Deep-Inelastic Scattering and Static Properties of the Baryons in the Quark-Gluon Model

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The longitudinal structure function $[W_L(q^2,\nu)]$ of inelastic neutrino and electron scattering is studied in the deep-inelastic limit in the canonical quark-gluon model. Although $W_L(q^2,\nu)$ vanishes asymptotically in this model, $\nu W_L(q^2,\nu)$ should scale. Sum rules are derived which relate integrals over the scaling limit of $\nu W_L(q^2,\nu)$ and the well-known structure function $F_2(-q^2/2\nu)$ to octet baryon masses, the Gell-Mann–Oakes–Renner parameter (c), and the pion-nucleon sigma term (σ_π). The sum rules are convergent, since leading Regge terms are to be subtracted off according to a well-known prescription and contain no arbitrary constants if the residues of $\alpha=0$ singularities in forward current-hadron scattering are polynomials in q^2. The sum rules are derived using light-cone techniques. It is shown that the parton model and Bjorken-Johnson-Low commutators yield identical results. Similar sum rules are presented for other interactions and scalar "quarks." Estimates of c and σ_π allow numerical evaluation of the sum rules indicating that the integrals over $\nu W_L(q^2,\nu)$ are small. The pattern of chiral-symmetry breaking in the vector-gluon model is discussed. It is shown that the dictum that scaling laws may be abstracted from free-field theory leads to difficulties (in that it generates too trivial a theory) if applied to the chiral-symmetry-breaking structure functions of neutrino scattering. Abstraction from gluon models, however, remains adequate.

I. INTRODUCTION

The scaling observed in the SLAC-MIT electron scattering experiments[1] has generated considerable interest in calculational approaches to field theory which ignore the renormalizations and attendant logarithms of perturbation theory. Light-cone expansions[2] with canonical singularities, Bjorken-Johnson-Low (BJL) expansions,[3-5] and parton models[6,7] all account for scaling in inelastic electroproduction, and are probably equivalent to leading order in the deep-inelastic limit.[5,8] For the sake of brevity we refer to all three approaches as canonical field theories. A feature of these theories is that interactions do not appear explicitly in calculations of inelastic electroproduction. Their effects are relegated to the otherwise unspecified matrix elements of bilocal operators or parton fields which determine the shape of the observed structure functions. An exception to this is the quark–vector-gluon model[9] in which interaction-dependent terms appear in leading order in the deep-inelastic limit, but only as gauge terms which do not alter scaling laws.

Recently it has become clear that interactions enter in a nontrivial way into the description of less well-known (and less easily measured) structure functions. It is well known[10] that in canonical theories in which the weak or electromagnetic currents are constructed from spin-$\frac{1}{2}$ fields the longitudinal structure function $W_L(q^2,\nu)$ vanishes in the deep-inelastic limit, although on the basis of naive dimensional analyses it would be expected to attain a finite limit. In most models similar behavior is discovered for the chiral-symmetry-violating structure functions $[W_4(q^2,\nu)$ and $W_5(q^2,\nu)]$ of neutrino scattering.[2] It originates in the scale dimension of interactions which violate chiral symmetry rather than the spin structure of currents.

In the context of canonical field theories a departure from the naive predictions of dimensional analysis must be governed by some dimensional parameter such as a mass or gluon field strength occurring in the canonical equations of motion. It is to be expected therefore that the scaling behavior of W_L, W_4, and W_5 will probe the structure of masses and of interaction terms in the theory.

This possibility has been realized for the chiral-symmetry-breaking structure functions in the quark–vector-gluon model.[11-13] Tests of the vector nature of the interaction are obtained[13] as well as sum rules which relate the structure functions W_4 and W_5 to low-energy parameters of chiral-symmetry breaking.[12,13] Unfortunately the predictions of this program are rather far removed from experimental confirmation, since they involve structure functions which are nearly impossible to measure.[14]

In this paper we study the Bjorken-limit behavior of the longitudinal structure function $W_L(q^2,\nu)$ in quark-gluon models. In the text we derive our results using coordinate-space light-cone expansions. We have checked that all of our results are also valid in the parton model[7] and in the equal-time[3,4] and light-cone[15] BJL limits, and we outline the derivations in the Appendixes.

Our intentions are primarily twofold: first, to relate W_L in the deep-inelastic limit (via sum rules) to low-energy properties of the baryons and parameters of chiral-symmetry breaking without involving the experimentally intractable structure functions W_4 and W_5; and second, to show that an attempt to extract too much from free-field theory near the light cone leads to too trivial a theory. Specifically, the free-field scaling laws for leading chiral-symmetry-breaking structure functions, when related to static properties of the baryons, necessitate the divergence of a (Regge-convergent) integral over a leading structure function $[\nu W_2(q^2,\nu)]$ in the $SU(2)\times SU(2)$ limit. Exactly such a divergence occurs in a totally free-field theory in which structure functions are δ functions. Abstraction from a vector-gluon model, on the other hand, generates no difficulties. It has long been believed[16] that there is a limit to the amount which can be abstracted from the light-cone structure of free-field theory. We confirm this even at the level of the leading chiral-symmetry-breaking structure functions.

The reader should be warned that all our considerations are formal. That is to say they are untrue in perturbation theory. We take the observation of scaling in the SLAC-MIT experiments as motivation for abandoning perturbation theory, although the data do not exclude $\ln Q^2$ terms with small coefficients. It should be stressed, however, that our techniques and conclusions will be invalidated by either a breakdown in scaling at higher energies or a failure of the more conventional predictions of quark-gluon models.[17]

Since the derivations of Secs. II and III are rather technical, we have summarized our results, in the interest of clarity, in Sec. V, to which the reader may wish to turn first. In the remainder of this section we review the previous work on $W_L(q^2,\nu)$, present the plan of the remainder of the paper, and discuss the assumptions on which our work is based.

Several years ago, Callan and Gross[10] pointed out that $W_L(q^2,\nu)$ vanishes in the deep-inelastic limit if the hadronic weak and electromagnetic currents are constructed of spin-$\frac{1}{2}$ fields. Subsequently several authors[18,19] pointed out that $\nu W_L(q^2,\nu)$ scales in the gluon model. Implicit in the work of Jackiw and Schnitzer[19] is a sum rule relating the scaling limit of $\nu W_L(q^2,\nu)$ for electroproduction to the proton matrix element of the quark density operator (weighted by charge squared) $\overline{\psi}Q^2\psi$. Their sum rule diverges linearly if, as expected, the Pomeranchukon couples to W_L.[20] Recently Fritzsch, Gell-Mann, and Schwimmer[21] independently discovered this sum rule.

In Sec. II we review the kinematics necessary for our derivation of this and other sum rules. Special attention is paid to the systematics of $\alpha=0$ fixed singularities in current scattering amplitudes since an understanding of this problem is crucial in avoiding possibly divergent sum rules.

In Sec. III we derive three finite, regulated sum rules for the matrix elements $\langle P|\overline{\psi}(0)\lambda_i\psi(0)|P\rangle$ $(i=0,3,8)$ in terms of bare quark masses and integrals over the scaling limit of $\nu W_L(q^2,\nu)$ and $\nu W_2(q^2,\nu)$ in electroproduction and (anti-) neutrino production. For $i=3$ our result is the properly regulated, convergent version of that of Refs. 19–21. These sum rules are valid in the canonical quark-vector-gluon model (and also in models with no interactions at all). They are convergent regardless of the Regge asymptotic behavior of the structure functions; they are free from unmeasurable constants so long as the residues of $\alpha=0$ fixed singularities are polynomials in q^2.

Also in Sec. III we discuss the expected scaling behavior of the chiral-symmetry-breaking structure function $W_5(q^2,\nu)$. In the vector-gluon model $\nu^2W_5(q^2,\nu)$ scales but is explicitly proportional to the vector-gluon field strength $F_{\mu\nu}$ and coupling constant g. If we follow the dictum that the leading behavior of structure functions should be given by *free-field theory*, $\nu^2W_5(q^2,\nu)$ vanishes in the deep-inelastic limit.

Lastly in Sec. III we list sum rules for $\nu W_L(q^2,\nu)$ which occur in scalar- and pseudoscalar-gluon models, and for $\nu W_1(q^2,\nu)$ in models with currents constructed from only spin-zero fields.

In Sec. IV we relate the matrix elements $\langle P|\overline{\psi}(0)\lambda_i\psi(0)|P\rangle$ to baryon mass differences and the σ term in pion-nucleon scattering (σ_π). Combined with the vector-gluon-model sum rules of Sec. III we obtain three relations among baryon masses, the ratio of the bare masses of $\mathcal{P}$- and λ-type quarks, σ_π, and integrals over the scaling limit of νW_2 and νW_L in inelastic electron and neutrino scattering. Similar relations valid for the free-field model, in which $\nu^2W_5(q^2,\nu)$ vanishes, were given in Ref. 12. In the $SU(2)\times SU(2)$ limit those relations imply that either the λ quark becomes infinitely massive or otherwise convergent integrals over νW_2 must diverge. Such pathologies are not encountered in the vector-gluon model and lead us to conclude that at this level free-field theories cease to be adequate guides to scaling phenomena.

Section V contains a summary of our conclusions. In several appendixes we outline derivations of our results in the BJL and parton languages. It is worth noting that these approaches continue to yield identical results even at the level of nonleading structure functions.

Before passing to the body of the paper we summarize the assumptions which underlie our work. We assume

(a) that the leading light-cone signularities of current-current, divergence-divergence, and current-divergence commutators are correctly given by canonical manipulation in the gluon model,

(b) that the operators which arise in light-cone [or equivalently equal-time (BJL)] commutators obey gluon-model equations of motion,

(c) that the residues of $\alpha = 0$ fixed singularities in the real parts of kinematic-singularity-free amplitudes are polynomials in q^2.

The consequences of these assumptions have been discussed in the literature. We note that our use of (a) and (b) requires finite bare-quark masses.[13] This is a reflection of the incompatibility of our approach with perturbation theory, where bare masses are infinite order by order. Assumption (c) is necessary to exclude arbitrary constants in the sum rules. It is best motivated in constituent models, where it follows from the assumption that the unitarity arguments which rule out $\alpha = 0$ fixed singularities in hadron-hadron amplitudes also apply to parton-hadron amplitudes. It then follows that $\alpha = 0$ fixed singularities in the Compton amplitude scale and have polynomial residues.[22] The handling of $\alpha = 0$ fixed poles in sum rules is correspondingly more direct in such models as illustrated in Appendix A.

Assumption (c) is subject to experimental test in the case of the electroproduction structure function $F_2^{ep}(x)$. The $\alpha = 0$ fixed pole in T_2/q^2 has residue

$$C^p = 1 + \frac{1}{2\pi^2\alpha}\int_0^\infty \tilde{\sigma}_{\gamma p}(\nu)\,d\nu$$

in photoproduction ($q^2 = 0$). If the residue is independent of q^2, as would follow from assumption (c), then we obtain the Cornwall-Corrigan-Norton-Rajaraman-Rajasakaran[23] sum rule

$$\int_0^\infty d\omega\, \tilde{F}_2^{ep}(\omega) = C^p, \quad \omega \equiv \frac{2P\cdot q}{-q^2}$$

where $\tilde{F}_2^{ep}(\omega)$ is the scaling limit of $\nu W_2(q^2, P\cdot q)$ with all leading Regge ($\alpha > 0$) terms removed. Estimates of C^p from recent photoproduction data suggest $C^p = 1$,[24] in which case Eq. (9) predicts a rather definite behavior for F_2^{ep} illustrated in Fig. 1 and discussed at length by Close and Gunion.[25]

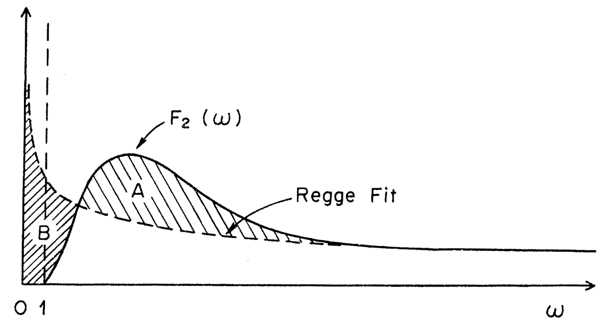

FIG. 1. Illustration of the behavior of F_2^{ep} implied by the assumption that fixed-pole residues are polynomials in Q^2. This assumption yields Area (A) − Area (B) $= C^P$.

II. KINEMATIC PRELIMINARIES AND FIXED POLES

Here we define the structure functions for inelastic lepton scattering on unpolarized nucleons and indicate their expected scaling and Regge behavior. In addition we analyze the constraints on possible $J = 0$ fixed singularities imposed by scaling and the assumption of polynomial residues in kinematic-singularity-free amplitudes.

