boson with a mass comparable to that of the pion? One possibility is that chiral U(1) *is* broken spontaneously, but not as badly as chiral $SU(2)\otimes SU(2)$, so that the isoscalar pseudoscalar particle is heavier than the pion, perhaps as heavy as the η. In this connection, see S. L. Glashow, R. Jackiw, and S.-S. Shei, Phys. Rev. 187, 1916 (1969).

[13]There is a possibility, although not a very attractive possibility, that SU(3) and/or chirality are only approximate symmetries even in zeroth order, and arise more or less by accident. For instance, if the fermion masses were all somewhat smaller than the gluon mass μ_B, then chirality might arise as an approximate symmetry, broken even in zeroth order by terms of order m/μ_B.

[14]From earliest times, it has been recognized that a symmetry can break down spontaneously, without the need for elementary scalar fields appearing in the Lagrangian. See, for example, Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122, 345 (1961); J. Goldstone, A. Salam, and S. Weinberg, Phys. Rev. 127, 965 (1962), Sec. III; K. Johnson, in *Nuclear Physics and Particle Physics: The Ninth Latin American School of Physics, Santiago, Chile, 1967,* edited by I. Saavedra (Benjamin, New York, 1968); H. Pagels, Phys. Rev. Lett. 28, 1482 (1972); H. Pagels, Phys. Rev. D 7, 3689 (1973).

[15]Approximate zeroth-order symmetries of m, of the sort mentioned in Ref. 13, may also suffer a further spontaneous breaking by the strong interactions, giving rise to pseudo-Goldstone bosons with masses that are small but finite even to zeroth order in e.

[16]The pseudo-Goldstone bosons dealt with in Refs. 3 and 5 are associated with scalar fields appearing in the Lagrangian, whose vacuum expectation values break symmetries of the polynomial $P(\phi)$. The pion mass has been studied in a model of this kind by I. Bars and K. Lane, Phys. Rev. D, to be published. The trouble with any such model, in which there appear strongly coupled fields for pseudo-Goldstone bosons, is that it is very difficult to see why parity and strangeness should be conserved to order α, when the weak interactions are taken into account. It is for this reason that I am here concentrating on pseudo-Goldstone bosons associated with a purely dynamical breakdown of strong-interaction symmetries. The paper by Dicus and Mathur, Ref. 7, reminded me that calculations involving Goldstone or pseudo-Goldstone bosons can be carried out using the methods of current algebra, without having to associate the bosons with elementary fields appearing in the Lagrangian. Of course, if the zeroth-order symmetries of the strong-interactions arise from an "accidental" symmetry of the polynomial $P(\phi)$ [these are called type 2 symmetries (see Ref. 9)] there will also appear weakly coupled pseudo-Goldstone bosons, of the sort discussed in Refs. 3 and 5.

[17]P. W. Higgs, Phys. Lett. 12, 132 (1964); Phys. Rev. Lett. 13, 508 (1964), Phys. Rev. 145, 1156 (1966); F. Englert and R. Brout, Phys. Rev. Lett. 13, 321 (1964); G. S. Guralnik, C. R. Hagen, and T. W. B. Kibble, *ibid.* 13, 585 (1964); T. W. B. Kibble, Phys. Rev. 155, 1554 (1967).

[18]There is a curious problem here, related to a phenomenon first noted by S. Coleman and E. Weinberg, Phys. Rev. D 7, 1888 (1973) and also discussed in Refs. 3 and 5. When the zeroth-order symmetries of m are spontaneously broken by the strong interactions, there is generally a continuous infinity of possible symmetry-breaking solutions, all perfectly equivalent as far as the strong interactions are concerned. [For instance, when chiral $SU(2)\otimes SU(2)$ is spontaneously broken, the vacuum expectation value of the chiral four-vector $\{\bar{N}N, \bar{N}\gamma_5\vec{\tau}N\}$ can point in any direction.] However, these solutions become physically inequivalent when we turn on the gauge interactions. How then do the strong interactions know in which direction to break the zeroth-order symmetries? The general answer is that given in Ref. 5; the zeroth-order symmetries must be spontaneously broken by the strong interactions in such a way that the matrix element of the weak and electromagnetic perturbation of order e^2 between the vacuum and a single pseudo-Goldstone boson state vanishes. This condition will be discussed in greater detail in future papers of this series.

[19]This has already been shown for certain special cases by R. N. Mohapatra and P. Vinciarelli, this issue, Phys. Rev. D 8, 481 (1973); Fayyazuddin and Riazuddin (unpublished). The deep relation between current algebra and the cancellations occurring in gauge theories was stressed earlier by W. Kummer and K. D. Lane, Phys. Rev. D 7, 1910 (1973).

[20]See Ref. 8. This is the same argument as was used in the days of the "unitarity cutoff" to show that the weak interactions would not produce parity violations of order unity in a neutral vector gluon theory of strong interactions; see M. Gell-Mann, M. L. Goldberger, N. M. Kroll, and F. E. Low, Phys. Rev. 179, 1518 (1969); S. L. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D 2, 1285 (1970).

[21]S. Coleman and S. L. Glashow, Phys. Rev. Lett. 6, 423 (1961); Phys. Rev. 134, B671 (1964).

[22]For an excellent survey, see A. Zee, Phys. Rep. 3C, 127 (1972).

[23]D. G. Sutherland, Phys. Lett. 23, 384 (1966); J. S. Bell and D. Sutherland, Nucl. Phys. B4, 315 (1968).

[24]A term in δm with the chiral transformation properties of τ_3 could invalidate the vanishing of the η-decay matrix element at zero neutral pion momentum, in agreement with observation. See K. G. Wilson, Phys. Rev. 179, 1499 (1969). However, it is not possible to rely on such calculations in neutral-gluon theories until we settle the questions raised in Ref. 12.

[25]R. Jackiw, R. Van Royen, and G. B. West, Phys. Rev. D 2, 2473 (1970); H. Pagels, Phys. Rev. 185, 1990 (1969).

[26]R. Jackiw and H. J. Schnitzer, Phys. Rev. D 5, 2008 (1972); 7, 3116 (1973); J. Gunion, this issue, Phys. Rev. D 8, 517 (1973).

[27]The general formalism used here is that introduced by S. Weinberg, Phys. Rev. D 7, 1068 (1973), and further developed in Ref. 5. (An error in the derivation of the unitarity gauge Feynman rules, which fortunately does not affect the answer, has been corrected by L.-F.

Li, Phys. Rev. D 7, 3815 (1973).
[28]The gauge used here was introduced by K. Fujikawa, B. W. Lee, and A. I. Sanda, Phys. Rev. D 6, 2923 (1972). Also see Y.-P. Yao, Phys. Rev. D 7, 1647 (1973); G. 't Hooft and M. Veltman, Nucl. Phys. B50, 318 (1972); T. Appelquist, J. Carazzone, T. Goldman, and H. R. Quinn, Phys. Rev. D, to be published. The Feynman rules for general theories are derived, in notation used here, in Appendix A of Ref. 5.
[29]B. W. Lee, Nucl. Phys. B9, 649 (1969); J.-L. Gervais and B. W. Lee, Nucl. Phys. B12, 627 (1969).
[30]S. Weinberg, Phys. Rev. 118, 838 (1960). A more readable exposition is given by J. D. Bjorken and S. D. Drell, *Relativistic Quantum Fields* (McGraw-Hill, New York, 1965), Sections 19.10, 19.11, and 19.14.
[31]K. Wilson, unpublished report; and Ref. 24. Also see W. Zimmermann, in *Lectures on Elementary Particles and Quantum Field Theory* (M.I.T. Press, Cambridge, Mass., 1970). The relation between the operator-product expansion and the bridge analysis of Ref. 30 was pointed out by C. Callan, Phys. Rev. D 5, 3202 (1972).
[32]These singularities were brought to my attention by R. Jackiw and K. Johnson.
[33]M. Gell-Mann and F. E. Low, Phys. Rev. 95, 1300 (1954). Also see N. N. Bogoliubov and D. V. Shirkov, *Introduction to the Theory of Quantized Fields* (Interscience, New York, 1959).
[34]The possibility of "superheavy" fermions is mentioned in Ref. 1.
[35]An example of "type 2" zeroth-order symmetry which does not depend on the quartic nature of the polynomial is given in Ref. 3. The fact that $T1$ contributions to δm vanish in such cases was discovered in the course of calculations by A. Duncan and P. Schattner.
[36]H. Georgi and S. L. Glashow, Phys. Rev. Lett. 26, 1494 (1972).

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A Class of Gauge Theories with Superweak CP Violation*

A. Pais
Rockefeller University, New York, New York 10021
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A class of gauge theories is discussed in which the $\Delta S=0$ and the $\Delta S=1$ semileptonic decays are mediated by distinct intermediate bosons, whose mass ratio is related to the Cabibbo angle θ. *Common* features of the models are as follows: θ is well defined only as the result of spontaneous symmetry breaking; μ decay and the semileptonic $\Delta S=0$, $\Delta S=1$ decays are in the ratio $1:\cos^2\theta:\sin^2\theta$ only if CP is maximally violated in the lepton sector; a breakdown of μe universality related to CP violation; a superweak impact of CP violation on K decays; the mediation through a neutral vector boson of nonleptonic decays which obey $|\Delta I|=\frac{1}{2}$; and an amplitude $\ll O(G)$ for $\nu_\mu e$ scattering. Two *distinct* types of theories are discussed in detail. (a) The gauge group O(4) reported before. Here the CP-violating parameter needs a renormalization. To $O(G)$, ν_μ-nucleon reactions are possible only if a heavy lepton is produced. (b) $O(4)\times\mathcal{G}$, where left- [right-] handed fermions are in O(4) [$\mathcal{G}$] but scalar with respect to $\mathcal{G}$ [O(4)]. Here the CP-violating parameter can be made finite if a constraint between electron and muon multiplets is satisfied. Further consequences for case (b) are: the $\bar{\nu}_e$-e and ν_e-e elastic cross sections are $(1+\sin 2|\theta|)$ times their respective $V-A$ values, and ν_μ-nucleon reactions are possible with or without production of a heavy lepton. However, the final hadronic state is necessarily "charmed." The example $\mathcal{G}=U(1)$ is discussed in detail. The role of discrete symmetries is emphasized.

I. INTRODUCTION

A surge of theoretical activity has been generated by the discovery of a new class of renormalizable theories in which the notion of spontaneous breakdown of a local symmetry plays a key role. This development opens the strongly attractive prospect of unifying weak interactions with electromagnetism. Current investigations are proceeding on two main fronts. First and foremost, work is going on to clarify further some difficult and obscure technical aspects of this new renormalization program. Secondly (and hopefully not too early) attempts are under way to close in on the local symmetry that is chosen by nature, and on the representations of the symmetry to be assigned to the particles.[1]

Features common to all these investigations are (1) the occurrence of a number of vector mesons with masses that appear to hover invariably in a region well over 10 GeV/c^2, (2) the appearance of scalar mesons mainly needed for the mass gen-