A. Kinematics

We adopt the usual decomposition of the forward Compton amplitude[26]:

$$\begin{aligned} T_{\mu\nu} &= i\int d^4y\, e^{iq\cdot y}\langle P|\,T^*(J_\mu(y)J_\nu(0))\,|P\rangle \\ &= -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)T_1(q^2,\nu) \\ &\quad + \frac{1}{M^2}\left(P_\mu - \frac{\nu}{q^2}q_\mu\right)\left(P_\nu - \frac{\nu}{q^2}q_\nu\right)T_2(q^2,\nu), \end{aligned} \tag{2.1}$$

where $\nu = P\cdot q$ and the matrix element is averaged over nucleon spin. The Bjorken limit[27] ($\lim_{\rm Bj}$) is $-q^2, \nu \to \infty$ with $x \equiv -q^2/2\nu$ fixed. The electromagnetic current in the quark model is

$$\begin{aligned} J_\mu(y) &= \bar{\psi}(y)\gamma_\mu Q\psi(y) \\ &= \tfrac{1}{2}\left(\lambda_3 + \frac{1}{\sqrt{3}}\lambda_8\right) \end{aligned}$$

(our λ matrices are normalized conventionally: $\mathrm{Tr}\,\lambda^2 = 2$). For neutrino scattering we define

$$T^{\bar{\nu},\nu}_{\mu\nu} = i\int d^4y\, e^{iq\cdot y}\langle P|\, T^*(J^{\pm}_{\mu}(y)J^{\mp}_{\nu}(0))\,|P\rangle\,,$$
$$T^{\bar{\nu},\nu}_{\mu\nu} = -\left(g_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right)T^{\bar{\nu},\nu}_1(q^2,\nu)+\frac{1}{M^2}\left(P_\mu-\frac{\nu}{q^2}q_\mu\right)\left(P_\nu-\frac{\nu}{q^2}q_\nu\right)T^{\bar{\nu},\nu}_2(q^2,\nu)-\frac{i\epsilon_{\mu\nu\alpha\beta}P^\alpha q^\beta}{2M^2}T^{\bar{\nu},\nu}_3(q^2,\nu)$$
$$+\frac{q_\mu q_\nu}{M^2}T^{\bar{\nu},\nu}_4(q^2,\nu)+\frac{1}{2M^2}(P_\mu q_\nu+P_\nu q_\mu)T^{\bar{\nu},\nu}_5(q^2,\nu)\,, \qquad (2.2)$$

where

$$J^{\pm}_{\mu}(y)=\bar{\psi}(y)\gamma_\mu(1-\gamma_5)\lambda^{\pm}\psi(y)\,,$$
$$\lambda^{\pm}\equiv\tfrac{1}{2}(\lambda_1\pm i\lambda_2)\,.$$

We have used T invariance to set the amplitude ($T^{\bar{\nu},\nu}_6$) to zero, and have ignored the strangeness-changing current. Generalization to the full Cabibbo current is straightforward.

The structure functions are defined by the absorptive part of the scattering amplitudes:

$$W_i(q^2,\nu)\equiv\frac{1}{2\pi}\,\mathrm{Im}\,T_i(q^2,\nu)\,.$$

The quark model (with scalar, pseudoscalar, or vector interactions) has the following scaling behavior:

$$\lim_{\mathrm{Bj}} W_1(q^2,\nu)\equiv F_1(x)\,,$$
$$\lim_{\mathrm{Bj}}\frac{\nu}{M^2}W_{2,3}(q^2,\nu)\equiv F_{2,3}(x)\,, \qquad (2.3)$$
$$\lim_{\mathrm{Bj}}\frac{\nu^2}{M^4}W_{4,5}(q^2,\nu)\equiv F_{4,5}(x)\,.$$

Note that $W_{4,5}(q^2,\nu)$ do not scale as expected from dimensional analysis [which predicts $\nu W_{4,5}(q^2,\nu)$ to scale]. This is because these theories are chirally invariant to leading order on the light cone. Also, quark-gluon models predict that

$$W_L(q^2,\nu)\equiv\left(1-\frac{\nu^2}{M^2q^2}\right)W_2(q^2,\nu)-W_1(q^2,\nu) \qquad (2.4)$$

vanishes in the Bjorken limit,[10] while $\nu W_L(q^2,\nu)$ scales.[18,19] Accordingly we define[28]

TABLE I. The large-ν behavior of T_i and small-x behavior of F_i with Pomeranchukon dominance.

$T_i(q^2,\nu) \underset{\nu\to\infty}{\sim} \nu^{D_i}$ Amplitude	D_i	$F_i(x) \underset{x\to 0}{\sim} x^{-N_i}$ Structure function	N_i
T_L	1	F_G	2
T_2	−1	F_2	0
T_3	0	F_3	1
T_4	1	F_4	3
T_5	0	F_5	2

$$\lim_{\mathrm{Bj}}\frac{\nu}{M^2}W_L(q^2,\nu)\equiv F_G(x)\,. \qquad (2.5)$$

For fixed spacelike q^2 a Regge trajectory with intercept α leads to the following asymptotic behavior:

$$W_{4,L}(q^2,\nu)\sim\nu^{\alpha}\,,$$
$$W_{3,5}(q^2,\nu)\sim\nu^{\alpha-1}\,, \qquad (2.6)$$
$$W_2(q^2,\nu)\sim\nu^{\alpha-2}\,.$$

Assuming that this behavior is reflected in the Bjorken limit as $x\to 0$ we find

$$F_4(x)\sim 1/x^{\alpha+2}\,,$$
$$F_{5,G}(x)\sim 1/x^{\alpha+1}\,, \qquad (2.7)$$
$$F_3(x)\sim 1/x^{\alpha}\,,$$
$$F_2(x)\sim 1/x^{\alpha-1}\,,$$

where the Pomeranchukon has $\alpha=1$. For future convenience we tabulate (Table I) the large-ν (small-x) behavior of T_i (F_i) assuming that the Pomeranchukon dominates.

The positivity of the matrix $W_{\mu\nu}$ requires[29]

$$F_2(x)\geq|xF_3(x)|\,,$$
$$F_G(x)\geq 0\,, \qquad (2.8)$$
$$4F_G(x)[2xF_4(x)-F_5(x)]\geq|F_5(x)|^2.$$

The sum rules of Sec. III involve the amplitudes $T_{2,4,5,L}(q^2,\nu)$, for which we write the following fixed-q^2 dispersion relations, consistent with the Regge behavior of Eq. (2.6) (and allowing for $\alpha=0$ fixed poles, which we shall discuss below):

$$T_2(q^2,\nu)=4\int_{-q^2/2}^{\infty}\frac{d\nu'\,\nu' W_2(q^2,\nu')}{\nu'^2-\nu^2}\,,$$
$$T_4(q^2,\nu)=T_4(q^2,0)+4\nu^2\int_{-q^2/2}^{\infty}\frac{d\nu' W_4(q^2,\nu')}{\nu'(\nu'^2-\nu^2)}\,,$$
$$T_5(q^2,\nu)=4\nu\int_{-q^2/2}^{\infty}\frac{d\nu' W_5(q^2,\nu')}{\nu'^2-\nu^2}\,, \qquad (2.9)$$
$$T_L(q^2,\nu)\equiv\left(1-\frac{\nu^2}{M^2q^2}\right)T_2(q^2,\nu')-T_1(q^2,\nu)$$
$$=T_L(q^2,0)+4\nu^2\int_{-q^2/2}^{\infty}\frac{d\nu' W_L(q^2,\nu')}{\nu'(\nu'^2-\nu^2)}\,.$$

In the case of neutrinos we take only the isospin-symmetric part

$$T^{+}_{\mu\nu}=T^{\bar\nu}_{\mu\nu}+T^{\nu}_{\mu\nu}\quad (W^{\pm}_i\equiv W^{\bar\nu}_i\pm W^{\nu}_i)\,,$$

and for electrons $T_{4,5}(q^2,\nu)=0$.

B. Fixed Poles

It is essential for the derivation of our sum rules to isolate the $\alpha=0$ terms in the real parts of the amplitudes of Eqs. (2.9). Many authors have discussed this problem (see, for example Refs. 22 and 23). A review of this subject may be found in Ref. 30. For any $T_i(q^2,\nu)$ with expected Regge behavior $\nu^{\alpha+D_i-1}$ (see Table I) an $\alpha=0$ fixed singularity is defined to be the term in $\mathrm{Re}T_i(q^2,\nu)$ which goes asymptotically like ν^{D_i-1}. Clearly Eqs. (2.9) allow for $\alpha=0$ fixed poles (which cannot be excluded *a priori*) according to the Regge dictates of Table I. To isolate the fixed pole in any $T_i(q^2,\nu)$ write

$$W_i(q^2,\nu)=\tilde W_i(q^2,\nu)+W^R_i(q^2,\nu)$$

such that

$$\lim_{\nu\to\infty}\nu^{1-D_i}\tilde W_i(q^2,\nu)=0.$$

[$W^R_i(q^2,\nu)$ must therefore parametrize the leading Regge behavior with $\alpha>0$.] For simplicity we shall assume this leading behavior to be a sum of poles with $\alpha>0$, which implies

$$W^R_i(q^2,\nu)=\sum_{\alpha>0}\gamma_i(\alpha,q^2)\,\nu^{\alpha+D_i-1}\quad\text{for }\nu>0.$$

The treatment of Regge cuts and terms in $W_i(q^2,\nu)$ with $\alpha=0$ is given in Ref. 30.

$T^R_i(q^2,\nu)$ is constructed from $W^R_i(q^2,\nu)$ by means of dispersion relations analogous to Eq. (2.9), in which the subtraction constants are equal to zero. (The threshold of the dispersion relations is now at $\nu=0$.) Fixed-pole residues are given by

$$T^{\mathrm{FP}}_i(q^2)\equiv\lim_{\nu\to\infty}\left(\frac{\nu}{M^2}\right)^{1-D_i}[T_i(q^2,\nu)-T^R_i(q^2,\nu)],\tag{2.10}$$

which yields

$$\begin{aligned}T^{\mathrm{FP}}_2(q^2)&=-\frac{4}{M^4}\int_0^\infty d\nu\,\nu\,\tilde W_2(q^2,\nu)\,,\\ T^{\mathrm{FP}}_4(q^2)&=T_4(q^2,0)-4\int_0^\infty\frac{d\nu}{\nu}\tilde W_4(q^2,\nu),\\ T^{\mathrm{FP}}_5(q^2)&=-\frac{4}{M^2}\int_0^\infty d\nu\,\tilde W_5(q^2,\nu)\,,\\ T^{\mathrm{FP}}_L(q^2)&=T_L(q^2,0)-4\int_0^\infty\frac{d\nu}{\nu}\tilde W_L(q^2,\nu)\,.\end{aligned}\tag{2.11}$$

As mentioned in Sec. I, there is good reason to suppose the residue of $\alpha=0$ fixed poles in kinematic-singularity-free current hadron amplitudes to be polynomials in q^2. Unfortunately the amplitudes $T_i(q^2,\nu)$ are not free of kinematic singularities. However, they are related as follows to amplitudes, $\hat T_i(q^2,\nu)$, which are kinematic-singularity-free [$\hat T_i(q^2,\nu)$ are defined in Ref. 17]:

$$\begin{aligned}T_i(q^2,\nu)&=\hat T_i(q^2,\nu),\quad i=1,2,3\\ T_4(q^2,\nu)&=\hat T_4(q^2,\nu)-\frac{\nu^2}{q^4}\hat T_2(q^2,\nu)-\frac{M^2}{q^2}\hat T_1(q^2,\nu)\,,\\ T_5(q^2,\nu)&=\hat T_5(q^2,\nu)+\frac{2\nu}{q^2}\hat T_2(q^2,\nu)\,.\end{aligned}\tag{2.12}$$

The fixed-pole residues $\hat T^{\mathrm{FP}}_i(q^2)$, defined in analogy to Eq. (2.10), are by assumption polynomials, $\boldsymbol{P}_i(q^2)$.

The fixed poles in the amplitudes T_i are then

$$\begin{aligned}T^{\mathrm{FP}}_1(q^2)&=-B\,,\\ T^{\mathrm{FP}}_2(q^2)&=A+B\frac{q^2}{M^2}\,,\\ T^{\mathrm{FP}}_4(q^2)&=-\frac{M^4}{q^4}A\,,\\ T^{\mathrm{FP}}_5(q^2)&=\frac{2M^2}{q^2}A\,,\end{aligned}\tag{2.13}$$

where A and B are the first two terms in $\boldsymbol{P}_2(q^2)\equiv A+(q^2/M^2)B+\cdots$. In writing Eq. (2.13) we have assumed that $T_i(q^2,\nu)$ scales in the Bjorken limit with the same power of ν as its absorptive part $W_i(q^2,\nu)$. This eliminates higher polynomials. Actually, this assumption is not necessary, as explained in Ref. 30.[31]

The over-all fixed-pole structure is then[32]

$$T^{\mathrm{FP}}_{\mu\nu}=\frac{B}{\nu^2}[\nu^2g_{\mu\nu}+q^2\boldsymbol{P}_\mu P_\nu-\nu(\boldsymbol{P}_\mu q_\nu+P_\nu q_\mu)]+\frac{AM^2}{\nu^2}\boldsymbol{P}_\mu P_\nu\,.\tag{2.14}$$

Combining Eqs. (2.11) and (2.13) and taking $q^2\to-\infty$ we obtain

$$\lim_{q^2\to-\infty}\frac{q^4}{M^4}T_4(q^2,0)=4\int_0^\infty dx[4x\tilde F_4(x)-\tilde F_5(x)],\tag{2.15a}$$

$$\lim_{q^2\to-\infty}\frac{q^2}{M^2}T_L(q^2,0)=-4\int_0^\infty dx[2\tilde F_G(x)+\tilde F_5(x)]\,,\tag{2.15b}$$

$$B=2\int_0^\infty\frac{dx}{x^2}\tilde F_2(x)\,,\tag{2.16a}$$

$$A=4\int_0^\infty dx\,\tilde F_5(x)\,.\tag{2.16b}$$

When applied to neutrino-induced production Eqs. (2.15) and (2.16) should be read with $F_i=F^{\nu}_i+F^{\bar\nu}_i$.

In electroproduction Eqs. (2.14)–(2.16) are to be read with A, T_4, $T_5=0$.

Note that the combinations of regulated structure functions which occur in the subtraction constants of Eqs. (2.15) differ from the absorptive part, due to the *nonpolynomial* fixed poles in $T_{4,L}(q^2,\nu)$ generated by the polynomial fixed poles of the kinematical-singularity-free amplitudes.

Taking the Bjorken limit of the dispersion relations of Eq. (2.9) and using the asymptotic form of the subtraction constants given by Eq. (2.15) we obtain

$$\lim_{\rm Bj}\frac{P^\mu P^\nu T_{\mu\nu}}{M^2}=\frac{2}{x^2}\left\{\int_0^\infty dx'[2x'\tilde{F}_4(x')+\tilde{F}_G(x')]+\int_0^1 dx'\frac{x'^2}{x^2-x'^2}[2x'F_4(x)+F_G(x')]\right\}, \tag{2.17}$$

$$\lim_{\rm Bj}\frac{P^\mu q^\nu T_{\mu\nu}}{M^2}=\frac{2}{x}\left\{\int_0^\infty dx'[\tilde{F}_5(x')-4x'\tilde{F}_4(x')]+\int_0^1 dx'\frac{x'^2}{x^2-x'^2}[F_5(x')-4x'F_4(x')]\right\}. \tag{2.18}$$

The combination of structure functions is the same in the subtraction constants and absorptive parts because both $P_\mu q_\nu T^{\mu\nu}$ and $P_\mu P_\nu T^{\mu\nu}$ have no $J=0$ fixed poles, as may be seen from Eq. (2.14). It is this fact that allows the simple coordinate-space derivation of the sum rules presented in Sec. III.

III. SUM RULES FOR QUARK DENSITIES

A. Vector-Gluon Model

In the vector-gluon model the quark triplet, from which weak and electromagnetic currents are constructed, satisfies the equation of motion

$$(i\partial\!\!\!/-gB\!\!\!/(y)-m)\psi(y)=0\ , \tag{3.1}$$

where $B^\mu(y)$ is the gluon field and m is the (bare) mass matrix

$$\begin{pmatrix} m_{\mathcal{P}} & 0 & 0\\ 0 & m_{\mathfrak{N}} & 0\\ 0 & 0 & m_\lambda\end{pmatrix}.$$

Assuming charge symmetry we will take $m_{\mathcal{P}}=m_{\mathfrak{N}}$. Canonically the leading light-cone singularity of the quark propagator is given by[9]

$$\{\psi(y),\overline{\psi}(x)\}=-I(y,x)\,\partial\!\!\!/ D(y-x)$$
$$+\text{less singular terms}, \tag{3.2}$$

where $D(y)\equiv-(1/2\pi)\,\epsilon(y_0)\,\delta(y^2)$, and

$$I(y,x)\equiv\exp\left[-ig\int_0^1 da\,(y-x)^\mu B_\mu\,(ay+(1-a)\,x)\right]. \tag{3.3}$$

That is, the leading light-cone singularity is that of free-field theory (being independent of the quark masses) modulated on the light cone by a phase depending on the gluon field. The phase factor of Eq. (3.3) is well defined on the light cone if we work with the gauge in which

$$(y-x)_\mu\,(y-x)_\nu\,[B^\mu(x),B^\nu(y)]=0$$

for $(x-y)^2=0$. The terms ignored in Eq. (3.2) are no more singular than $\delta(y^2)$, as is the anticommutator $\{\psi(y),\psi(x)\}$ which vanishes in free-field theory.

1. *Derivation of Sum Rules*

Consider first electroproduction. From Eq. (3.2) we obtain

$$\langle P|[J_\mu(y),J_\nu(0)]|P\rangle$$
$$=-\tfrac{1}{2}\,\mathrm{Tr}[\gamma_\mu\gamma_\lambda\gamma_\nu\gamma_\sigma]\,V^\lambda(P,y)\,\partial^\sigma D(y)+R_{\mu\nu}(P,y)\ , \tag{3.4}$$

where $V^\lambda(P,y)$ is the antisymmetric vector bilocal,

$$V^\lambda(P,y)\equiv\tfrac{1}{2}\langle P|\overline{\psi}(y)\gamma^\lambda Q^2 I(y,0)\,\psi(0)-(y\leftrightarrow -y)|P\rangle$$
$$\equiv F(y\cdot P,y^2)P^\lambda+M^2G(y\cdot P,y^2)\,y^\lambda, \tag{3.5}$$

and $R_{\mu\nu}(P,y)$ is no more singular than $\delta(y^2)$.

Taking the Fourier transform of Eq. (3.4) and projecting out $F_2(x)$ and $F_G(x)$ we find ($\lambda\equiv P\cdot y$)

$$F_2(x)=-x\lim_{\rm Bj}W_{\mu\nu}g^{\mu\nu}$$
$$=\frac{x}{2\pi}\int_{-\infty}^{\infty}d\lambda\,e^{-i\lambda x}\,F(\lambda,0) \tag{3.6}$$

and

$$F_G(x)=\frac{2x}{M^2}\lim_{\rm Bj}W_{\mu\nu}P^\mu P^\nu$$
$$=\frac{x}{2\pi}\int_{-\infty}^{\infty}d\lambda\,e^{-i\lambda x}[F(\lambda,0)+2\lambda G(\lambda,0)]. \tag{3.7}$$

At $\lambda=0$, $F(\lambda,0)$ and $G(\lambda,0)$ are related, by virtue of the equation of motion (3.1), to the symmetric scalar bilocal:

$$S(y\cdot P,y^2)\equiv\tfrac{1}{2}\langle P|\overline{\psi}(y)\,m\,Q^2 I(y,0)\psi(0)+(y\leftrightarrow -y)|P\rangle\ . \tag{3.8}$$

In general, by application of Eq. (3.1),

$$S(y\cdot P,y^2)=-i\partial^\mu V_\mu(y\cdot P,y^2)+gy^\mu C_\mu(P,y)\,, \tag{3.9}$$

where $C_\mu(P,y)$ is a bilocal involving explicitly the curl of the gluon field. At $y\cdot P=y^2=0$, Eqs. (3.5), (3.8), and (3.9) give

$$\langle P|\overline{\psi}(0)m\,Q^2\psi(0)|P\rangle = -iM^2\left[\frac{\partial}{\partial\lambda}F(\lambda,0)+4G(y,0)\right]_{\lambda=0}. \tag{3.10}$$

A sum rule is obtained by inverting the Fourier transforms of Eqs. (3.6) and (3.7). Inverting Eq. (3.6) gives

$$\frac{\partial}{\partial\lambda}F(\lambda,0)=2i\int_0^1 dx\cos\lambda x\,F_2(x)\,, \tag{3.11}$$

which is convergent by Regge arguments. However, the inverse of Eq. (3.7),

$$G(\lambda,0)=\frac{1}{2\lambda}\int_{-\infty}^{\infty}\frac{dx}{x}\,e^{i\lambda x}[F_G(x)-F_2(x)]\,, \tag{3.12}$$

must be interpreted as a distribution, since we expect $F_G(x)\to 1/x^2$ as $x\to 0$. Parametrizing the leading Regge behavior by

$$F_G(x)=\tilde{F}_G(x)+\sum_{\alpha>0}\gamma_G(\alpha)\,|x|^{-(\alpha+1)}$$

we obtain

$$G(\lambda,0)=i\int_0^\infty dx\left(\frac{\sin\lambda x}{\lambda x}\right)[\tilde{F}_G(x)-F_2(x)] +\tfrac{1}{2}i\pi\sum_{\alpha>0}\frac{\gamma_G(\alpha)|\lambda|^\alpha}{\Gamma(\alpha+2)\cos\frac{1}{2}\pi(\alpha+1)}\,. \tag{3.13}$$

As $\lambda\to 0$, $G(\lambda,0)$, so defined, remains finite.

A priori either $F_G(x)$ or $F_2(x)$ could contain a term $\delta(x)$ which would add to $G(\lambda,0)$ or $\partial F(\lambda,0)/\partial\lambda$ an unknown constant unmeasurable in electroproduction. In Ref. 30 such terms were shown to correspond in general to $\alpha=0$ fixed poles. They are forbidden in $F_G(x)$ since it was shown [cf. Eq. (2.17)] in Sec. II that given the polynomial residue assumption $P_\mu P_\nu W^{\mu\nu}$ has no $\alpha=0$ fixed poles. A term $\frac{1}{2}B\delta(x)$ in $F_2(x)/x^2$ corresponds to the fixed pole Bq^2/M^2 in $T_2(q^2,\nu)$ [cf. Eq. (2.16a)] but does not contribute to either (3.11) or (3.13) since $x^2\delta(x)=0$.

Finally we note that the possibility of Regge cuts with $\alpha_c>0$ and a possible $\alpha=0$ term in $F_G(x)$ were treated in Ref. 30. Neither is an essential complication. For cuts the reader is referred to Ref. 30. An $\alpha=0$ term in $F_G(x)$ should simply be subtracted off before performing the integral of Eq. (3.13). No counterterm [analogous to $\sum_\alpha$ in Eq. (3.13)] is generated. In any case $G(\lambda,0)$ remains well defined and finite regardless of the behavior of $F_G(x)$ near $x=0$ so long as F_G scales and the polynomial residue assumption is valid.[30]

Combining Eqs. (3.10). (3.11), and (3.13) we obtain the sum rule

$$\langle P|\overline{\psi}(0)m\,Q^2\psi(0)|P\rangle=2M^2\left[2\int_0^\infty\tilde{F}_G(x)\,dx-\int_0^1F_2(x)\,dx\right]. \tag{3.14}$$

The analogous sum rule for neutrino scattering is obtained from the leading singularity

$$\{\langle P|[J^+_\mu(y),J^-_\nu(0)]|P\rangle+(+\leftrightarrow-)\}_{\rm sym}=-\operatorname{Tr}[\gamma_\mu\gamma^\lambda\gamma_\nu\gamma^\sigma]\,V^+_\lambda(P,y)\partial_\sigma D(y)+R^+_{\mu\nu}(P,y)\,. \tag{3.15}$$

$V^+_\lambda(P,y)$ and $S^+(P\cdot y,y^2)$ are defined in analogy with Eqs. (3.5) and (3.8) with Q^2 replaced by $\{\lambda^+,\lambda^-\}$. $R^+_{\mu\nu}(P,y)$ is again no more singular than $D(y)$. Note that we have taken the sum of ν and $\overline{\nu}$ scattering and symmetrized in μ and ν since only these terms enter our sum rules. There is an extra factor of 2 on the right-hand side of Eq. (3.15), compared with Eq. (3.4), since vector and axial-vector currents contribute equally to the leading light-cone singularity.

We now consider projections of $W^+_{\mu\nu}$:

$$\lim_{\rm Bj}W^+_{\mu\nu}g^{\mu\nu}=-\frac{1}{x}F^+_2(x)\,, \tag{3.16a}$$

$$\lim_{\rm Bj}W^+_{\mu\nu}\frac{P^\mu P^\nu}{M^2}=F^+_4(x)+\frac{1}{2x}F^+_G(x), \tag{3.16b}$$

$$\lim_{\rm Bj}W^+_{\mu\nu}\frac{P^\mu q^\nu}{M^2}=-\tfrac{1}{2}[4xF^+_4(x)-F^+_5(x)]\,, \tag{3.16c}$$

$$\lim_{\rm Bj}W^+_{\mu\nu}\frac{q^\mu q^\nu}{M^2}=2x[2xF^+_4(x)-F^+_5(x)]\,. \tag{3.16d}$$

From Eqs. (3.16a) and (3.16b) we obtain the neutrino analog of Eq. (3.14):

$$\langle P|\bar{\psi}(0)m\{\lambda^+,\lambda^-\}\psi(0)|P\rangle = M^2\left\{2\int_0^\infty dx[\tilde{F}_G^+(x)+2x\tilde{F}_4^+(x)] - \int_0^1 dx\,F_2^+(x)\right\}, \tag{3.17}$$

where we argue as before for the absence of δ functions in these combinations of structure functions [cf. Eq. (2.17)] and for the regularization of the integrals.

Two more results involving $F_{4,5}^+(x)$ may be obtained using the relation

$$\partial_\mu J^{\mu\pm}(y) = -2im_\wp\bar{\psi}(y)\gamma_5\lambda^\pm\psi(y)\ , \tag{3.18}$$

which follows from the canonical equation of motion. These results were derived in Ref. 13 using a combination of light-cone and BJL techniques. For completeness we derive them here from a coordinate-space approach. From Eq. (3.18) we obtain

$$\langle P|[\partial^\mu J_\mu^+(y), J_\nu^-(0)]|P\rangle + (+\leftrightarrow -) = -4i\partial_\nu[S^+(y\cdot P, y^2)D(y)] + R_\nu^+(P,y)\ , \tag{3.19}$$

$$\langle P|[\partial^\mu J_\mu^+(y), \partial^\nu J_\nu^-(0)]|P\rangle + (+\leftrightarrow -) = -2m_\wp^2 V_\lambda^+(P,y)\partial^\lambda D(y) + R^+(y\cdot P, y^2)\ , \tag{3.20}$$

where both R_ν^+ and R^+ are no more singular than $D(y)$. Equation (3.19) gives

$$\lim_{\text{Bj}} \frac{P^\mu q^\nu W_{\mu\nu}^+}{M^2} = -\frac{1}{2\pi}\int_{-\infty}^\infty d\lambda\, e^{-i\lambda x} S^+(\lambda, 0)\ .$$

Combining this with Eq. (3.16c) we obtain the sum rule

$$\langle P|\bar{\psi}(0)m\{\lambda^+,\lambda^-\}\psi(0)|P\rangle = M^2\int_0^\infty dx[4x\tilde{F}_4^+(x) - \tilde{F}_5^+(x)]\ . \tag{3.21}$$

We rely on the absence of a fixed pole in this combination of amplitudes [cf. Eq. (2.18)] to rule out terms like $\delta(x)$.

Comparing Eqs. (3.15) and (3.20) we find

$$\lim_{\text{Bj}} q^\mu q^\nu W_{\mu\nu}^\pm = -m_\wp^2 \lim_{\text{Bj}} g^{\mu\nu} W_{\mu\nu}^\pm\ . \tag{3.22}$$

Combining with Eqs. (3.16a) and (3.16d) we obtain the relation

$$F_4^\pm(x) - \frac{1}{2x}F_5^\pm(x) = \frac{m_\wp^2}{4M^2x^3}F_2^\pm(x)\ . \tag{3.23}$$

2. *Sum Rules*

We now present our sum rules in a compact notation. We let $J_i(pt)$ denote the regulated integrals of the scaling functions $F_i(x)\,x^{N_i-2}$ (where N_i is given in Table I) which occur in the sum rules, for a projectile p (e or ν) on a target t (p or n):

$$J_i \equiv \int_0^\infty dx\, x^{N_i-2}\tilde{F}_i(x)$$

$$\equiv \int_0^1 dx\left[x^{N_i-2}F_i(x) - \sum_{\alpha\ge 0}\gamma_i(\alpha)x^{-\alpha-1}\right] - \sum_{\alpha>0}\gamma_i(\alpha)/\alpha\,. \tag{3.24}$$

For simplicity we continue to assume the leading Regge behavior in $F_i(x)$ to be a sum of poles. Cuts are treated in Ref. 30. Note that an $\alpha = 0$ term is to be subtracted from $F_i(x)$ should it occur, but does not contribute a counterterm.[30]

We also require the simple integrals $I_2(pt)$;

$$I_2 = \int_0^1 dx\, F_2(x)\ . \tag{3.25}$$

Combining Eqs. (3.14) for ep and en scattering and Eqs. (3.17), (3.21), and (3.23) with the definitions of J_i and I_i we obtain three sum rules, which we choose to write as follows:

$$\frac{m_\wp}{6M^2}\left\langle P\left|\bar{\psi}\begin{pmatrix}1 & & \\ & -1 & \\ & & 0\end{pmatrix}\psi(0)\right|P\right\rangle = 2J_G(ep-en) - I_2(ep-en), \tag{3.26}$$

$$\frac{m_{\wp}}{M^2}\left\langle P\left|\bar\psi(0)\begin{pmatrix}1&&\\&1&\\&&0\end{pmatrix}\psi(0)\right|P\right\rangle=-2J_G(\nu p+\nu n)+I_2(\nu p+\nu n)+\frac{m_{\wp}^2}{M^2}J_2(\nu p+\nu n), \tag{3.27}$$

$$\frac{m_\lambda}{9M^2}\left\langle P\left|\bar\psi(0)\begin{pmatrix}0&&\\&0&\\&&1\end{pmatrix}\psi(0)\right|P\right\rangle=2J_G(ep+en)+\tfrac{5}{9}J_G(\nu p+\nu n)-I_2(ep+en)-\tfrac{5}{18}I_2(\nu p+\nu n)-\frac{5}{18}\frac{m_{\wp}^2}{M^2}J_2(\nu p+\nu n)\,. \tag{3.28}$$

Note that we have expressed everything in terms of proton matrix elements and neutrino or electron scattering using charge symmetry.

An alternative form of Eqs. (3.26)–(3.28) in terms of J_2 and J_5 may be obtained by using Eqs. (3.21) and (3.23):

$$\frac{m_{\wp}}{6M^2}\left\langle P\left|\bar\psi(0)\begin{pmatrix}1&&\\&-1&\\&&0\end{pmatrix}\psi(0)\right|P\right\rangle=\frac{m_{\wp}^2}{M^2}J_2(ep-en)+J_5(ep-en)\,, \tag{3.29}$$

$$\frac{m_{\wp}}{M^2}\left\langle P\left|\bar\psi(0)\begin{pmatrix}1&&\\&1&\\&&0\end{pmatrix}\psi(0)\right|P\right\rangle=\frac{m_{\wp}^2}{M^2}J_2(\nu p+\nu n)+J_5(\nu p+\nu n)\,, \tag{3.30}$$

$$\frac{m_\lambda}{9M^2}\left\langle P\left|\bar\psi(0)\begin{pmatrix}0&&\\&0&\\&&1\end{pmatrix}\psi(0)\right|P\right\rangle=\frac{m_\lambda^2}{M^2}\left[J_2(ep+en)-\tfrac{5}{18}J_2(\nu p+\nu n)\right]+\tfrac{1}{2}J_5(ep+en)-\tfrac{5}{18}J_5(\nu p+\nu n)\,. \tag{3.31}$$

Equations (3.29)–(3.31) were derived and discussed in Ref. 13. Their utility is spoiled by the appearance of J_5, not only because $F_5^{\nu p}$ and $F_5^{\nu n}$ are hard to measure but also because F_5^{ep} and F_5^{en} are the structure functions of an experimentally unknown chiral current with the quantum numbers of the photon. Only in models where J_5 vanishes will Eqs. (3.29)–(3.31) be of any use.

3. $F_5(x)$ in the Gluon Model

We first consider the behavior of $F_5(x)$ in the limit $m_{\wp}\to 0$. From the positivity conditions, Eqs. (2.8), and from Eq. (3.23) we obtain

$$2F_G(x)\,F_2(x)\geq\left[\frac{MxF_5(x)}{m_{\wp}}\right]^2\,. \tag{3.32}$$

As long as $F_G(x)$ and $F_2(x)$ remain finite in the limit (as expected) $F_5(x)$ must vanish at least linearly with $m_{\wp}$.

$F_5(x)$ is given entirely by R_ν^+ in Eq. (3.19). This may be seen by considering $q^\mu W_{\mu\nu}^+$. The first term in Eq. (3.19) when Fourier-transformed is proportional to q_ν, whereas W_5 is the coefficient of P_ν in $q^\mu W_{\mu\nu}^+$. From the equation of motion (3.1) we deduce that R_ν^+ must have the form

$$R_\nu^+(P,y)=-4iD(y)m_{\wp}gy^\mu\,C_{\mu\nu}^+(P,y)+\bar R_\nu^+(P,y)\,, \tag{3.33}$$

where $\bar R_\nu^+(P,y)$ is regular as $y^2\to 0$ and $C_{\mu\nu}^+(P,y)$ is a bilocal (even in y^σ) involving the curl of the gluon field. The term in $C_{\mu\nu}^+(P,y)$ proportional to $P_\mu P_\nu$ determines $F_5(x)$. From Eq. (3.33) we obtain the convergent sum rules[33]:

$$\int_0^1 dx\,x\,F_5^-(x)=0\,, \tag{3.34}$$

$$\int_0^1 dx\,x^2\,F_5^+(x)=\frac{m_{\wp}g\alpha}{2M^2}\,, \tag{3.35}$$

where

$$\langle P|\bar\psi(y)\sigma_{\mu\lambda}[\partial^\lambda B_\nu(y)-\partial^\nu B_\lambda(y)]\,\{\lambda^+,\lambda^-\}\psi(y)|P\rangle\equiv\alpha P_\mu P_\nu+\beta g_{\mu\nu}\,.$$

There is no reason for α to vanish as $m_{\wp}\to 0$, so from Eq. (3.35) $F_5^+(x)$ will in general vanish linearly with $m_{\wp}$.[34]

In summary, we have found that $F_5(x)$ is proportional to the quark-gluon coupling constant g and in general vanishes linearly with $m_{\wp}$ as $m_{\wp}\to 0$.

B. Free-Field Model

To obtain free-field quark-model predictions from the results of the previous section we have only to set the quark-gluon coupling constant to zero. Since $F_5(x)$ is proportional to g it vanishes in the free-field case. In other structure functions, the gluon phase integral reduces to unity, but their scaling behavior is not affected. With $F_5(x)=0$ for all chiral currents, the sum rules of Eqs. (3.29)–

(3.31) are accessible to experiment. They become

$$\left\langle P \left| \bar{\psi}(0) \begin{pmatrix} 1 & & \\ & -1 & \\ & & 0 \end{pmatrix} \psi(0) \right| P \right\rangle = 6 m_{\mathcal{P}} J_2(ep - en), \quad (3.36)$$

$$\left\langle P \left| \bar{\psi}(0) \begin{pmatrix} 1 & & \\ & 1 & \\ & & 0 \end{pmatrix} \psi(0) \right| P \right\rangle = m_{\mathcal{P}} J_2(\nu p + \nu n), \quad (3.37)$$

$$\left\langle P \left| \bar{\psi}(0) \begin{pmatrix} 0 & & \\ & 0 & \\ & & 1 \end{pmatrix} \psi(0) \right| P \right\rangle = 9 m_\lambda [J_2(ep + en) - \tfrac{5}{18} J_2(\nu p + \nu n)]. \quad (3.38)$$

It should be emphasized that these sum rules are only true in models in which interactions are ignored near the light cone. As shown in Appendix A, $F_5(x)$ is not zero in a parton model with vector-gluon interactions. In BJL-limit calculations (Appendix B) $J_5(pt)$ enters as a fixed-pole residue which cannot be shown necessarily to vanish.

In models where $F_5(x) = 0$ bilocal operators presumably obey free-field equations of motion. Actual free-field theory—where deep-inelastic structure functions are δ functions—is an example of such a model. However, we know no reason to suppose that all models where bilocal operators obey free-field equations of motion are trivial, and we do not exclude them *a priori*.

C. Other Models

We list here sum rules analogous to the electroproduction sum rule (3.14) of the vector-gluon model which obtain in models with other interactions and currents constructed from other than spinor fields. Neutrino-induced production analogs are not easily derived in these models because the divergences of axial-vector currents are dynamically more complex than in the vector-gluon model.

Our primary interest in such sum rules is the role they play in interpreting[19,35] the divergence of the Cottingham formula for electromagnetic mass differences. The derivations are entirely analogous to those of Sec. III A.

(1) (Pseudo-) scalar-gluon model:

$$\langle P | \bar{\psi}(0)\, Q^2 [m + g(\gamma_5)\, \phi(0)] \psi(0) | P \rangle = 2M^2 [2 J_G(ep) - I_2(ep)] ,$$

where $\phi(x)$ is the (pseudo-) scalar-gluon field (the γ_5 is to be read only in the pseudoscalar case).

(2) Scalar "partons" interacting via vector gluons:

$$\langle P | \phi^*(0) m^2 Q^2 \phi(0) | P \rangle = M^2 [I_2(ep) - 4 J_S(ep)] ,$$

where $J_S(ep)$ is defined in analogy with Eq. (3.24) ($N_S = 2$) for the structure function

$$F_S(x) = \lim_{\text{Bj}} \frac{\nu}{M^2} W_1(q^2, \nu) \, [(\lim_{\text{Bj}} W_1(q^2, \nu) = 0]$$

in models with currents constructed only of scalar fields.

(3) Scalar "partons" interacting via (neutral) scalar gluons:

$$\langle P | \phi^*(0) [m^2 + g\chi(0)] Q^2 \phi(0) | P \rangle = M^2 [I_2(ep) - 4 J_S(ep)] .$$

Note that analogous sum rules hold for electroproduction off neutrons.

IV. SCALAR DENSITIES AND STATIC PROPERTIES OF THE BARYONS

The scalar densities which appear in the sum rules of the previous section control various low-energy parameters of SU(2) × SU(2), SU(3), and SU(3) × SU(3) symmetry breaking. Via these sum rules we shall relate deep-inelastic scattering data to baryon masses, the Gell-Mann–Oakes–Renner parameter (c), and the σ term in pion-nucleon scattering (σ_π). Although difficult to test experimentally [the relations involve integrals over the Regge-subtracted structure function $F_G(x)$] it is interesting in principle that such relations exist. An experimental test of these relations is far easier than for those of Ref. 13 which involve $F_4(x)$ and $F_5(x)$. Moreover, our results allow us to explore the pattern of chiral-symmetry breaking near the SU(2) × SU(2) limit.

We consider first the results of canonical vector-gluon theory as summarized in the three sum rules of Eqs. (3.26)–(3.28). Later we consider the free-field model where $F_5(x) = 0$, which was discussed in the previous section. Our analysis follows that of Ref. 12.

A. Vector-Gluon Model

1. Low-Energy Symmetry Breaking

In this model the Hamiltonian has the form

$$H = H_0 + \sum_i \int d^3x \bar{\psi}_i(x)\, m_i\, \psi_i(x) = H_0 + \int d^3x \left[\frac{2m_{\mathcal{P}} + m_\lambda}{\sqrt{6}} u_0(x) + \frac{m_{\mathcal{P}} - m_\lambda}{\sqrt{3}} u_8(x) \right], \quad (4.1)$$

where $u_i(x) = \bar{\psi}(x)\, \lambda_i \psi(x)$ [Tr $\lambda^2 = 2$]. The sum extends over $\mathcal{P}$-, $\mathcal{N}$-, and λ-type quarks. H_0 is SU(3)- and SU(3) × SU(3)-invariant. It contains all kinetic-energy terms, gluon mass terms, and the quark-gluon interaction. Both SU(3) and SU(3) × SU(3) are broken by quark mass terms.

The σ term in pion-nucleon scattering (σ_π) which occurs in various PCAC (partially conserved axial-vector current) low-energy theorems is defined as follows:

$$\sigma_\pi \equiv \frac{1}{4Mi}\int d^3x \langle P|[\partial^\mu J_\mu^-(\vec{x},0), J_0^+(0)]|P\rangle \quad . \qquad (4.2)$$

This is the conventional σ term, the value of which is still subject to considerable controversy[36] but is suspected to lie in the range 10–100 MeV. (The factor of $2M$ arises because our states are normalized covariantly.) In the vector-gluon model Eq. (4.2) reduces to[37]

$$\sigma_\pi = \frac{m_{\mathcal{P}}}{2M_p}\left\langle P\left|\bar{\psi}(0)\begin{pmatrix}1 & & \\ & 1 & \\ & & 0\end{pmatrix}\psi(0)\right|P\right\rangle . \qquad (4.3)$$

Comparing this with Eq. (3.27) we conclude

$$\sigma_\pi = \tfrac{1}{2}M_p\left[I_2(\nu p+\nu n) - 2J_G(\nu p+\nu n) + \frac{m_{\mathcal{P}}^2}{M_p^2}J_2(\nu p+\nu n)\right]. \qquad (4.4)$$

The operators $u_i(0)$ $(i=1,\ldots,8)$ form an SU(3) octet. The matrix elements of $-\delta m u_8(0)/\sqrt{3}$ generate SU(3) mass splittings in this model, where $\delta m \equiv m_\lambda - m_{\mathcal{P}}$. We may estimate the proton matrix elements of $u_3(0)$ and $u_8(0)$ in terms of ratios of baryon masses to δm using first-order perturbation theory—a procedure which would be exact only in the SU(3) limit. The success of the Gell-Mann–Okubo mass formula suggests that errors introduced by this approximation are less than 10%. In terms of baryon masses then[38]

$$\begin{aligned}\delta m\langle P|u_3(0)|P\rangle &\cong \tfrac{1}{2}M_p[2(M_\Xi - M_p) + 3(M_\Lambda - M_\Sigma)],\\ \delta m\langle P|u_8(0)|P\rangle &\cong \tfrac{1}{2}\sqrt{3}M_p[2(M_\Xi - M_p) - (M_\Lambda - M_\Sigma)] \ .\end{aligned} \qquad (4.5)$$

Appropriate combinations of Eqs. (3.26)–(3.28) yield

$$\begin{aligned}\langle P|u_3(0)|P\rangle &= 6M_p^2\left[-\frac{1}{m_{\mathcal{P}}}I_2(ep-en) + \frac{2}{m_{\mathcal{P}}}J_G(ep-en)\right],\\ \langle P|u_8(0)|P\rangle &= \frac{2M_p^2}{\sqrt{3}}\left\{\frac{m_\lambda + 5m_{\mathcal{P}}}{m_\lambda m_{\mathcal{P}}}\frac{\sigma_\pi}{M_p} - \frac{9}{m_\lambda}[-I_2(ep+en) + 2J_G(ep+en)]\right\}\end{aligned} \qquad (4.6)$$

Equations (4.6) are exact in the gluon model. To obtain a comparison with the data we shall approximate the matrix elements by means of Eqs. (4.5). It would seem at first sight that to be consistent we should assume exact SU(3) symmetry in Eqs. (4.6). However, the success of the Gell-Mann–Okubo formula which we invoked to motivate the approximation of Eqs. (4.5) does not reflect on $\delta m/m_{\mathcal{P}}$ or $\delta m/m_\lambda$ at all. That is to say, the small magnitude of the corrections to the Gell-Mann–Okubo formula is evidence only for the approximate SU(3) symmetry of the states—in this case the baryon octet. The symmetry breaking in the current divergences, reflected in the quark masses, need not be small. Just such a situation was envisioned by Gell-Mann, Oakes, and Renner.[39] We shall persist with $m_{\mathcal{P}} \neq m_\lambda$ throughout and derive results valid for any ratio $m_{\mathcal{P}}/m_\lambda$.

Combining Eqs. (4.5) and (4.6) we obtain then

$$2J_G(ep-en) - I_2(ep-en) \cong -\frac{c+\sqrt{2}}{3c}\left[\frac{M_\Xi - M_p + \frac{3}{2}(M_\Lambda - M_\Sigma)}{6M_p}\right], \qquad (4.7)$$

$$2J_G(ep+en) - I_2(ep+en) \cong \frac{c+2\sqrt{2}}{3(c+\sqrt{2})}\frac{\sigma_\pi}{M_p} + \frac{\sqrt{2}-2c}{3c}\left[\frac{2(M_\Xi - M_p) + M_\Sigma - M_\Lambda}{12M_p}\right], \qquad (4.8)$$

where c is the parameter introduced by Gell-Mann, Oakes, and Renner[39]:

$$c \equiv -\sqrt{2}\frac{m_\lambda - m_{\mathcal{P}}}{m_\lambda + 2m_{\mathcal{P}}}$$

in this model. $c \cong -1.25$ was estimated in Ref. 36 from the kaon-pion mass ratio.

We may estimate the baryon matrix element of the SU(3) singlet term which breaks SU(3) × SU(3) symmetry, defined by

$$\mu_0 \equiv \frac{2m_{\mathcal{P}} + m_\lambda}{6M_p}\left\langle P\left|\bar{\psi}(0)\begin{pmatrix}1 & & \\ & 1 & \\ & & 1\end{pmatrix}\psi(0)\right|P\right\rangle \quad .$$

μ_0 measures the baryon mass shift due to the average quark mass $\bar{m}\equiv\frac{1}{3}(2m_{\mathcal{P}}+m_\lambda)$. μ_0 may be written in the approximate form

$$\mu_0=\frac{3\sigma_\pi}{\sqrt{2}\,(c+\sqrt{2})}-\frac{2m_{\mathcal{P}}+m_\lambda}{2\sqrt{3}}\,\frac{\langle P|\bar{\psi}(0)\,\lambda_8\,\psi(0)\,|P\rangle}{2M_p}$$

$$\cong\frac{3\sigma_\pi}{\sqrt{2}\,(c+\sqrt{2})}+\frac{1}{4\sqrt{2}\,c}\,[2(M_\Xi-M_p)+M_\Sigma-M_\Lambda]$$

$$\cong\frac{9M_p}{\sqrt{2}\,(c+2\sqrt{2})}\,[2J_G(ep+en)-I_2(ep+en)]+\frac{3c+\sqrt{2}}{4\sqrt{2}\,c(c+2\sqrt{2})}[2(M_\Xi-M_p)+M_\Sigma-M_\Lambda]\ , \tag{4.9}$$

where we have used Eq. (4.8).

Equations (4.7) and (4.8) may be evaluated numerically. We use the data[39,1]

$$c\cong-1.25\ ,$$

$$I_2(ep)=\int_0^1 dx\,F_2^{ep}(x)=0.16\pm0.02\ ,$$

$$I_2(en)=\int_0^1 dx\,F_2^{en}(x)=0.12\pm0.02\ ,$$

and leave $J_G(ep+en)$ and σ_π as unknowns. Equation (4.7) implies

$$J_G(ep-en)\cong\tfrac{1}{2}I_2(ep-en)+0.001$$
$$\cong0.02\pm0.01\ , \tag{4.10}$$

while Eq. (4.8) yields

$$J_G(ep+en)\cong\tfrac{1}{2}I_2(ep+en)+0.0017\,\sigma_\pi-0.04$$
$$\cong0.10+0.0017\,\sigma_\pi\ \ (\text{MeV})\ . \tag{4.11}$$

Were it not for the possibility of asymptotic Regge behavior, Eqs. (4.10) and (4.11) [or equivalently Eqs. (4.7) and (4.8)] would be quite useful in anticipating measurements of $F_G(x)$. Unfortunately the necessity of subtraction of Regge asymptotic behavior means $F_G(x)$ will have to be rather well known before these predictions can be tested.

Although it is not currently possible to experimentally verify the sum rules obtained above for $F_G(\lim_{\text{Bj}}(\nu/M^2)W_L)$, it is of interest to see if the available data for $R=\sigma_S/\sigma_T$ are at least consistent with the existence of a scaling limit for νW_L. In terms of $F_G(x)$ and $F_2(x)$, R is given by

$$R=\frac{-M^2q^2}{\nu^2}\,\frac{F_G(x)}{F_2(x)}$$

in the scaling limit. This ratio is plotted against x in Fig. 2 using the data of Miller *et al.*[1] It is apparent that to within the errors the ratio could be a function of x only and that, for small x, F_G/F_2 increases as $x\to0$. This is to be expected since on the basis of Regge asymptotics (Table I) $F_G/F_2\sim1/x^2$. This description is as consistent with the data as the more usual prescriptions of $R=$ constant and $R=q^2/\nu^2$.[1]

2. SU(2)×SU(2) Limit

Here we explore the vector-gluon-model sum rules of the previous section in the SU(2)×SU(2) limit. We expect the symmetry limit to be realized by the appearance of massless pions, while the nucleons retain finite masses. In models such as the vector-gluon model the limit is achieved when c is $-\sqrt{2}$.[36] This corresponds to $m_{\mathcal{P}}=0$ while m_λ remains finite, and $m_\lambda=\delta m$ generates SU(3) mass splittings.

In the SU(2)×SU(2) limit we expect

$$F_4(x),\ F_5(x)\to0$$
and
$$\sigma_\pi\to0\ , \tag{4.12}$$

whereas $F_2^{\nu p}(x)$ and $F_2^{\bar{\nu} n}(x)$ should be unaffected by chiral-symmetry breaking (they are determined by chirally symmetric, leading light-cone singularities). As discussed in the previous section, $F_5(x)$ vanishes linearly in the limit $m_{\mathcal{P}}\to0$. Equation (3.23) implies that $F_4(x)$ also vanishes linearly. The σ-term sum rule of Ref. 13, Eq. (3.21), indicates that σ_π also vanishes linearly in the SU(2) × SU(2) limit:

$$\sigma_\pi\underset{c\to-\sqrt{2}}{\sim}\beta(c+\sqrt{2}). \tag{4.13}$$

As $c\to-\sqrt{2}$, Eqs. (4.4) and (4.7) reduce to

$$I_2(\nu p+\nu n)=2J_G(\nu p+\nu n)\ ,$$
$$I_2(ep+en)=2J_G(ep-en)\ . \tag{4.14}$$

To take the SU(2)×SU(2) limit of Eq. (4.8) it is necessary to use Eq. (4.13):

$$2J_G(ep+en)-I_2(ep+en)$$
$$\cong\frac{\sqrt{2}}{3M_p}\,\beta-\frac{2(M_\Xi-M_p)+M_\Sigma-M_\Lambda}{12M_p}\ . \tag{4.15}$$

Similarly the SU(2)×SU(2) limit of Eq. (4.9) is

$$\mu_0=\tfrac{9}{2}M_p[2J_G(ep+en)-I_2(ep+en)]$$
$$+\tfrac{1}{4}[2(M_\Xi-M_p)+M_\Sigma-M_\Lambda]\ . \tag{4.16}$$

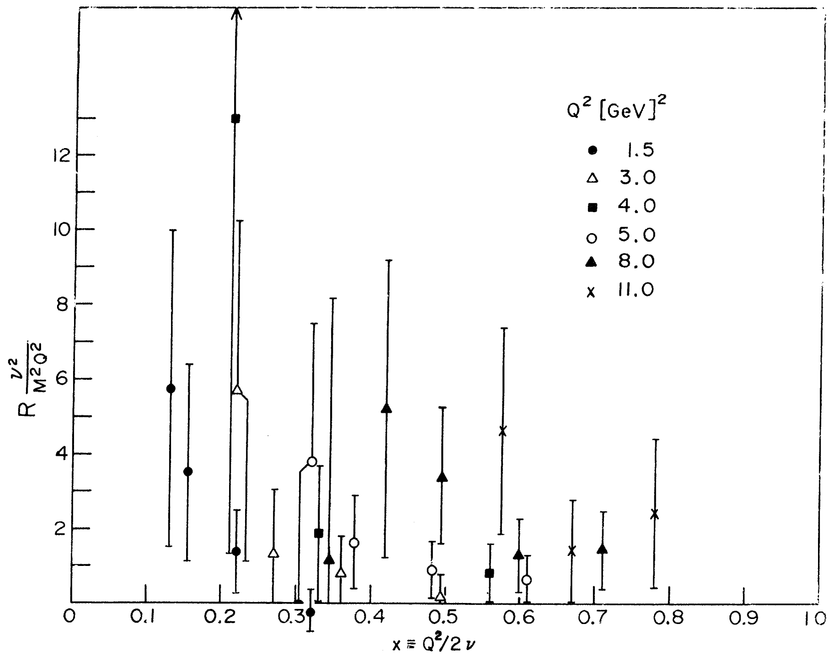

FIG. 2. $(\nu^2/M^2Q^2)R$ plotted vs x, where $R=\sigma_L/\sigma_T$. If canonical quark-gluon theories are correct this quantity should scale.

Equations (4.13)–(4.16) comprise the SU(2) × SU(2) limit of the vector-gluon model. Models in which $F_5 \equiv 0$ for finite $m_\mathcal{P}$ have rather different behavior.

B. Models in Which $F_5(x)=0$

As discussed in Sec. III, $F_5(x)$ vanishes if we let the quark-gluon coupling constant vanish. In this event Eqs. (3.36)–(3.38) are valid. Their consequences for chiral-symmetry breaking were explored in Ref. 12, where the following results were obtained:

$$6m_\mathcal{P}(m_\lambda - m_\mathcal{P})J_2(ep-en) \cong M_p[M_\Xi - M_p + \tfrac{3}{2}(M_\Lambda - M_\Sigma)] \quad \text{[cf. Eqs. (4.5) and (3.36)]}, \quad (4.17)$$

$$\sigma_\pi \underset{m_\mathcal{P}\to 0}{\sim} m_\mathcal{P}^2 J_2(\nu p + \nu n) \quad \text{[cf. Eqs. (4.3) and (3.37)]}, \quad (4.18)$$

$$\mu_0 \underset{m_\mathcal{P}\to 0}{\sim} \frac{\sigma_\pi m_\lambda}{2m_\mathcal{P}} - \tfrac{1}{8}[2(M_\Xi - M_p) + M_\Sigma - M_\Lambda] \quad \text{[cf. Eq. (4.9)]}. \quad (4.19)$$

Note that unless m_λ or $J_2(\nu p+\nu n)$ diverges as $m_\mathcal{P}$ vanishes, μ_0 will be negative (≈ -145 MeV) in the limit. Equation (4.17) evidences a rather strange behavior in the SU(2) × SU(2) limit. Since the mass differences in the right-hand side do not vanish as $m_\mathcal{P} \to 0$ we require either

$$m_\lambda \underset{m_\mathcal{P}\to 0}{\sim} M^2/m_\mathcal{P}$$

or

$$J_2(ep-en) \underset{m_\mathcal{P}\to 0}{\sim} M/m_\mathcal{P}.$$

In the first case ($m_\lambda \to \infty$), σ_π vanishes quadratically but μ_0 is not necessarily negative in the limit since the first term in Eq. (4.19) no longer vanishes. In the second case $[J_2(ep-en)\to\infty]$, $J_2(\nu p+\nu n)$ would be expected to behave similarly since it too depends only on the SU(2) × SU(2) currents. σ_π would then vanish linearly in the limit and μ_0 would not necessarily be negative. It would indeed be bizarre if m_λ had to diverge as $m_\mathcal{P}\to 0$, and in fact the other alternative seems likely to occur in such theories. To see why $J_2(pt)$ might be expected to diverge as $m_\mathcal{P}\to 0$ in models where $F_5(x)=0$ consider the case that $\psi(x)$ obeys the free Dirac equation

$$(i\partial\!\!\!/ - m)\psi(x) = 0. \quad (4.20)$$

Equations (3.36)–(3.38) are valid in such a model. By virtue of Eq. (4.20), the bilocal operator which determines $F_2(x)^{ep-en}$ obeys the Klein-Gordon equation

$$(\Box + m_\mathcal{P}^2)\langle P|\bar\psi(0)\lambda_3\psi(x)|P\rangle = 0.$$

A simple (but not unique) solution to this equation is[40]

$$\langle P|\bar{\psi}(0)\lambda_3\psi(x)|P\rangle = Ae^{i(m_{\mathcal{P}}/M)P\cdot x}. \qquad (4.21)$$

Equation (4.24) defines a suitable model and yields[41]

$$F_2^{ep-en}(x) \propto x\delta(x - m_{\mathcal{P}}/M);$$

likewise

$$F_2^{\nu p+\nu n}(x) \propto x\delta(x - m_{\mathcal{P}}/M),$$

from which it follows that $J_2(ep-en)$ and $J_2(\nu p+\nu n)$ are proportional to $M/m_{\mathcal{P}}$ and diverge as $m_{\mathcal{P}} \to 0$.

We consider the divergence of either m_λ or $J_2(ep-en)$ and $J_2(\nu p+\nu n)$ in the SU(2)×SU(2) limit unattractive. These pathologies arose when we set $F_5 = 0$ [they also occur if $J_5 = 0$, since it is J_5 which spoils the sum rules of Eqs. (3.36)–(3.38)]. We have emphasized that $F_5(x)$ depends explicitly on the gluon field strength and coupling constant. $F_5(x)$ vanishes when the leading behavior of divergence current scattering is assumed to be given by the leading light-cone singularities of free-field theory.

If one attempts to abstract $F_5(x) = 0$ from free-field theory on the light cone, rather bizarre behavior of structure functions or quark masses must be countenanced. This confirms the prejudice expressed by Gell-Mann[42] that results which would be spoiled by gluon fields and coupling constants should not be extracted from free-field theories on the light cone.

V. SUMMARY

In order for our description of the relation between deep-inelastic scattering and static properties of the baryons to be valid it is necessary that

(a) the leading scaling behavior of $W_{L,2,4,5}$ is correctly described by (results abstracted from) "canonical" field theory,

(b) the relevant field theory is the quark–vector-gluon model, and

(c) the residues of $\alpha = 0$ fixed singularities in kinematic-singularity-free amplitudes are polynomials in q^2.

Our results are only relevant if the well-known quark-model sum rules and scaling laws derived by previous authors[17] [which rely heavily on (a) and (b)] prove correct. If (a) holds but (b) fails, analogous results may be obtained in other models—some of which are listed in Sec. III C. If (c) fails, as indicated by a failure of the Cornwall-Corrigan-Norton-Rajaraman-Rajasakaran sum rule, our results may involve unknown constants, unmeasurable in lepton scattering.

Our main results are the following.

(1) We obtain a set of three sum rules for the proton matrix elements of the local quark operators $u_0(0)$, $u_3(0)$, and $u_8(0)$ [$u_i \equiv \bar{\psi}(0)\lambda_i\psi(0)$] which are exact in the vector-gluon model.[43] The sum rules are given in Eqs. (3.26)–(3.28) in terms of finite, regulated integrals over the scaling limits of the structure functions $\nu W_2(q^2,\nu)$ and $\nu W_L(q^2,\nu)$ in inelastic electron and neutrino scattering.

(2) We also obtain [cf. Eqs. (3.29)–(3.31)] the sum rules of Ref. 13 for the matrix elements of u_0, u_3, and u_8 in terms of the Bjorken limit of the chiral-symmetry-breaking structure functions, $\nu^2 W_4(q^2,\nu)$ and $\nu^2 W_5(q^2,\nu)$.

(3) We find that $F_5(x)$ [$\equiv \lim_{\rm Bj}(\nu^2/M^4)W_5(q^2,\nu)$] vanishes linearly in the proton-quark bare mass in the SU(2)×SU(2) limit and is explicitly proportional to the quark-gluon coupling constant g and the gluon field strengths $F_{\mu\nu}$. $F_5(x)$ therefore vanishes if we accept the dictum that scaling laws are to be abstracted from free-field theory on the light cone.

(4) We then relate the proton matrix elements of u_i to static SU(3)- and SU(3)×SU(3)-breaking effects in the vector-gluon model. Specifically the matrix element $\langle P|u_0 + \sqrt{2}u_8|P\rangle$ determines the σ term in pion-nucleon scattering (σ_π). The proton matrix elements of the octet operators u_3 and u_8 may be approximated as ratios of baryon mass differences to quark mass differences. From these and the sum rules of (1) we obtain two relations among deep-inelastic electron scattering data, octet baryon masses, σ_π, and the Gell-Mann–Oakes–Renner parameter c [$= -\sqrt{2}(m_\lambda - m_{\mathcal{P}})/(m_\lambda + 2m_{\mathcal{P}})$], and another relation among σ_π, neutrino scattering structure functions, and the $\mathcal{P}$-type quark bare mass. The relations are to be found in Eqs. (4.7), (4.8), and (4.4), respectively.

(5) Assuming $c \cong -1.25$ and $\sigma_\pi \cong 40$ MeV,[37] we evaluate Eqs. (4.7) and (4.8) numerically, and obtain

$$\int_0^\infty dx\, \tilde{F}_G^{ep-en}(x) \cong 0.02 \pm 0.01$$

and

$$\int_0^\infty dx\, \tilde{F}_G^{ep+en}(x) \cong 0.17$$

[see Eqs. (3.24) for a precise definition of these regulated integrals].

(6) We find that present data on $W_L(q^2,\nu)$ are insufficient to test the results of (5). However, we show that the data are as consistent with

$$\lim_{\rm Bj} \nu W_L(q^2,\nu) = F_G(x)$$

as with the more conventional parametrizations of W_L (discussed in Sec. IV).

(7) We show that abstraction from free-field theory of $F_5(x)=0$ [see (3) above] leads to too trivial a theory. Specifically, if $F_5(x)=0$ the integral

$$\int_0^\infty \frac{dx}{x^2}\,\bar{F}_2^{ep-en}(x)$$

(which is Regge-convergent) must diverge as the $\mathcal{P}$-type quark bare mass goes to zero, or else the λ-type bare quark mass must simultaneously become infinite.

In a truly free-field model where $F_2(x) = x\,\delta(x-m_{\mathcal{P}}/M)$ the integral indeed diverges as $m_{\mathcal{P}}\to 0$, from which we conclude that models with $F_5(x)=0$ are indeed too trivial. Abstraction from the gluon model $[F_5(x)\neq 0]$ encounters no problems.

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APPENDIX A: PARTON MODEL

Here we present a parton-model derivation of the electroproduction sum rule [Eq. (3.14)]. Derivations of all the other sum rules in the text—for different quantum numbers, spins, and interactions—are straightforward generalizations. We discuss also the relations involving F_4 and F_5.

In an attempt to make the present paper self-contained we shall review the basic formalism required. We use the notation established by the Cambridge school (see, for example, Hughes and Osborn[44]).

To isolate the structure functions F_G and F_2 in $T_{\mu\nu}$ [see Eqs. (2.1), (2.3), (2.5)] it is convenient to choose a frame in which $q_-\to\infty$ with

$$q_-P_+=\nu\,,\qquad \frac{q_+}{P_+}=\frac{q^2}{2\nu}\equiv -x\,, \tag{A1}$$

whence it follows that

$$\lim_{\text{Bj}}\frac{1}{2\pi}\,\text{Im}T_{--}P_+{}^2=\frac{M^2}{2x}F_G(x) \tag{A2}$$

and

$$\lim_{\text{Bj}}\frac{1}{2\pi}\,\text{Im}T_{\perp\perp}=\frac{1}{2x}F_2(x)\,. \tag{A3}$$

[We have already used the fact that $\lim_{\text{Bj}}W_L(q^2,\nu)=0$, which is, of course, valid in spin-$\frac{1}{2}$ parton models.]

Both $T_{\perp\perp}$ and T_{--} receive contributions only from from the contiguous vertex diagram. (For the moment we ignore the soft vector-gluon interactions, which if present modify the contiguous vertex diagram just as they modify the leading bilocal operator in the light-cone derivation. We return to them presently.) To see this we note that the contiguous vertex contribution to $T_{\mu\nu}$ may be written as

$$T_{\mu\nu}=\frac{1}{(2\pi)^4}\int d^4k\,\text{Tr}\left\{T(P,k)\,\gamma_\mu S_F'(k+q)\gamma_\nu\right\}, \tag{A4}$$

where $T(P,k)$ is the parton-proton amplitude and

$$S_F'(k+q)\approx\frac{\gamma_+}{2P_+(\xi-x)}+O\left(\frac{1}{\nu}\right)\,. \tag{A5}$$

We have taken

$$k_+=\xi P_+\,,\qquad k_-=\frac{\bar{y}}{2P_+}\,,\qquad k^2=\xi\bar{y}-k_\perp{}^2\,. \tag{A6}$$

So long as the leading term in Eq. (A5) contributes in the trace we need keep only the contiguous photon vertex diagrams. An exception to this occurs in the vector-gluon model, where the spin structure of quark-gluon vertex promotes the diagrams in which interactions take place between the absorption and emission of the photon. Since

$$\gamma_-\gamma_+\gamma_-=2\gamma_-\quad\text{and}\quad \gamma_\perp\gamma_+\gamma_\perp=\gamma_+\,, \tag{A7}$$

the leading term in Eq. (A5) does contribute to T_{--} and $T_{\perp\perp}$.

Using Eqs. (A2)–(A4) we obtain

$$\frac{M^2}{2x}F_G(x)=\frac{i}{2(2\pi)^4}P_+{}^2\int d^4k\,\delta(\xi-x)\,\text{Tr}\left\{T(P,k)\frac{\gamma_-}{P_+}\right\},$$

$$\frac{1}{2x}F_2(x)=\frac{i}{2(2\pi)^4}\int d^4k\,\delta(\xi-x)\,\text{Tr}\left\{T(P,k)\,\frac{\gamma_+}{2P_+}\right\}, \tag{A8}$$

where $d^4k=\frac{1}{2}\,dx\,dy\,d^2k_\perp$. To proceed we close the $\bar{y}$ contour about either the u- or the s-channel discontinuity of the four-part parton-proton amplitude $T(p,k)$ according to whether $0<\xi<1$ or $-1<\xi<0$. The regions $|\xi|>1$ do not contribute since the contour may then be closed without encountering any singularities. The result is best expressed in terms of the imaginary part of the parton-proton scattering amplitude

$$V_{\alpha\beta}(P,k) \equiv \int d^4y\, e^{ik\cdot y} \langle P|\overline{\psi}_\beta(0)\, Q^2 \psi_\alpha(y)|\boldsymbol{P}\rangle \quad \text{(A9)}$$

(the charge-squared operator, Q^2, has been incorporated in V for convenience) as

$$\frac{M^2 F_G(x)}{2x} = \frac{P_+{}^2}{2(2\pi)^4}\int dk_-\, d^2k_\perp \frac{(\gamma_-)^{\alpha\beta}}{P_+}\, V_{\alpha\beta}(P,k)\Big|_{\xi=x}, \quad \text{(A10)}$$

$$\frac{F_2(x)}{2x} = \frac{1}{2(2\pi)^4}\int dk_-\, d^2k_\perp \frac{(\gamma_+)^{\alpha\beta}}{2P_+}\, V_{\alpha\beta}(\boldsymbol{P},k)\Big|_{\xi=x}. \quad \text{(A11)}$$

Note that Eqs. (A10) and (A11) are the complete answer only if no unknown subtraction constants (i.e., $\alpha = 0$ fixed poles or Kronecker δ functions) enter into the dispersive representation of the parton-proton amplitude. That strong-interaction amplitudes are determined entirely by their imaginary parts is a common assumption based on unitarity arguments. This same assumption ensures that any fixed poles in Compton scattering have polynomial residues and is therefore the parton-model equivalent to the assumptions made in the text.

To continue, it is convenient to expand $V_{\alpha\beta}(P,k)$ in a spinor basis:

$$V_{\alpha\beta}(\boldsymbol{P},k) = V_1(P\cdot\gamma)_{\alpha\beta} + V_2(k\cdot\gamma)_{\alpha\beta} + R_{\alpha\beta}\,, \quad \text{(A12)}$$

where $R_{\alpha\beta}$ vanishes when contracted with $(\gamma_\pm)_{\alpha\beta}$ and spin-averaged. V_1 and V_2 are functions of the invariants $p\cdot k$ and k^2. Combining Eqs. (A10)–(A12), we have

$$\frac{F_2(x)}{x} = \frac{1}{4(2\pi)^4}\int d\overline{y}\int d^2k_\perp (V_1 + x V_2)\,, \quad \text{(A13)}$$

$$\frac{F_G(x) - F_2(x)}{x} = \frac{1}{4(2\pi)^4}\int d\overline{y}\int d^2k_\perp \left(\frac{\overline{y}}{\boldsymbol{M}^2} - x\right) V_2\,.$$

To obtain the required sum rule we examine

$$K_{\alpha\beta} \equiv i\langle P|\overline{\psi}(0)\, Q^2\gamma_\alpha\partial_\beta\psi(y)|\boldsymbol{P}\rangle\,|_{y=0}$$
$$= P_\alpha P_\beta K + g_{\alpha\beta}K' \quad \text{(A14)}$$

so that

$$K_\alpha{}^\alpha = M^2 K + 4K'. \quad \text{(A15)}$$

From Eqs. (A9) and (A12) we obtain by partial integration

$$i\int d^4y\, e^{ik\cdot y}\langle P|\overline{\psi}(0)\, Q^2\gamma_\alpha\partial_\beta\psi(y)|\boldsymbol{P}\rangle = V_1 P_\alpha k_\beta + V_2 k_\alpha k_\beta\,, \quad \text{(A16)}$$

which implies

$$\int\frac{d^4k}{(2\pi)^4}(V_1 P_\alpha k_\beta + V_2 k_\alpha k_\beta) = \boldsymbol{P}_\alpha P_\beta K + g_{\alpha\beta}\boldsymbol{K}'\,. \quad \text{(A17)}$$

From the $++$ and $-+$ components of this equation we obtain

$$\frac{1}{2}\frac{1}{(2\pi)^4}\int dx\int d\overline{y}\int d^2k_\perp (xV_1 + x^2V_2) = K \quad \text{(A18)}$$

and

$$\frac{1}{2}\frac{1}{(2\pi)^4}\int dx\int d\overline{y}\int d^2k_\perp \left(xV_1 + \frac{x\overline{y}}{M^2}V_2\right) = K + \frac{2K'}{M^2}. \quad \text{(A19)}$$

Comparing Eqs. (A18) and (A19) with (A13) and (A15) we conclude

$$2M^2\int dx[2F_G(x) - F_2(x)] = K_\alpha^\alpha\,, \quad \text{(A20)}$$

with

$$K_\alpha^\alpha = \langle P|\overline{\psi}(0)\, Q^2 i\,\partial\!\!\!/\psi(y)|P\rangle\,|_{y=0}\,. \quad \text{(A21)}$$

The vector-gluon case is more complicated. We begin by rewriting the contiguous-vertex results (A10) and (A11) for F_G and F_2 as

$$\frac{M^2F_G(x)}{2x} = \frac{P_+{}^2}{2(2\pi)^4}\int dy_-\, e^{ixP_+y_-}\left\langle P\left|\overline{\psi}(0)\frac{\gamma_-}{P_+}\psi(y_-)\right|P\right\rangle, \quad \text{(A22)}$$

$$\frac{F_2(x)}{2x} = \frac{1}{2(2\pi)^4}\int dy_-\, e^{ixP_+y_-}\left\langle P\left|\overline{\psi}(0)\frac{\gamma_+}{2P_+}\psi(y_-)\right|P\right\rangle.$$

Consider, next, the diagram in which one vector-gluon interaction occurs between absorption and emission of the photon. In momentum space the leading contribution to $T_{\mu\nu}$ is proportional to

$$T_{\mu\nu} \propto \int\frac{d^4k}{\xi - x}\int\frac{d^4k'}{\xi' - x} \times \mathrm{Tr}\left\{T_5^\alpha(P,k,k')\gamma_\mu\frac{\gamma_+}{2P_+}\gamma_\alpha\frac{\gamma_+}{2P_+}\gamma_\nu\right\}, \quad \text{(A23)}$$

where T_5^α is the five-point parton-proton vector-gluon amplitude depicted in Fig. 3. (α is the Lorentz index of the B field.) As before, the k and k' integrations are performed by closing the $\overline{y}$ and $\overline{y}'$ contours about the appropriate discontinuities of the five-point amplitude. For $\mu = \nu = \perp$ or $-$ only $\alpha = +$ survives in the trace and the result is

$$\frac{1}{2\pi}\operatorname{Im}T_{\perp\perp}\propto g\int d\xi\,\delta(x-\xi)\int\frac{d\xi'}{\xi'-x}\,e^{i\xi'P_+y_-+i(\xi-\xi')P_+y'_-}\otimes\langle P|\overline{\psi}(0)\,\gamma_+\,B_+(y'_-)\psi(y_-)|P\rangle\ , \tag{A24a}$$

$$\frac{1}{2\pi}\operatorname{Im}T_{--}\ \propto g\int d\xi\,\delta(x-\xi)\int\frac{d\xi'}{\xi'-x}\,e^{i\xi'P_+y_-+i(\xi-\xi')P_+y'_-}\otimes\langle P|\overline{\psi}(0)\,\gamma_-B_+(y'_-)\psi(y_-)|P\rangle\ , \tag{A24b}$$

where we have used the representation

$$(T_5^\alpha)_{\delta\epsilon}=g\int d^4y\int d^4y'\,e^{ik'\cdot y+i(k-k')\cdot y'}\,\langle P|\overline{\psi}_\delta(0)\,B^\alpha(y')\,\psi_\epsilon(y)|P\rangle \tag{A25}$$

for the five-point amplitude. The ξ' integral may be performed using the identity

$$\int\frac{d\xi}{\xi}\,e^{i\xi a}=\theta(a)$$

to yield, for example,

$$\frac{M^2F_G(x)}{2x}=\frac{P_+{}^2}{2(2\pi)^4}\int dy_-\,e^{ixP_+y_-}\left\langle P\middle|\overline{\psi}(0)\,\frac{\gamma_-}{P_+}\,g\int_0^{y_-}B_+(y'_-)\,dy'_-\,\psi(y_-)\middle|P\right\rangle\ , \tag{A26}$$

with a similar result for $F_2(x)$. Multiple interactions simply exponentiate the phase, so that in (A22) $\psi(y_-)$ should in general be replaced by

$$\exp\left[ig\int_0^{y_-}dy'_-\,B_+(y'_-)\right]\psi(y_-)\ .$$

Making the replacement and performing an integration by parts in y_- we obtain

$$\tfrac{1}{2}M^2\int_0^1F_G(x)\,dx=\frac{1}{2(2\pi)^4}\left\langle P\middle|\overline{\psi}(0)\,\gamma_-\left(i\frac{\partial}{\partial y_-}-gB_+\right)\psi(y_-)\middle|P\right\rangle\bigg|_{y_-=0}\ , \tag{A27}$$

with a corresponding result for $\int F_2dx$ which corresponds as well to the replacement

$$i\partial_\alpha\to i\partial_\alpha-gB_\alpha\ . \tag{A28}$$

Thus the sum rule (A20) is modified to read

$$\begin{aligned}M^2\int_0^1[2F_G(x)-F_2(x)]\,dx&=\tfrac{1}{2}\langle P|\overline{\psi}(0)\,Q^2(i\partial\!\!\!/-gB\!\!\!/)\,\psi(y)|P\rangle\,|_{y=0}\\&=\tfrac{1}{2}\langle P|\overline{\psi}(0)\,Q^2m\,\psi(0)|P\rangle\ .\end{aligned} \tag{A29}$$

It remains to discuss the treatment of leading Regge behavior which naively would lead to a divergence of the sum rule. In the parton model the leading ($\alpha>0$) Regge behavior of $F_G(x)$ arises from the $\alpha>0$ Regge behavior of the parton-proton amplitude. The dispersive representation of this amplitude required in deriving (A10) thus requires a subtraction for the Regge contribution,

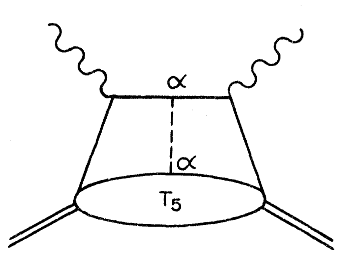

FIG. 3. One of the class of diagrams whose sum generates the gluon phase in the parton model.

$$T^R(P,k)=\int_0^\infty\frac{dm^2(m^2)^\alpha}{m^2-(P-k)^2}-\int_0^\infty(m^2)^{\alpha-1}\,dm^2\ . \tag{A30}$$

The first term of (A30) in combination with the remainder of the parton-proton amplitude leads to a sum rule of exactly the form given in (A20) or (A29). The subtraction term in (A30) does not contribute to the structure functions F_G and F_2, since it is real, but does contribute to the local expectation value $\langle P|\overline{\psi}(0)m\,Q^2\psi(0)|P\rangle$. Its contribution serves to regulate the sum rule in precisely the fashion discussed in the text. Details of these considerations may be found in Refs. 25 and 30.

Finally we discuss briefly the structure function $F_5(x)$ in parton models. As shown above, soft gluon interactions enter parton-model calculations in the same way they enter a coordinate-space approach. In particular the operations which are necessary to show $F_5(x)=0$ in free-field theories are thwarted in vector-gluon–parton models by the appearance of the curl of the gluon field after differentiating the gluon phase integral. $F_5(x)$ is then in general not zero in parton models.

This has an interesting consequence for familiar parton "intuition." The relations of Eqs. (3.21) and (3.23) are easily derived in parton models:

$$M^2\int_0^\infty dx[4x\tilde{F}_4^{+i}(x)-\tilde{F}_5^{+i}(x)]=m_i\langle P|\bar{\psi}_i(0)\psi_i(0)|P\rangle,$$

$$x\,F_4^{+i}(x)-\tfrac{1}{2}F_5^{+i}(x)=\frac{m_i^2}{4M^2}\,\frac{F_2^{+i}(x)}{x^2}\;,$$

where the relations hold for each parton species i individually [begin with hypothetical currents $\bar{\psi}_i(x)\gamma_\mu(1-\gamma_5)\psi_i(x)$ and proceed as usual]. It is clear that we obtain the naive result

$$\langle P|\bar{\psi}_i(0)\psi_i(0)|P\rangle=m_\varphi\int_0^\infty\frac{\tilde{F}_2^{+i}(x)\,dx}{x^2}\,, \qquad (A31)$$

i.e., $\bar{\psi}\psi$ simply measures the distribution function $f(x)/x$ of the partons only if $\int F_5(x)\,dx=0$. In general, however, $F_5(x)\neq 0$ (though it may be small) in vector-gluon models.

APPENDIX B: DERIVATION OF SUM RULES FROM EQUAL-TIME AND LIGHT-CONE COMMUTATORS

In Ref. 30 it was shown how to obtain regulated sum rules from equal-time commutators (ETCs) via the BJL limit, $q_0\to i\infty$, and from light-cone commutators via the limit $q_-=(1/\sqrt{2})(q_0-q_3)\to\infty$. Here we apply these methods to derive the sum rule for the electromagnetic structure functions, Eq. (3.14).

First, we consider $T_{\mu\nu}$ in the limit $q_0\to i\infty$, $\vec{q}$ and P fixed (BJL limit), and use the BJL expansion[3-5]

$$\lim_{\text{BJL}} T_{\mu\nu}=-\frac{i}{q_0^2}\int d^4y\,e^{iq\cdot y}\langle P|[J_\mu(y),J_\nu(0)]|P\rangle\,\delta(y_0)+\frac{1}{q_0}\,(\text{Schwinger term})+(\text{polynomials})\,. \qquad (B1)$$

We shall assume that the operator Schwinger term is zero, consistent with our posited scaling behavior [Eq. (2.5)].

It is sufficient to consider the trace, $T^\mu{}_\mu$. We choose for convenience a frame with $\vec{q}=0$, so that

$$T_\mu{}^\mu=2\left(\frac{P_0^2}{M^2}-1\right)T_2(q^2,\nu)+3T_L(q^2,\nu)\;, \qquad (B2)$$

and from Eqs. (2.3), (2.5), (2.9), and (2.15b) of Sec. II we obtain

$$\lim_{\text{BJL}} q^2\,T_2(q^2,\nu)=-8M^2\int_0^1 dx\,F_2(x)\;, \qquad (B3)$$

$$\lim_{\text{BJL}} q^2\,T_L(q^2,\nu)=\lim_{q^2\to-\infty} q^2T_L(q^2,0)=-8M^2\int_0^\infty dx\,\tilde{F}_G(x)\;. \qquad (B4)$$

To evaluate the ETC of Eq. (B1) we use the equation of motion, Eq. (3.1), and the canonical ETC

$$\delta(y_0)\{\psi(y),\psi^+(0)\}=\delta^4(y)$$

to obtain

$$-i\int d^4y\langle P|[\dot{J}_\mu(y),J^\mu(0)]|P\rangle\,\delta(y_0)=-4(S+2\Theta^{00}), \qquad (B5)$$

where

$$\Theta_{\mu\nu}\equiv\langle P|\bar{\psi}(0)\,Q^2\gamma_\mu\,[i\partial_\nu-gB_\nu(0)]\psi(0)|P\rangle$$

and

$$S\equiv\langle P|\bar{\psi}(0)m\,Q^2\psi(0)|P\rangle\;.$$

We write

$$\Theta_{\mu\nu}=A\,g_{\mu\nu}+\frac{1}{M^2}\,BP_\mu\,P_\nu \qquad (B6)$$

so that

$$S=\Theta^\mu{}_\mu=A+4B$$

and from Eqs. (B1)–(B5) we find

$$\lim_{\text{BJL}} q^2\,T^\mu_\mu=-8M^2\left[2\left(\frac{P_0^2}{M^2}-1\right)\int_0^1 dx\,F_2(x)+3\int_0^\infty dx\,\tilde{F}_G(x)\right]=-4\left(S+2A+2\frac{P_0^2}{M^2}\,B\right). \qquad (B7)$$

Equation (B7) is true for arbitrary P_0. Thus

$$2M^2\int_0^1 dx\,F_2(x)=B\;, \qquad (B8)$$

$$2M^2\left[3\int_0^\infty dx\,\tilde{F}_G(x)-2\int_0^1 dx\,F_2(x)\right]=S+2A\;. \qquad (B9)$$

Eliminating A and B from Eqs. (B6), (B8), and (B9) we have

$$S=2M^2\left[2\int_0^\infty dx\,\tilde{F}_G(x)-\int_0^1 dx\,F_2(x)\right]\,, \qquad (B10)$$

which is the result of Eq. (3.14). A corresponding sum rule is obtained from $T^\mu{}_\mu$ in neutrino scattering. The σ-term sum rule of Eq. (3.21) has a very simple BJL derivation, given in Ref. 13.

The light-cone derivation proceeds from taking the limit $q_-\to\infty$ with all other components of momenta fixed (LC limit) and relates light-cone commutators to amplitudes $T_i(q^2,\nu)$ in the Bjorken limit ($q^2 \to -\infty$, $x = -q^2/2\nu = -q_+/P_+$ fixed).

We have[45]

$$\lim_{\rm LC} T_{\mu\nu} = -\frac{1}{q_-} \int d^4y\, e^{iq\cdot y} \langle P|[J_\mu(y), J_\nu(0)]|P\rangle\, \delta(x_+) + (\text{polynomials}) ,$$

and obtain

$$x \lim_{\rm Bj} \frac{\nu}{M^2} [2\,T_L(q^2,\nu) - T_2(q^2,\nu)] = ix^2 \int_{-\infty}^{\infty} d\lambda\, e^{-i\lambda x} \epsilon(\lambda) [F(\lambda,0) + 4\lambda G(\lambda,0)] \tag{B11}$$

$$= x^2 \lim_{\rm LC} \left(\frac{4 P_\mu P_\nu}{M^2} + g_{\mu\nu}\right) T^{\mu\nu} , \tag{B12}$$

where F and G are the form factors of the vector bilocal on the light cone, defined in Eq. (3.5). To obtain the sum rule we write *subtracted* dispersion relations for each side of Eq. (B11). From Eqs. (2.3), (2.5), (2.9), and (2.15b) of Sec. II we obtain

$$x \lim_{\rm Bj} \frac{\nu}{M^2} [2\,T_L(q^2,\nu) - T_2(q^2,\nu)] = 2\left\{\int_{-\infty}^{\infty} dx' [2\,\tilde{F}_G(x') - F_2(x')] + \int_{-1}^{1} \frac{x'^2 dx'}{x^2 - x'^2} [2F_G(x') - F_2(x')]\right\} , \tag{B13}$$

and for the right-hand side of Eq. (B11) we use the representation of the distribution $\epsilon(\lambda)$ which gives

$$ix^2 \int_{-\infty}^{\infty} d\lambda\, e^{-i\lambda x} \epsilon(\lambda)\, \lambda g(\lambda) = -2ig(0) + \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{x'^3 dx'}{x^2 - x'^2} \int_{-\infty}^{\infty} d\lambda\, e^{-i\lambda x'} \lambda g(\lambda) \tag{B14}$$

when $g(\lambda) = g(-\lambda)$. Choosing

$$g(\lambda) = \frac{1}{\lambda} [F(\lambda,0) + 4\lambda G(\lambda,0)] \tag{B15}$$

from Eqs. (B11)–(B15) we find

$$\int_{-\infty}^{\infty} dx [2\tilde{F}_G(x) - F_2(x)] = -i\, g(0) , \tag{B16}$$

$$2F_G(x) - F_2(x) = \frac{x}{2\pi} \int_{-\infty}^{\infty} d\lambda\, e^{-i\lambda x} \lambda g(\lambda) . \tag{B17}$$

Combining (B16) with the expression for the scalar density of Eq. (3.10) we obtain the sum rule

$$\langle P|\overline{\psi}(0)\, m\, Q^2 \psi(0)|P\rangle = -iM^2 g(0)$$

$$= 2M^2 \left[2\int_0^{\infty} dx\, \tilde{F}_G(x) - \int_0^1 dx\, F_2(x)\right] , \tag{B18}$$

which agrees with Eqs. (B10) and (3.14).

It can be seen that both derivations presented here relate the scalar density to the subtraction constant in the dispersion relation for $T_L(q^2,\nu)$ and thus rely upon the analysis of Sec. II, where the subtraction constant was related to *regulated* integrals, *assuming polynomial fixed-pole residues*.

Finally we remark the connection with the coordinate-space derivation of Sec. III, which used Eq. (B17). The Fourier transform of Eq. (B17) gives a relation between $g(0)$ and the structure functions, provided these are interpreted as distributions, so that leading Regge terms may be accommodated. However, *as distributions* they may contain δ functions corresponding to nonpolynomial fixed-pole residues.

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[32]For a quick derivation of this form in the electromagnetic case $(A=0)$ and the generalization to the nonforward direction non-spin-averaged case, see Brodsky, Close, and Gunion, Ref. 22.

[33]$F_5 \propto (1/x)\int e^{iy\cdot p} y\cdot p\alpha(y\cdot p)d(y\cdot p)$. Hence

$$\int_{-1}^{1} dx\, x F_5^- = \int \delta(\lambda)\lambda\alpha^-(\lambda)d\lambda = 0,$$

and

$$\int_{-1}^{1} dx\, x^2 F_5^+ = \int \delta(\lambda)\frac{\partial}{\partial\lambda}[\lambda\alpha^+(\lambda)]d\lambda \sim \alpha^+(0).$$

[34]Note that from (3.34) and (3.23) we can obtain a somewhat esoteric version of the Adler sum rule:

$$\int_0^1 dx\, x^2[F_4^{\nu n}(x) - F_4^{\nu p}(x)] = m_{\mathcal{O}}^2/2M^2.$$

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